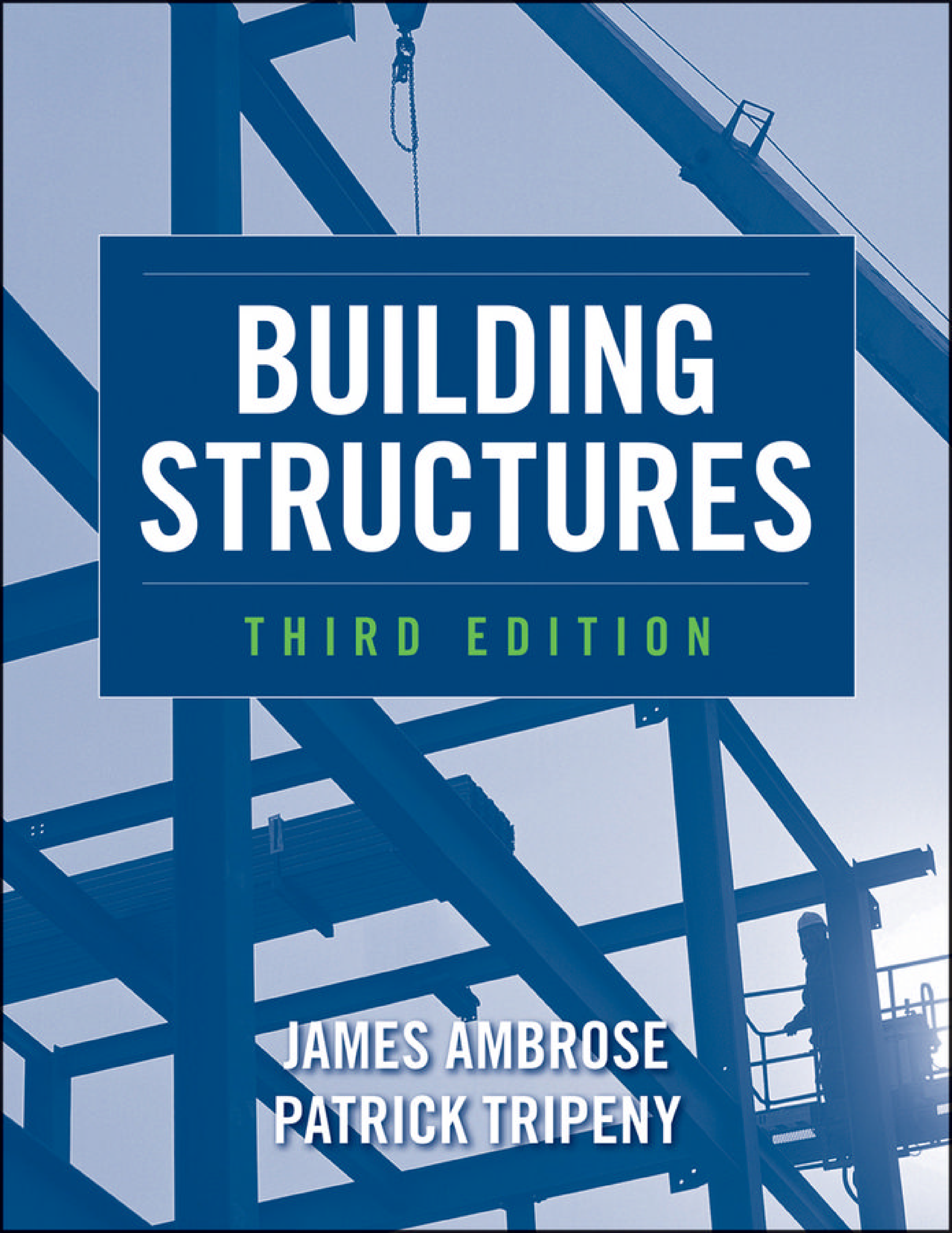

BUILDING STRUCTURES

THIRD EDITION

JAMES AMBROSE
PATRICK TRIPENY

BUILDING STRUCTURES

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Third Edition

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PATRICK TRIPENY

WILEY

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Contents

Preface

This book covers the topic of structures for buildings in a broad scope and from multiple points of view. The primary purpose is to provide a reference for study for persons with limited experience in the field and with interest in the general problems of design of buildings. Presentations in the book are intended to be accessible to persons with limited backgrounds in mathematics, science, and engineering.

The materials in this book are developed to serve two primary needs of readers. The first is that of a text for study for courses within a collegiate program in building design. The second is that of a study reference for preparation to take the exam for architectural registration (ARE), as currently prepared by the National Conference of Architectural Registration Boards (NCARB).

Because of the broad scope of the book, it is unlikely that its content can be covered in a single course of instruction in a typical college-level term of 12–14 weeks. This depends, however, on the type of course work. Traditional development of courses with example computations for structural elements and systems requires considerable time if a range of structural materials and types of structural elements are to be treated. If the purpose of the study is limited to a general acquisition of understanding of basic concepts, issues, and design problems—with no involvement in structural computations—more of the book topics can be covered in a shorter time. The latter form of study may be undertaken in a collegiate program and is the general case for those preparing for the ARE. A guide for course instructors with suggestions for course organization and operation is provided on the publisher's website.

The first edition of this book was quite large in number of pages. The second edition was trimmed down a bit and this edition is further reduced in size. Trimming has resulted in some reduction of materials but has been mostly accomplished by careful elimination of repetitions and redundancies and by stricter concentration on the specific aims for the book.

Of critical importance for all readers are the study materials at the back of the book. These may be used as a guide to the reader's accomplishment of general knowledge.

James Ambrose
Patrick Tripeny

Introduction

This book deals broadly with the topic of structures related to buildings. Emphasis is placed on the concerns of the working, professional designers who must cope with the practical problems of figuring out how to make plans for the construction of good, practical, and sensible buildings. Designers' concerns range from a basic understanding of structural behaviors to the determination of the construction details for a specific type of building.

The materials in this book are arranged to present a logical sequence of study. However, it is to be expected that few readers will start at page 1 and proceed to the end, as if reading a novel. The separate book chapters are therefore developed as reasonably freestanding, with appropriate referencing to other chapters for those readers who need some reinforcement. Additionally, at any time, the reader can use the Table of Contents, the Index, or the Glossary to seek help in understanding unfamiliar terms or ideas.

This book is intended for possible use as a course text but is also prepared to be used for individual self-study. In fact, even in a classroom situation where time is limited, students may well require considerable time for self-study outside the classroom.

Whether required as homework assignments or not, the exercise problems provided for individual book sections should be used by readers to test their own comprehension and problem-solving skills. For this self-study effort, answers to the problems are given, although readers should first attempt to solve the problems without recourse to the answers. Skill in performing computational work cannot be achieved by simply following a text example; the problems must be faced by the unassisted reader.

COMPUTATIONS

Structures for buildings are seldom produced with a high degree of dimensional precision. Exact dimensions of some parts of the construction—such as window frames and elevator rails—must be reasonably precise; however, the basic structural framework is ordinarily achieved with only a very limited dimensional precision. Add this to various considerations for the lack of precision in predicting loads for any structure, and the significance of highly precise structural computations becomes moot. This makes a case for not being highly concerned with any numbers beyond about the second digit (103 or 104; either will do).

While most professional design work these days is likely to be done with computer support, most of the work illustrated here is quite simple and was actually performed with a hand calculator (the eight-digit, scientific type is quite adequate).

SYMBOLS

The following symbols are used in this book.

Symbol	Reading
$>$	Is greater than
$<$	Is less than
$\geq$	Equal to or greater than
$\leq$	Equal to or less than
$6'$	Six feet
$6''$	Six inches
$\sum$	The sum of
ΔL	Change in L

STANDARD NOTATION

Notation used in this book complies generally with that used in the design and construction fields and the latest editions of standard specifications. The following list includes the notation used in this book and is compiled from more extensive lists in the references. Additional notation is explained in various chapters in this book.

A	Area, general
A_g	Gross area of a section, defined by the outer dimensions
A_n	Net area (gross area less area removed by holes or notches)
C	Compressive force
C_D	Load duration factor
C_F	Size factor for sawn lumber
C_M	Wet (moisture) service factor
C_P	Column stability factor
C_T	Buckling stiffness factor for dimension lumber
C_r	Repetitive member factor for dimension lumber
D	Diameter
E	Reference modulus of elasticity
E'	Adjusted modulus of elasticity
$E_{\min}$	Modulus of elasticity for stability investigation
F_b	Reference bending design value
F'_b	Adjusted bending design value
F_c	Reference compressive design value parallel to the grain, due to axial load only
F'_c	Adjusted compressive design value parallel to the grain, due to axial load only
F_{cE}	Design value for critical buckling in compression members
$F_{c\perp}$	Reference compression design value perpendicular to the grain
$F'_{c\perp}$	Adjusted compression design value perpendicular to the grain
F_v	Reference shear design value parallel to the grain
F'_v	Adjusted shear design value parallel to the grain
G	Specific gravity
I	Moment of inertia, or importance factor (wind and earthquakes)
L	Length (usually of a span), or unbraced height of a column
M	Bending moment
M_r	Reference design moment
M'_r	Adjusted design moment

P	Concentrated load or axial compression load
Q	Statical moment of an area about the neutral axis
R	Radius of curvature
S	Section modulus
T	Temperature in degrees Fahrenheit, or tension force
V	Shear force, or vertical component of a force
V_r	Reference design shear
V_r'	Adjusted design shear
W	Total gravity load, or weight, or dead load of an object, or total wind load force, or total of a uniformly distributed load or pressure due to gravity
b	Width or breadth of bending member
c	In bending: distance from extreme fiber stress to the neutral axis
d	Overall beam depth, or pennyweight of nail
e	Eccentricity of a nonaxial load, from the point of application of the load to the centroid of the section
f_b	Actual computed bending stress
f_c	Actual computed compressive stress due to axial load
f_c'	Specified compressive strength of concrete
f_m'	Specified compressive strength of masonry
f_p	Actual computed bearing stress
f_t	Actual computed stress in tension parallel to the grain
f_v	Actual computed shear stress
r	Radius of gyration
s	Spacing of objects, center to center
t	Thickness, general
w	Unit of a distributed load on a beam (lb/ft, etc.)

Greek Letters

Δ (delta)	1. Deflection, usually maximum vertical deflection of a beam; 2. indication of "change of" in mathematical expression
$\sum$ (sigma)	Sum of
λ (lambda)	1. Time effect factor (LRFD); 2. adjustment factor for building height (wind)
ϕ (phi)	Resistance factor (LRFD)

UNITS OF MEASUREMENT

Previous editions of this book have used U.S. units (feet, inches, pounds, etc.) for the basic presentation. In this edition the basic work is developed with U.S. units with equivalent metric (SI) unit values in brackets [thus].

Table 1 lists the standard units of measurement in the U.S. system with the abbreviations used in this work and a description of common usage in structural design work. In similar form, Table 2 gives the corresponding units in the metric system. Conversion factors to be used for shifting from one unit system to the other are given in Table 3. Direct use of the conversion factors will produce what is called a *hard conversion* of a reasonably precise form.

In the work in this book many of the unit conversions presented are *soft conversions*, meaning ones in which the converted value is rounded off to produce an approximate equivalent value of some slightly more relevant numerical significance to the unit system. Thus a wood 2 by 4 (actually 1.5 × 3.5 in. in the U.S. system) is precisely 38.1 mm × 88.9 mm in the metric system. However, the metric equivalent "2 by 4" is more likely to be made 40 × 90 mm—close enough for most purposes in construction work.

Table 1 Units of Measurement: U.S. System

Name of Unit	Abbreviation	Use in Building Design
Length		
Foot	ft	Large dimensions, building plans, beam spans
Inch	in.	Small dimensions, size of member cross sections
Area		
Square feet	ft^2	Large areas
Square inches	$in.^2$	Small areas, properties of cross sections
Volume		
Cubic yards	yd^3	Large volumes, of soil or concrete (commonly called simply "yards")
Cubic feet	ft^3	Quantities of materials
Cubic inches	$in.^3$	Small volumes
Force, Mass		
Pound	lb	Specific weight, force, load
Kip	kip, k	1000 lb
Ton	ton	2000 lb
Pounds per foot	lb/ft, plf	Linear load (as on a beam)
Kips per foot	kips/ft, klf	Linear load (as on a beam)
Pounds per square foot	lb/ft^2, psf	Distributed load on a surface, pressure
Kips per square foot	k/ft^2, ksf	Distributed load on a surface, pressure
Pounds per cubic foot	lb/ft^3	Relative density, unit weight
Moment		
Foot-pounds	ft-lb	Rotational or bending moment
Inch-pounds	in.-lb	Rotational or bending moment
Kip-feet	kip-ft	Rotational or bending moment
Kip-inches	kip-in.	Rotational or bending moment
Stress		
Pounds per square foot	lb/ft^2, psf	Soil pressure
Pounds per square inch	$lb/in.^2$, psi	Stresses in structures
Kips per square foot	$kips/ft^2$, ksf	Soil pressure
Kips per square inch	$kips/in.^2$, ksi	Stresses in structures
Temperature		
Degree Fahrenheit	°F	Temperature

Table 2 Units of Measurement: SI System

Name of Unit	Abbreviation	Use in Building Design
Length		
Meter	m	Large dimensions, building plans, beam spans
Millimeter	mm	Small dimensions, size of member cross sections
Area		
Square meters	m^2	Large areas
Square millimeters	mm^2	Small areas, properties of member cross sections
Volume		
Cubic meters	m^3	Large volumes
Cubic millimeters	mm^3	Small volumes
Mass		
Kilogram	kg	Mass of material (equivalent to weight in U.S. units)
Kilograms per cubic meter	kg/m^3	Density (unit weight)
Force, Load		
Newton	N	Force or load on structure
Kilonewton	kN	1000 N
Stress		
Pascal	Pa	Stress or pressure (1 Pa = 1 N/m^2)
Kilopascal	kPa	1000 Pa
Megapascal	MPa	1,000,000 Pa
Gigapascal	GPa	1,000,000,000 Pa
Temperature		
Degree Celcius	°C	Temperature

Table 3 Factors for Conversion of Units

To Convert from U.S. Units to SI Units, Multiply by:	U.S. Unit	SI Unit	To Convert from SI Units to U.S. Units, Multiply by:
25.4	in.	mm	0.03937
0.3048	ft	m	3.281
645.2	$in.^2$	mm^2	1.550×10^{-3}
16.39×10^3	$in.^3$	mm^3	61.02×10^{-6}
416.2×10^3	$in.^4$	mm^4	2.403×10^{-6}
0.09290	ft^2	m^2	10.76
0.02832	ft^3	m^3	35.31
0.4536	lb (mass)	kg	2.205
4.448	lb (force)	N	0.2248
4.448	kip (force)	kN	0.2248
1.356	ft-lb (moment)	N-m	0.7376
1.356	kip-ft (moment)	kN-m	0.7376
16.0185	lb/ft^3 (density)	kg/m^3	0.06243
14.59	lb/ft (load)	N/m	0.06853
14.59	kips/ft (load)	kN/m	0.06853
6.895	psi (stress)	kPa	0.1450
6.895	ksi (stress)	MPa	0.1450
0.04788	psf (load or pressure)	kPa	20.93
47.88	ksf (load or pressure)	kPa	0.02093
$0.566 \times (°F - 32)$	°F	°C	$(1.8 \times °C) + 32$

CHAPTER

1

Basic Concepts

This chapter presents basic issues that affect the design of building structures and presents an overall view of the materials, products, and systems used to achieve them.

1.1 BASIC CONCERNS

All physical objects have structures. Consequently, the design of structures is part of the general problem of design for all physical objects. It is not possible to understand fully why buildings are built the way they are without some knowledge and understanding of the problems of their structures. Building designers cannot function in an intelligent manner in making decisions about the form and fabric of a building without some comprehension of basic concepts of structures.

Safety

Life safety is a major concern in the design of structures. Two critical considerations are for fire resistance and for a low likelihood of collapse under load. Major elements of fire resistance are:

- *Combustibility of the Structure.* If structural materials are combustible, they will contribute fuel to the fire as well as hasten the collapse of the structure.
- *Loss of Strength at High Temperature.* This consists of a race against time, from the moment of inception of the fire to the failure of the structure—a long interval increasing the chance for occupants to escape the building.
- *Containment of the Fire.* Fires usually start at a single location, and preventing their spread is highly desirable. Walls, floors, and roofs should resist burn-through by the fire.

Major portions of building code regulations have to do with aspects of fire safety. Materials, systems, and details of construction are rated for fire resistance on the basis of experience and tests. These regulations constitute restraints on building design with regard to selection of materials and use of details for building construction.

Building fire safety involves much more than structural behavior. Clear exit paths, proper exits, detection and alarm systems, firefighting devices (sprinklers, hose cabinets, etc.), and lack of toxic or highly flammable materials are also important. All of these factors will contribute to the race against time, as illustrated in Figure 1.1.

The structure must also sustain loads. Safety in this case consists of having some margin of structural capacity beyond that strictly required for the actual task. This margin of safety is defined by the safety factor, SF, as follows:

$$SF = \frac{\text{actual capacity of the structure}}{\text{required capacity of the structure}}$$

Thus, if a structure is required to carry 40,000 lb and is actually able to carry 70,000 lb before collapsing, the safety factor is expressed as SF = 70,000/40,000 = 1.75. Desire for safety must be tempered by practical concerns. The user of a structure may take comfort in a safety factor as high as 10, but the cost or gross size of the structure may be undesirable. Building structures are generally designed with an average safety factor of about 2. There is no particular reason for this other than experience.

For many reasons, structural design is a highly imprecise undertaking. One should not assume, therefore, that the true safety factor in a given situation can be established with great accuracy. What the designer strives for is simply a general level of assurance of a reasonably adequate performance without

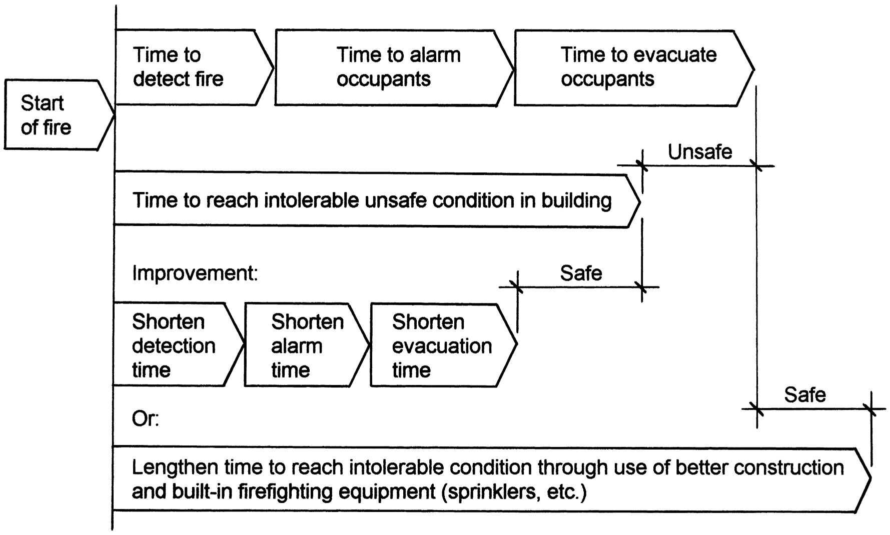


Figure 1.1 Concept of fire safety.

pushing the limits of the structure too close to the edge of failure.

There are two basic techniques for assuring the margin of safety. The method once used most widely is called the *allowable stress design* or *service load method.* With this method stress conditions under actual usage (with service loads) are determined and limits for stresses are set at some percentage of the predetermined ultimate capacity of the materials. The margin of safety is inferred from the specific percentage used for the allowable stresses.

A problem encountered with the allowable stress method is that many materials do not behave in the same manner near their ultimate failure limits as they do at service load levels. Thus prediction of failure from a stress evaluation cannot be made on the basis of only a simple linear proportionality; thus using an allowable stress of one-half the ultimate stress limit does not truly guarantee a safety factor of 2.

The other principal method for assuring safety is called the *strength design* or *load and resistance factor* (LRFD) method. The basis of this method is simple. The total load capacity of the structure at failure is determined and its design resistance is established as a percentage (factored) of the ultimate resistance. This factored resistance is then compared to an ultimate design failure loading, determined as a magnified (factored) value of the service load. In this case the margin of safety is inferred by the selected design factors.

Although life safety is certainly important, the structural designer must also deal with many other concerns in establishing a satisfactory solution for any building structure.

General Concerns

Feasibility

Structures are real and thus must use materials and products that are available and can be handled by existing craftspeople and production organizations. Building designers must have a reasonable grasp of the current inventory of available materials and products and of the usual processes for building construction. Keeping abreast of this body of knowledge is a challenge in the face of the growth of knowledge, the ever-changing state of technology, and the market competition among suppliers and builders.

Feasibility is not just a matter of present technological possibilities but relates to the overall practicality of a structure. Just because something *can* be built is no reason that it *should* be built. Consideration must be given to the complexity of the design, dollar cost, construction time, acceptability by code-enforcing agencies, and so on.

Economy

Buildings cost a lot of money, and investors are seldom carefree, especially about the cost of the structure. Except for the condition of a highly exposed structure that constitutes a major design feature, structures are usually appreciated as little as the buried piping, wiring, and other mundane hidden service elements. Expensive structures do not often add value in the way that expensive hardware or carpet may. What is usually desired is simple adequacy, and the hard-working, low-cost structure is much appreciated.

However vital, the building structure usually represents a minor part of the total construction cost. When comparing alternative structures, the cost of the structure itself may be less important than the effects of the structure on other building costs.

Optimization

Building designers often are motivated by desires for originality and individual expression. However, they are also usually pressured to produce a practical design in terms of function and feasibility. In many instances this requires making decisions that constitute compromises between conflicting or opposing considerations. The best or optimal solution is often elusive. Obvious conflicts are those between desires for safety, quality of finishes, grandeur of spaces, and general sumptuousness on the one hand and practical feasibility and economy on the other. All of these attributes may be important, but often changes that tend to improve

one factor work to degrade others. Some rank ordering of the various attributes is generally necessary, with dollar cost usually ending up high on the list.

Integration

Good structural design requires integration of the structure into the whole physical system of the building. It is necessary to recognize the potential influences of the structural design decisions on the general architectural design and on the development of the systems for power, lighting, thermal control, ventilation, water supply, waste handling, vertical transportation, firefighting, and so on. The most popular structural systems have become so in many cases largely because of their ability to accommodate the other subsystems of the building and to facilitate popular architectural forms and details.

1.2 ARCHITECTURAL CONSIDERATIONS

Primary architectural functions that relate to the structure are:

Need for shelter and enclosure
Need for spatial definition, subdivision, and separation
Need for unobstructed interior space

In addition to its basic force-resistive purpose, the structure must serve to generate the forms that relate to these basic usage functions.

Shelter and Enclosure

Exterior building surfaces usually form a barrier between the building interior and the exterior environment. This is generally required for security and privacy and often in order to protect against various hostile external conditions (thermal, acoustic, air quality, precipitation, etc.). Figure 1.2 shows many potential requirements of the building skin. The skin is viewed as a selective filter that must block some things while permitting the passage of others.

In some instances, elements that serve a structural purpose must also fulfill some of the filter functions of the building skin, and properties other than strictly structural ones must be considered in the choice of the materials and details of the structure. Basic structural requirements cannot be ignored but frequently can be relatively minor as final decision criteria.

When need exists for complete enclosure, the structure must either provide it directly or facilitate the addition of other elements to provide it. Solid masonry walls, shell domes, and tents are examples of structures that provide naturally closed surfaces. It may be necessary to enhance the bare structure with insulation, waterproofing, and so on, to generate all the required skin functions, but the enclosure function is inherent in the structural system.

Frame systems, however, generate open structures that must be provided with separate skin elements to develop the enclosure function. In some cases, the skin may interact with the frame; in other cases it may add little to the basic structural

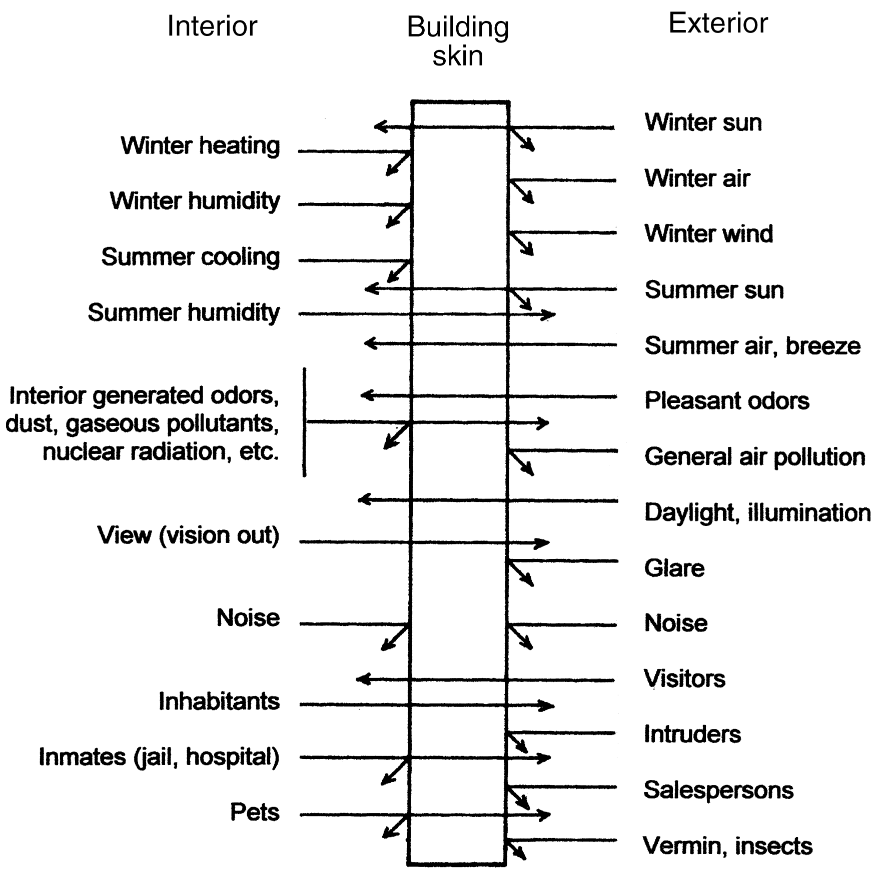


Figure 1.2 Functions of the building skin as a selective filter.

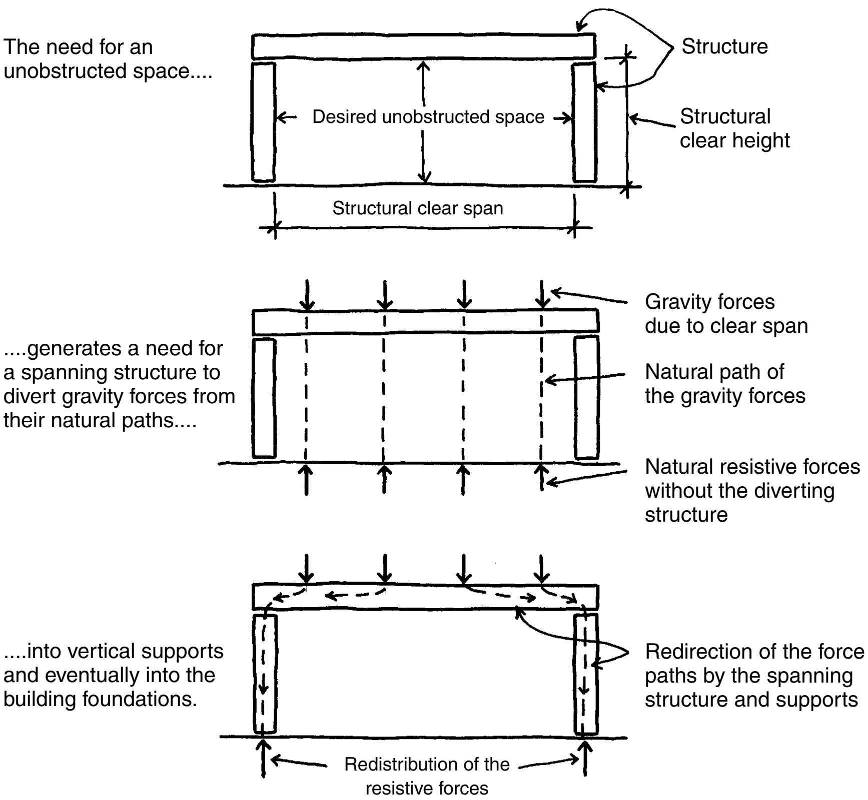


Figure 1.3 Structural task of generating unobstructed interior space.

behavior. An example of the latter is a highrise building with a thin curtain wall of light metal and glass elements.

Interior Space Division

Most buildings have interior space division with separate rooms and often separate levels. Structural elements used to develop the interior must relate to the usage requirements of the individual spaces and to the needs for separation and privacy. In multistory buildings structural systems that form the floor for one level may simultaneously form the ceiling for the space below. These two functions generate separate form restrictions, surface treatments, attachments, or incorporation of elements such as light fixtures, air ducts, power outlets, and plumbing pipes and fixtures. In addition, the floor–ceiling structure must provide a barrier to the transmission of sound and fire. As in the case of the building skin, the choice of construction must be made with all necessary functions in mind.

Generating Unobstructed Space

Housing of activities typically creates the need for producing open, unobstructed interior spaces. The spaces may be very small (for bathrooms) or very large (for sports arenas). Generating open space involves the structural task of spanning for overhead roofs or floors, as illustrated in Figure 1.3. The magnitude of the spanning problem is determined by the loads and the span. As the length of the span increases, the required structural effort increases rapidly, and options for the structural system narrow.

A particularly difficult problem is that of developing a large open space in the lower portion of a multistory building.

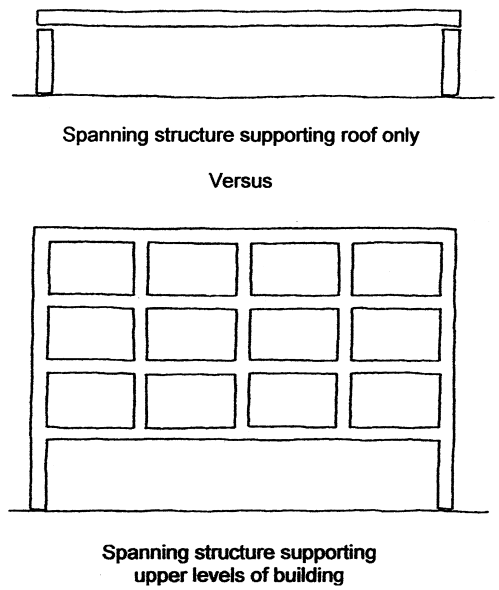


Figure 1.4 Load conditions for the spanning structure.

As shown in Figure 1.4, this generates a major load on the transitional spanning structure. This situation is unusual, however, and most large spanning structures consist only of roofs, for which the loads are relatively light.

Architectural Elements

Most buildings consist of combinations of three basic elements: walls, roofs, and floors. These elements are arranged to create both space division and unobstructed space.

Walls

Walls are usually vertical and potentially lend themselves to the task of supporting roofs and floors. Even when they do not serve as supports, they often incorporate the columns that do serve this function. Thus the design development of the spanning roof and floor systems must begin with consideration of the locations of wall systems over which they span.

Walls may be classified on the basis of their functions, which affects many of the design conditions regarding materials and details. Some basic categories are:

Structural Walls. These serve one or both of two functions in the general building structural system. *Bearing walls* support roofs, floors, and other walls. *Shear walls* brace the building against horizontal forces, utilizing the stiffness of the wall in its own plane, as shown in Figure 1.5.

Nonstructural Walls. Actually, there is no such thing as a nonstructural wall, since the least that any wall must do is hold itself up. However, the term *nonstructural* is used to describe walls that do not contribute to the general structural system of the building. When they occur on the exterior, they are called *curtain walls*; on the interior they are called *partitions*.

Exterior Walls. As part of the building skin, exterior walls have a number of required functions, including the barrier and filter functions described previously. Wind produces both inward and outward pressures that the wall surface must transmit to the bracing system. Exterior walls are usually permanent, as opposed to interior walls that can be relocated if they are nonstructural.

Interior Walls. Although some barrier functions are required of any wall, interior walls need not separate interior and exterior environments or resist wind. They may be permanent, as when they enclose stairs, elevators, or toilets, but are often essentially only partitions and can be built as such.

Many walls must incorporate doors or windows or provide space for wiring, pipes, or ducts. Walls of hollow construction provide convenient hiding places, whereas those of solid construction can present problems in this regard. Walls that are not flat and vertical can create problems with hanging objects. Walls that are not straight in plan and walls that intersect at other than right angles can also present problems. (See Figure 1.6.)

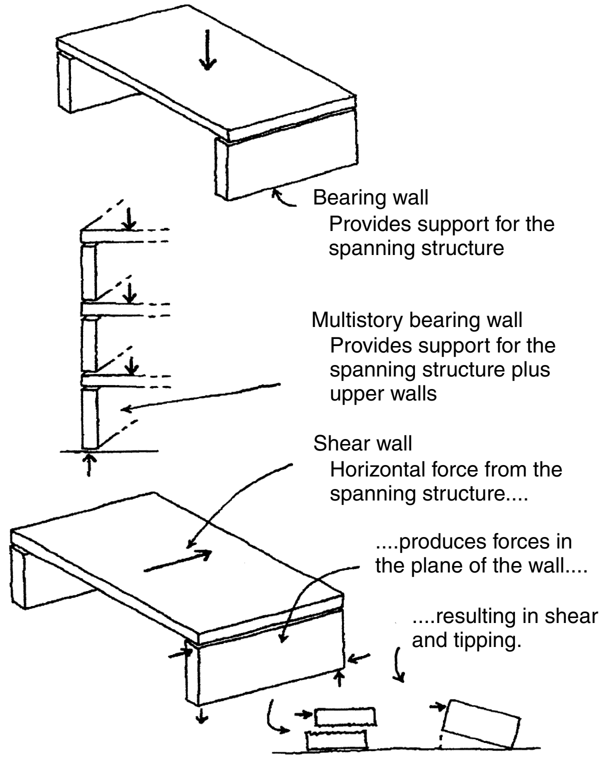


Figure 1.5 Structural functions of walls.

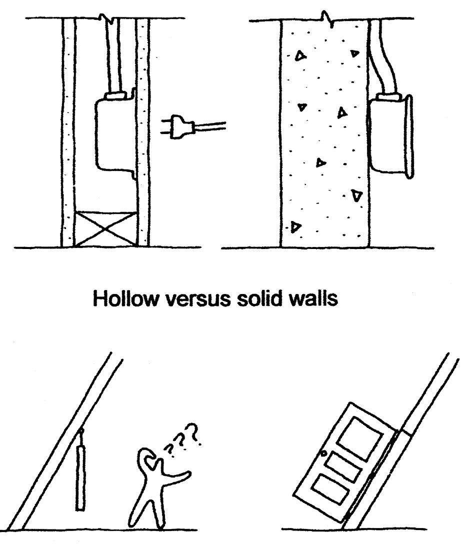


Figure 1.6 Problems of wall form.

Roofs

Roofs have two primary functions: They must serve as skin elements and must facilitate the runoff of water from precipitation. Barrier functions must be met and the roof geometry and surface must relate to the drainage problem. Whereas floors must generally be flat, roofs generally must not be, as some slope is required for drainage. The so-called flat roof must actually have some slope, typically a minimum of 1/4 inch per foot, or approximately 2%. In addition, the complete drainage operation must be controlled so that runoff water is collected and removed from the roof.

Floors are meant to be walked on; roofs generally are not. Thus, in addition to not being horizontal, roofs may be constructed of materials or systems that are not rigid, the ultimate possibility being a fabric or membrane surface held in position by tension.

Because of the freedom of geometry and lack of need for rigidity or solidity, the structural options for roofs are more numerous than those for floors. The largest enclosed, unobstructed spaces are those spanned only by roofs. Thus most of the dramatic and exotic spanning structures for buildings are roof structures.

Floors

Floor structures may serve as both a floor for upper spaces and a ceiling for lower spaces. Floors usually require a flat, horizontal form, limiting the choice to a flat-spanning system. Barrier requirements derive from the needs for separation of the spaces above and below.

Most floor structures are relatively short in span, since flat-spanning systems are relatively inefficient and load magnitudes for floors are generally higher than those for roofs. Achieving large open spaces under floors is considerably more difficult than achieving such spaces under roofs.

Form–Scale Relationships

There is a great variety of types of architectural space and associated structural problems. Figure 1.7 illustrates variables in terms of interior space division and of scale in terms of span and clear height. Other variables include the number of levels or adjacent spaces. The following discussion deals with some of the structural problems inherent in the situations represented in Figure 1.7.

Single Space

This ordinarily represents the greatest degree of freedom in the choice of the structural system. The building basically requires only walls and a roof, although a floor structure other than a paving slab may be required if the building is elevated above the ground. Some possible uses for such buildings and the potential structural systems follow.

Small Scale (10 ft high, 15 ft-span). This includes small sheds, cabins, and residential garages. The range of possible structural systems is considerable, including tents, air-inflated bubbles, ice block igloos, and mud huts, as well as more ordinary construction.

Medium Scale (15 ft high, 30 ft-span). This includes small stores, classrooms, and commercial or industrial buildings. The 15-ft wall height is just above the limit for 2 × 4 wood studs and the 30-ft span is beyond the limit of solid wood rafters on a horizontal span. Use of a truss, gabled frame, or some other efficient spanning system becomes feasible at this scale, although flat deck and beam systems are also possible.

Large Scale (30 ft high, 100+-ft span). This includes gyms, theaters, and large showrooms. The 30-ft wall height represents a significant structural problem, usually requiring braced construction of some form. The 100-ft span is generally beyond the feasible limit for a flat-spanning beam system, and the use of a truss, arch, or some other system is usually required. Because of the size of the spanning elements, they are usually widely spaced, requiring a secondary spanning system to fill in between the major spanning elements. Loads from the major spanning elements place heavy concentrated forces on walls, often requiring columns or piers. If the columns or piers are incorporated in the wall construction, they may serve the dual function of bracing the tall walls.

Super Large Scale (50 ft high, 300+-ft span). This includes large convention centers and sports arenas. Walls become major structural elements, requiring considerable bracing. The spanning structure itself requires considerable height in the form of spacing of truss elements, rise of arches, or sag of cables. Use of superefficient spanning systems becomes a necessity.

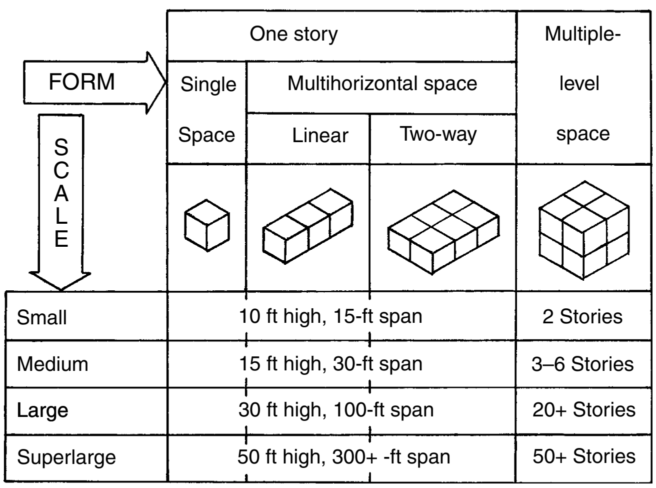


Figure 1.7 Form–scale relationships.

Multiple Horizontal Spaces—Linear Array

This category includes motels, small shopping centers, and school classroom wings. Multiplication may be done with walls that serve the dual function of supporting the roof and separating interior spaces, or it may be done only with multiples of the roof system with the possibility of no interior supports. The roof system has somewhat less geometric freedom than that for the single-space building, and a modular system of some kind may be indicated. (See Figure 1.8.)

Although space utilization and construction simplicity generally will be obtained with the linear multiplication of rectangular plan units, there are some other possibilities, as shown in Figure 1.9. If units are spaced by separate connecting links, more freedom can be obtained for the roof geometry of the individual units.

Structural options remain essentially the same as for the single-space building. If adjacent spaces are significantly different in height or span, it may be desirable to change the system of construction using systems appropriate to the scale of the individual spaces.

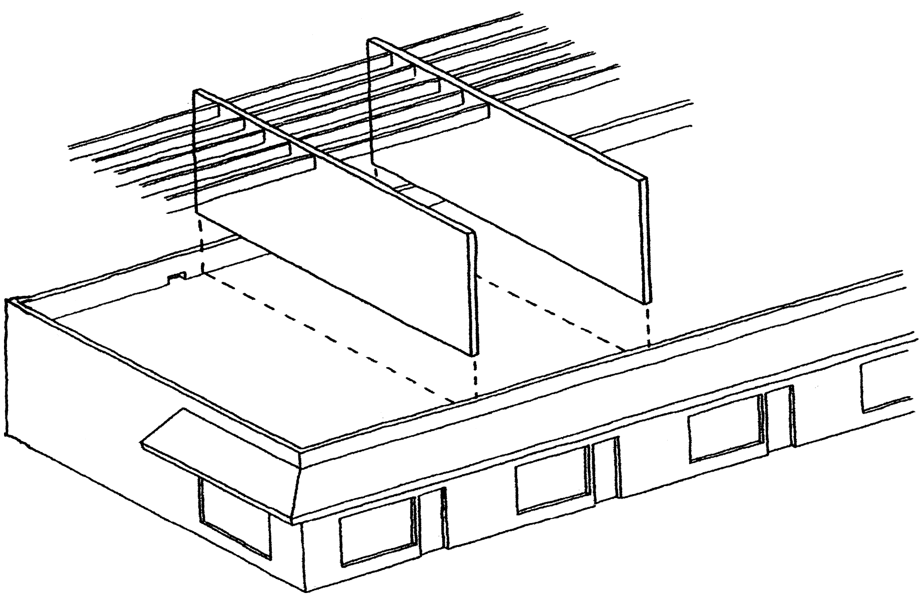


Figure 1.8 Multiple horizontal spaces in a linear array can be produced by a large number of optional structural modules, one of the simplest being the repetition of ordinary bearing walls and rafters.

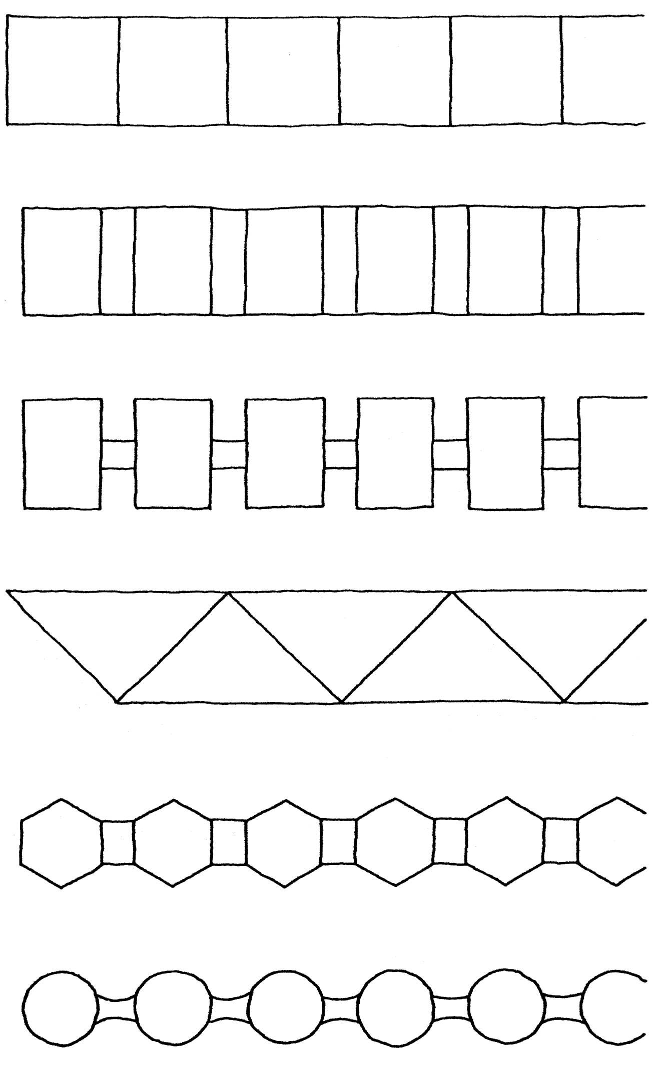


Figure 1.9 Linear plan multiplication.

Multiple Horizontal Spaces—Two-Way Array

This category includes factories, stores, warehouses, and large single-story offices. As with linear multiplication, the unit repetition may be done with or without interior walls, utilizing columns as supporting elements (see Figure 1.10).

Constraints on geometry are greater here than with linear unit multiplication. Modular organization and coordination become increasingly logical for the structure.

Although still possible using linear multiplication, roof structures that are other than flat and horizontal become increasingly less feasible for two-way multiplication. Roof drainage becomes a major problem when the distance from the center to the edge of the building is great. The pitch required for water runoff to an edge is often not feasible, in which case more costly and complex interior roof drains are required.

Multilevel Spaces

The jump from single to multiple levels has significant structural implications.

Need for a Framed Floor Structure. This is a spanning, separating element, not inherently required for the single-story building.

Need for Stacking of Support Elements. Lower elements (bearing walls and columns) must support upper elements as well as the spanning elements immediately above them. This works best if support elements are aligned vertically and imposes a need to coordinate the building plans at the various levels.

Increased Concern for Lateral Loads. As the building becomes taller, wind and earthquake loads impose greater overturning effects as well as greater horizontal forces in general, and the design of lateral bracing becomes a major problem.

Figure 1.10 Two-way multiplication of structural units in a single-story building. The structure is achieved here at a medium scale with a common system: steel posts and beams, light steel truss rafters, and a light-gauge formed sheet steel deck.

Vertical Penetration of the Structure. Elevators, stairs, ducts, chimneys, piping, and wiring must be carried upward through the horizontal structure at each level, and spanning systems must accommodate the penetrations.

Increased Foundation Loads. As building height increases without an increase in plan size, the total vertical gravity load for each unit of the plan area increases, eventually creating a need for a very strong foundation.

The existence of many levels also creates a problem involving the depth or thickness of the spanning structure at each floor level. As shown in Figure 1.11, the depth of the structure (*A* in the figure) is the distance from top to bottom of the complete structure, including structural decking and fireproofing. In many buildings a ceiling is hung below the floor structure, and the space between the ceiling and the underside of the floor contains various items, such as ducts, wiring, sprinkler piping, and recessed light fixtures. Architecturally, the critical depth dimension is the total distance from the top of the floor finish to the underside of the ceiling (*B* in the figure).

The floor-to-floor height, from finish floor level to finish floor level, is the total construction depth (*B*) plus the clear floor-to-ceiling height in each story. Repeated as required, the sum of these dimensions equals the total building height and volume, although only the clear space is of real value to the occupants. There is thus an efficiency ratio of clear height to total story height that works to constrain the depth allowed for the floor construction. While structural efficiency and reduced cost for the structure are generally obtained with increased structural depth, they are often compromised in favor of other cost considerations. Reduction of the building height and allowance for the air-handling ducts will probably push structural efficiency aside. Thus the net dimension allowed for the structure itself—dimension *C* in Figure 1.11—is a hard-fought one.

Sometimes it is possible to avoid placing the largest of the contained elements (usually air ducts) under the largest of the spanning structure's elements. Some techniques for accomplishing this are shown in Figure 1.12.

An important aspect of the multilevel building is the plan of the vertical supporting elements, since these represent fixed objects around which interior spaces must be arranged.

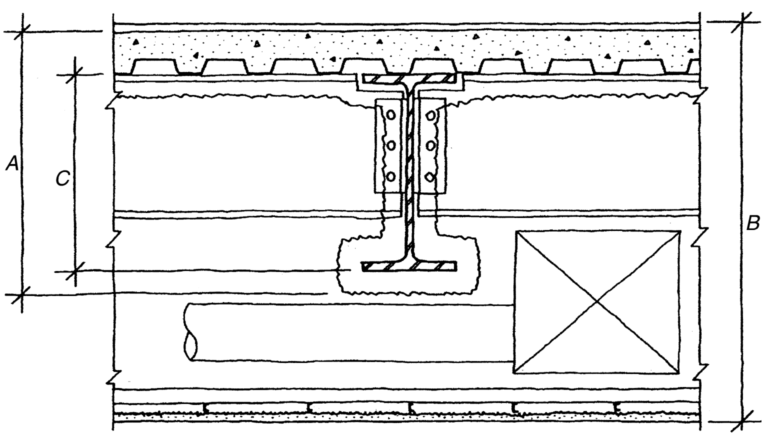


Figure 1.11 Dimensional relationships in the floor–ceiling systems for multistory buildings.

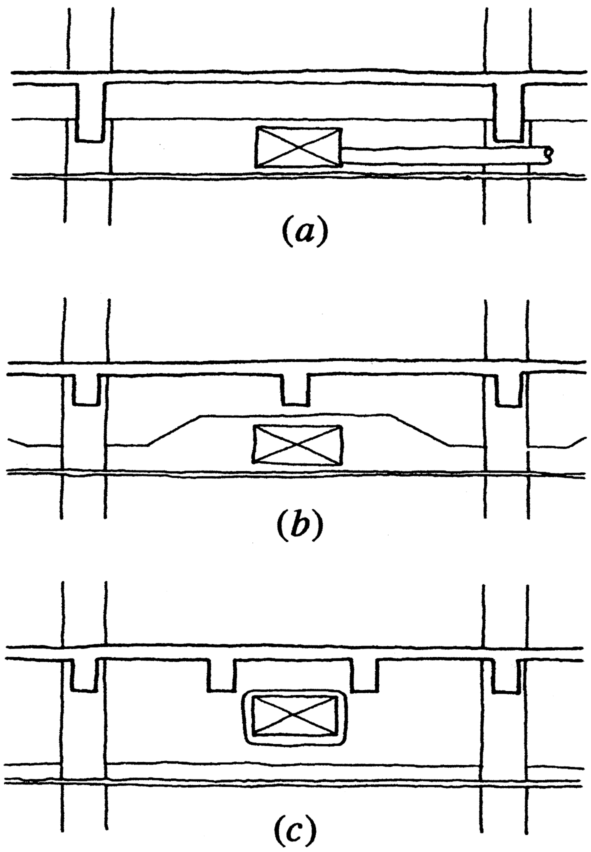


Figure 1.12 Accommodation of air ducts in the floor–ceiling construction: (*a*) by running major ducts parallel to major beams; (*b*) by varying beam depth; (*c*) by penetrating exceptionally deep beams.

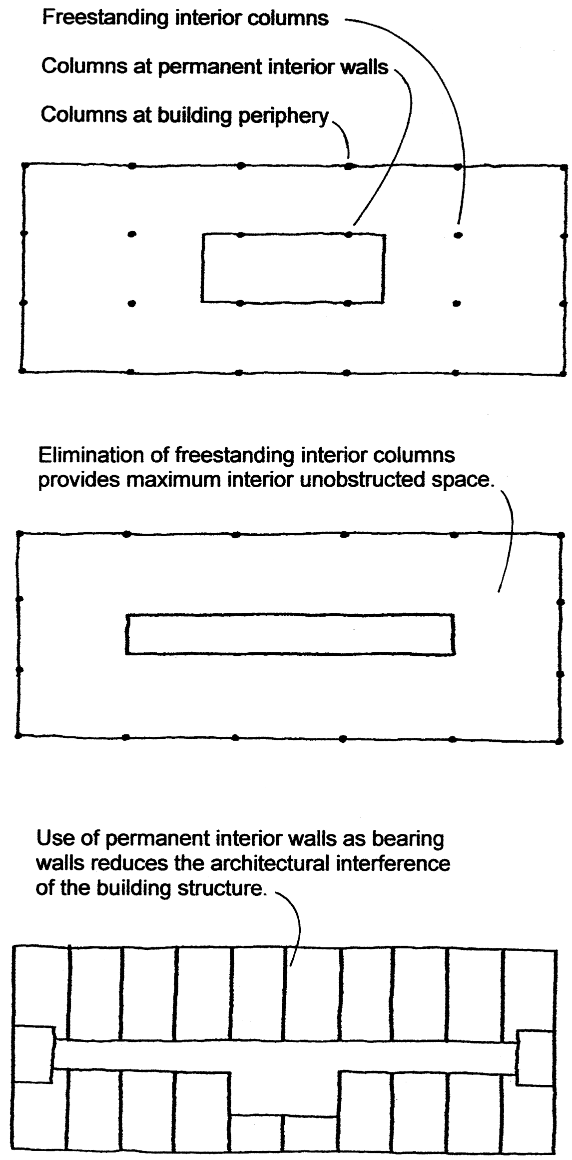


Figure 1.13 Development of vertical supports in multistory buildings.

Because of the stacking required, vertical structural elements are often a constant plan condition for each level, despite possible changes in architectural requirements at the various levels. An apartment building with parking in basement levels presents the problem of developing plans containing fixed locations of vertical structural elements that accommodate both the multiple parking spaces and the rooms of the apartments.

Vertical structural elements are usually walls or columns situated in one of three possible ways, as shown in Figure 1.13:

1. As isolated and freestanding columns or wall units in the interior of the building
2. As columns or walls at the location of permanent features such as stairs, elevators, toilets, or duct shafts
3. As columns or walls at the building periphery

Freestanding interior columns tend to be annoying for planning, because they restrict placement of doors and walls and are usually not desired within rooms. They are clumsy to incorporate into thin walls, as shown in Figure 1.14. This annoyance has motivated some designers to plan buildings with very few, if any, freestanding interior columns. The middle plan in Figure 1.13 shows such a solution, with interior supports only at the location of permanent construction. For buildings with fixed plan modules, such as hotels, dormitories, and jails, a plan with fixed interior bearing walls may be possible, as shown in the lower figure in Figure 1.13.

When columns are placed at the building periphery, their relationship to the building skin has a great bearing on the exterior appearance as well as interior planning. Figure 1.15 shows various locations for columns relative to the building skin wall.

Although freestanding columns (Figure 1.15*a*) are usually the least desirable, they may be tolerated if they are small (as in a low-rise building) and are of an unobtrusive shape (round, octagonal, etc.). The cantilevered edge of the horizontal structure is difficult to achieve with wood or steel framing but may actually be an advantage with some concrete systems.

Placing columns totally outside the wall (Figure 1.15*e*) eliminates both the interior planning intrusion and the

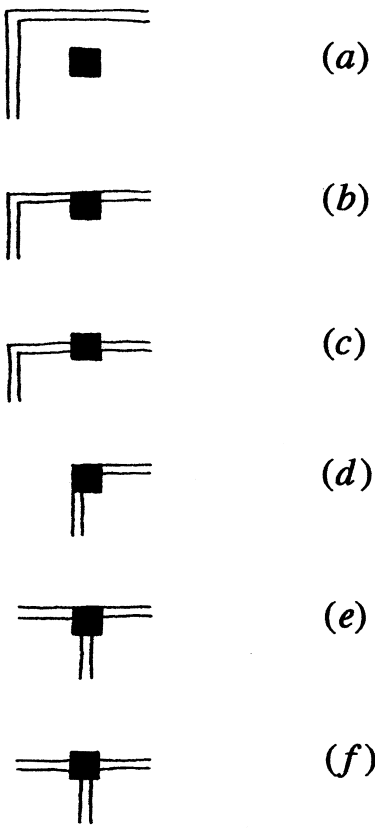


Figure 1.14 Interior column–wall relationships.

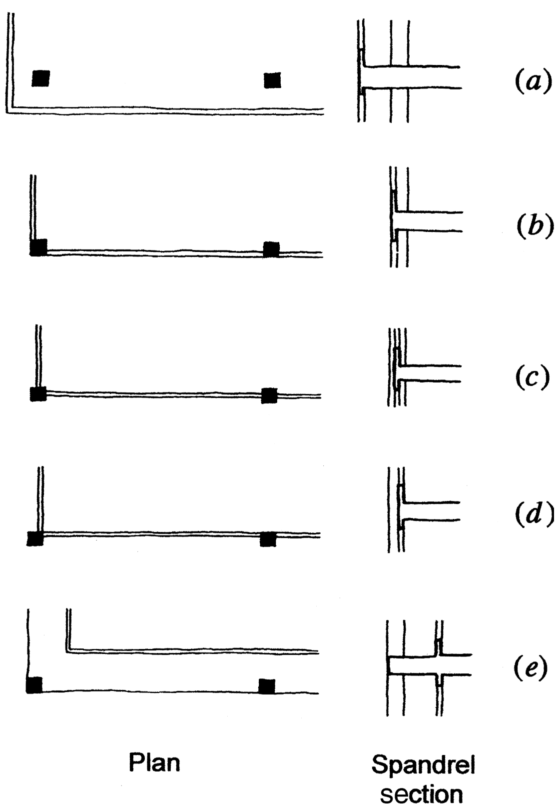


Figure 1.15 Relation of the structure to the building skin.

cantilevered edge. A continuous exterior ledge is produced and may be used as a sun shield, for window washing, as a balcony, or as an exterior balcony corridor. However, unless some such use justifies it, the ledge may be a nuisance, creating water runoff and dirt accumulation problems. The totally exterior columns also create a potential problem with thermal expansion.

If the wall and column are joined, three possibilities exist for the usually thick columns and usually thin wall, as shown in Figures 1.15*b*–*d*. For a smooth exterior surface, the column may be flush with the outside of the wall, although the interior lump may interfere with space use. If the wall is aligned with the interior edge of the column, the interior surface will be smooth (for ease of interior planning) but the outside will be dominated by the strong vertical ridges of the columns. The least useful scheme would seem to be to place the column midway in the wall plane (Figure 1.15*c*). Another solution, of course, is to thicken the wall sufficiently to accommodate the column—a neat architectural trick, but generally resulting in considerable wasted space in the building plan.

In tall buildings, column sizes usually vary from top to bottom of the building, although it is possible to achieve considerable range of strength within a fixed dimension, as shown in Figure 1.16. Although some designers prefer the more honest expression of function represented by varying column size, planning is often simplified by the use of a constant column size.

Planning problems usually make it desirable to reduce column size as much as possible. If size changes for interior columns are required, the usual procedure is to have the column grow concentrically, as shown in Figure 1.17. Exceptions are columns at the edges of stairwells or elevator shafts, where it is usually desirable to keep the inside surface of the shaft aligned vertically, as shown for the corner exterior columns in Figure 1.17.

For exterior columns, size changes relate to the column-to-skin relationship. If the wall is aligned with the inside of the column, there are several ways to let it grow in size without changing this alignment.

In very tall buildings, lateral bracing often constitutes a major concern. In regions of high risk for earthquakes or windstorms, this issue may dominate planning even for small and low-rise buildings.

Building–Ground Relationship

As shown in Figure 1.18, there are five variations of this relationship.

Subterranean Building

Figure 1.18*a* illustrates a situation that includes subway stations, underground parking structures, and bomb shelters. The insulating effect of the ground can be useful in extreme climates. Building surfaces must deal with soil pressures, water, and deterioration in general. Constant contact with the soil limits choices for construction materials.

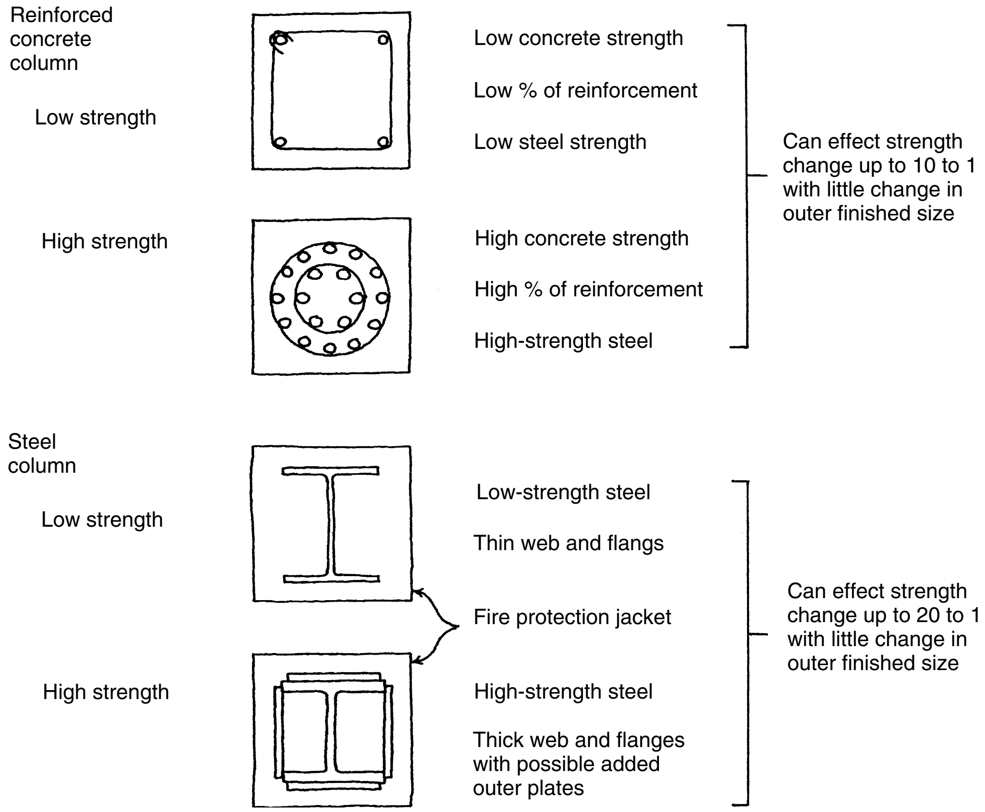


Figure 1.16 Variation in column strength without change in architectural finished size.

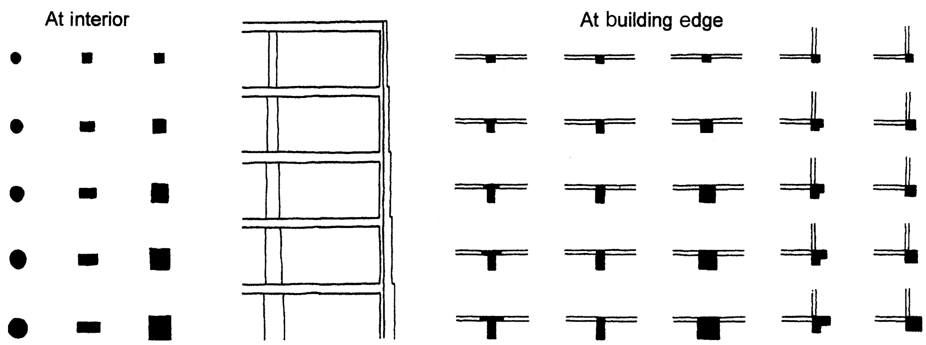


Figure 1.17 Patterns of size increase for columns in multistory buildings.

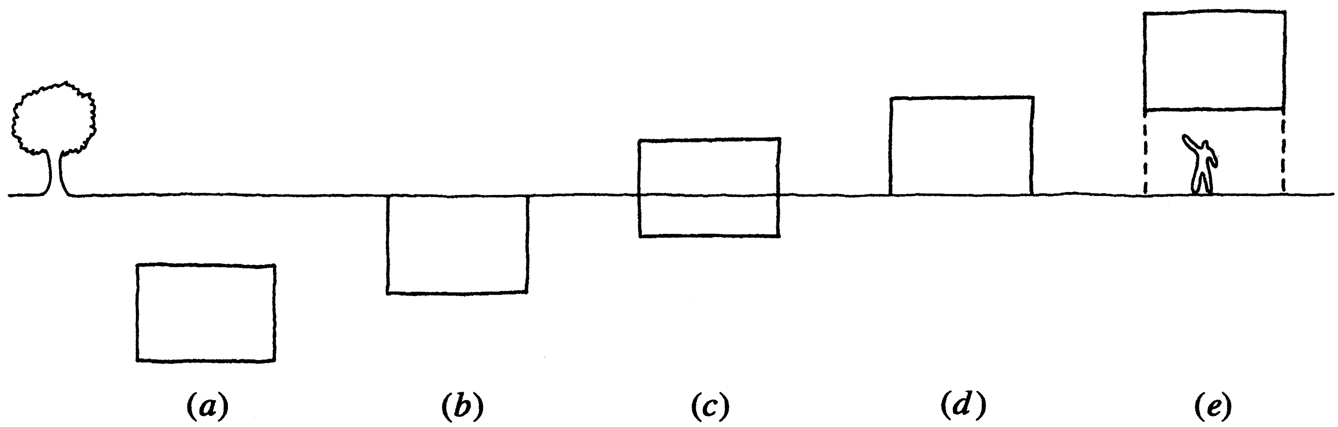


Figure 1.18 Building–ground relationships.

Ground-Level Roof

Figure 1.18*b* illustrates a situation similar to that of the submerged building, except that the exposed top offers some possibilities for light and air. Roof loading is less critical, although some traffic is likely and paving may be required. Some possibilities of opening up the building to dissipate the buried feeling are shown in Figure 1.19.

Partially Submerged Building

In this case the building often consists of two structural elements: the superstructure (above ground) and the substructure (below ground). The structure below ground has all the problems of the submerged building and in addition must support the superstructure. For very tall buildings the loads on the substructure will be very large. Horizontal wind and earthquake forces must also be resolved by the substructure. The superstructure is highly visible, whereas the substructure is not; thus the form of the superstructure is usually of much greater concern in architectural design.

Grade-Level Floor

Years ago basements were common, often required for gravity heating systems and storage of fuel. They were also useful for food storage before refrigeration and for general storage of junk. The advent of forced air heating systems and refrigeration and the high cost of construction have limited the use of basements unless they are needed for parking or housing of equipment.

If there is no basement, the building is reduced to a superstructure and a foundation, with the first floor often consisting of a simple paving slab. A building with no basement and only a shallow foundation system may have a problem regarding anchorage by the substructure for wind and earthquakes.

Above-Ground Building

As shown in Figure 1.20, buildings are sometimes built on legs, are cantilevered, or are suspended so that they are literally in midair. The support structures must be built into the ground, but the building proper may have little or no contact with the ground. The bottom floor of such a building must be designed for the barrier and filter functions usually associated only with roofs and exterior walls. If the floor underside is visible, it becomes an architectural design feature.

1.3 STRUCTURAL FUNCTIONS

Understanding of the work performed by structures requires the consideration of various issues; basic questions involve the following:

- Load sources and their effects
- What the structure actually does in performing its tasks of supporting, spanning, or bracing
- What happens internally in the structure
- What are the specific needs of the parts of the structure

Load Sources

The term *load* refers to any effect that results in a need for some resistive effort on the part of the structure. There are many sources for loads and many ways in which they can be classified. The principal kinds and sources of loads on building structures are discussed below.

Gravity

Source. Weight of the structure and other parts of the building, of occupants and contents, and of ice, snow, or water on the roof.

Computation. By determining the volume, density, and type of dispersion of items.

Application. Vertically downward and constant in magnitude.

Wind

Source. Moving air, in fluid flow action.

Computation. From anticipated maximum wind velocities as established by local weather history.

Application. As pressure perpendicular to exterior surfaces or as frictional drag parallel to surfaces. As a total horizontal force on the building or as action on any single surface.

Earthquakes (Seismic Shock)

Source. Movement or acceleration of the ground as a result of violent subterranean faults, volcanic eruptions, underground blasts, and so on.

Computation. By prediction of the probability of occurrence on the basis of the history of the region

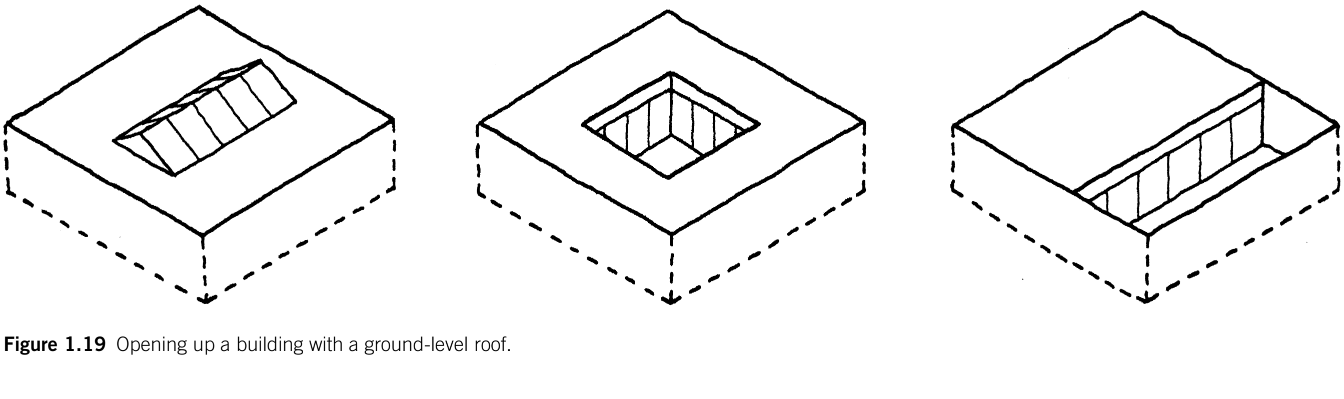


Figure 1.19 Opening up a building with a ground-level roof.

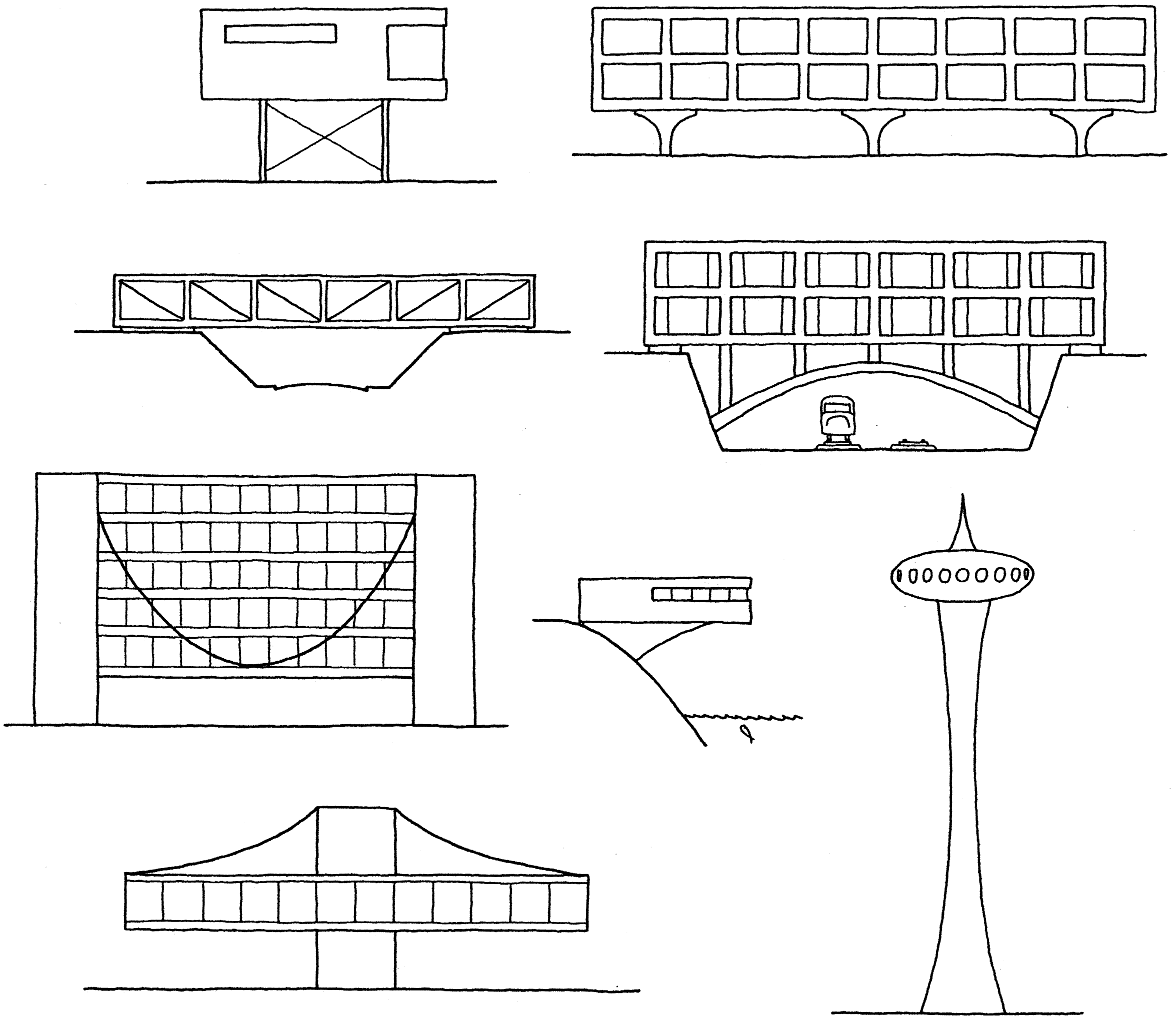


Figure 1.20 Buildings above ground.

and records of previous seismic activity. A principal force on the building structure is generated by the momentum of the building mass once it is moved.

Application. Consideration of the building mass as a horizontal or vertical force with the necessary resistance of the building's bracing system.

Hydraulic Pressure

Source. Principally from groundwater when the free water level in the soil is above the bottom of the building basement floor.

Computation. As fluid pressure proportional to the depth of the fluid.

Application. As horizontal pressure on walls or upward pressure on floors.

Soil Pressure

Source. Action of soil as a semifluid exerting pressure on buried objects or vertical retaining structures.

Computation. By considering the soil as a fluid with the typical hydraulic action of the fluid pressures.

Application. As for hydraulic pressure; horizontally on walls.

Thermal Change

Source. Temperature variations in building materials from fluctuations in outdoor temperature.

Computation. From weather histories, indoor design temperatures, and the coefficients of expansion of the materials.

Application. Forces on the structure if free movement due to expansion or contraction is prevented; stresses within the structure if connected parts have different temperatures or different rates of thermal expansion.

Other Potential Load Sources

Shrinkage. Volume reduction in concrete, plaster, stucco, or mortar in masonry joints as the materials dry out and harden. Dimensional and form changes in large timber members as the wood dries out from the green, freshly cut condition.

Vibration. Oscillations caused by machinery, vehicles, high-intensity sounds, and people walking.

Internal Actions. Movements within the structure due to settlement of supports, slippage of connections, warping of materials, and so on.

Secondary Structural Actions. Horizontal force effects from arches, gabled rafters, or tension structures. Soil pressures from nearby loads on the ground.

Handling of Construction. Forces generated during production, transportation, and erection of structural elements and extra loads from stored materials during construction.

Live and Dead Loads

A distinction is made between *live* and *dead loads*. A dead load is a permanent load, such as the weight of the building construction. A live load is anything that is not permanent, although the term is usually used to refer to loads on building floors.

Static versus Dynamic Loads

A distinction is also made between static and dynamic loads. This has to do with the time-dependent character of the load. As shown in Figure 1.21*a*, the weight of an object is considered static (essentially not moving); however, if the object is impelled, its weight becomes a potential dynamic effect. Dynamic effects include those from ocean waves, wind gusts, and seismic shocks.

The effects of dynamic loads are different for the building and the structure. A steel frame may adequately resist a dynamic load, but the distortion of the frame may result in cracked finishes or perception of movement by the building occupants (see Figure 1.21*b*). A masonry structure, although possibly not as strong as the steel frame, has considerable mass and resistance and may absorb a dynamic load with little perceptible movement.

Load Dispersion

Forces from loads may be distinguished by the manner of their dispersion. Gas under pressure in a container exerts a pressure effect that is uniform in all directions at all points. The dead load of roofing, the weight of snow, and the weight of water on the bottom of a tank are all loads that are

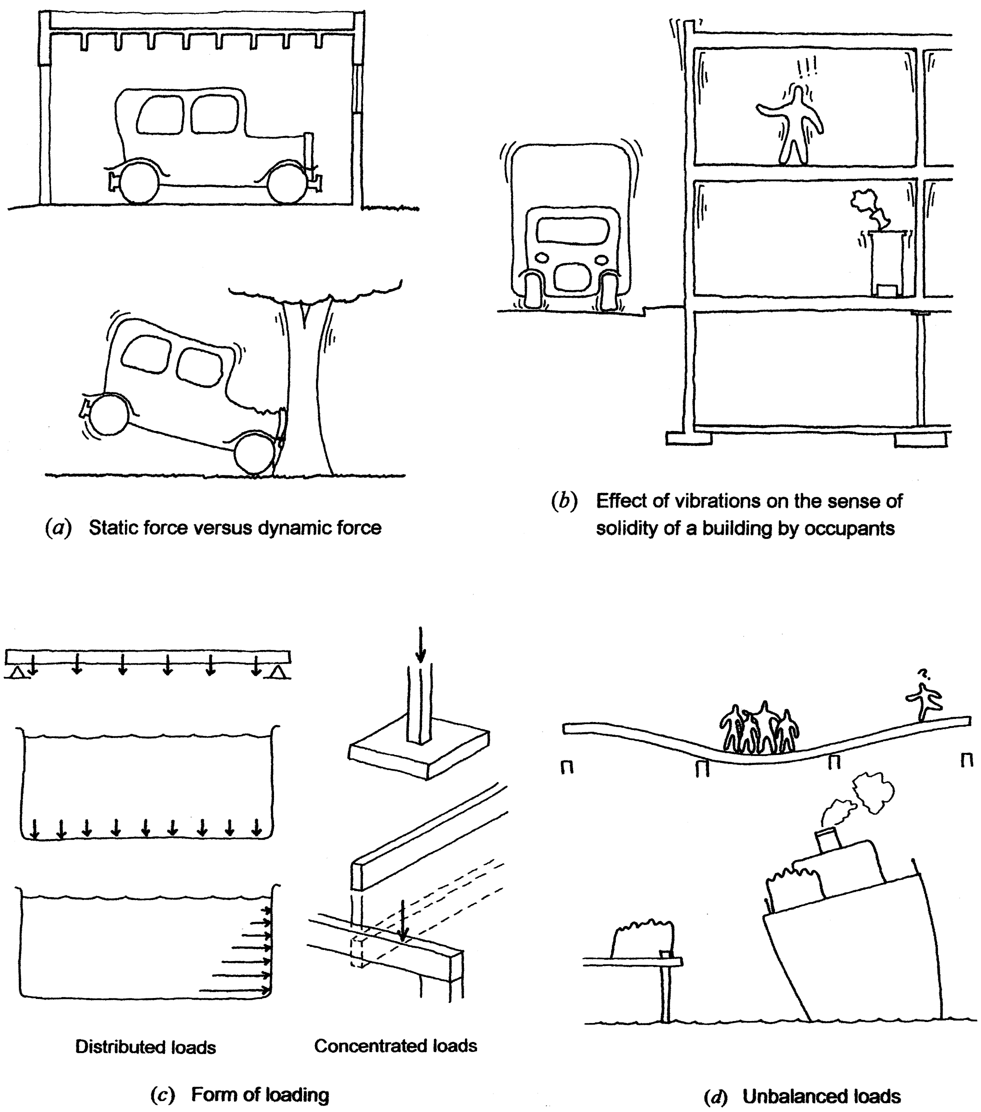


Figure 1.21 Load effects: (*a, b*) static and dynamic effects; (*c*) dispersion of loads; (*d*) unbalanced loads.

uniformly distributed on a surface. The weight of a beam is a load that is uniformly distributed along a line. The foot of a column or the end of a beam represents loads that are concentrated at a relatively small location. (See Figure 1.21*c*.)

Randomly dispersed live loads may result in unbalanced conditions or in reversals of internal conditions in the structure (see Figure 1.21*d*). The shifting of all the passengers to one side of a ship can cause the ship to capsize. A large load in one span of a beam that is continuous through several spans may result in upward deflection in other spans and possibly lifting of the beam from some supports. Because live loads are generally variable in occurrence, magnitude, location, and even direction, several different combinations of them must often be considered in order to determine the worst effects on a structure. Directions for performance of such investigations are given by building design codes.

Load Combinations

A difficult judgment for the designer is that of the likelihood of the simultaneous occurrence of various force effects. Combinations must be considered carefully to determine those that cause critical situations and that have some reasonable possibility of simultaneous occurrence. In most cases, directions for required combinations given by design codes are used, but many designers use their own judgment for other possible concerns.

Reactions

Successful functioning of a structure in resisting various loads involves two considerations. The structure must have sufficient internal strength and stiffness to redirect the loads to the supports without developing undue stress on the materials or an undesirable amount of deformation. In addition, the supports must develop the necessary forces—called *reactions*—to keep the structure from moving or collapsing.

The balancing of the loads from the structure and the reaction forces produces the necessary static condition for the structure. This condition is described as one of *static equilibrium*. Both the magnitudes and form of the support reactions must provide for this balanced condition. The form of the structure is one factor in establishing the character required for the reactions.

For the column in Figure 1.22, the reaction force generated by the support must be aligned with and be equal to the column load and must act upward in response to the downward column load.

Figure 1.22 also shows the reaction forces required for various spanning structures. For the beam with two supports, the two reaction forces must combine to develop a total force equal in magnitude to the beam load and must act upward. For the gable frame, the reactions must also develop vertical forces equal to the loads; however, the supports must also develop horizontal forces to keep the bottoms of the rafters from moving sideways. The net effect for the gabled frame is to have reactions that are not simply directed vertically.

Horizontal-force reaction components are also required for arches and cables. The means for achieving these horizontal reactions becomes more challenging when the spanning structure is supported on the top of tall columns or walls. Options for these supports are discussed in the next section.

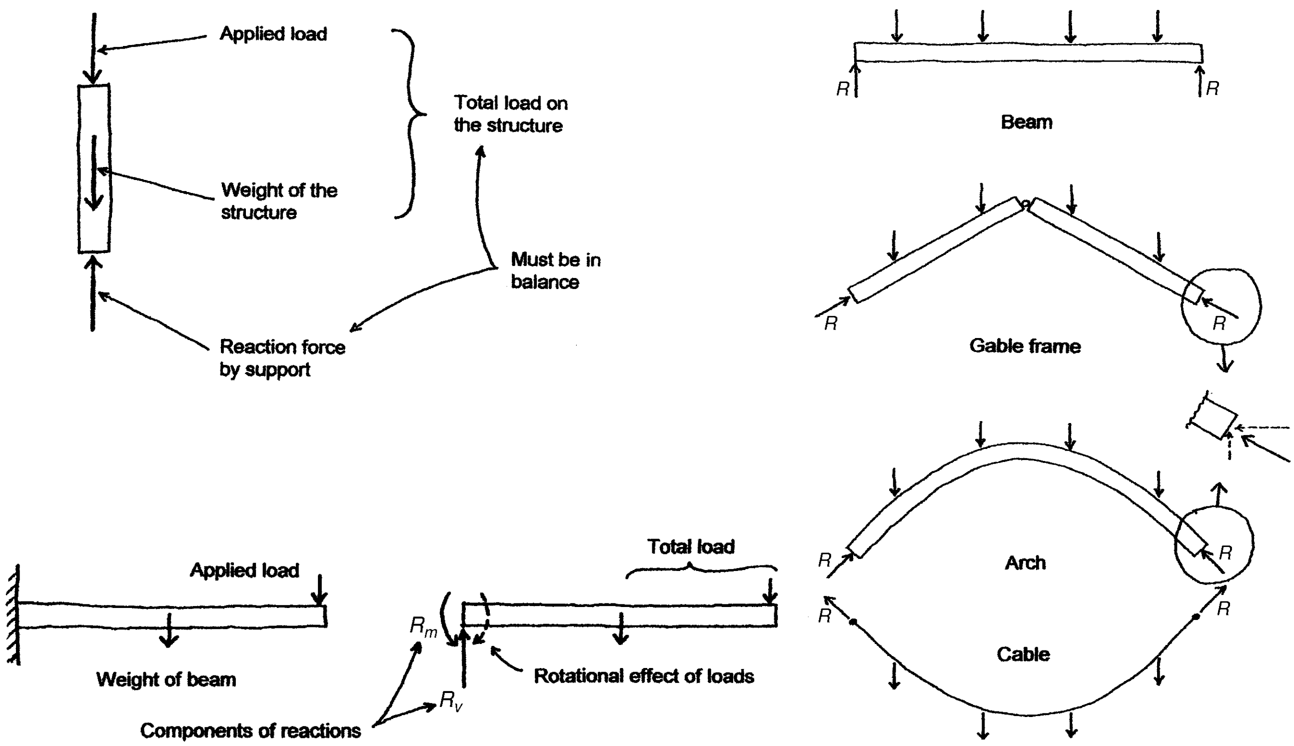


Figure 1.22 Development of reactions.

Another type of reaction force is required for the supported end of a cantilever beam. In this case, it is not sufficient to have simple, direct forces; another effort is required to keep the supported end of the cantilever from rotating. Thus the complete reaction system consists of a vertical force and a rotational resistance—called a *moment*. This combination may be developed for other structures, such as a flagpole.

For the rigidly connected frame shown in Figure 1.23 there are three possible components of the reactions. If vertical force alone is resisted, the bottoms of the columns will rotate and move outward, as shown in Figure 1.23*a*. If horizontal resistance is developed, the column bottoms can be pushed back to their original locations but will still rotate, as shown in Figure 1.23*b*. If a moment resistance is developed at the supports, the column bottoms can be held entirely in their original position, as shown in Figure 1.23*c*.

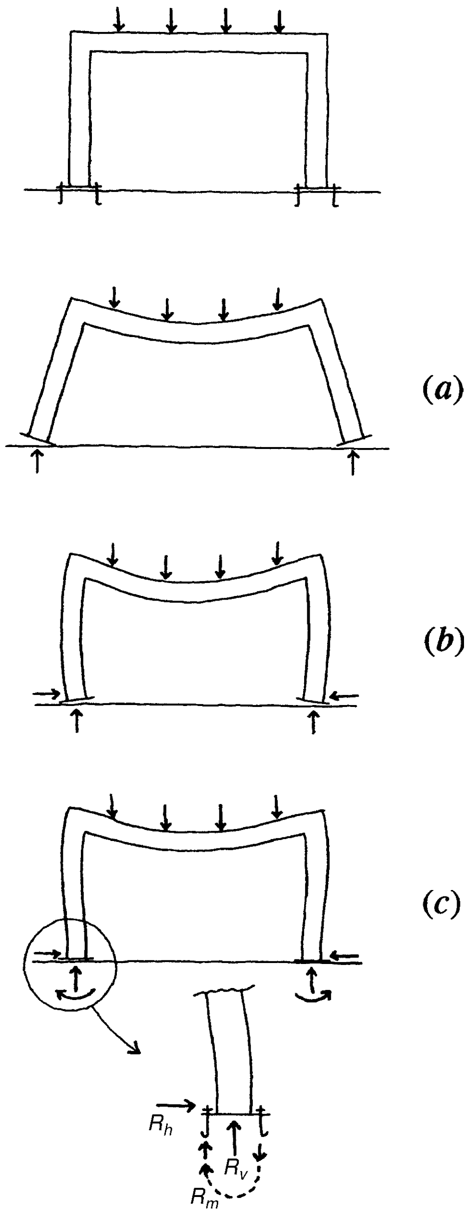


Figure 1.23 Reactions for a rigid frame.

The applied loads and support reactions for a structure constitute the external forces on the structure. This system of forces is in some ways independent of the structure's ability to respond. That is, the external forces must be in equilibrium if the structure is to be functional, regardless of the materials, strength, stiffness, and so on, of the structure itself. However, as has been shown, the form of the structure may affect the nature of the required reactions.

Internal Forces

In response to the external effects of loads and reactions, certain internal forces are generated within a structure as the material of the structure strives to resist the deformations induced by the external effects. These internal forces are generated by *stresses* in the material. The stresses are actually incremental forces within the material, and they result in incremental deformations, called *strains*.

When subjected to external forces, a structure sags, twists, stretches, shortens, and so on; or, to be more technical, it stresses and strains. It thus assumes some new shape as the incremental strains accumulate into overall dimensional changes. Whereas stresses are not visually apparent, their accompanying strains often are.

As shown in Figure 1.24, a person standing on a wooden plank that spans two supports will cause the plank to sag downward and assume a curved profile. The sag may be visualized as the manifestation of a strain phenomenon accompanying a stress phenomenon. In this example the principal cause of the structure's deformation is bending resistance, called *internal bending moment*. The stresses associated with this internal force action are horizontally directed compression in the upper portion of the plank and horizontally directed tension in the lower portion. Anyone could have predicted that the plank would assume a sagged profile when the person stepped on it. However, we can also predict the deformation as an accumulation of the strains, resulting in the shortening of the upper portion and the lengthening of the lower portion of the plank.

We would not, of course, want a building floor to sag like the example plank. However, it is useful for investigations to understand internal force actions by the device of visualizing exaggerated deformations.

Because stress and strain are inseparable, it is possible to infer one from the other. This allows us to visualize the nature of internal force effects by imagining the exaggerated form of the deformed structure under load. Thus, although stresses cannot be seen, strains can, and the nature of the accompanying stresses can be inferred. This relationship can be used in simple visualization or it can be used in laboratory tests where quantified stresses are determined by careful measurement of observed strains.

Any structure must have certain characteristics in order to function. It must have adequate strength for an acceptable margin of safety and must have reasonable

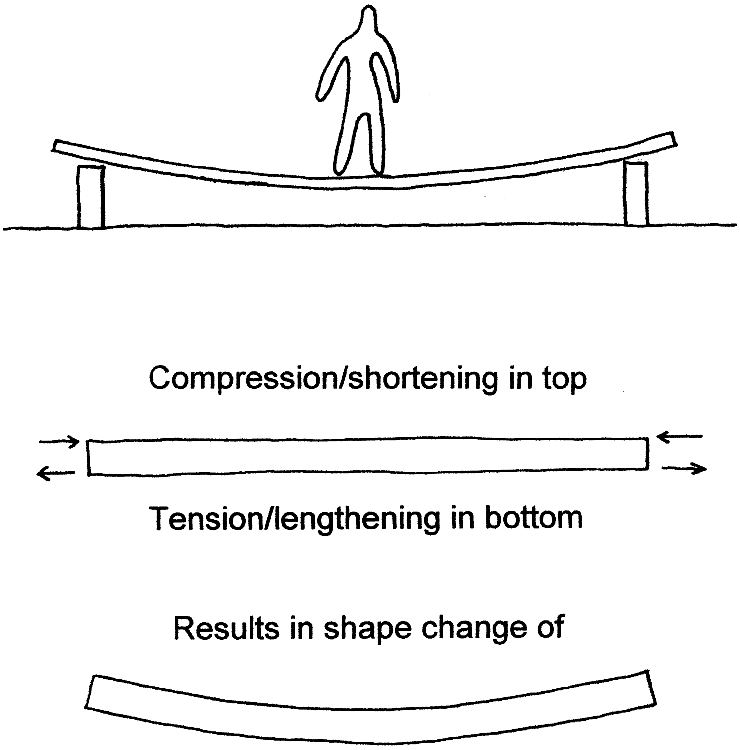


Figure 1.24 Development of bending.

resistance to dimensional deformation. It must also be inherently stable, both internally and externally. These three characteristics—strength, stiffness, and stability—are the principal functional requirements of structures.

Stresses and Strains

There are three basic types of stress: tension, compression, and shear. Tension and compression are similar in nature although opposite in sign or direction. Both tension and compression produce a linear type of strain and can be visualized as pressure effects perpendicular to the surface of a stressed cross section. Because of these similarities, tension and compression are referred to as *direct stresses*; one is considered positive and the other negative.

Shear stress occurring in the plane of a cross section is similar to the frictional sliding effect. Strain due to shear takes the form of angular distortion, rather than the lengthening or shortening due to direct stress.

Dynamic Effects

Vibrations, moving loads, and sudden changes in the state of motion, such as the jolt of braking or rapid acceleration of vehicles, cause force effects that result in stresses and strains in structures. The study of dynamic forces and their effects is complex, although some of the basic concepts can be illustrated simply.

For structural investigation and design the significant distinction between static and dynamic effects has to do with response of the structure to the loading. If the principal response of the structure can be effectively evaluated in static terms (force, stress, linear deformation, etc.), the effect on the structure is essentially static. If, however, the structure's response can be evaluated effectively only in terms such as energy capacity, work accomplished, cyclic movement, and so on, the effect of the loading is of a true dynamic character. Judgments made in this regard must be made in consideration of the adequate performance of the structure in its role in the building system. Performance relates to both structural responses and effects on the building and its occupants.

A critical factor in the evaluation of a structure's response to dynamic loads is the *fundamental period* of the structure's cyclic motion or vibration. This is the time required for one full cycle of motion, in the form of a bounce or a continuing vibration. The relation of this time to the time of buildup of the load is a major factor in the determination of the relative degree of the dynamic effect on the structure. A structure's fundamental period may vary from a fraction of a second to several seconds, depending on the size, shape, mass, materials, stiffness, support conditions, and possible presence of damping effects.

Design for dynamic effects begins with an evaluation of possible dynamic load sources and their potential actions. The response of the structure is then considered using the variables of its dynamic character. Once the dynamic behavior is understood, the designer can consider how to manipulate the variables to improve the structure's behavior or to reduce the load effects.

Design for Structural Behavior

In professional design practice the investigation of structural behavior is an important part of the design process. To

incorporate this investigation into design work, the designer needs to develop a number of capabilities, including the following:

The ability to visualize and evaluate the sources that produce the loads on the structure
The ability to quantify the loads and the effects they produce on the structure
The ability to analyze the structure's response to the loads in terms of internal forces, stresses, and strains
The ability to determine the structure's limits of load-carrying capacity
The ability to manipulate the variables of material, form, and construction details for the structure in order to optimize its responses to loads

Although analysis of stresses and strains is necessary in the design process, there is a sort of chicken-and-egg relationship between analysis and design. To analyze some of the structure's responses, one needs to know some of its properties. However, these properties are not known until the designed object is established. In some simple cases it is possible to derive expressions for desired properties by simple inversion of analytical formulas. For example, a simple formula for stress in a compression member is

$$\text{Stress} = \frac{\text{total load on the member}}{\text{area of the member cross section}}$$

If the load is known and the stress limit for the material is established, this formula can be easily converted to one for finding the required area of the cross section, as follows:

$$\text{Required area} = \frac{\text{total load on the member}}{\text{stress limit for the material}}$$

Most structural situations are more complex, however, and involve variables and relationships that are not so simply converted for design use. In the case of the compression member, for example, if the member is a slender column, its load capacity will be limited to some degree by the tendency to buckle. The relative stiffness of the column in resisting buckling can be determined only after the geometry of its cross section is known. Therefore, the design of such an element is a hit-or-miss situation, consisting of guessing at a possible cross-sectional shape and size, analyzing for its performance, and refining the choice as necessary until a reasonable fit is established.

Professional designers use their experience together with various design aids, such as tabulations of capacities of common elements, to shorten the design process. Even so, final choices often require some progressive effort.

Investigation of Structural Behavior

Whether for design purposes, for research, or for study of structural behaviors as a learning experience, analysis of stresses and strains is important. Analysis may be performed as a testing procedure on the actual structure with a loading applied to simulate actual usage conditions. If carefully done, this is a highly reliable procedure. However, except for some of the widely used elements of construction, it is generally not feasible to perform destructive load testing on building structures built to full scale. The behavior of building structures must usually be anticipated speculatively on the basis of demonstrated performance of similar structures or on a modeling of the actions involved. The modeling can be done in the form of physical tests on scaled-down structures but is most often done mathematically using the current state of knowledge in the form of formulas for analysis. When the structure, the loading conditions, and the necessary formulizations are relatively simple, computations may be done by "hand." More commonly, however, computations of even routine nature are done by professional designers using computer-assisted techniques. While the computer is an extremely useful tool, it is imperative that the designer keep an upper hand in this process by knowing reasonably well what the computer is doing, a knowledge often gained from a lot of "hand" investigations and the follow-up to applications in design decision making. Otherwise, there is often the danger of garbage in, garbage out.

1.4 STRUCTURAL MATERIALS

All materials—solid, liquid, or gaseous—have some structural nature. The air we breathe has a structural nature: It resists compression when contained. Every time you ride in a car you are sitting on an air-supported structure. Water supports the largest human-made vehicles: huge ships. Oil resists compression so strongly that it is used as the resisting element in hydraulic presses and jacks capable of developing tremendous force.

In the design of building structures, use is made of the available structural materials and the products formed from them. The discussion in this section deals with common structural materials and their typical uses in contemporary construction.

General Considerations

Broad classifications of materials can be made, such as distinctions among animal, vegetable, and mineral; between organic and inorganic; and the physical states of solid, liquid, and gaseous. Various chemical and physical properties distinguish individual materials from others. In studying or designing structures, particular properties of materials are of concern. These may be split between essential structural properties and general properties.

Essential properties for building structures include the following:

Strength. May vary for different types of force, in different directions, at different ages, or at different amounts of temperature or of moisture content.

Deformation Resistance. Degree of rigidity, elasticity, ductility; variation with time, temperature, and so on.
Hardness. Resistance to surface indentation, scratching, abrasion, and general wear.
Fatigue Resistance. Time loss of strength, progressive fracture, and shape change with time.
Uniformity of Physical Structure. Grain and knots in wood, cracks in concrete, shear planes in stone, and effects of crystallization in metals.

Some general properties of interest in using and evaluating structural materials include the following:

Form. Natural, reshaped, reconstituted.
Weight. Contributing to gravity loads.
Fire Resistance. Combustibility, conductivity, melting or softening point, and general behavior at high temperature.
Thermal Expansion. Relating to dimensional change.
Availability and Cost.
Green Concerns. Toxicity, renewable source, energy use for production, potential for reuse.

In any given situation choices of materials must often be made on the basis of several properties—both structural and general. There is seldom a material that is superior in all respects, and the importance of various properties must often be ranked.

Wood

Technical innovations have overcome some of the long-standing limitations of wood. Size and form limits have been extended by various processes, including lamination and reconstitution as fiber products. Special fastenings have made some structures possible through better performing jointing. Combustibility, rot, and insect infestation can be retarded by chemical impregnations.

Dimensional movements from changes in temperature or moisture content remain a problem with wood. Although easily worked, wood elements are soft and readily damaged; thus damage during production, transportation, and construction and even some uses are a problem.

Although hundreds of species (different trees) exist, structural use is limited mostly to a few softwoods: Douglas fir, southern pine, northern white pine, spruce, redwood, cedar, and hemlock. Regional availability and cost are major concerns in selection of a particular species.

Economy is generally achieved by using the lowest grade (quality) of material suitable for the work. Grade is influenced by lack of knots, splits, and pitch pockets and by the particular grain character of individual pieces.

Fabricated products are increasingly used in place of solid-sawn wood pieces. Plywood and glued laminated timbers have been used for some time. More recently items fabricated from wood fibers and strands are being used to replace plywood panels and light framing elements.

Fabricated compound structural elements are also widely used. The light wood truss with wood top and bottom chords and metal interior members is in direct competition with the steel open web joist for medium- to long-span roof and floor structures. A newer product is the wood I-joist, composed of solid wood or laminated top and bottom pieces and a web of plywood or fiber board.

Because of its availability, low cost, and simple working possibilities, wood is used extensively for secondary and temporary construction. However, it is also widely used for permanent construction and is generally the material of choice for light construction unless its limitations preclude its use. It is a renewable resource, although the best wood comes from very slow-growth trees. However, the most extensive use of wood is as fiber for the paper industry, which has become a major commercial institution in the United States. The fiber users can use small, fast-growth trees and they routinely plant and harvest trees for quick turnover. This is a major factor in the rapid expansion of use of fiber products for building construction.

Steel

Steel is used in a variety of forms in nearly every building. From its use for huge towers to the smallest nails, steel is the most versatile of traditional materials. It is also one of the strongest, the most resistive to aging, and generally the most reliable in its quality control. Steel is a highly industrialized material and is subjected to tight control of its content and of the details of forming and fabrication. It has the additional desirable qualities of being noncombustible, nonrotting, and dimensionally stable with time and moisture change.

Although the bulk material is expensive, steel can be used in small quantities because of its high strength and its forming processes; thus the completed steel structure is competitive with structures produced with materials of much cheaper bulk cost. Economy can also be produced with mass production of standardized items. Choosing the parts for a steel structure is done mostly by picking items from standard documented references.

Two principal disadvantages of using steel for building construction are inherent in the basic material. These are its rapid heat gain and resultant loss of strength when exposed to fire and its rusting when exposed to moisture and air or to corrosive conditions (such as salty water). A variety of techniques can be used to overcome these limitations, two common ones being special coatings and the encasing of the steel in construction of a protective nature.

Concrete

The word *concrete* is used to describe a number of materials having something in common: the use of a binder to form a solid mass from a loose, inert aggregate. The three basic ingredients of ordinary structural concrete are water, a binder (cement), and a large volume of loose aggregate (sand and gravel). Variation of the end product is endless through the use of different binders and aggregates and with

the use of chemical additives and air-producing foaming agents.

Ordinary cementitious concrete has several attributes, chief among which are its low bulk cost and its resistance to moisture, rot, insects, fire, and wear. Being formless in its initial mixed condition, it can be made to assume a large variety of forms.

One of concrete's chief shortcomings is its lack of tensile strength. The use of inert reinforcement or prestressing is imperative for any structural functions involving bending or torsion. Recent use of imbedded fibers is another means for enhancing resistance to tension. Because the material is formless, its forming and finishing are major expenses in its use. Precasting in permanent forms is one means for reducing forming cost.

Aluminum

In alloyed form, aluminum is used for a large variety of structural, decorative, and hardware elements in building construction. Principal advantages are its light weight (one-third that of steel) and its high resistance to corrosion. Some disadvantages are its softness, its low stiffness, its high rate of thermal expansion, its low resistance to fire, and its relatively high cost.

Large-scale structural use in buildings is limited by cost and its increased dimensional distortion due to its lack of stiffness. Low stiffness also reduces its resistance to buckling. Minor structural use is considerable, however, for window and door frames, wall panels, trim, and various hardware items.

Masonry

The term *masonry* is used to describe a variety of formations consisting of separate, inert objects bonded together by some binding joint filler. Elements may be rough or cut stone, fired clay tile or bricks, or cast units of concrete. The binder is usually a cement and lime mortar. The resulting assemblage is similar in weight and bulkiness to concrete construction and possesses many of the same properties. Assemblage typically involves considerable hand labor, making it highly subject to the skill of individual craftspersons. Reinforcing can be used to increase strength, as is commonly done for increased resistance to windstorms and earthquakes. Shrinkage of the mortar and thermal-expansion cracking are two major concerns that necessitate care in detailing, material quality control, and field inspection during construction.

Plastics

Plastic elements represent the widest variety of usage in building construction. The great variation of material content, properties, and formation processes yields an unlimited field for the designer's imagination. Some of the principal problems with plastics are lack of fire resistance, low stiffness, high rate of thermal expansion, and some cases of chemical or physical instability with time.

Some of the uses in building construction are:

Glazing. For windowpanes, skylights, and sheet-form or corrugated panels.
Coatings and Laminates. Sprayed, painted, or rolled on or applied as laminates in composite panels.
Formed elements. For frames, trim, and hardware.
Foamed. In preformed or foamed-in-place applications, as insulation and filler for various purposes.

Design developments in recent years include pneumatic and tension-sustained surface structures using various plastic membranes and fabric products. Small structures may use thin plastic membranes, but for larger structures the surface material is usually a coated fabric with enhanced resistance to tension and tearing. The plastic-surfaced structure can also be created by using plastic elements on a framework.

Miscellaneous Materials

Glass

Ordinary glass possesses considerable strength but has the undesirable characteristic of being brittle and subject to shattering under shock. Special glass products are produced with higher strength, but a more widely used technique is to produce laminated panels with alternating layers of plastic and glass—like good old "safety glass," which has been in use for car windows for a long time.

Fiber-Reinforced Products

Glass fiber and other stranded elements are used to strengthen paper, plastic membranes, and various panel materials. This notably increases tension and tear resistance.

Paper

Paper—that is, sheet material of basically rag or wood fiber content—is used considerably in building construction, although for some uses it has been replaced by plastics. Various coatings, laminations, impregnations, and reinforcing can be used to produce a tougher or more moisture-resisting material. A widely used product is the "drywall" panel, consisting of a thin slab of plaster sandwiched between two thick paper sheets.

Mixed Materials

Buildings use a large mixture of materials for their construction. This also applies to building structures. Just about every building has concrete foundations, regardless of the materials of the rest of the structure. For structures of wood, concrete, and masonry, many steel elements will be used for fastenings, reinforcement, and other purposes. Nevertheless, despite the typical material mixture that designers must use, the industries that produce structural products are very material specific. Thus major concentrations exist in terms of primary structural materials: wood, steel, concrete, and masonry. Information for design comes primarily from these sources.

1.5 STRUCTURAL SYSTEMS

The materials, products, and systems available for the construction of building structures constitute a vast inventory through which the designer must sift carefully for the appropriate selection in each case. The material in this section presents some of the issues relating to this inventory and its applications.

Attributes of Structural Systems

A specific structural system derives its unique character from any number of considerations—and probably from many of them simultaneously. Considered separately, some of these factors are the following:

Structural Functions or Tasks. These include support in compression (piers, footings, columns); support in tension (vertical hangers, guy wires, suspended cables); spanning—horizontally (beams, arches), vertically (window glass, basement walls), or inclined (rafters); cantilevering—vertically (flagpoles) or horizontally (balconies, canopies). A single element or system may be required to perform more than one of these functions, simultaneously or for different loading conditions.

Geometric Form or Orientation. Note the difference between the flat beam and the curved arch, both of which can be used for the same basic task of spanning horizontally. The difference is one of structural form. Also compare the arch and the suspended cable—similar in form but different in orientation to the loads.

Material of Elements. May possibly be all the same or of different materials in complex systems.

Manner of Joining of Elements. A major concern for systems with many assembled parts.

Loading Conditions. Sources, static or dynamic, in various combinations.

Usage. Structures usually serve some purpose (wall, roof, floor, bridge, etc.) and must be appropriate to the task.

Limits of Form and Scale. Many factors establish both upper and lower limits of size. These may have to do with material sources, with joining methods, or with inherent performance characteristics of particular systems or elements.

Special Requirements. Performance may be conditioned by need for light weight, visual exposure, demountability, portability, multifunctions, and so on.

Structural systems occur in almost endless variety. The designer, in attempting to find the ideal structure for a specific purpose, is faced with an exhaustive process of comparative "shopping." The ideal solution is mostly illusive, but careful shopping can narrow the field of acceptable solutions. For shopping, a checklist can be used to rate the available known systems for a given purpose. Items to be considered are as follows:

Economy. This includes the cost of the structure itself as well as its influence on the total cost of construction. Some special considerations may be required for factors such as slow construction time, adaptability to modifications, and first cost versus maintenance cost over the life of the structure.

Special Structural Requirements. These may include unique aspects of the structure's action, details required for development of its strength and stability, adaptability to special loadings, and need for symmetry or modular development. Thus arches require horizontal restraints at their bases; tension elements must be hung from something. Structures with very thin parts must be braced or stiffened against buckling, and domes must have some degree of symmetry and a concentric continuity.

Problems of Design. Possible concerns include difficulty of performing reliable investigation of behaviors, ease of detailing of the structure, and ease of integrating the structure with the other elements of the complete building.

Problems of Construction. Possible issues include availability of materials—especially ones that are difficult to transport, availability of skilled labor or equipment, speed of erection, requirements for temporary bracing or forming, and need for on-site storage of large inventories of parts.

Material and Scale Limitations. There are feasible ranges of size for most systems; for example, beams cannot span nearly as far as trusses, arches, or cables.

Form Constraints. Arches and domes produce curved shapes, which may or may not be acceptable for the building form. Cables sag and produce low points at the center of their spans. For efficient performance, trusses need some significant height of the truss structure itself, generating space that may not be usable on the building interior.

Historic Precedent. Many structural systems have been developed over long periods of time and are so classical in both their structural performance and their accommodation of desired architectural forms that they well may be considered as permanent features of construction. However, materials and construction processes change, and the means for achieving some classic forms are often not the same as they were in previous times. Figure 1.25 shows a centuries-old structure (Santa Ines Mission Church in Santa Ynez, California) with construction primarily of adobe bricks and hand-hewn timbers. Neither of these construction methods is likely to be possible today, there being no hand-hewers and seismic design criteria pretty much ruling out the unreinforced adobe. While it might be desirable to recapture the charm of the old building, it would have to be faked.

Figure 1.25 Historic construction with hand-hewn timbers and adobe brick walls and arcades.

Categorization of Structural Systems

Structural systems can be categorized in a variety of ways. One broad differentiation is that made between solid structures, framed structures, and surface structures.

Solid Structures

Solid structures are structures in which strength and stability are achieved through mass, even though the structure is not completely solid (Figure 1.26*a*). Large piers, abutments, dams and seawalls, retaining walls, and ancient burial pyramids are examples. These are highly resistive to forces such as those created by blasts, violent winds, wave actions, and intense vibrations. Although exact analysis is usually highly indeterminate, distribution of stresses may be diffuse enough to allow simple approximations with a reasonable assurance of safety.

Framed Structures

Framed structures consist of networks of assembled elements (Figure 1.26*b*). Construction is completed by filling in the voids as required between the spaced frame members. Although the infilling elements may also have a structural character—often serving to brace the frame members, for example—they do not serve as essential components of the frame itself. Animal skeletons, beam and column systems, and trussed towers are examples.

These structures are generally very adaptable to variations in form, dissymmetry of layouts, and the carrying of special loads. They can be cumbersome, however, if the complexity of their assembly becomes excessive. Attachment of infilling elements must also be accommodated. Construction can be achieved in wood, steel, or concrete, and a great range of scale is possible, from the most modest light wood frames (2 by 4s, etc.) to the tallest high-rise structures.

Surface Structures

Surface structures can be very efficient because of their simultaneous twofold function as structure and enclosure (Figure 1.26*c*). They may also be very stable and strong, especially in the case of three-dimensional forms. They are, however, somewhat limited in resisting most concentrated loads and in facilitating any sudden discontinuities such as openings. They are also quite dominant as building form givers, which may be useful or not for the building's functions.

Other categorizations can be made for particular types of construction or configuration of the structure. Thus we describe certain family groups such as structural walls, post and beam, arch, suspension, pneumatic, trussed folded plate,

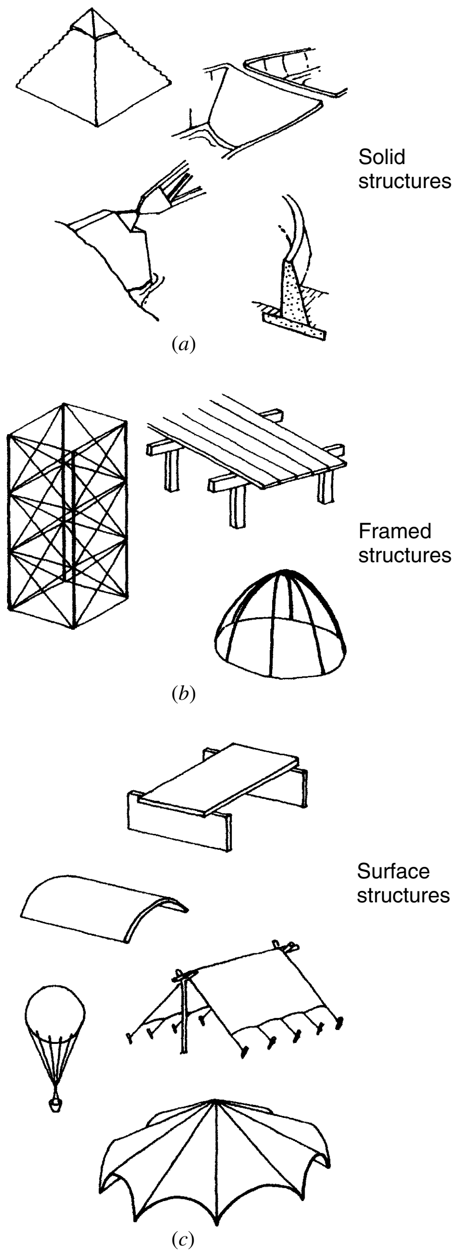


Figure 1.26 Categorization of structures by basic constructive nature.

or thin shell systems. Each of these has certain characteristics and is subject to specific material–scale limitations. Each lends itself most aptly to certain uses. A knowledge of the specific attributes of the various systems is essential to the designer but can be gained only by exhaustive study and some design experience. The inventive designer can, of course, consider new variations of the basic systems and possibly invent systems with no existing categorical identification.

A complete presentation of all structural materials and systems and a discussion of their relative merits, potentialities, and limitations would undoubtedly fill a volume several times the size of this book. Nevertheless, a short survey of traditional systems with some commentary follows. The categorization used—for example, post and beam—has no particular significance; it is merely a convenient one for discussion.

Structural Walls

It seems to be a direct structural development to use the enclosing and dividing walls of a building for support and bracing. When this system is utilized, there are usually two distinct elements in the total building structure:

Walls. Used to provide lateral stability for the building as well as to support the spanning elements.

Spanning Elements. Functioning as roofs and floors and as definers of clear-span interior spaces.

The spanning elements are usually structurally distinct from the walls and can be considered separately. They may consist of a variety of assemblies, from simple wooden joists and decks to complex precast concrete or trussed systems. The flat-spanning system is discussed as a separate category. However, if either the supporting walls or the spanning systems have some modular, repeating dimension or form, some codependent relations must be considered for the two systems. They will be joined, and the joining must be considered.

Bearing walls are essentially compression elements. They may be of monolithic form (like a brick wall) or may actually be frameworks assembled of many pieces (like a wood stud wall). They may be uninterrupted, or they may be pierced in a variety of ways (see Figure 1.27). Holes for windows and doors may be punched in the solid wall, and as long as their heads are framed and they are arranged so as not to destroy the structural potential of the wall, the structure remains intact.

Even when not used for support of vertical loads, walls are often used to provide lateral stability for the building. This can be achieved by having the wall acting independently or in combined interaction with the structural frame. An example of the latter is a plywood sheet surfacing attached to wooden studs. Even if it does not share in vertical-load development, the plywood attached to the studs will prevent collapse of the studs in the direction of the wall plane. This lateral bracing potential of the rigid wall plane (called *shear wall action*) is extensively used in bracing wood frame structures against wind and earthquake forces.

Consider the simple structure shown in Figure 1.28, which consists of a single space bounded by four walls and a flat roof. The two end walls in the upper illustration are capable of resisting horizontal forces in a direction parallel to the plane of the walls. However, horizontal forces in a direction perpendicular to the plane of the walls would not be resisted easily. If the other two walls of the building are also rigid, they may, of course, function to resist forces parallel to their planes. However, if these walls lack the structural capacity to function as shear walls, one device that may be used is to turn the ends of the end walls slightly around the

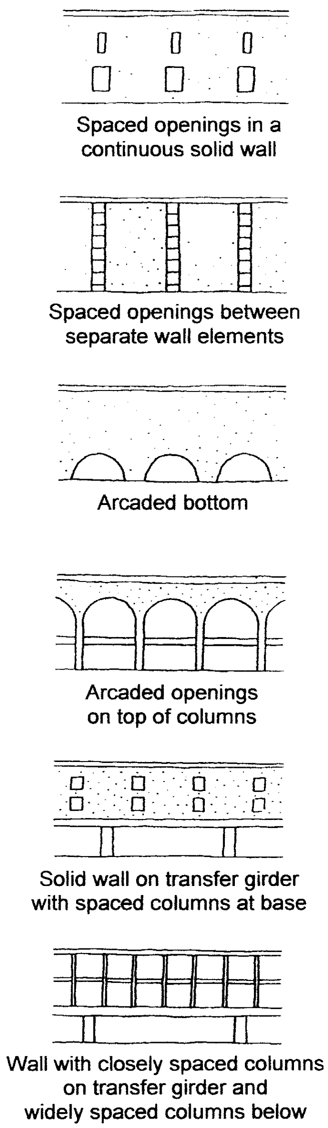


Figure 1.27 Opening up the structural wall.

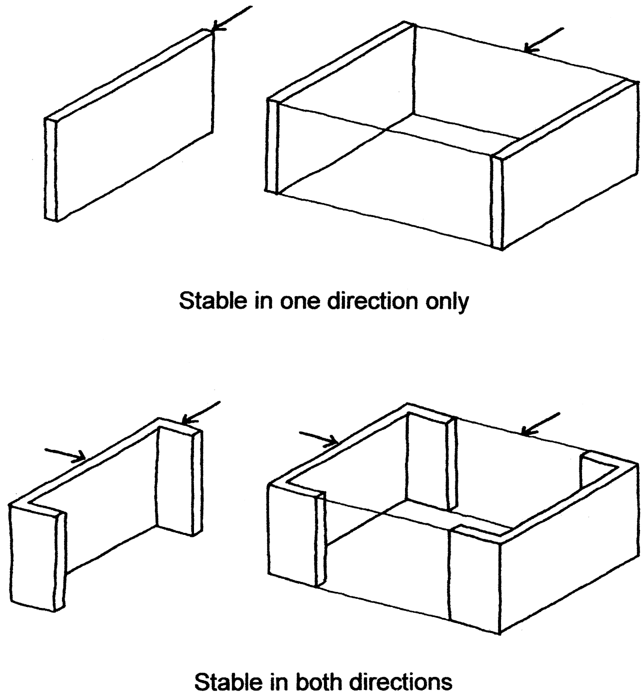


Figure 1.28 Lateral stability of walls.

corners, as shown in the lower illustration. This makes the walls independently stable against horizontal forces from all direction.

The device just illustrated is one technique for stabilizing the flat wall against horizontal forces perpendicular to the wall. It may also be necessary to stabilize the wall against buckling under vertical loads if it is very tall and thin. In addition to folding or curving the wall in plan, some other means for achieving transverse resistance are as follows (see Figure 1.29):

Spreading the Base. This can be done by thickening the wall toward its base, as with a gravity dam, or by attaching the wall rigidly to a broad footing.

Stiffening the Wall with Ribs. This is done most often to brace tall walls against buckling or to provide localized strength for heavy concentrated loads on top of the wall.

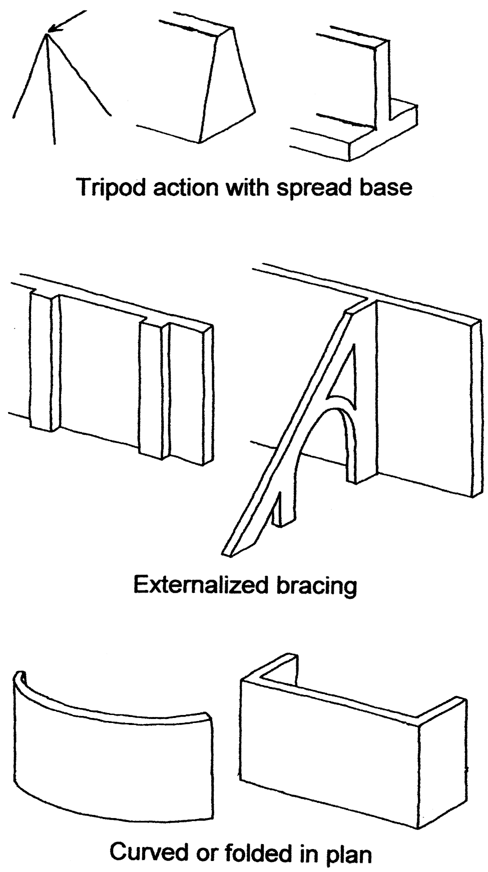


Figure 1.29 Means of stabilizing walls.

Figure 1.30 Simply detailed wood framing produces a structure with a clear lineage of historical development. Close inspection, however, reveals quite evidently that the elements are of contemporary industrialized origin, as opposed to the rough-hewn elements seen in Figure 1.25.

Providing Externalized Bracing. In addition to large vertical ribs, various bracing may be provided for the walls in the direction perpendicular to their planes. An example, as shown in Figure 1.29, is the flying buttress, extended at some distance outside the wall plane.

Exterior walls may also be braced by interior walls that intersect them, a common situation in buildings with multiple interior spaces.

Post and Beam Systems

Primitive cultures' use of tree trunks as building elements was the origin of this basic system. Later expansion of the vocabulary of construction materials into stone, masonry, concrete, and metals carried over the experience and tradition of form and detail established with wood. This same tradition, plus the real potentialities inherent in the system, keeps this building technique a major part of our structural repertoire. (See Figure 1.30.)

The two basic elements of the system are the post (column) and beam (lintel):

Post. Essentially a linear compression member subject to crushing or to lateral buckling depending on its relative slenderness.

Beam. Essentially a linear member subjected to transverse loading; must develop resistance to shear, bending, and excessive deflection (sag).

Critical aspects of the system are the ratio of height to thickness of the post and the ratio of span to depth of the beam. Efficiency of the beam cross-sectional shape in resisting bending is also critical.

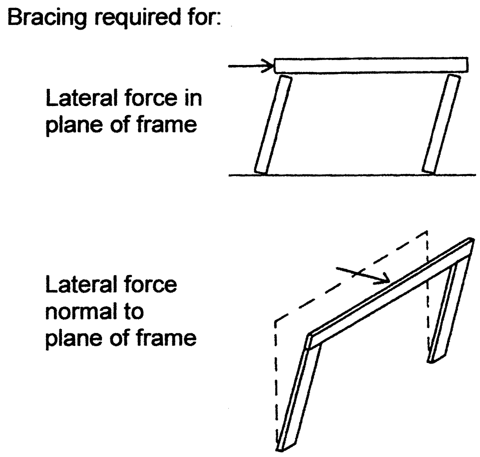


Figure 1.31 Bracing of framed structures against horizontal forces. See also the considerations for walls as shown in Figures 1.28 and 1.29.

The stability of the system under horizontal load is critical in two different ways (see Figure 1.31). Consider first the resistance to horizontal force in the same plane as the frame. This resistance can be provided in a number of ways, for instance, by fixing the posts rigidly to the base supports, connecting the posts rigidly to the beams, or using trussing or a shear wall paneling.

Stability against horizontal force perpendicular to the plane of the frame is a slightly different situation. This is similar in many ways to the wall in the same situation. In real design of whole building structures, horizontal forces (especially those due to wind or earthquakes) must be considered by studying the entire, three-dimensional building structure. In buildings of very complex form this is no small task.

A critical design problem with all framework systems is that of generating the infilling systems that turn the frame into a building. Thus, the development of wall, roof, and floor surfaces must be considered as an addition to the frame design itself. Many possible combinations must be considered. Design criteria and performance needs vary for the many elements of the whole building construction.

As with the situation illustrated in Figure 1.30, parts of frame systems are often exposed to view, adding additional design concerns regarding architectural issues to those of the structural design. Choice of materials, details of frame connections, and shape of individual elements will be given much consideration beyond simple structural performance.

Since walls provide only vertical elements of the building construction, various other elements must be considered for completion of the building roof and floors. Thus, since walls need some framing for completion of the system, and frames need infilling walls, many negotiated solutions are offered.

Some variation on the basic post and beam system are the following (see Figure 1.32):

Use of Extended Beam Ends. Produces beam overhangs, or cantilevers. This serves to reduce the degree of bending and sag at the center of the span, thus increasing the relative efficiency of the spanning element.

Rigid Attachment of Beam and Posts. One device for producing stability in the plane of the frame. It achieves some reduction of bending and sag at the center of the beam span but does so at the expense of the post—in contrast with the extended beam ends. It also produces an outward kick at the base of the posts.

Rigid Attachment with Extended Beams. Combines the two previous variations; also reduces the outward kick of the post bases.

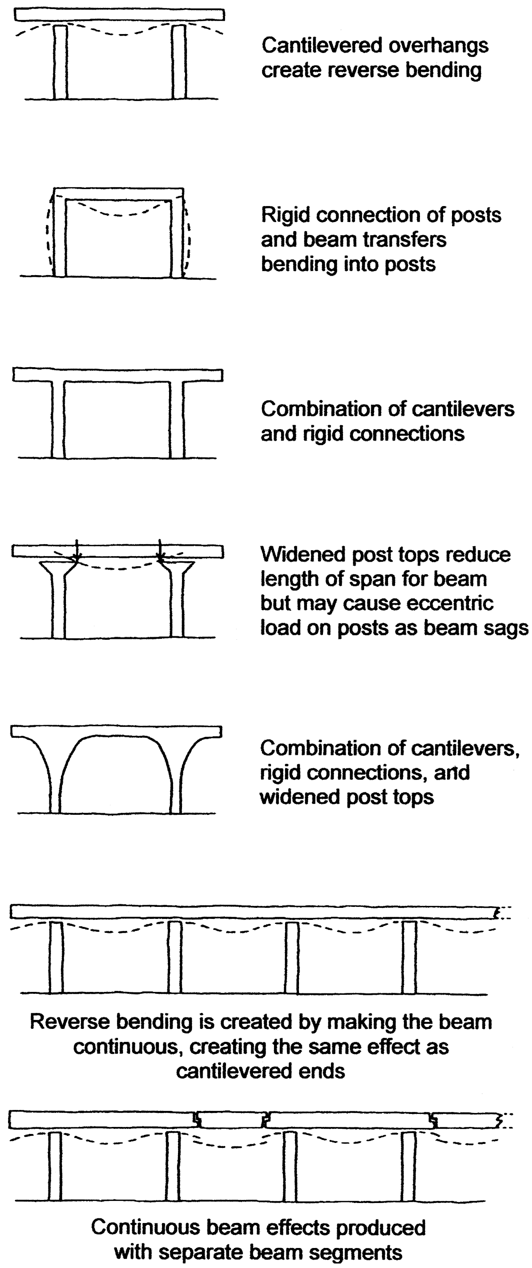


Figure 1.32 Variations of the post and beam.

Widened Top of Post. Serves to reduce the span of the beam. As the beam deflects, however, its load becomes concentrated at the edge of the widened top of the post, thus causing bending in the post. V-shaped or Y-shaped posts are possible variations.

Widened Post Top with Rigid Attachment and Extended Beam Ends. Combines the advantages of all three techniques.

Continuous Beams. Produces beneficial effects similar to those gained by the extended ends of the single span beam.

Additional gain is in the tying together of the system. A variation in which internal beam joints are placed off the columns preserves the advantages of the continuous beam but allows shorter beam segments. The latter is an advantage with beams of wood, steel, and precast concrete. Poured-in-place concrete can achieve virtually any length in a single piece form, although practical limits exist.

As with the framed wall-bearing structure, the post-and-beam frame requires the use of secondary systems for infilling to produce solid surfaces of walls, roofs, and floors (see Figure 1.33). A great variety is possible for these systems, as described in the discussion of structural walls. One possible variation with masonry or cast concrete systems is to combine the post and wall monolithically, producing a series of pilasters. Similarly, the beam and flat deck may be combined monolithically, producing a continuous ribbed deck or a series of T-shaped beams.

The post-and-beam system suggests the development of rectilinear architectural forms and spaces. The beams may, however, be curved in plan, tilted from the horizontal (as commonly done with roof rafters), or have other than a flat top or bottom. Posts can be T-shaped, Y-shaped, V-shaped, or multitiered. The system, in fact, lends itself to a greater degree of variation than practically any other system, which is one reason for its continuing widespread use (see Figure 1.34).

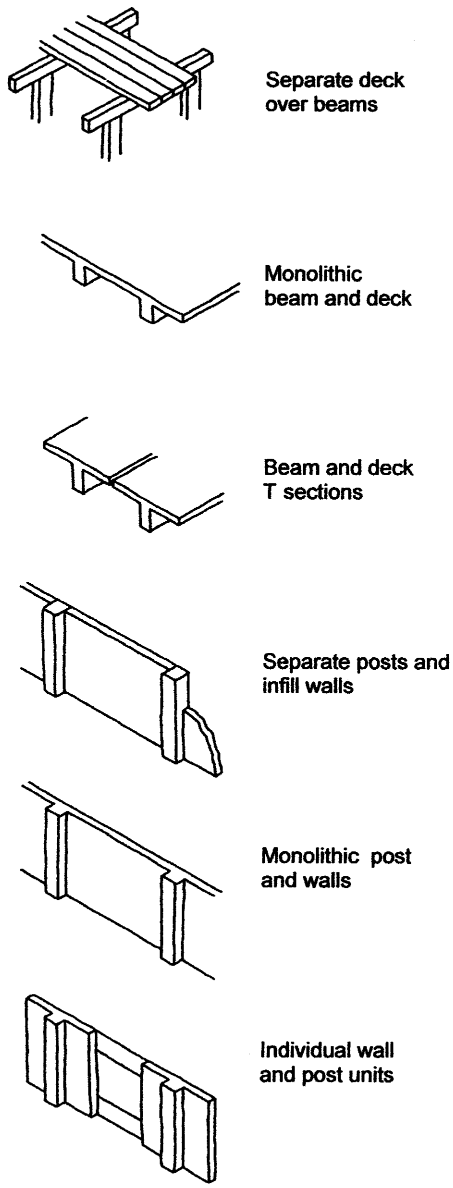


Figure 1.33 Infilling the post-and-beam system.

Rigid Frames

When the members of a linear framework are rigidly attached—that is, when the joints are capable of transferring bending between members—the system assumes a particular character. If all joints are rigid, it is impossible to load any one member transversely without causing bending in all members. This property and the inherent stability and redundancy of the system are its unique aspects in comparison to the simple post-and-beam system. The rigid-frame action may be restricted to a single plane or it may be extended in all directions in the three-dimensional framework. (See Figure 1.35.)

The joints take on a high degree of importance in this system. In fact, in the usual case, the highest magnitude of stresses and internal forces are concentrated at the joints. If the frame is assembled from separate members, the jointing must be developed carefully for structural function and feasibility.

A popular form of rigid frame is the gabled frame, in which two slanted elements are joined at the peak. When the slanted members sit on top of columns, one method for preventing the tops of the columns from spreading outward is to make one column and one sloping member into a single piece. This simple rigid frame can be executed in steel, laminated wood timber, or precast concrete.

Occasionally, the rigid-frame action is objectionable—for instance, when the beam transfers major bending to a small column or causes large curvature or an outward movement at the base of the column. It is sometimes necessary to avoid the rigid-frame action deliberately or to control it by using special joint detailing that controls the magnitude of bending or turning of the joints.

Flat-Spanning Systems

Compared to the arch, the dome, or the draped cable, the flat-spanning structure is very hardworking. In fact, it is exceeded only by the cantilever in this respect. Consequently,

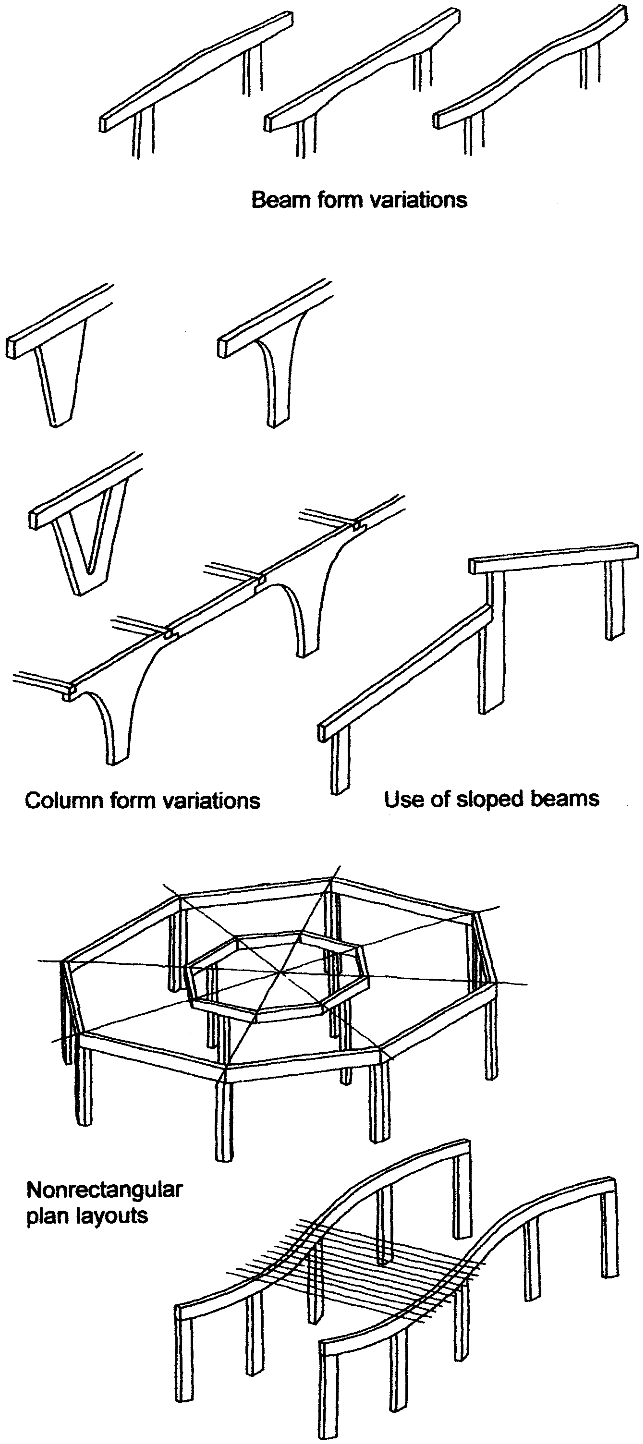


Figure 1.34 Form variations with the post-and-beam system.

scale limits for spans can be overcome by various techniques that improve the efficiency of the spanning elements. One of these is to develop the system as a two-way rather than a one-way spanning system (see Figure 1.36). This sharing of the spanning effort reduces the magnitude of bending as well as the deflection of the system.

Maximum benefit is derived from two-way spanning if the spans are equal. The more different they become, the less the work accomplished in the longer span. At a 2 : 1 ratio, less than 10% of the resistance will be offered by the longer span.

The other chief device for increasing efficiency is to improve the bending character of the spanning elements. A simple example is the difference in effectiveness illustrated by a flat sheet of paper and one that has been pleated or corrugated. The concept involved is that of increasing the depth of the element. A critical relationship in the flat span is the ratio of the span to the depth. Load capacity falls off rapidly as this ratio is pushed to its limits. Resistance to deflection is often more critical than bending stress in this situation.

Efficiency can also be increased by extending the elements beyond their supports, by using monolithic elements continuous over several supports, or by developing bending transfer between the elements and the supports.

Trussed Systems

A framework of linear elements connected by joints can be stabilized independently by guys, struts, rigid infilling, or rigid-frame action. Another means of achieving stability is through the arrangement of the elements in triangular forms (see Figure 1.37). This is called *trussing*, and when the structural element produced is a flat-spanning form, it is called simply a *truss*.

The triangulated frame can also be used to produce other structural forms, such as towers, rigid frames, arches, and two-way flat-spanning systems. Or, virtually any form of structure can be produced, such as the Statue of Liberty in New York Harbor.

There are two basic principles that work to make the truss an efficient system. The first is the self-stabilizing nature of the triangle, which cannot be changed in form without changing the length of a side. The other is the high efficiency of placing interacting elements at the greatest distance apart.

The multiplicity of joints in trussed systems makes their detailing a major item in truss design. The logic of form of the linear truss members derives as much, if not more, from the jointing as from their function as tension-resistive or compression-resistive elements. The elimination of bending and shear in the truss members is, by the way, another basic concept for the truss and is actually or essentially achieved in most trusses. One design aim in truss design is to avoid loading truss members directly; instead, loads and supports are located at truss joints. However, in developing roof, floor, or ceiling infill systems, it is sometimes necessary to make attachments to the top or bottom members directly. In the latter case, the members have a dual function—first as truss members and then as beams, with the two functions occurring at the same time.

An almost infinite variety of truss configurations is possible. The particular configuration, the loads sustained, the dimensional scale, the truss materials, member forms, and jointing methods are all design considerations.

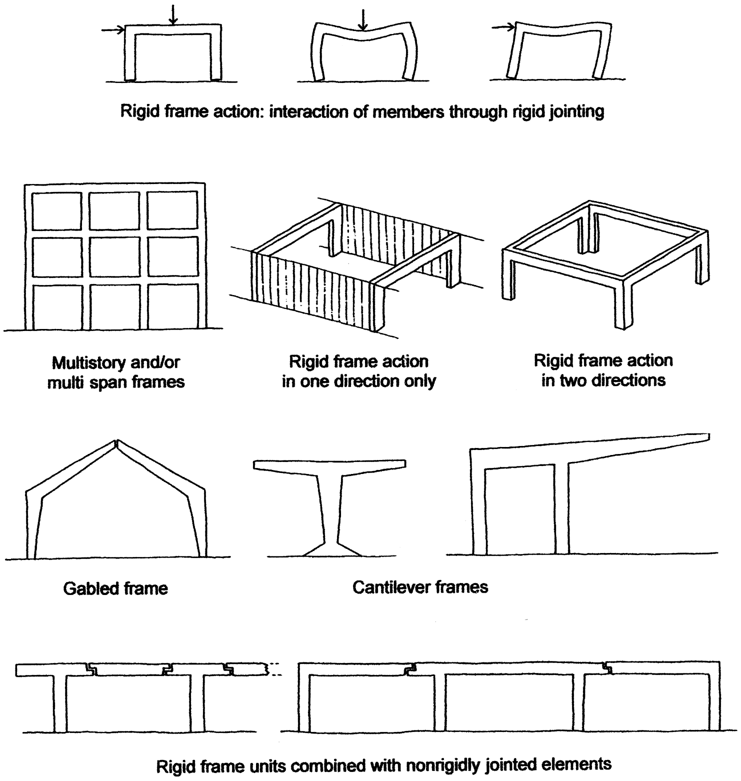


Figure 1.35 Rigid frames.

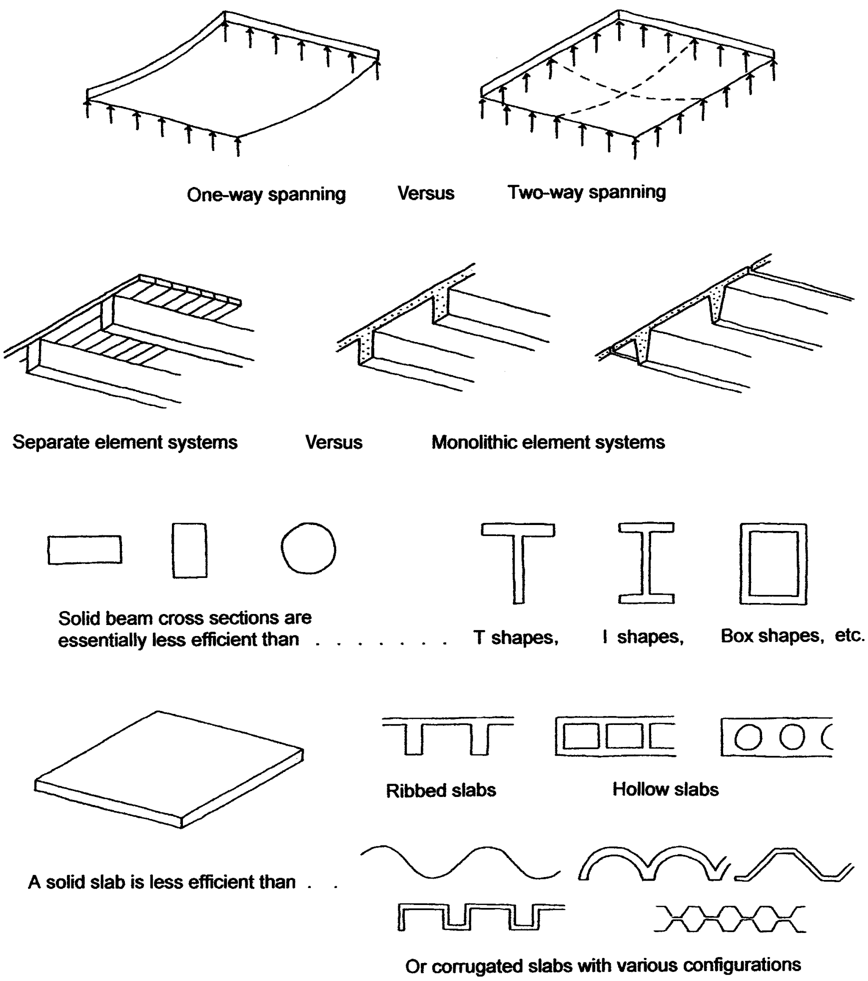


Figure 1.36 Basic aspects of flat-spanning systems.

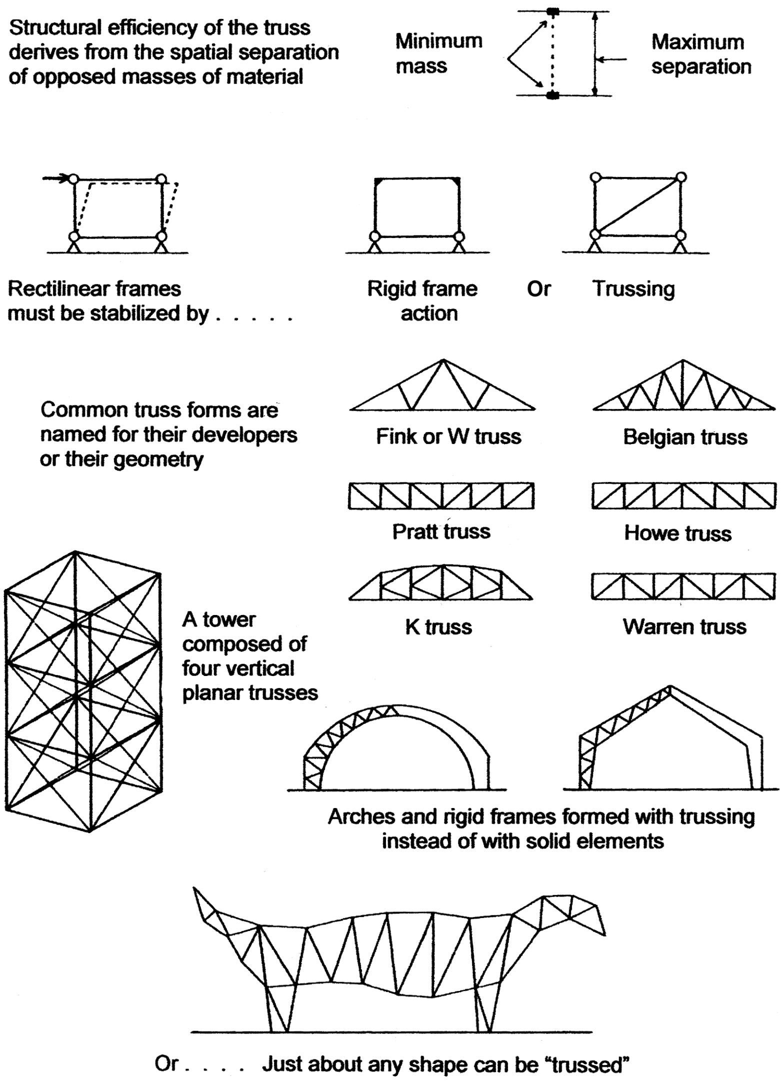


Figure 1.37 Basic aspects of trussed structures.

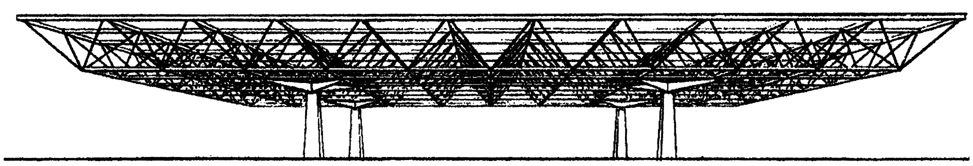


Figure 1.38 Two-way spanning truss. Roof for a high school basketball arena.

The two-way spanning truss—often called a *space frame*, although the term is rather confusing—has been developed with various configurations and at a range of span sizes, mostly with steel truss members. The structure shown in Figure 1.38 uses a form of configuration, first developed by the Unistrut Company in the 1950s, consisting of a series of square-based pyramids that relates well to rectangular plan layouts. This system, and other options for two-way spanning trusses, is discussed in Section 10.10 for Building Nine of the case studies.

Arch, Vault, and Dome Systems

The basic concept in the arch is the development of a spanning structure through the use of only internal compression (see Figure 1.39). The profile of the "pure" arch may actually be derived from the loading and support conditions. For a single-span arch with no fixity at the base in the form of moment resistance, with supports at the same level, and with a uniformly distributed load on the entire span, the resulting form is that of a parabola. Most arches, however, are circular in profile, which works alright as long as the arch is reasonably thick.

Basic considerations are the necessary horizontal forces at the base and the ratio of span to rise. As this ratio increases, the arch form becomes flatter and the thrust increases, producing greater compression in the arch and larger horizontal forces at the supports. Thus, most old stone arches have considerable rise with the familiar tall curved form.

In the great stone arches of old the principal load was the weight of the heavy stone itself. Although other forces existed,

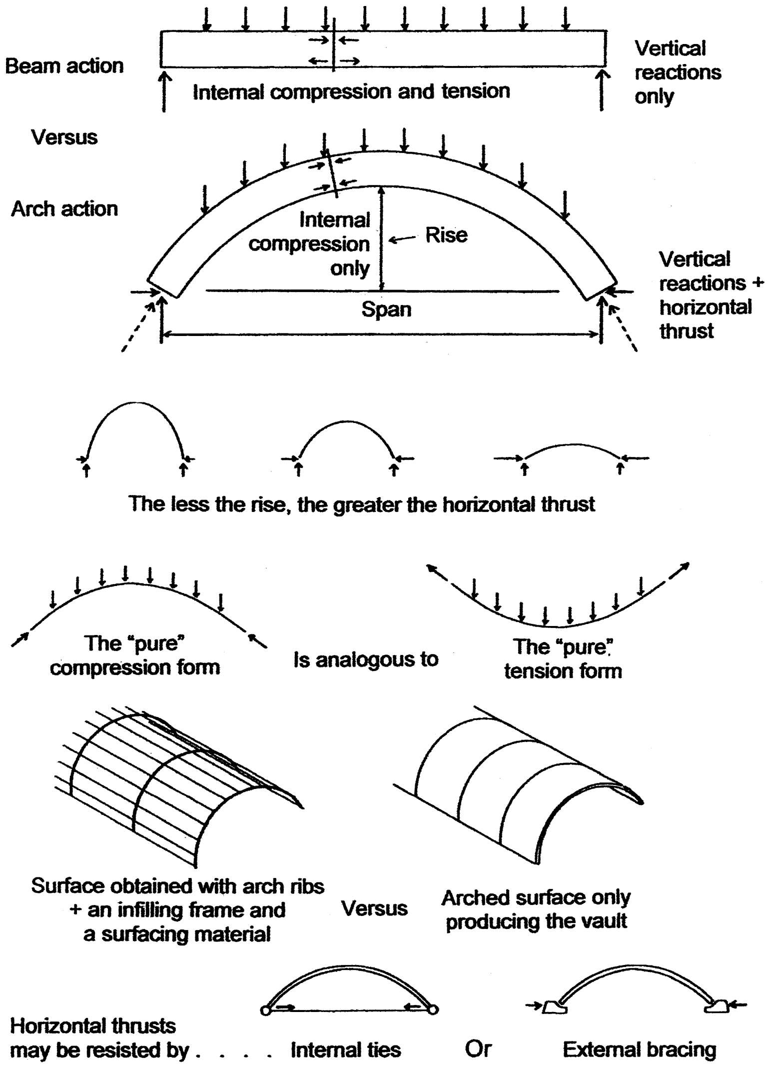


Figure 1.39 Basic aspects of arches.

they were usually incidental compared to the gravity force. In most contemporary construction, thinner and lighter arches are produced and the pure arch action is seldom achieved.

The thrust of the arch—that is, the horizontal component—is resolved in one of two ways. The most direct way is to balance the force at the supports against each other by using a tension tie across the base of the arch. This very possibly, however, destroys the interior vaulted space defined by the arch and is therefore not always acceptable. The second way is to resolve the outward kick at each support outside the arch. This means creating a heavy abutment or, if the arch rests on top of a wall or column, creating a strut or a buttress for the wall or column.

A major consideration in the structural behavior of an arch is the nature of its configuration. The three most common cases are those shown in Figure 1.40, consisting of the fixed arch (Figure 1.40*a*), the two-hinged arch (Figure 1.40*b*), and the three-hinged arch (Figure 1.40*c*).

The fixed arch occurs most commonly with reinforced concrete bridge and tunnel construction. Maintaining the fixed condition (no rotation) at the base is generally not feasible for long-span arches, so this form is more often used for short to medium spans. It may occur in the action of a series of arches built continuously with their supporting piers, as shown in Figure 1.40*a*. The fixed arch is highly indeterminate in its actions and is subject to internal stress and abutment forces as a result of thermal expansion.

The two-hinged arch is most common for long spans. The pinned base is feasibly developed for a large arch and is not subject to forces as a result of thermal change to the degree of the fixed-arch base. This arch is also indeterminate, although not to the degree as the fixed arch.

The three-hinged arch is a popular form for medium-span building roof structures. The principal reasons for this popularity are the following:

The pinned bases are more easily developed than fixed ones, making shallow bearing-type foundations reasonable for the medium-span structure.

Thermal expansion and contraction of the arch will cause vertical movements at the peak pin joint but will have no appreciable effect on the bases. This further simplifies the foundation design.

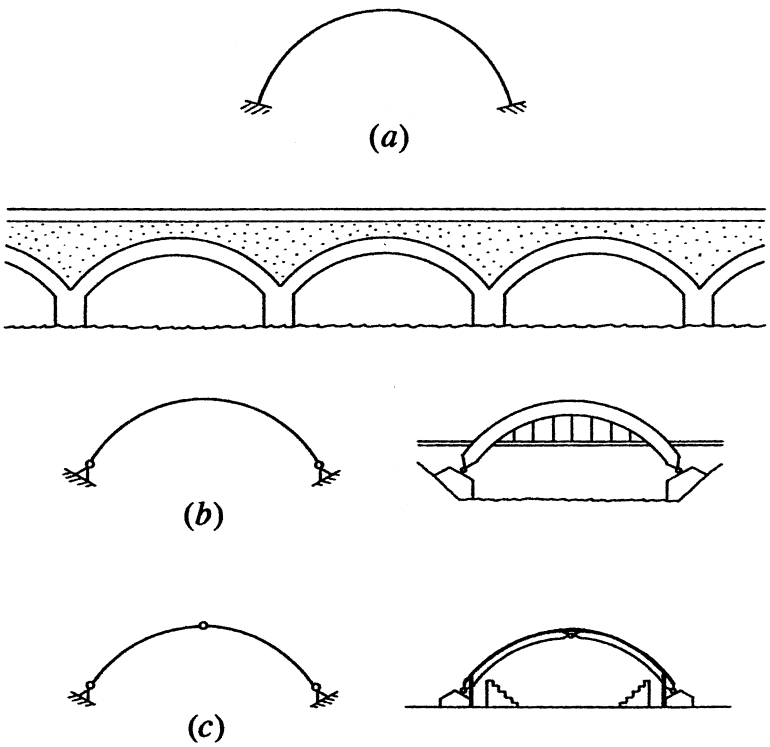


Figure 1.40 Types of arches.

For small spans, construction can often be achieved by use of prefabrication for the two arch sections and connecting them at the peak in the field. The pin joint is much easier to achieve under these circumstances.

For a uniformly distributed loading applied over the horizontal span of the arch, the net internal compression follows a parabolic profile. However, uneven loadings and effects of wind and earthquakes will result in other internal force effects of bending and shear. If the arch itself is very heavy, the parabolic form may have some significance. Otherwise, with construction that can resist bending and shear (laminated wood, steel, or reinforced concrete), the arch profile can be less than geometrically "pure." The most popular form is a circular one, which was used most often by the builders of ancient, heavy masonry arches.

If adjacent arches are assembled side by side, the vault is produced, describing an essentially cylindrical form. If vaults intersect, complex forms are produced with the intersections of the vaults describing three-dimensional shapes. The forms resulting from intersecting vaults and the strongly expressed ribs at the vault intersections were dominant architectural features of Gothic cathedrals.

If a single arch is rotated in plan about its peak, the form generated is a dome. This structural form relates to a circular plan, in contrast to the vault, which relates to a rectangular or cross-shaped plan.

Tension Structures

The tension suspension structure was highly developed by some primitive societies through the use of vines or strands woven from grass or shredded bamboo. These structures achieved impressive spans; foot bridges spanning 100 ft have been recorded. The development of steel, however, heralded the great span capability of this system; at first in chain and link, and later in the cable woven of drawn wire, the suspension structure quickly took over as the long-span champion.

Structurally, the single draped cable is merely the inverse of the arch in both geometry and internal force (see Figure 1.41). The compression arch parabola is flipped over to produce the tension cable. Sag-to-span ratio and horizontal inward thrust at the supports have their parallels in the arch behavior.

A problem to be dealt with is the usual lack of stiffness of the suspended structure, which results in reforming under load changes. Fluttering or flapping is possible with wind load. Also, resistance to tension at supports is usually more difficult than resistance to compression.

Steel is obviously the principal material for this system, and the cable produced from multiple thin wires is the logical form. Actually, the largest spans use clusters of cables—up to 3 ft in. diameter for the Golden Gate Bridge with its 4000+-ft span. Although a virtually solid steel element 3 ft in diameter hardly seems flexible, one must consider the span-to-thickness ratio—approximately 1330 : 1. This is like a 1-in.-diameter rod over 100 ft long. One cannot anchor this size element by tying a clove hitch around a tent stake.

Structures can also be hung simply by tension elements. The deck of the suspension bridge, for example, is not placed directly on the spanning cables but is hung from them by another system of cables. Cantilevers or spanning systems

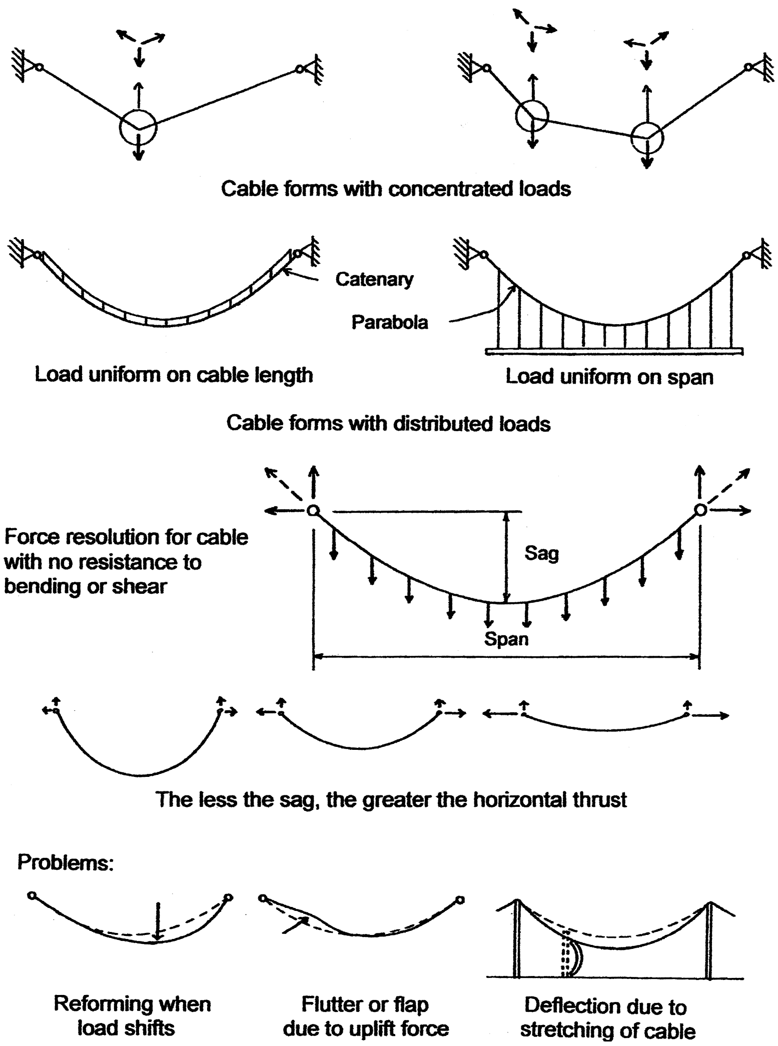


Figure 1.41 Basic aspects of tension systems.

may thus be supported by hanging as well as by columns, piers, or walls.

There are many possibilities for the utilization of tension elements in structures in addition to the simple draped or vertically hung cables. Cables can be arranged in a circular radiating pattern with an inner tension hub and an outer compression ring similar to that in a bicycle wheel (see Figure 1.42).

Cables can also be arranged in crisscrossing networks, as draped systems, or as restraining elements for air-inflated membrane surfaces. Membrane surfaces can be produced by air inflation, by edge stretching, or by simple draping.

Tension elements can also be used in combinations with compression elements, as they are in trussed structures. For a spanning truss, the bottom chords and end diagonals actually constitute a continuous string of tension elements, which might actually be developed as such. Tension ties for rafters and arches are another example of this type of system.

Surface Structures

The neatness of any categorization method for structural systems eventually breaks down, since variations within one system tend to produce different systems, and overlapping between categories exists. Thus the rigidly connected posts and beams become the rigid frame and the vault and dome become surface structures. As a general category, surface structures consist of any thin, extensive surfaces functioning primarily by resolving only internal forces within their surfaces (see Figure 1.43).

We have already discussed several surface structures. The wall in resisting compression or in acting as a shear wall acts like a surface structure. As with other rigid surface elements, the wall can also resist force actions perpendicular to the wall surface, developing bending and out-of-plane shear.

The purest surface structures are flexible tension surfaces, since they are usually made of materials with no out-of-plane resistance. Thus the canvas tent, the rubber balloon, and the plastic bag are all limited in function to tension resistance within the planes of their surfaces. The forms they assume, then, must be completely "pure." In fact, the pure compression surface is sometimes derived by simulating it in reverse with a tension surface. There are, however, other structural elements within the surface structure category

Figure 1.42 New York State's "Tent of Tomorrow" pavilion at the 1964 World's Fair. The 100-ft-high concrete columns carry an elliptical steel compression ring 350 × 250 ft in plan. Suspended from the ring, a double layer of steel cables converges toward a steel tension ring at the center. The roof surface consists of translucent sandwich panels formed with two sheets of reinforced plastic separated by an aluminum grid. The panels, approximately 3000 in number, are trapezoidal in shape.

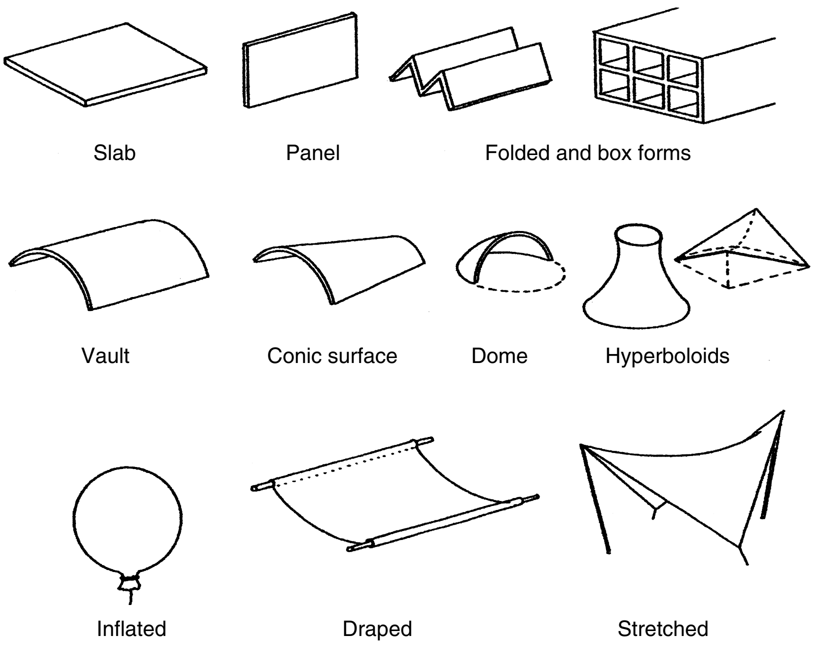


Figure 1.43 Basic forms of surface structures.

that develop structural actions other than those of simple surface tension or compression.

Compression surfaces must be more rigid than tension ones because of the possibility of buckling. This increased stiffness makes them difficult to use in a way that avoids developing out-of-plane bending and shear.

Compression-resistive surface structures of curved form are also called *shells*. The egg, the light bulb, the plastic bubble, and the auto fender are all examples of shells. At the building scale the most extensively utilized material for this system has been reinforced concrete.

Both simple and complex geometries are possible with shells. Edges, corners, openings, and points of support are locations for potential high stress and out-of-plane bending and shear; consequently, reinforcement of the shell is often created at these locations by monolithically cast ribs. While the general shell behavior may still occur between ribs, the ribs themselves become a system of beams, arches, or rigid frames. The ribs are also highly visible and are often exploited for architectural design purposes, as in the Gothic cathedrals.

A special variation of the shell is the system produced by multiple folds or pleats of surfaces. The folds may be flat or curved. A curved example is the simple corrugated sheet of metal or plastic. If the surfaces are flat, the system is referred to as a *folded plate*. Although possible to achieve with concrete, these structures can also be made with wood, metal, or plastic elements.

A special variation of the surface system uses hollow-core, sandwich-form elements to develop the surface components.

Special Systems

Innumerable special systems are possible, each creating a new category by its unique aspects. Some of these are described as follows.

Inflated Structures

Inflation, or air pressure, can be used as a structural device in a variety of ways (see Figure 1.44). Simple internal inflation of a totally enclosing membrane surface, for instance, in the rubber balloon, is the most direct. This requires about the least structural material imaginable for spanning. The structure is unavoidably highly flexible, however, and dependent on the constant differential between inside and outside pressure. It is

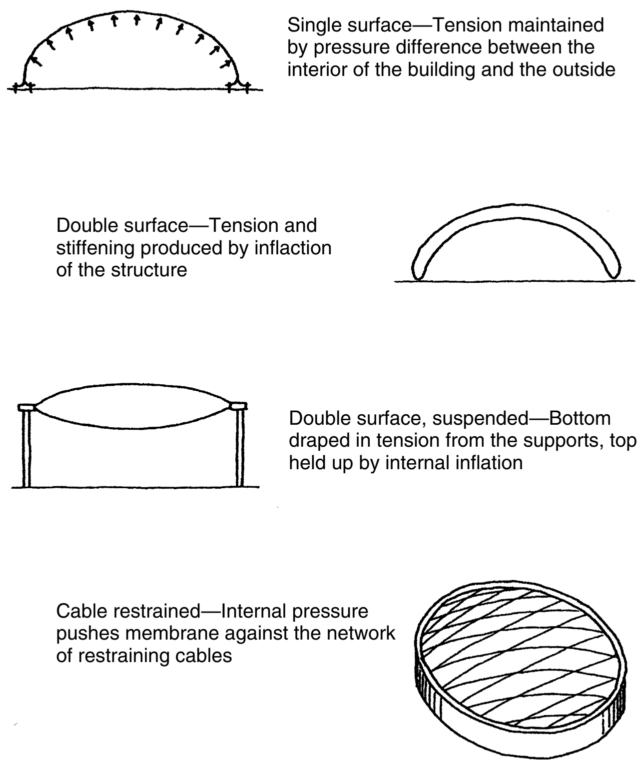


Figure 1.44 Basic forms of air-supported structures.

also necessarily lumpy in form because the surface is stretched. It has nevertheless been utilized for buildings of considerable size.

A second use of inflation is the stiffening of a structural element. This can be a sandwich or hollow ribbed structure of tension membrane material given a rigid-frame character by inflation of the voids within the structural element, for example, the inflated inner tube or air mattress. The need for sealing the space enclosed by the structure is thus eliminated.

Another possibility is that of a combination of inflation and tension, like that of a suspended pillow. A possible advantage in this case is the elimination of the cupped water pocket formed by draped membranes.

Lamella Frameworks

A unique method for forming arched or domed surfaces utilizes a network of perpendicular ribs that appear to be diagonal in plan (see Figure 1.45). It has been used at both modest and great spans and has been executed in wood, steel, and concrete. One great advantage—in addition to an economy of materials—is the use of the repetition of similar elements and joint details. Another advantage is in the use of straight, linear elements to produce the curved vaulted surface.

Geodesic Domes

A few lines can scarcely do justice to this unique system. Developed from ideas innovated by R. Buckminster Fuller, this technique for transforming of hemispherical surfaces is based on spherical triangulation (see Figure 1.46). It is also useful at both large and small scales and subject to endless variations of detail, member configuration, and materials. In addition to ordinary wood, steel, and concrete, it has also been executed in plywood, plastic, cardboard, bamboo, and aluminum.

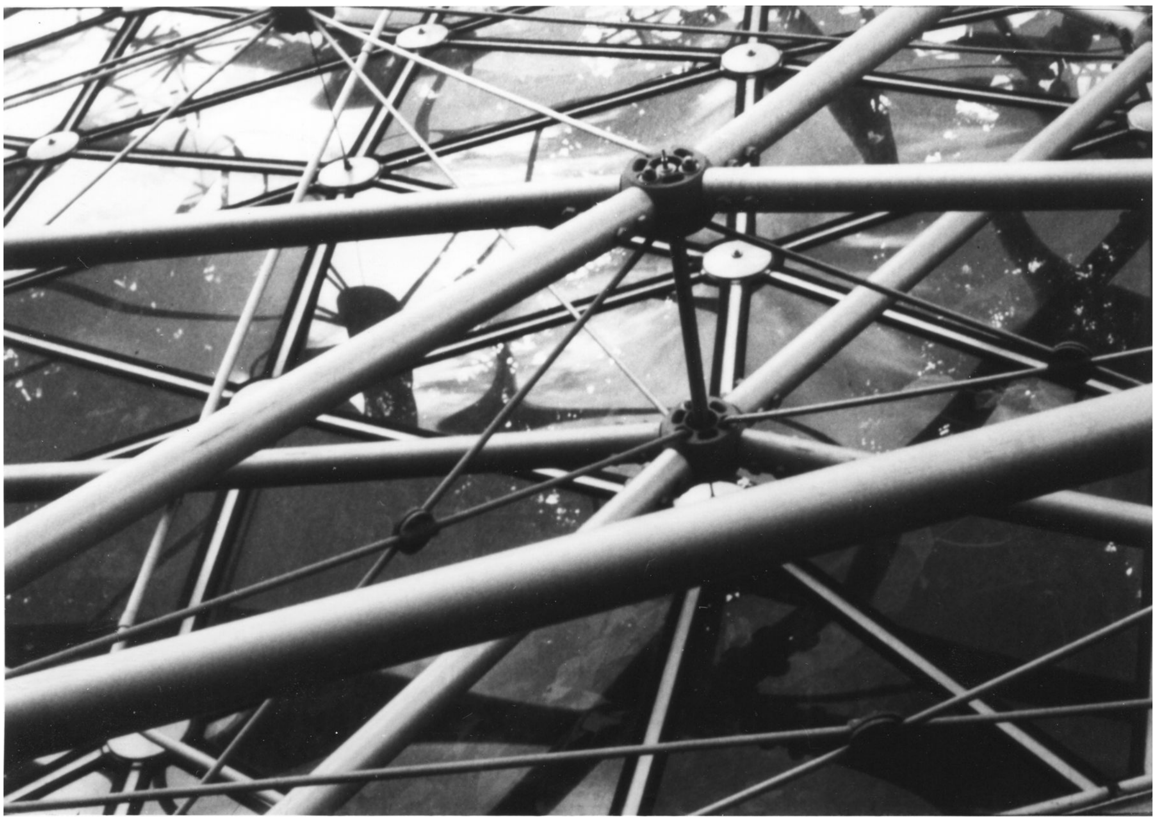
The chief attributes of the system are its multiplication of basic units and joints and the efficiency of its resolution of internal forces. The claim has been made that its efficiency increases with size, making it difficult to see any basis for establishing a limiting scale.

Mast Structures

These are structures similar to trees, having single legs for vertical support and supporting one level or a series of "branches." They obviously require very stable bases, well anchored against the overturning effect of horizontal forces. A chief advantage is the minimum space occupied by the ground-level base. (See Figure 1.47.)

Figure 1.45 Wood lamella structure. Simple wood elements in a diagonal lamella pattern form this roof for a bowling alley.

(*a*)



(*b*)

Figure 1.46 Geodesic dome structure, Climatron, Missouri Botanical Gardens, St. Louis. Plastic glazing suspended from an exposed aluminum frame.

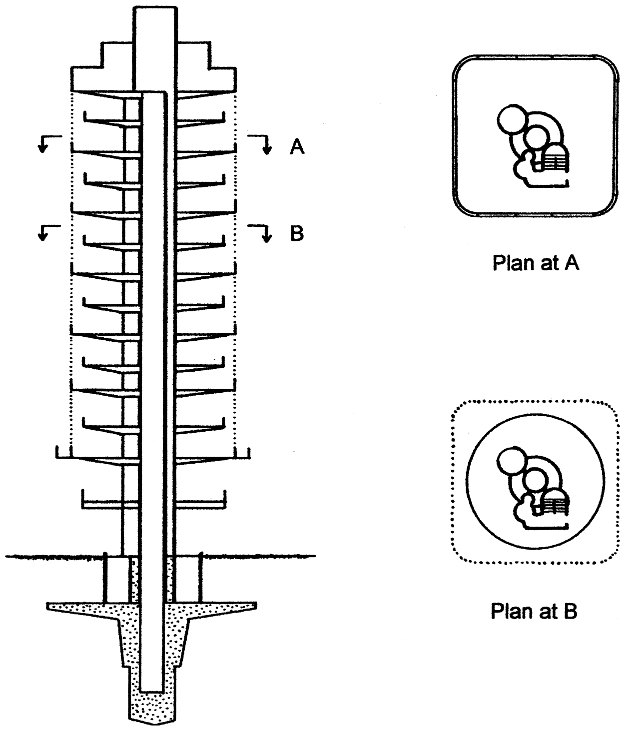


Figure 1.47 Mast form tower structure. Laboratory tower, Johnson Wax Company, Racine, Wisconsin. A central concrete core supports alternating square and round floors. Architect: Frank Lloyd Wright.

Multiple Monopod Units

Multiple mushroom, lily-pad, or morning glory shaped elements can be used to produce one-story buildings with multiple horizontal plan units. Principally developed with reinforced concrete shell forms, this system offers savings in the repetitive use of a single form (see Figure 1.48).

This brief sampling does not pretend to present the complete repertoire of contemporary structural systems for buildings. The continual development of new materials, products, and systems and new methods of construction keep this a dynamic area of endeavor. New systems are added; established ones become outmoded. Modern techniques of analysis and design make the rational, reliable design of complex systems feasible.

One continuing trend is the industrialization of the building process. This tends to emphasize those materials, systems, and processes that lend themselves to industrial production. Use of prefabrication, modular coordination, component systems, and machine-produced details steadily dominate building structures. Highly craft-dependent means of production fade continuously into history, except in the form of visual imitations.

What we build and the means we use to produce it have always been in a dynamic state of change. It is important to have some awareness of the current status of things but also important to understand that change is inevitable. We can choose to be agents of change or settle for being observers of it.

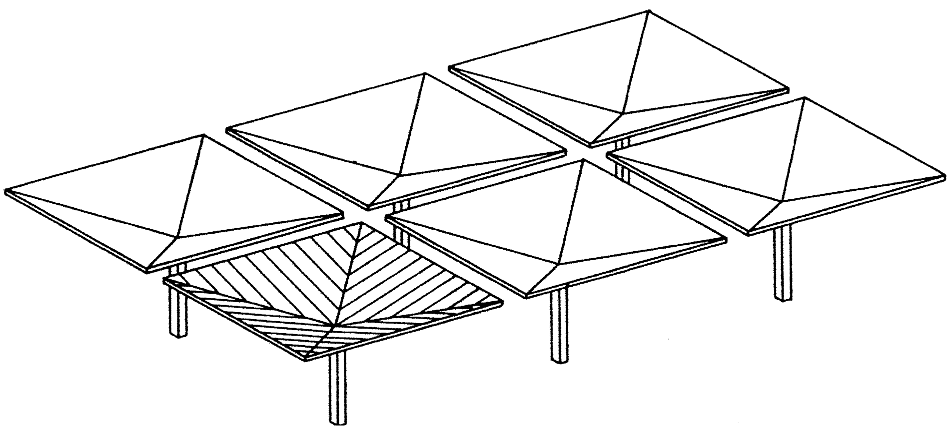


Figure 1.48 Monopod structural units.

CHAPTER

2

Investigation of Structures

This chapter consists of a survey of basic concepts and procedures from the field of applied mechanics as they have evolved in the process of the investigation of the behavior of structures. The purpose of studying this material is twofold. First is the general need for an understanding of what structures must do and how they do it. Second is the need for some factual, quantified basis for the exercise of judgment in the process of structural design. If it is accepted that the understanding of a problem is the necessary first step in its solution, this essentially analytical study should be seen as the basic cornerstone of any successful design process.

2.1 INTRODUCTION TO STRUCTURAL INVESTIGATION

The material in this section consists of discussions of the nature, purposes, and various techniques of the work of the investigation of structures. As in all of the work in this book, the primary focus is on material relevant to the tasks of structural design.

Purpose of Investigation

Most structures exist because of some usage need. Their evaluation must therefore begin with consideration of the effectiveness with which they facilitate or satisfy the usage requirements. Three factors of this effectiveness may be considered: the structure's functionality, feasibility, and safety.

Functionality deals with the physical relationships of the structure's form, detail, durability, fire resistance, and so on, as these relate to its intended use. Feasibility includes considerations of cost, the availability of materials, and the practicality of production. Safety in terms of structural actions is generally obtained in the form of some margin between the structure's capacity for resistance and the demands placed on it.

Analysis of structural behaviors serves to establish the nature of the structure's deformations (pertinent to its usage) and to relate its performance to its requirements. There are two critical phases of the structure's behavior: its working condition in service and its ultimate response or limit at failure.

Means of Investigation

Analysis for investigation may progress with the following considerations.

- Determination of the structure's physical being with regard to material, form, detail, scale, orientation, location, support conditions, and internal character
- Determination of the demands placed on the structure, that is, the loads and the manner of their application and any usage limits on deformation
- Determination of the structure's responses in terms of deformations and development of internal stresses
- Determination of the limits of the structure's capabilities
- Evaluation of the structure's effectiveness

Analysis may be performed in several ways. One can visualize the nature of the structure's deformation under load—through mental images or with sketches. Using available theories and techniques, one can manipulate mathematical models of the structure. Finally, one can load and measure responses of the structure itself or of a scaled model of the actual structure.

When reasonably precise quantitative evaluations are required, the most useful tools are direct measurements of physical responses or careful mathematical modeling with

theories that have been demonstrated to be reliable. Some amount of mathematical modeling generally precedes the actual construction—even that of a test model. Direct measurement is usually limited to experimental studies or to efforts to verify unproven theories or behaviors of unique structures.

Aspects of Investigation

The subject of structural investigation is traditionally divided into three areas of study: mechanics (statics and dynamics), strength of materials, and analysis of structural elements and systems. Mechanics is the branch of physics that deals with the motion of physical objects and the forces that cause their motion. It is divided into the topics of statics and dynamics. Statics deals with the condition of bodies at rest; that is, motion is implied or impending due to the presence of forces, but at present no motion occurs. Dynamics treats the general case of forces and motions, a significant variable being time.

Although dynamics is the general field of mechanics, most of structural engineering for buildings deals with statics. Dynamics is necessary only for treatment of effects caused by blasts, vibrations, and shocks due to earthquakes.

Strength of materials deals with the resistance of materials and structural elements to deformations caused by forces. This involves relationships between the external forces (loads and support reactions) and the stresses that develop in the materials to generate the necessary internal resisting forces.

Realism in Investigation

Investigation is essentially a fact-finding mission. For study purposes, the usual procedure is to deal with isolated problems that allow for concentrated efforts and somewhat simplified procedures. This is actually useful for a learning process, but there is some danger in not being aware of the broad context of the problems. In most of this book we deal with concentration on limited problems. Only in Chapter 10 is there some attempt to deal with the broad consideration of design.

Indeed, in most real design situations, the first step in the work is to define the problems. Once "seen," the isolated problems can usually be solved by simple and direct work. However, the seeing must be done by experienced designers.

Techniques and Aids for Investigation

The professional designer or investigator uses all the means available for accomplishment of the work. At this time mathematical modeling is aided greatly by use of the computer. However, routine problems are still often treated by use of simple hand computations or reference to data in handbook tables or graphs. Our purpose here is essentially educational, so an emphasis is placed on visualization and understanding, not necessarily on efficiency of computational means.

In this book, major use is made of graphic visualization, and we would like to encourage this habit on the part of the reader. The use of sketches as learning tools and as aids for problem-solving work cannot be overemphasized. Three types of graphical devices are most useful: the free-body diagram, the profile of the load-deformed structure, and the cut section.

A free-body diagram consists of a picture of any isolated physical element together with representations of all of the forces that act externally on the element. The isolated element may be a whole structure or any fractional part of it. Consider the structure shown in Figure 2.1. Figure 2.1*a* shows the entire structure, consisting of attached horizontal and vertical elements (beams and columns) that produce a planar rigid frame bent. This may be one of a set of such bents comprising a building structural system. The free-body diagram in Figure 2.1*a* represents the entire structure, with forces external to it represented by arrows. The forces include the weight of the structure and its supported gravity loads, the horizontal forces of wind, and the reaction forces acting at the supports of the frame, consisting of the building foundations.

Shown in Figure 2.1*b* is a free-body diagram of a single beam from the framed bent. Operating on the beam are the applied gravity loads plus the forces of interaction between the ends of the beam and the columns to which it is attached. These actions are not visible in the free-body diagram of the whole frame, so one purpose of the diagram of the single beam

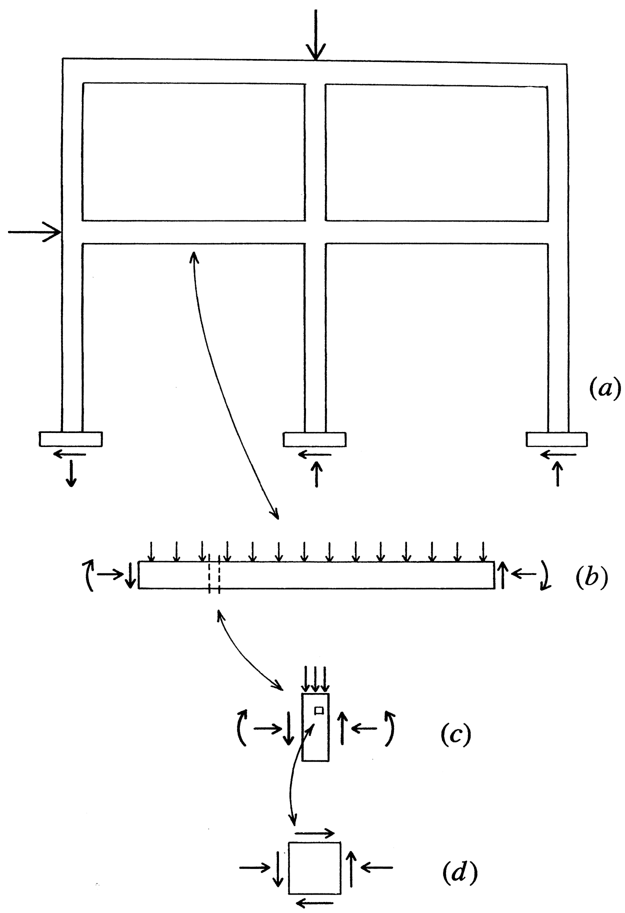


Figure 2.1 Free-body diagrams.

is simply the visualization of the nature of these interactions. We can now see that the columns transmit horizontal and vertical forces to the ends of the beams as well as rotational bending actions.

In Figure 2.1*c* we see an isolated portion of the beam length, which is produced by slicing vertical planes a short distance apart and removing the portion between them. Operating on this free body are the applied gravity loads and the actions of the beam segments on the opposite sides of the slicing planes, since it is these actions that hold the removed portion in place in the uncut beam. This slicing device, called a *cut section*, is used to visualize internal actions in the beam, since they are not visible in the whole beam.

Finally, in Figure 2.1*d*, we isolate a tiny segment, or particle, of the material of the beam and visualize the external effects consisting of the interactions between this particle and those adjacent to it. This is a basic device for the visualization of stress; in this case, due to its location in the beam, the particle is subject to a combination of shear and linear compression stresses.

This visualization of free-body diagrams will be used for all of the structural investigation work in this book. The ability to generate these free bodies will eventually be aquired by the reader.

Figure 2.2*a* shows the exaggerated deformation of the bent under wind load alone. The movement of the structure and the character of bending in each member of the frame can be visualized from this figure. As shown in Figure 2.2*b*, the character of deformation of segments or particles can also be visualized. These diagrams are very helpful in establishing the qualitative nature of the relationships between force actions and shape changes or between stresses and strains.

Another useful graphic device is the scaled plot of some mathematical relationship or of the data from some observed physical phenomenon. Considerable use is made of this technique in presenting ideas in this book. The graph in Figure 2.3 represents the form of damped vibration of an elastic spring. It consists of the plot of the displacement ($\pm s$) against elapsed time t and represents the graph of a mathematical formula of the general form

$$s = \frac{1}{e^t} P \sin(Qt + R)$$

Although the equation is technically sufficient for the description of the phenomenon, the graph helps in the visualization of many aspects of the relationship, such as the rate of decay of the displacement, the interval (period) of the vibration, the specific position at some exact amount of elapsed time, and so on.

All of the devices illustrated here can be used for a complete hand computation effort. However, they can also be incorporated into a computer-aided investigation, where they may be used as parts of the display of results of the investigation. As with the equation for the damped vibration, the formula itself may speak to those who are fluent in the language of mathematics, but for us lesser humans, pictures usually help a lot.

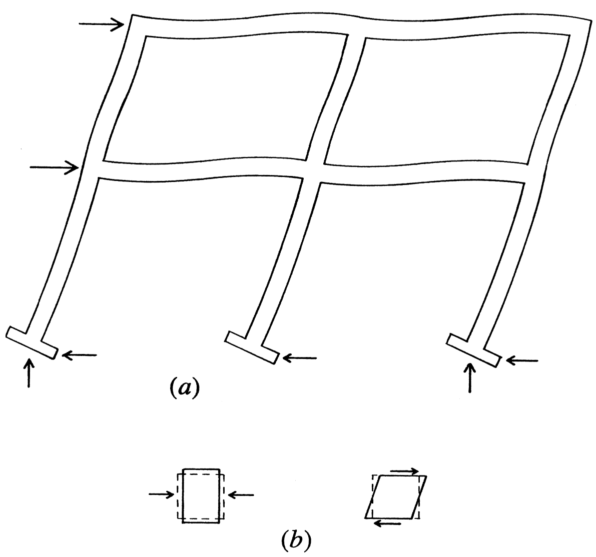


Figure 2.2 Visualization of structural deformation.

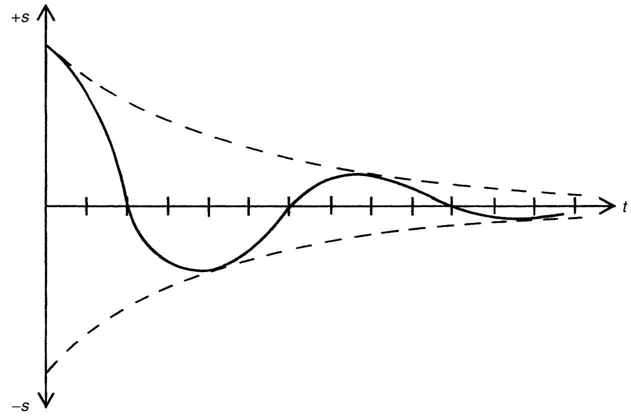


Figure 2.3 Graphic plot of a cyclic motion.

2.2 STATIC FORCES

This section presents basic concepts and procedures that are used in the analysis of the effects of static forces. Topics selected and procedures illustrated are limited to those that relate directly to the problems of designing most ordinary building structures.

Properties of Forces

Static forces are those that can be dealt with without the need for consideration of the time-dependent aspects of their actions. This limits necessary considerations to those dealing with the following properties:

Magnitude, or the amount, of the force, measured in weight units, such as pounds.

Direction of the force, which refers to the orientation of its line of action, usually described by the angle that the line makes with some reference, such as the horizontal.

Sense of the force, which refers to the manner in which it acts along its line of action (e.g., up or down, right or left). Sense is expressed algebraically in terms of the sign of the force, either plus or minus.

These force properties can be represented graphically by the use of an arrow, as shown in Figure 2.4. Drawn to scale, the length of the arrow represents the magnitude. The angle of inclination of the arrow represents direction. The location of the arrowhead determines sense. This form of representation can be more than merely symbolic. Actual mathematical manipulations can be performed using the vector representation that the arrows constitute. In this book arrows are used in a symbolic way for visual reference when performing algebraic computations and in a truly representative way when performing graphical analyses.

In addition to the basic properties of magnitude, sense, and direction, two other concerns that may be significant for certain investigations are:

The position of the line of action of the force with respect to the lines of action of other forces or to some object on which the force operates, as shown in Figure 2.5

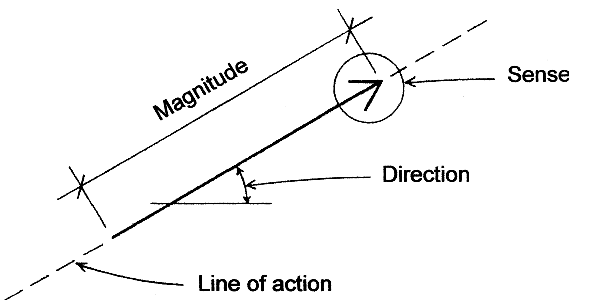


Figure 2.4 Properties of a force.

The point of application of the force along its line of action, which may be of concern regarding specific effects on an object, as shown in Figure 2.6

When forces are not counteracted by opposing forces, they tend to cause motion. An inherent aspect of static forces is that they exist in a state of *static equilibrium*, that is, with no motion occurring. In order for static equilibrium to exist, it is necessary to have a balanced system of forces. An important consideration in the analysis of static forces is the nature of the geometric arrangement of the forces in a given set of forces that constitute a single system. The usual technique for classifying force systems involves considerations of whether the forces are or are not:

Coplanar. All acting in a single plane, such as the plane of a flat vertical wall.

Parallel. All having the same direction.

Concurrent. All having their lines of action intersect at a common point.

Using these considerations, the possible variations are given in Table 2.1 and illustrated in Figure 2.7

Composition and Resolution of Forces

For structural analysis it is sometimes necessary to perform either addition or subtraction of force vectors. The process of vector addition is called *composition,* or the combining of forces. The process of subtraction is called *resolution,* or the resolving of forces into *components.* A component is any force that represents part, but not all, of the effect of the original force.

Resolution

In Figure 2.8 a single force is shown, acting upward toward the right. One type of component of such a force is the net horizontal effect, which is shown as F_h in Figure 2.8*a*. It can

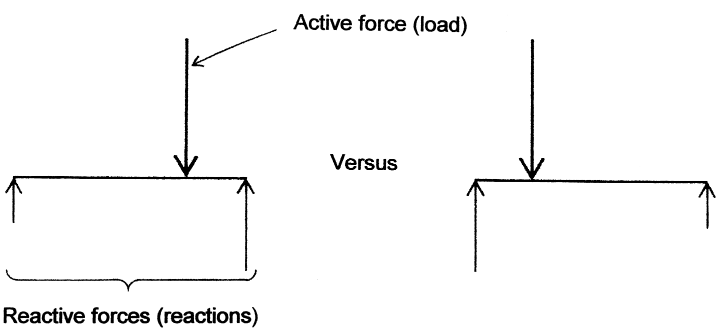


Figure 2.5 Effect of location of the line of action of a force.

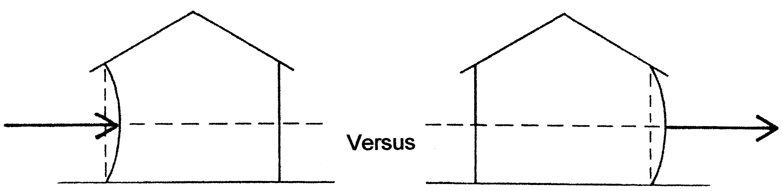


Figure 2.6 Effect of the position of a force along its line of action.

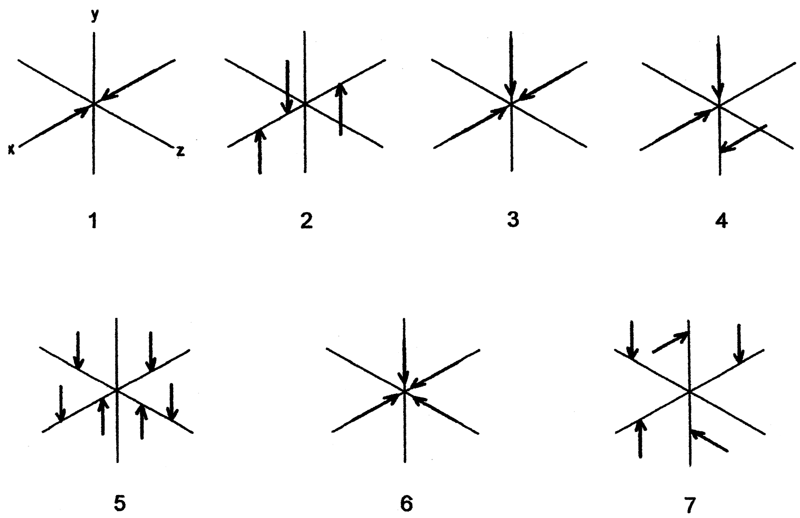


Figure 2.7 Classification of force systems with reference to orthogonal axes.

Table 2.1 Classification of Force Systems[a]

System Variation	Qualifications		
	Coplanar	Parallel	Concurrent
1	Yes	Yes	Yes
2	Yes	Yes	No
3	Yes	No	Yes
4	Yes	No	No
5	No	Yes	No
6	No	No	Yes
7	No	No	No

[a]See Fig. 2.7.

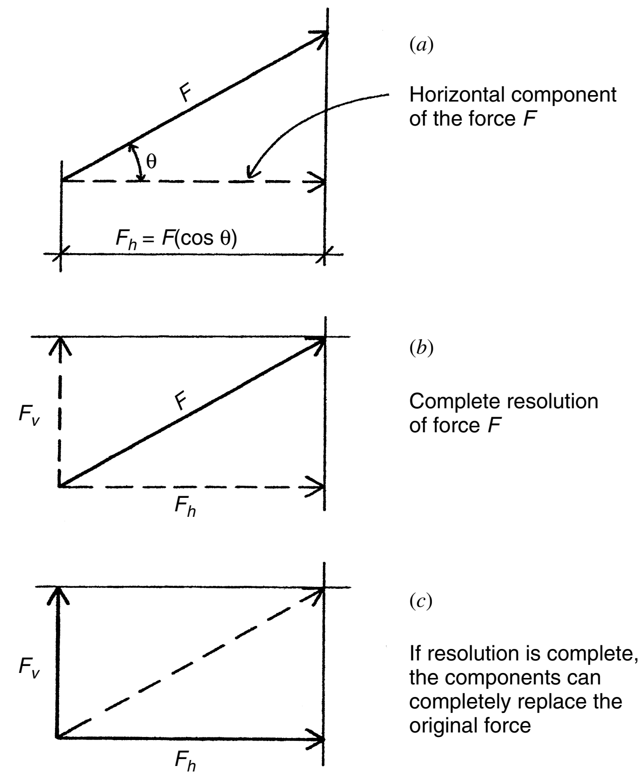


Figure 2.8 Resolution of a force into components.

be found by determining the side of the right-angled triangle, as shown in the illustration, and its magnitude can be scaled with the same scale used to lay out the force vector F. It may also be computed algebraically as $F(\cos\theta)$.

A common form of force resolution is that of determining the vertical and horizontal components of an inclined force, as shown in Figure 2.8. However, a single force can be resolved into an infinite number of components.

Composition

Whether performed algebraically or graphically, the combining of forces is essentially the reverse of that just demonstrated for resolution. Consider the two forces shown in Figure 2.9*a*. Their combined effect can be determined graphically by the parallelogram in Figure 2.9*b* or the triangle in Figure 2.9*c*. The product R of this addition is called the *resultant* of the forces.

When the addition of force vectors is performed by the use of mathematical computation, a common procedure is to first resolve the forces into their vertical and horizontal components. The components of the resultant can then be determined by simple addition of the components of the forces. Thus

$$R = \sqrt{\left(\sum F_v\right)^2 + \left(\sum F_h\right)^2}$$

$$\tan\theta = \frac{\sum F_v}{\sum F_h}$$

When performing algebraic summations, it is necessary to use the sign (or sense) of the forces. Note in Figure 2.9 that the horizontal components of the two forces are both in the same direction and thus have the same sense (or algebraic sign) and the summation is thus one of addition of the two

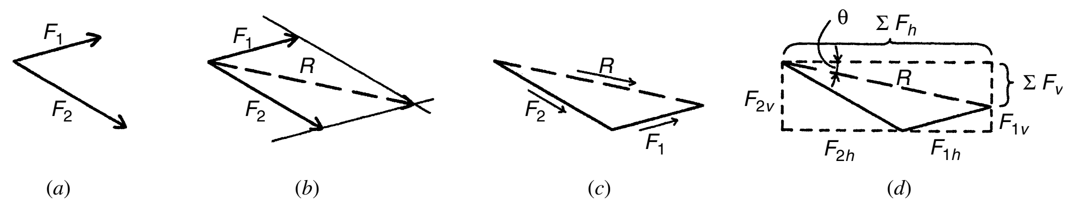


Figure 2.9 Composition of forces by vector addition.

components. However, the vertical components of the two components are opposite in sense and the summation consists of finding their difference by subtraction.

When more than two forces must be added, the graphic process consists of the construction of a force polygon. This process may be visualized as the successive addition of pairs of forces in a series of force triangles, as shown in Figure 2.10. The first pair of forces is added to produce their resultant; this resultant is then added to the third force; and so on. The process is continued until the last force is added, producing the final resultant of the complete force system. The process, as a sequence of additions of two forces at a time, is shown in Figure 2.10. In fact, the intermediate resultants need not be found, and the process can be continued in a single construction, as shown.

The sequence of the addition of the forces is arbitrary. Thus there is not only one polygon that must be constructed, but rather a whole series of polygons, all producing the same resultant.

Motion and Static Equilibrium

The natural state of a static force system is one of static equilibrium. This means that the resultant of any complete interactive set of static forces must be zero. For various purposes it is sometimes desirable to find the resultant combined effect of a limited number of forces, which may indeed be a net force. If a condition of equilibrium is then desired, it may be visualized in terms of producing the *equilibrant*, which is the force that will totally cancel the resultant. The equilibrant, therefore, is the force that is equal in magnitude and direction, but opposite in sense, to the resultant.

In structural design the typical force analysis problem begins with the assumption that the net effect is one of static equilibrium. Therefore, if some forces in a system are known (e.g., the loads on a structure), and some are unknown (e.g., the forces generated in members of the structure in resisting loads), the determination of the unknown forces consists of finding out what is required to keep the whole system in equilibrium. The relationships and procedures that can be utilized for such analyses depend on the geometry or arrangement of the forces.

For a simple concentric, coplanar force system, the conditions necessary for static equilibrium can be stated as follows:

$$\sum F_v = 0 \text{ (sum of the vertical components equals zero)}$$

$$\sum F_h = 0 \text{ (sum of the horizontal components equals zero)}$$

In other words, if both components of the resultant are zero, the resultant is zero and the system is in equilibrium. In a graphic solution the resultant will be zero if the force

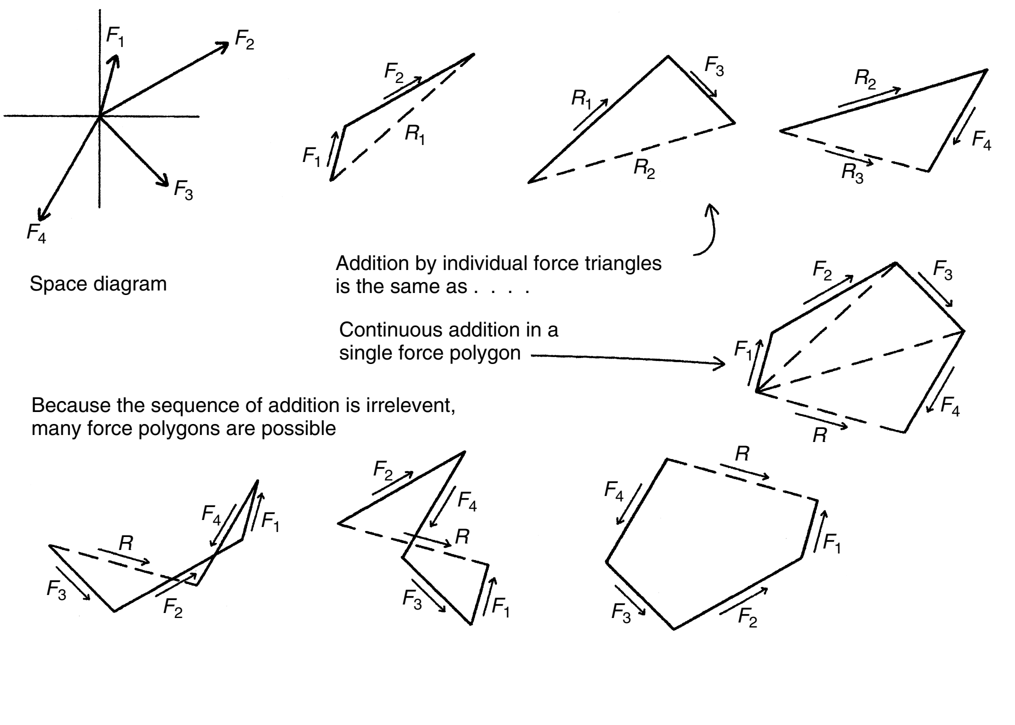


Figure 2.10 Composition of forces by graphic construction.

polygon closes on itself, that is, if the head of the last force vector coincides with the tail of the first force vector.

Although a basic concept of statics is that no motion occurs, it is often helpful in investigation of forces to consider the potential motion that is impending due to the effect of a force. This is especially true when considering the necessary requirements for maintaining equilibrium of a structure.

Analysis of Coplanar, Concurrent Forces

The forces that operate on individual joints in planar trusses constitute sets of coplanar, concurrent forces. The following discussion deals with the analysis of such systems, both graphically and algebraically, and introduces some of the procedures that will be used in the examples of truss analysis in this book.

In the preceding examples, forces have been identified as F_1, F_2, F_3, and so on. However, a different system of notation (Bow's notation) will be used in the work that follows. This method consists of placing a letter in each space that occurs between the forces or their lines of action, each force then being identified by the two letters that appear in the adjacent spaces. A set of five forces is shown in Figure 2.11*a*. The common intersection point is identified as *BCGFE* and the forces are *BC, CG, GF, FE,* and *EB.* Note that the forces have been identified by reading around the intersection point in a clockwise manner. This is a convention that will be used throughout this book because it has some relevance to the methods of graphic analysis that will be explained later.

In Figure 2.11*b* a portion of a truss is shown. Reading around the left upper chord joint designated as *BCGFE* in a clockwise manner, the upper chord member is read as force *CG*. Reading around the joint *CDIHG*, the same member is read as *GC*. Either designation may be used when referring to the member itself. However, if the effect of the force in the member on a joint is being identified, it is important to use the proper sequence for the two-letter designation.

In Figure 2.12*a* a weight is shown hanging from two wires that are attached at separate points to the ceiling. The two sloping wires and the vertical wire that supports the weight directly meet at joint *CAB*. The "problem" in this case is to find the tension forces in the three wires. We refer to these forces that exist within the members of a structure as *internal forces*. In this example it is obvious that the force in the vertical wire will be the same as the magnitude of the weight:

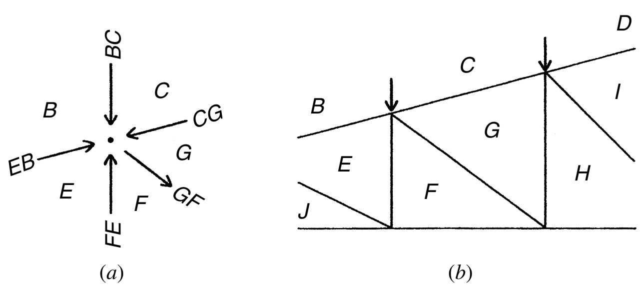


Figure 2.11 Force notation.

50 lb. Thus the solution is reduced to the determination of the tension forces in the two sloping wires. This problem is presented in Figure 2.12*b*, where the force in the vertical wire is identified in terms of both direction and magnitude, while the other two forces are identified only in terms of their directions, which must be parallel to the wires. The senses of the forces in this example are obvious, although this will not always be true in such problems.

A graphic solution of this problem can be performed by using the available information to construct a force polygon consisting of the vectors for the three forces: *BC*, *CA*, and *AB*. The process for this construction is as follows:

The vector for *AB* is totally known and can be shown as indicated by the vertical arrow in Figure 2.12*c*, with its head down and its length measured in some scale to be 50.

The vector for force *BC* is known as to direction and must pass through the point *b* on the force polygon, as shown in Figure 2.12*d*.

Similarly, the vector for force *CA* will pass through the point *a*, as shown in Figure 2.12*e*.

The point *c* is thus located by the intersection of these two lines, and the completed force polygon is as shown in Figure 2.12*f*. Sense of the forces in the polygon (that is, the location of the arrowheads) is established by the "flow" of the forces, beginning and ending at point *a* on the polygon. We thus read the direction of the arrows as flowing from *a* to *b* to *c* to *a*.

With the force polygon completed, we can determine the magnitudes for forces *BC* and *CA* by measuring their lengths on the polygon using the same scale that was used to lay out force *AB*.

For an algebraic solution of this problem, we first resolve the two unknown forces into their vertical and horizontal forces, as shown in Figure 2.12*g*. This increases the number of unknowns from two to four. However, we have two extra relationships that may be used in addition to the two conditions for static equilibrium, because the directions of forces *BC* and *CA* are known.

As shown in Figure 2.12*a*, force *BC* is at an angle with a slope of 1 vertical to 2 horizontal. Using the rule that the hypotenuse of a right triangle is related to the sides of the triangle such that the square of the hypotenuse is equal to the sum of the squares of the sides, we can determine that the length of the hypotenuse of the slope triangle is

$$l = \sqrt{(1)^2 + (2)^2} = \sqrt{5} = 2.236$$

We can now use the relationships of this triangle to express the relationships of the force *BC* to its components. Thus, referring to Figure 2.13,

$$\frac{BC_v}{BC} = \frac{1}{2.236}, \quad BC_v = \frac{1}{2.236}(BC) = 0.447(BC)$$

$$\frac{BC_h}{BC} = \frac{2}{2.236}, \quad BC_h = \frac{2}{2.236}(BC) = 0.894(BC)$$

These relationships are shown in Figure 2.12*g* by indicating the dimensions of the slope triangle with the hypotenuse having a value of 1. Similar calculations will produce the values shown for the force *CA*. We can now express the conditions for equilibrium. (Directions up and right are considered positive.)

$$\sum F_v = 0 = -50 + BC_v + CA_v$$
$$0 = -50 + 0.447(BC) + 0.707(CA) \tag{2.1}$$

and

$$\sum F_h = 0 = -BC_h + CA_h$$
$$0 = -0.894(BC) + 0.707(CA) \tag{2.2}$$

Simultaneous solution of these two equations produces the following:

$$BC = 37.29\,\text{lb}, \quad CA = 47.15\,\text{lb}$$

When answers obtained from an algebraic solution are compared to those obtained from a graphic solution, the level of correlation may be low unless great care is exercised in the graphic work and a very large scale is used for the graphic

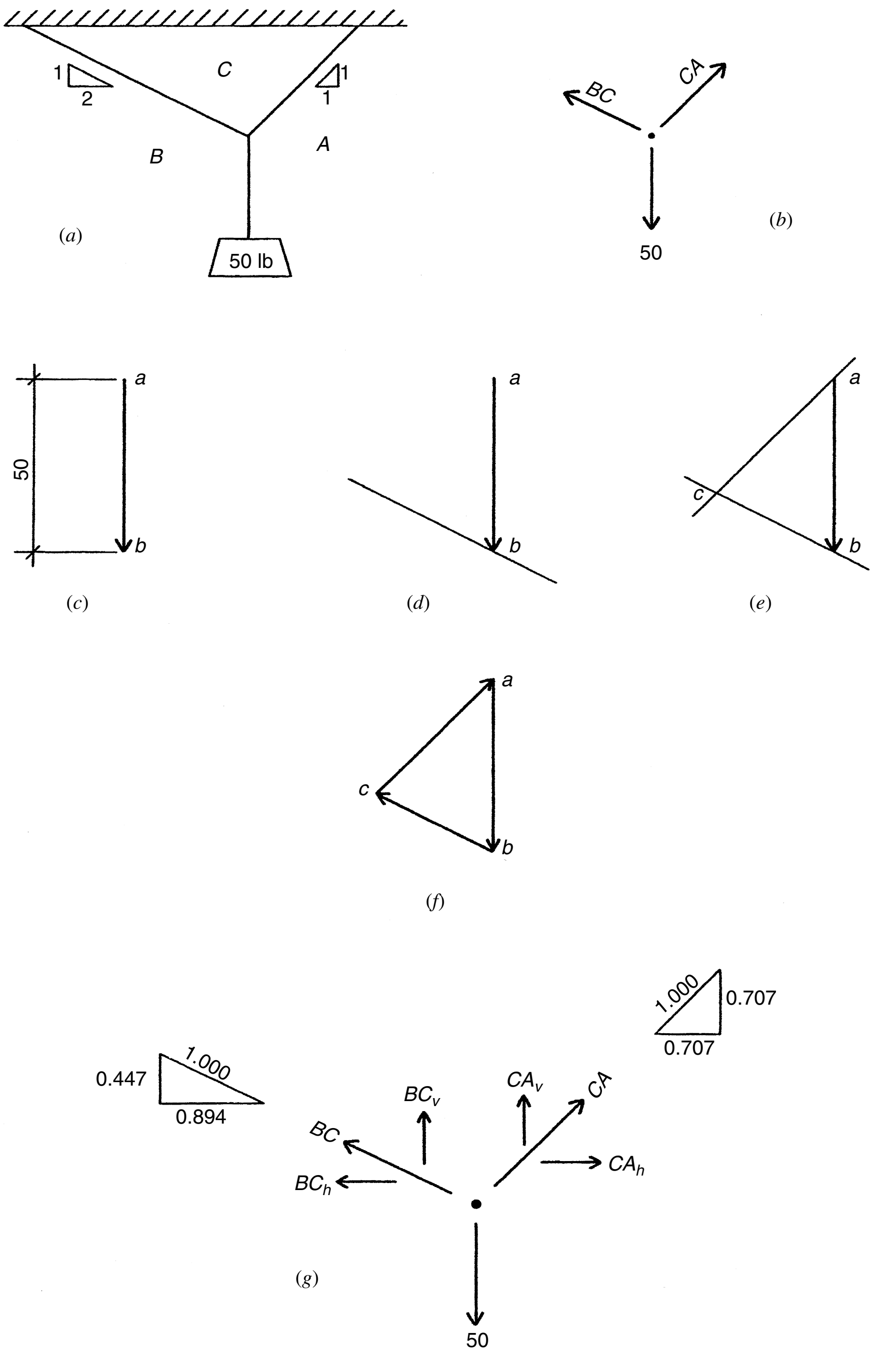


Figure 2.12 Simple concentric force system.

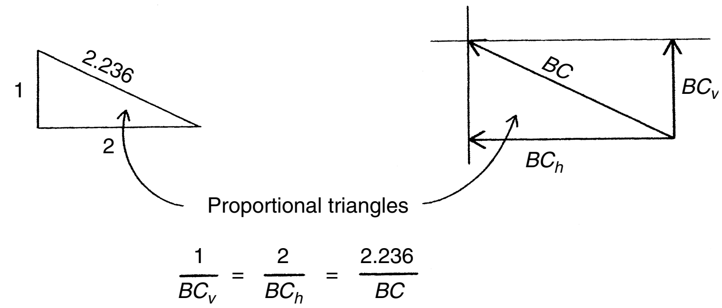


Figure 2.13 Determination of force components.

construction. If the scale used for the graphic solution is actually as small as that shown on the printed page in Figure 2.12, it is unreasonable to expect accuracy beyond the second digit.

For an algebraic solution, all forces are typically reduced to their horizontal and vertical components. Thus, for the structure in Figure 2.12, The member BC may be represented as shown in Figure 2.13, with its known direction angle used to express the horizontal and vertical components. The proportions of these forces are shown in the figure, and if any one of the forces is known, either of the other two may be determined. Thus, if the value of BC_v is known, then the value of BC may be determined as

$$\frac{1}{BC_v} = \frac{2.236}{BC}, \quad BC = \frac{2.236}{1}(BC_v)$$

The procedures shown here for solution of the problem illustrated in Figure 2.12 can be used for any structure for which the members interact only as sets of related systems of concurrent forces. The members of the structure are thus limited in capability to the resistance of simple tension or compression forces. The planar truss is the most common form of such a structure.

When the so-called *method of joints* is used, finding the internal forces in the members of a planar truss consists of solving a series of concurrent force systems. Figure 2.14 shows a truss with the form, the loads, and the reactions displayed in a *space diagram*. Below the space diagram is a figure consisting of the free-body diagrams of the individual joints of the truss. These are arranged in the same manner as they are in the truss in order to show their interrelationships. Each joint constitutes a complete force system that must have its own equilibrium. "Solving" the problem consists of determining the equilibrium conditions for all the joints.

Graphical Analysis of Planar Trusses

Figure 2.15 shows the space diagram for a single-span planar truss that is subjected to vertical gravity loads. We will use this example to illustrate the procedures for determining the internal forces in the truss, that is, the tension and compression forces in the individual members of the truss. The letters on the space diagram identify the individual forces at the truss joints. The placement of these letters is arbitrary, the only necessary consideration being to place a letter in each space between the loads and the truss members so that each force at a joint can be identified by a two-letter symbol.

The separated force diagram in Figure 2.15 shows the sets of forces operating at each joint. The individual forces are designated by two-letter symbols that are determined by reading around the joint in the space diagram.

The third diagram in Figure 2.15 is a composite figure containing the force polygons for each joint as well as a polygon for the eternal forces. This is called a *Maxwell*

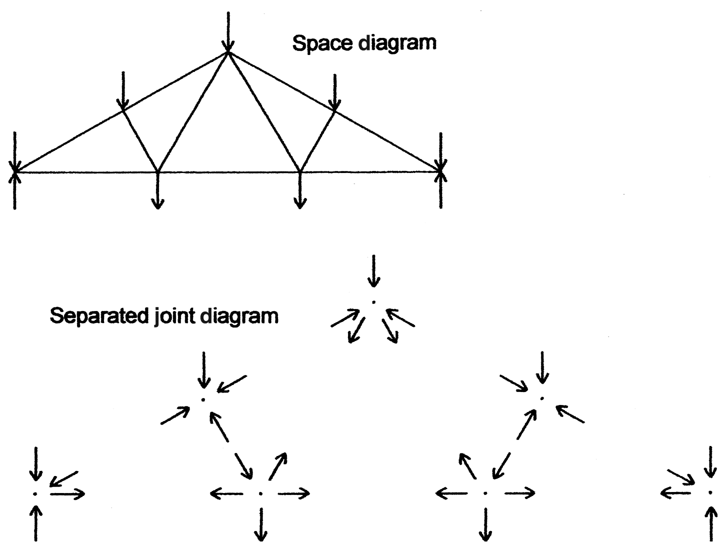


Figure 2.14 Examples of diagrams used to represent trusses and their actions.

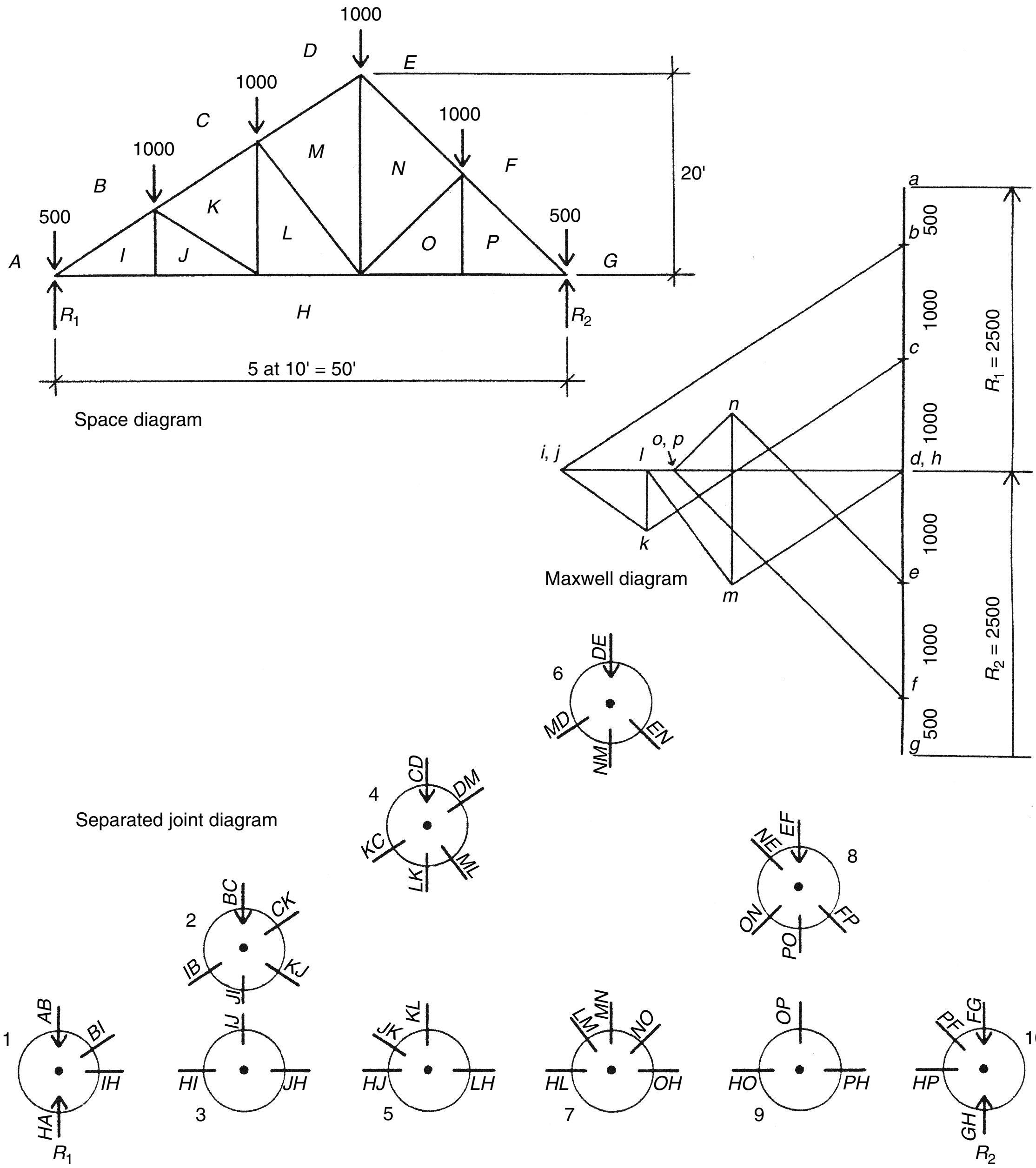


Figure 2.15 Graphical diagrams for the sample problem.

diagram after one of its early users, James Maxwell. The construction of this diagram constitutes a complete solution for the magnitudes and senses of the internal forces in the truss. The procedure for this construction is as follows.

1. *Construct the force polygon for the external forces.* Before this can be done, the values of the reactions must be found. There are graphical techniques for finding the reactions, but it is usually much faster and simpler to find them with an algebraic solution. In this example, although the truss is not symmetrical, the loading is, and it may be observed that the reactions are each equal to one-half of the total load on the truss, or $5000/2 = 2500$ lb.

Because the external forces in this case are all in a single direction, the force polygon for the external forces is actually a straight line. Using the two-letter symbols for the forces and starting with the letter *A* at the left end, we read the force sequence by moving in a clockwise direction around the outside of the truss. The loads are thus read as *AB, BC, CD, DE, EF,* and *FG,* and the two reactions are read *GH* and *HA*.

Beginning at *a* on the Maxwell diagram, the force vector sequence for the external forces is read as *a* to *b*, *b* to *c*, *c* to *d*, and so on, ending back at *a*, which shows that the force polygon closes and the external forces are in the necessary state of static equilibrium. Note that we have pulled the vectors for the reactions off to the side of the diagram to indicate them

more clearly. Vectors on a single line are superimposed on each other and their identity is not very clear on the diagram. Nevertheless, the vector for each of the external forces can actually be read directly from the force polygon.

Note also that we have used lowercase letters for the vector ends in the Maxwell diagram, whereas uppercase letters are used on the space diagram. The letters on the space diagram designate spaces, while the letters on the Maxwell diagram designate points of intersection of lines. The alphabetic correlation is retained (*A* to *a*, etc.), while confusion between the two diagrams is prevented.

2. *Construct the force polygons for the individual joints*. The graphic procedure for this consists of locating the points on the Maxwell diagram that correspond to the remaining letters, *I* through *P*, on the space diagram. When all the letters are located, the complete force polygon for each joint may be read on the diagram. To locate these points, we use two relationships. The first is that the truss members can resist only forces that are parallel to the member's positioned directions. Thus we know the directions of all the internal forces.

The second relationship is a simple one from plane geometry: A point may be located at the intersection of two lines. Consider the forces at joint 1, as shown in the separated joint diagram in Figure 2.15. Note that there are four forces and that two of them are known (the load and the reaction) and two are unknown (the internal forces in the two truss members).

The force polygon for this joint, as shown on the Maxwell diagram, is read as $a-b-i-h-a$, where $a-b$ represents the load, $b-i$ the force in the upper chord member, $i-h$ the force in the lower chord member, and $h-a$ the reaction. Thus the location of point *i* on the Maxwell diagram is determined by noting that *i* must be in a horizontal direction from *h* (corresponding to the horizontal position of the lower chord) and in a direction from *b* that is parallel to the position of the upper chord.

The remaining points on the Maxwell diagram are found by the same process using two known points on the diagram to project lines of known direction whose intersection will determine the location of another point. Once all the points are located, the diagram is complete and can be used to find the magnitude and sense of all the internal forces.

The process for construction of the Maxwell diagram typically consists of moving from joint to joint along the truss. As described previously, it begins with a plot of the known points as defined by the known values of the loads and reactions. Then, once one of the letters for an interior space is located, it can be used as a known point for finding the locations for other letters relating to adjacent interior spaces. The only limitation of the process is that it is not possible to find more than one unknown point on the Maxwell diagram by constructing the force polygon for a single joint, since each point corresponds to two unknown forces at the joint.

Consider joint 7 on the separated joint diagram in Figure 2.15. If we attempt to solve this joint first, knowing only the locations of letters *a* through *h* on the Maxwell diagram, we must locate four unknown points: *l*, *m*, *n*, and *o*. This is three more unknown points than we can determine in a single step, so we must first solve for three of the unknowns by using other joints. For this truss, the only possible starting points are joint 1 or joint 10, at each of which there is only a single unknown point to be found to complete the force polygon for the joint. From that start, additional joints can be solved until all points are found.

When the Maxwell diagram is completed, the internal forces can be read from the diagram as follows:

The magnitude is determined by measuring the length of the line in the diagram using the scale that was used to plot the vectors for the exterior forces.

The sense (or sign) of individual forces is determined by reading the forces in a clockwise sequence around a single joint in the space diagram and tracing the same letter sequences on the Maxwell diagram.

Figure 2.16 shows the force system at joint 1 and the force polygon for these forces as taken from the Maxwell diagram. The forces known initially are shown as solid lines of the force polygon, and the unknown forces are shown as dashed lines. Starting with letter *A* on the force system, we read the forces in a clockwise sequence as *AB, BI, IH,* and *HA.* On the Maxwell diagram we note that moving from *a* to *b* is moving in the order of the sense of the force, that is, from the tail to the head of the force vector that represents the external load on the joint.

If we continue in this sequence on the Maxwell diagram, this force sense flow will be a continuous one. Thus reading from *b* to *i* on the Maxwell diagram is reading from tail to head of the force vector, which tells us that force *BI* has its head at the left end. Transferring this sense indication from the Maxwell diagram to the joint diagram indicates that force *BI* is in compression; that is, it is pushing on the joint. Reading from *i* to *h* on the Maxwell diagram shows that the arrowhead for this vector is on the right, indicating a tension effect on the joint diagram.

Having solved for the forces at joint 1 as described, we may use the fact that we now know the forces in truss members *BI* and *IH* when we proceed to consideration of the adjacent joints, 2 and 3. However, we must be careful to note that the sense reverses at the opposite ends of the members in the joint diagrams. Referring to the separated joint diagram in Figure 2.15, if the upper chord member shown as force *BI* in joint 1 is in compression, its arrowhead is at the lower left end in the diagram for joint 1, as shown in Figure 2.16*a*. However, when the same force is shown as *IB* at joint 2, its pushing effect on the joint will be indicated by having the arrowhead at the upper right end in the diagram for joint 2. Similarly, the tension effect of the lower chord is shown in joint 1 by placing the arrowhead on the right end of the force *IH*, but the same force will be indicated in joint 3 by placing the arrowhead on the left end of the vector for force *HI*.

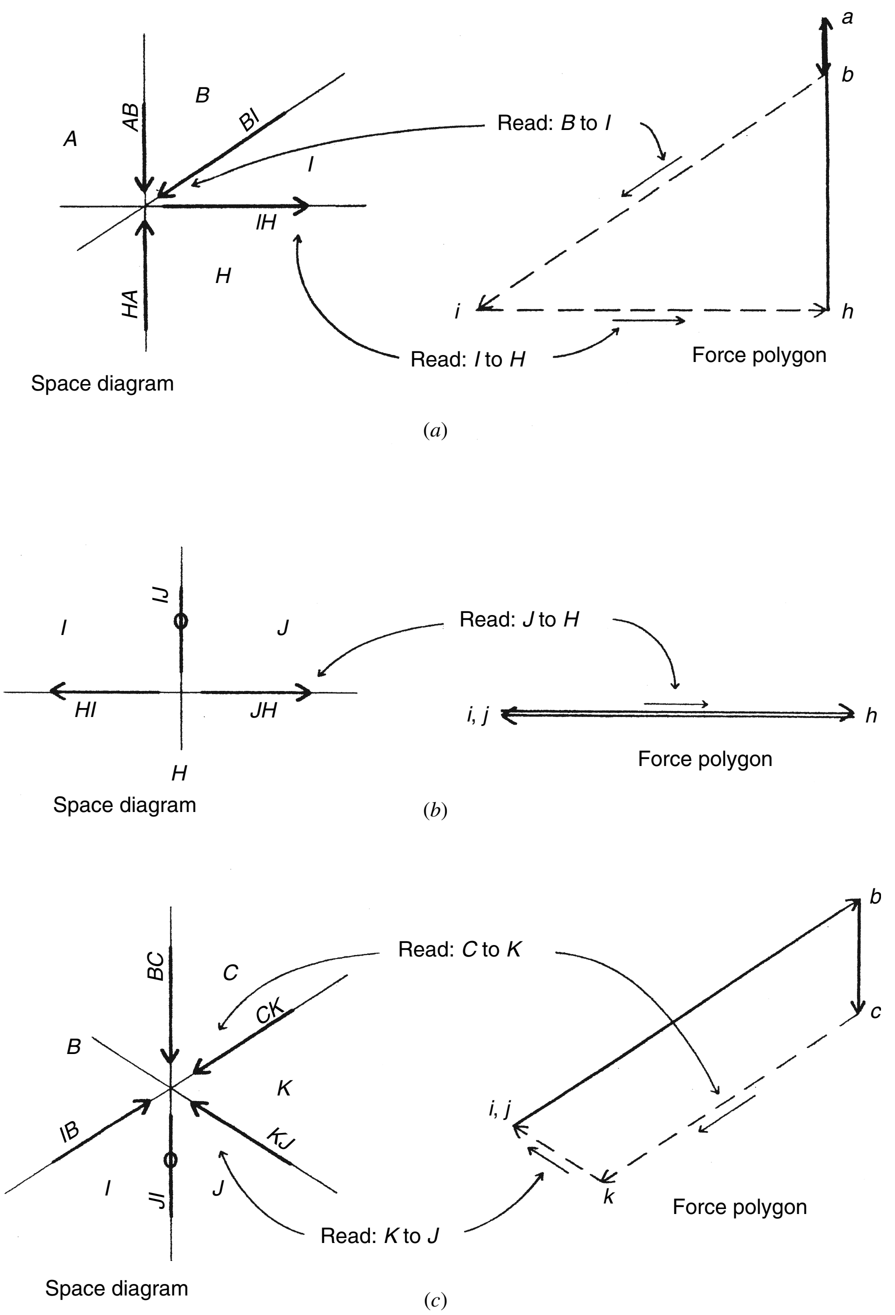


Figure 2.16 Analysis of truss joints. (a) joint 1; (2) joint 3; (c) joint 2.

If we choose to solve for the forces at joint 2 next, the known force found for joint 1 in the top chord can be used as a known at this joint. However, there remain three unknown forces at the joint corresponding to the unknown locations of points *k* and *j*. It is therefore necessary to solve for the forces at joint 3, which involves finding only the unknown point *j*.

On the Maxwell diagram we can find the unknown point *j* by projecting vector *IJ* vertically from point *i* and by projecting vector *JH* horizontally from point *h*. Since point *i* is also located horizontally from point *h*, we thus find that the vector *IJ* has zero magnitude. Thus, the truss member identified as *IJ* is a so-called *zero-stress member*, which is to say it is not stressed by this loading condition. For the Maxwell diagram, the condition thus determined is that points *i* and *j* are concurrent or coincident on the diagram. There are then only two effective members in this joint having forces equal in magnitude and opposite in sense.

The joint force diagram and the force polygon for joint 3 are as shown in Figure 2.16*b*. In the joint force diagram we place a zero, rather than an arrowhead, on the vector line for *IJ* to indicate the zero-stress condition. In the force polygon, we have slightly separated the two force vectors for clarity, although they are actually coincident on the same line.

Having solved for the forces at joint 3, we can next proceed to joint 2, since there now remain only two unknown forces at this joint. The forces at the joint and the force polygon

for joint 2 are shown in Figure 2.16*c*. As explained for joint 1, we read the force polygon in a sequence determined by reading in a clockwise direction around the joint: *BCKJIB*. Following the continuous direction of the force arrows on the force polygon in this sequence, we can establish the sense for the two forces CK and *KJ*.

This demonstrates the general procedure for the graphic analysis of planar trusses. The process may be simpler or more complex, depending on the layout of individual trusses.

It is possible to proceed from one end and to work continuously across the truss from joint to joint to construct the Maxwell diagram in this example. The sequence in terms of locating points on the Maxwell diagram would be *i–j–k–l–m–n–o–p*, which would be accomplished by solving the joints in the following sequence: 1–3–2–5–4–6–7–9–8. However, in performing the graphic construction, accuracy in the drawing limits the ability to precisely locate points. Thus, by the time point *p* is located, the drawing errors have been accumulated by eight constructions. For somewhat greater accuracy, a method used is to work separately from each end, thus locating two positions for one middle letter—the separation of the two indicating the degree of accuracy of the drawing.

Algebraic Analysis of Planar Trusses

Graphic solution by the use of a Maxwell diagram corresponds to an algebraic solution by what is called the *method of joints*. This method consists of solving the concentric force systems at the joints using force equilibrium conditions. We will now demonstrate the use of the method for the preceding example.

As with the graphic solution, the first step is the determination of the reactions. With the external forces defined, we proceed to solution of the individual joints, following the sequence used for the graphic solution.

The problem to be solved at joint 1 is as shown in Figure 2.17*a*. In Figure 2.17*b* the same system is shown with all forces expressed as vertical and horizontal components. Although this increases the number of unknowns to 3, an additional condition is available consisting of the relationship of the two components for force *BI*.

The condition for vertical equilibrium is shown in Figure 2.17*c*. The balance of vertical forces is limited to the load, the reaction, and the vertical component in *BI*, shown as force BI_v. Simple observation makes it obvious that the sense of BI_v must be downward, indicating a compressive force in member *BI*. The algebraic equation for vertical equilibrium (with upward force considered positive) is

$$\sum F_v = 0 = +2500 - 500 - BI_v$$

From this equation we determine BI_v to have a magnitude of 2000 lb. Using the known relationships between *BI*, BI_v, and BI_h, we can determine the values of these three quantities if any one is known. Thus, using the indication for the slope of *BI* as shown in Figure 2.17*a*,

$$\frac{BI}{1.000} = \frac{BI_v}{0.555} = \frac{BI_h}{0.832}$$

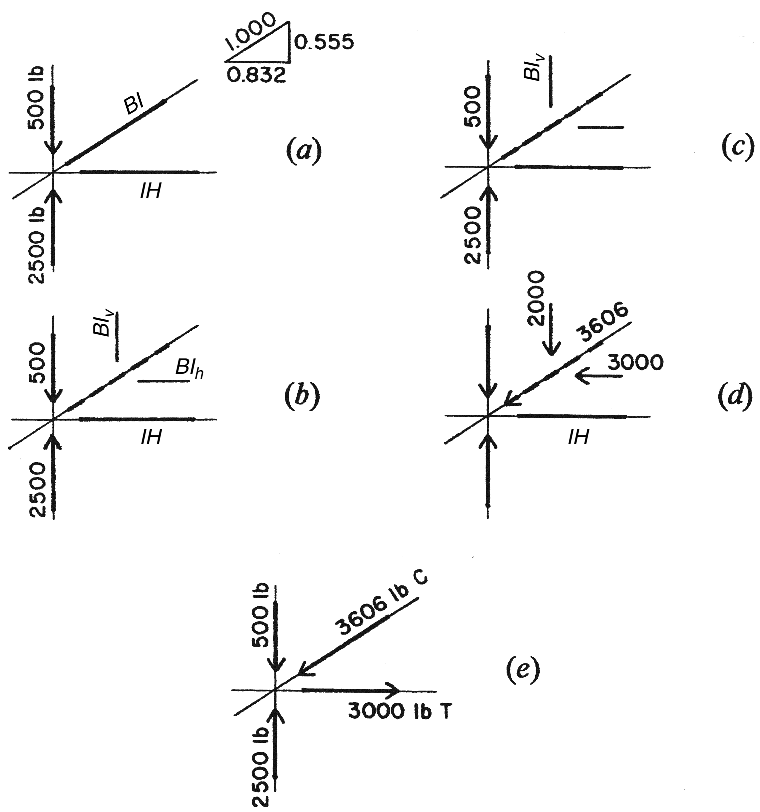


Figure 2.17 Algebraic solution of joint 1.

$$BI_h = \frac{0.832}{0.555}(2000) = 3000\,\text{lb}$$

$$BI = \frac{1.000}{0.555}(2000) = 3606\,\text{lb}$$

The results of the analysis to this point are shown in Figure 2.17*d*, from which we can observe the conditions for equilibrium of horizontal forces. Stated algebraically (with force sense to the right considered positive) the condition is

$$\sum F_h = 0 = +IH - 3000$$

from which we establish that the force in *IH* is 3000 lb. The final solution for the joint is shown in Figure 2.17*e* using *C* to indicate compression and *T* to indicate tension in the truss members.

As with the graphic solution, we now proceed to consider joint 3. The initial condition is as shown in Figure 2.18*a*, with the single known force in member *HI* and the unknown forces in *IJ* and *JH*. Since the forces at this joint are all vertical and horizontal, there is no need to use components. Consideration of vertical equilibrium makes it obvious that it is not possible to have force in member *IJ*, and the member is therefore classified as a zero-stress member for this loading.

The final answer for the forces at joint 3 is as shown in Figure 2.18*b*. Note the convention for indicating the condition of a truss member with no internal force.

If we now consider joint 2, the initial condition is as shown in Figure 2.19*a*. Of the five forces at the joint only two remain unknown. Following the procedure for joint 1, we first resolve the forces into their vertical and horizontal components, as shown in Figure 2.19*b*.

Because we do not know the sense of the forces *CK* and *KJ*, we may use the procedure of considering them to be positive until proven otherwise. That is, if we enter them into the algebraic equations with an assumed sense, and the solution produces a negative answer, then our assumption was wrong. However, we must be careful to be consistent with the sense of the force vectors, as the following solution will illustrate.

Let us arbitrarily assume that force *CK* is in compression and force *KJ* is in tension. If this is so, the forces and their components will be as shown in Figure 2.19*c*. If we then consider the conditions for vertical equilibrium, the forces involved will be those shown in Figure 2.19*d*, and the equation for vertical equilibrium will be

$$\begin{aligned}\sum F_v &= 0 = -1000 + 2000 - CK_v - KJ_v \\ 0 &= +1000 - 0.555CK - 0.555KJ\end{aligned} \tag{2.3}$$

If we consider the conditions for horizontal equilibrium, the forces will be as shown in Figure 2.19*e*, and the equation will be

$$\begin{aligned}\sum F_h &= 0 = +3000 - CK_h + KJ_h \\ 0 &= +3000 - 0.832CK + 0.832KJ\end{aligned} \tag{2.4}$$

Note the consistency of the algebraic signs and the sense of the force vectors, with positive forces considered as upward and toward the right. Simultaneous solution of these two equations yields the following answers:

$CK = +2704$ lb, indicating that the assumed sense was correct

$KJ = -901$ lb, indicating that the assumed sense was incorrect and member KJ is actually in compression

The final answers for the forces at joint 2 are thus as shown in Figure 2.19*g*. To verify that equilibrium exists, however, the forces are shown in the form of their vertical and horizontal components in Figure 2.19*f*.

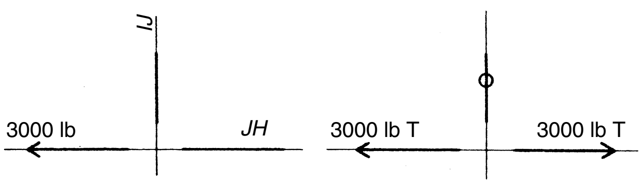
When all of the internal forces have been determined for the truss, the results may be displayed in a number of ways. The most direct way is to display them on a scaled drawing of the truss, as shown in the upper part of Figure 2.20. The force magnitudes are displayed next to each member with the sense shown as *T* for tension and *C* for compression. Zero-stress members are indicated by the conventional symbol of a zero placed directly on the member.

When solving by the algebraic method of joints, the results may be recorded on a separated joint diagram, as shown in the lower portion of Figure 2.20. If the values for the vertical and horizontal components of sloping members are shown, it is a simple matter to verify the equilibrium of the individual joints.

Moments

In the analysis of concurrent force systems, it is sufficient to consider only the basic force vector properties: magnitude, direction, and sense. However, when forces in a system are not concurrent, it is necessary to include the consideration of another type of force action called the *moment*, or *rotational effect.*

Consider the two interacting vertical forces shown in Figure 2.21*a*. Since the forces are concurrent, the condition of equilibrium is fully established by satisfying the single algebraic equation $\sum F_v = 0$. However, if the same two forces are not concurrent, as shown in Figure 2.21*b*, the single force summation is not sufficient to establish equilibrium. In this case, the force summation establishes the same fact



(*a*) (*b*)

Figure 2.18 Algebraic solution of joint 3.

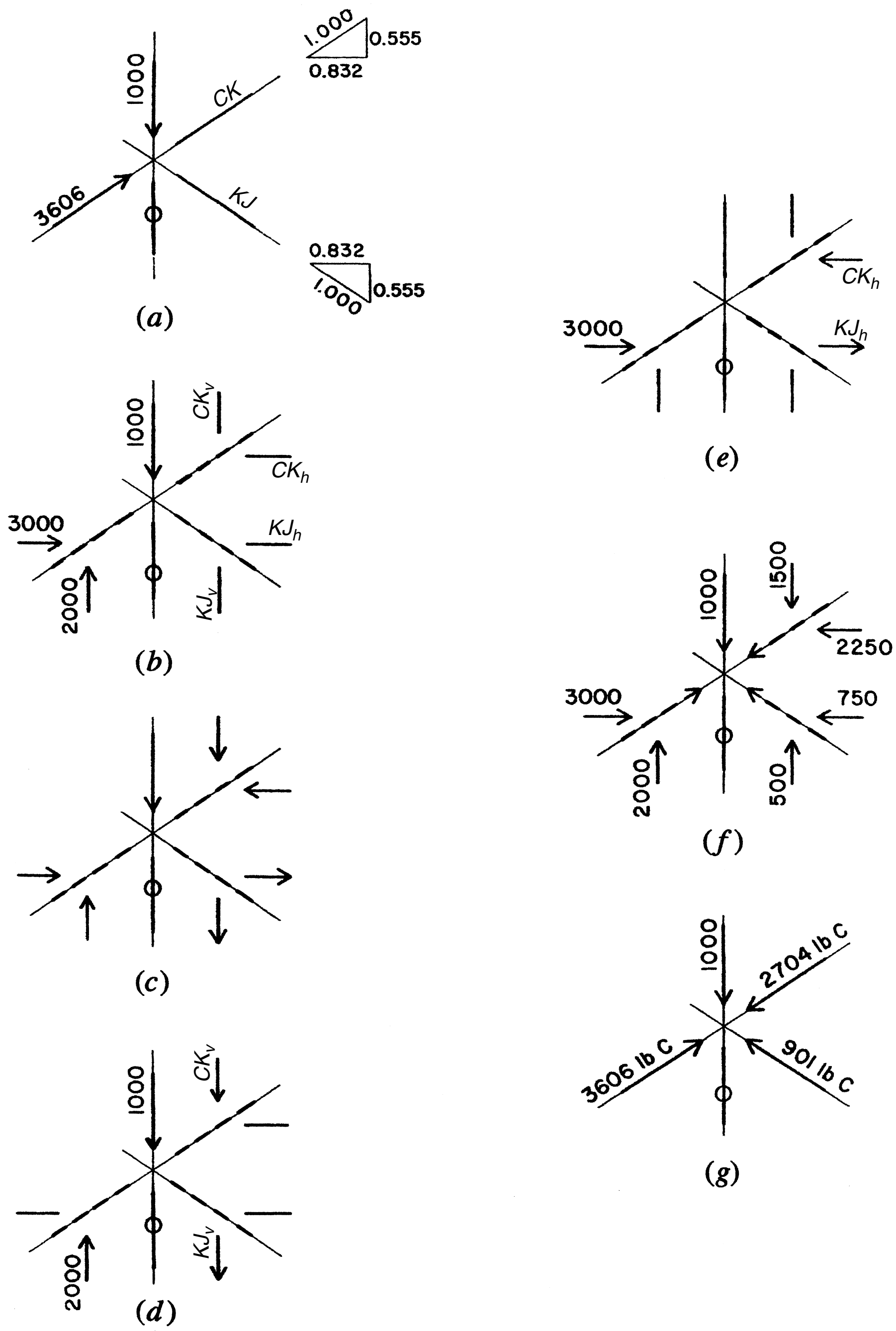


Figure 2.19 Algebraic solution of joint 2.

as before: There is no net tendency for vertical motion. However, because of their separation, the forces tend to cause a counterclockwise motion in the form of a rotational effect, called *a moment.* This creates necessity for the consideration of an additional condition for equilibrium, stated as

$$\sum M = 0$$

A moment has three basic properties:

It exists in a particular plane—in this case the plane defined by the two force vectors.

It has a magnitude, which is expressed as the product of the force magnitude times the distance between the two vectors. In the example shown in Figure 2.21*b*, the magnitude of the moment is $10 \times a$. The unit for this quantity becomes a compound of the force unit and the distance unit, for example, lb-ft.

It has a sense of rotational direction. In the example the sense is counterclockwise.

Rotational equilibrium can be established in a number of ways. One way is shown in Figure 2.21*c*. In this example a second set of forces whose rotational effect counteracts that

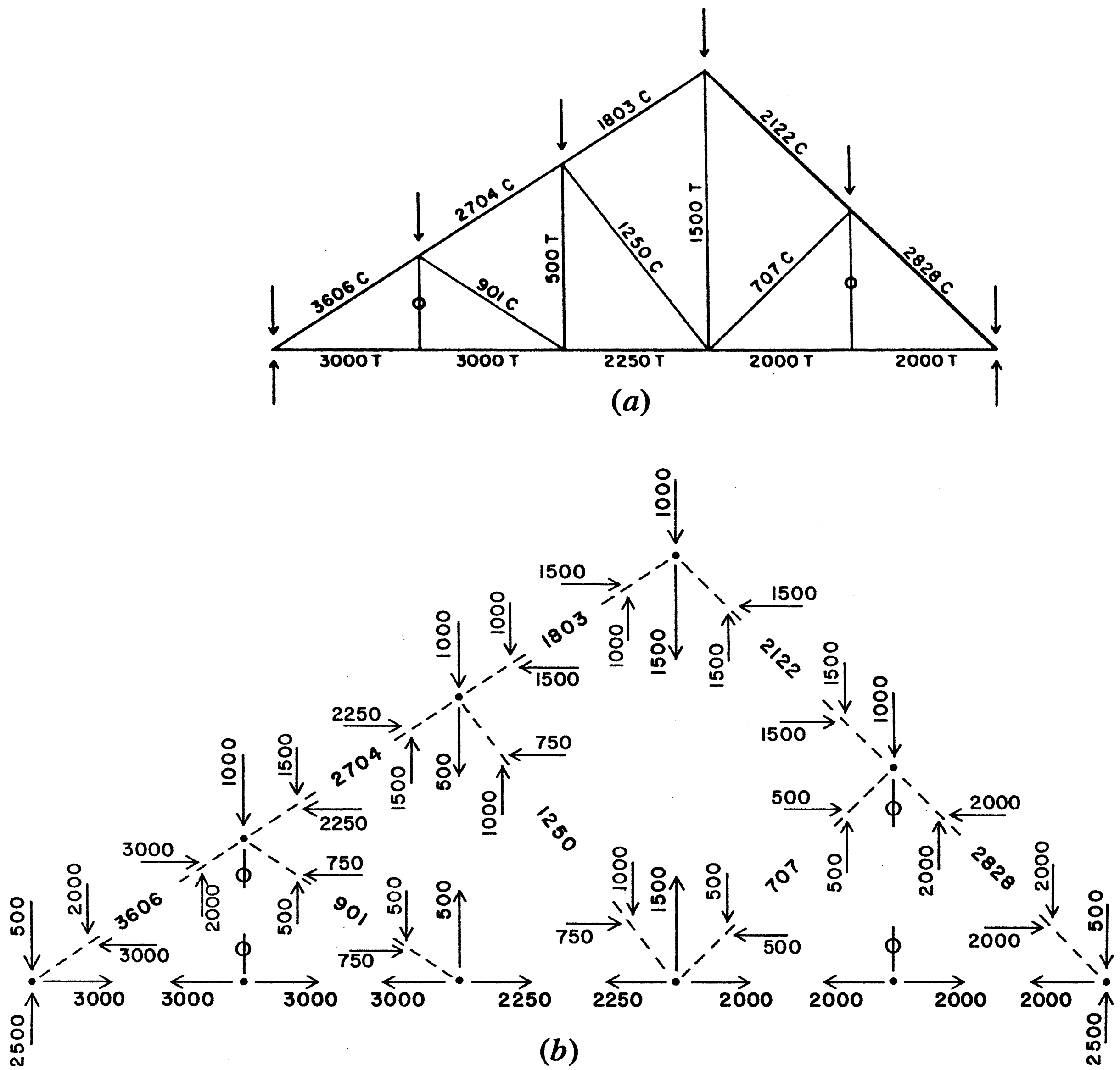


Figure 2.20 Display of internal forces in the truss and forces at individual joints.

of the first set has been added. This group of forces forms a general coplanar system, and its analysis typically requires consideration of both the equilibrium of the forces and a separate equilibrium analysis of the complete moment effects of all the forces.

For equilibrium of the system in Figure 2.21*c*, the three considerations are

$$\sum F_v = 0 = +10 - 10$$

$$\sum F_h = 0 = +4 - 4$$

$$\sum M = 0 = +10(a) - 4(2.5a)$$

$$= +10a - 10a$$

Because all of these summations total zero, the system is in equilibrium.

Analysis of Coplanar, Nonconcurrent Forces

Analysis procedures for coplanar, nonconcurrent force systems are usually qualified by some type of special conditions of the systems. A common example is that of the horizontal beam subject to vertical forces, for which the special condition is that all the forces are parallel. The following example illustrates this situation.

Figure 2.22*a* shows a 20-kip (k) force applied to a beam at a point between the supports. The supports must generate the two vertical, reaction forces, R_1 and R_2, in order to oppose this load. (In this case we will ignore the weight of the beam itself, which also adds load to the supports, and we will consider only the effect of the added load—also called the *superimposed load* on the beam). Since there are no horizontal forces, the complete equilibrium of this system can be established by the satisfaction of two equations: $\sum F_v = 0$

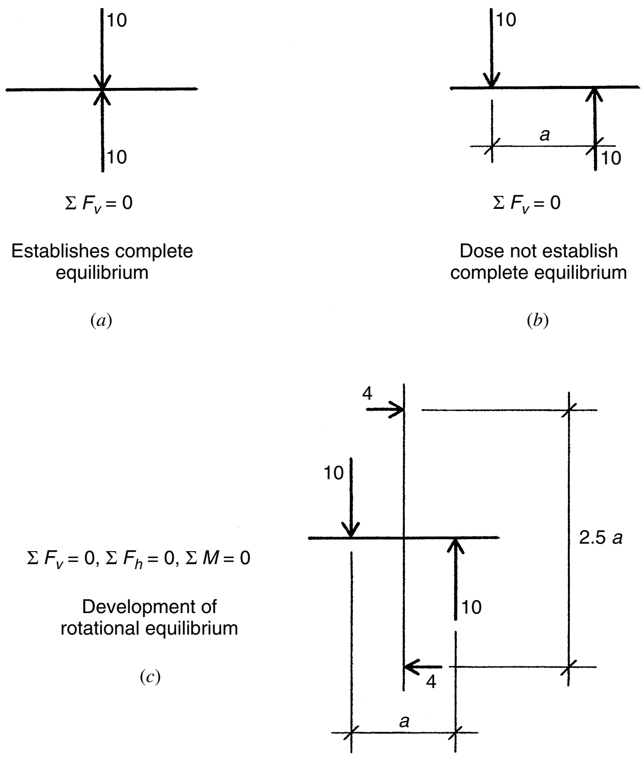


Figure 2.21 Consideration of rotational equilibrium.

and $\sum M = 0$. Considering the force summation first, with up considered as positive,

$$\sum F_v = 0 = -20 + (R_1 + R_2)$$

$$R_1 + R_2 = 20$$

This yields one equation involving the two unknown forces. If we proceed to write a moment summation involving the same two forces, we will then have two equations that can be solved simultaneously. However, we can simplify the algebraic task by making the moment summation in a way that eliminates one unknown from the equation. This is accomplished by simply choosing the location of one of the supports as the reference for the moments, which eliminates the reaction force at that location, its moment about a point on its own line of action being zero. Referring to Figure 2.22*b*, if we choose the point for moments at the right support, the summation of moments is as follows:

$$\sum M = 0 = -20(7) + R_1(10)$$

$$R_1(10) = 140 \quad \text{and} \quad R_1 = \frac{140}{10} = 14\,\text{kips}$$

Using the relationship established by the previous force summation, we have

$$R_1 + R_2 = 20$$
$$14 + R_2 = 20$$
$$R_2 = 6\,\text{kips}$$

The solution is thus as shown in Figure 2.22*c*.

In the structure shown in Figure 2.23*a*, the forces consist of a vertical load, a horizontal load, and some unknown reactions at the supports. Since the forces are not all parallel, we may use all equilibrium conditions for the general coplanar system in the determination of the unknown reactions. Although it is not strictly necessary, we will use the technique of finding the reactions separately for the two loads and then adding the two results to find the true reactions for the combined load.

The vertical load and its reactions are shown in Figure 2.23*b*. In this case, with the symmetrically placed load, each reaction is simply one-half the total load.

For the horizontal load, the reactions will have the components shown in Figure 2.23*c*, with the vertical reaction components developing resistance to the moment effect of the load and the horizontal reaction components combining to resist the actual horizontal-force effect.

Let us first consider a moment summation, choosing the location of the right support as the point of rotation. Since the action lines of V_2, H_1, and H_2 all pass through this point, their moments will be zero and the summation is reduced to dealing with the forces shown in Figure 2.23*d*. Thus, with the clockwise moment considered positive

$$\sum M = 0 = +6(12) - V_1(10), \quad V_1 = 72/10 = 7.2\,\text{kips}$$

We next consider the summation of vertical forces, which involves only V_1 and V_2, as shown in Figure 2.23*e*. Thus,

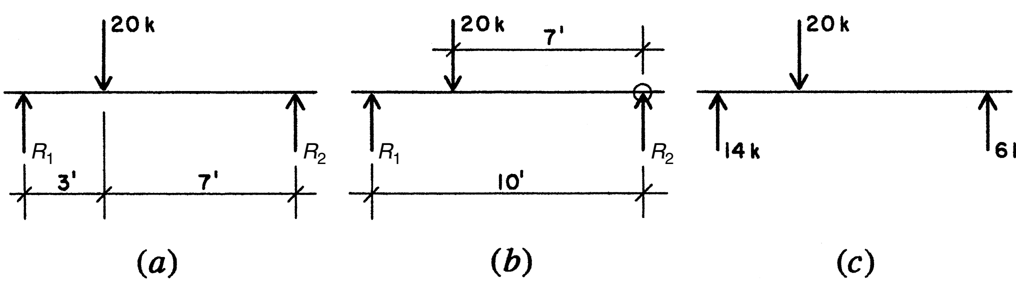


Figure 2.22 Analysis of a simple parallel force system.

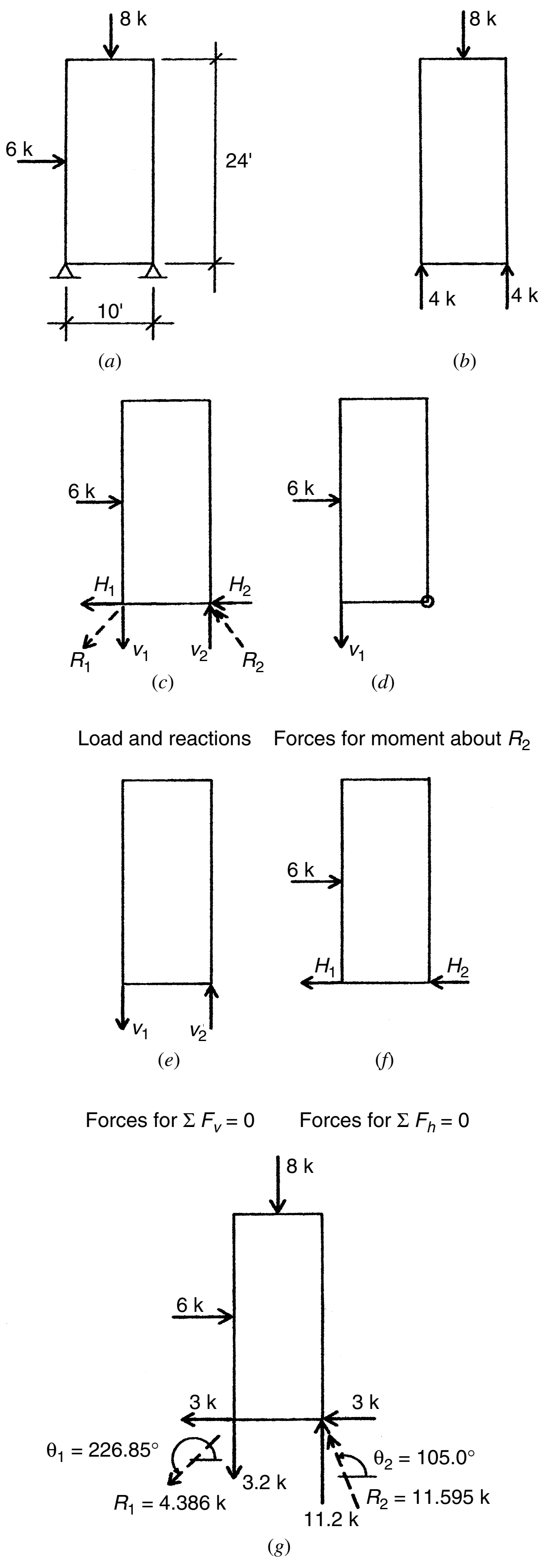


Figure 2.23 Analysis of a general coplanar force system.

with force sense up considered positive

$$\sum F_v = 0 = -V_1 + V_2 = -7.2 + V_2, \quad V_2 = +7.2 \text{ kips}$$

For the summation of horizontal forces, the forces involved are those shown in Figure 2.23f. Thus, with force to the right considered positive

$$\sum F_h = 0 = +6 - H_1 - H_2, \quad H_1 + H_2 = 6 \text{ kips}$$

This presents an indeterminate situation that cannot be solved unless some additional relationships can be established. Some possibilities are:

R_1 offers resistance to horizontal force, but R_2 does not. In this case $H_1 = 6$ kips and $H_2 = 0$.

The reverse of the preceding: R_2 offers resistance, but R_1 does not. Thus $H_2 = 6$ kips and $H_1 = 0$.

Details of the construction indicate a symmetrical condition for the two supports. In this case it may be reasonable to assume that the two reactions are equal: $H_1 = H_2 = 3$ kips.

For this example we will assume the symmetrical condition for the supports with the horizontal force being shared equally by the two supports. Adding the results of the separate analyses, we obtain the results for the combined reactions as shown in Figure 2.23g. The reactions are shown both in terms of their components and in their resultant form as single forces. The resultant forces are simply composed from the reaction components and the tangent of the resultant angle is determined by the components.

2.3 STRESSES AND STRAINS

Limitations on developed stresses and strains are primary devices for the control of structural behavior. This is most true with the allowable stress method of design, in which assigned limits are placed on stresses under service load conditions. It is less directly, but just as effectively, true with the strength design method, in which the stress and strain considerations at failure are used as a limit. This section presents the various stress and strain considerations that are encountered in structural investigation and some of the basic techniques for their computation in ordinary situations of design.

Development of Internal Forces

Although stresses and strains result from the actions of external forces, we visualize them directly as the products of internal force actions. Thus the individual actions of tension, compression, shear, bending, and torsion are each visualized as the manifestations of a characteristic set of internal stresses and strains in the material of the structure. The free-body diagram and the cut section are essential tools for these visualizations.

At any particular location within a structure under load there is usually not a single internal force action, but rather some combination of actions. Consider the actions of the simple, axially loaded column and the simple tension rod as shown in Figures 2.24*a* and *b*. In these the internal force actions can be visualized in the form of simple, single effects of compression or tension.

We do indeed make use of such simple elements on some occasions but more often need to consider more complex actions. For example, for the vertical support for the sign shown in Figure 2.24*c*, the internal actions developed by a combination of gravity plus wind include compression, shear, torsion, and two-way bending. These occur all at once, so a true analysis must somehow combine all the effects into some net stressed and strained condition.

Although true service conditions must be considered for a realistic design, it is the usual practice to visualize and analyze for individual internal actions separately. This is not necessary for the computer, but most human beings find it more feasible to handle individual actions one at a time before attempting to visualize their net effect. For initial study in particular, we will follow this procedure of divide and conquer in the following sections, before considering combined and net effects.

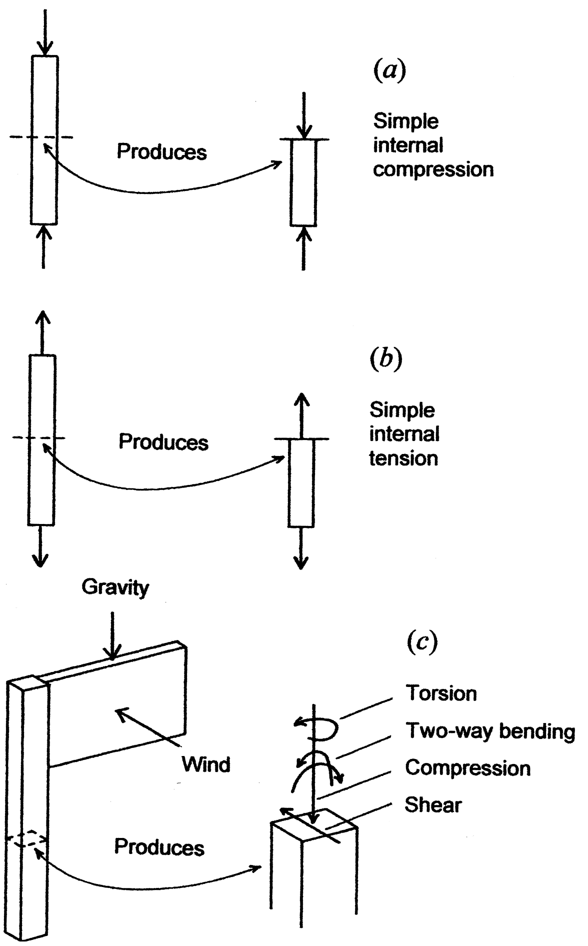


Figure 2.24 Development of internal forces.

Stress and Strain

Stress is not truly one dimensional or two dimensional; it is always three dimensional, as is the shape change in the three-dimensional material that experiences the stress. This idea was discussed and illustrated in Chapter 1, and it is important to keep the true nature of stress and strain in mind. For purposes of computation in investigation and design work, however, we routinely make determinations of some simplified stress conditions. These are not done as the true representations of the complete stress and strain actions, but simply as indicators of general conditions.

Direct Stress

Direct stress—tension or compression—results from the action of a direct force. It is visualized as operating at right angles to a surface, produced by the force action of tension or compression that acts at right angles to the surface. The "surface" referred to here may be an actual one, as in the action of a footing on the soil that supports it. More often, however, the surface is that of a visualized cut section, produced in the attempt to "see" into the solid material of some structural element. In both cases, the type of stress visualized is the same.

The two examples in Figure 2.25 illustrate this basic development and the simple form of the stress formulas used for computation. In Figure 2.25*a* a block is being pressed down against an unyielding surface, with a pressure on the bottom of the block developed by the crushing force C. If the crushing force acts with a symmetrical orientation on the block (called an *axial-force action*), the compressive pressure effect may be visualized as compression stress, with its unit value expressed as

$$f_c = \frac{C}{A}$$

where

f_c = unit stress expressed as force per unit area: pounds per square inch (psi), newtons per square meter (N/m^2), etc.

C = total force in pounds, newtons, etc.

A = area on which stress is developed

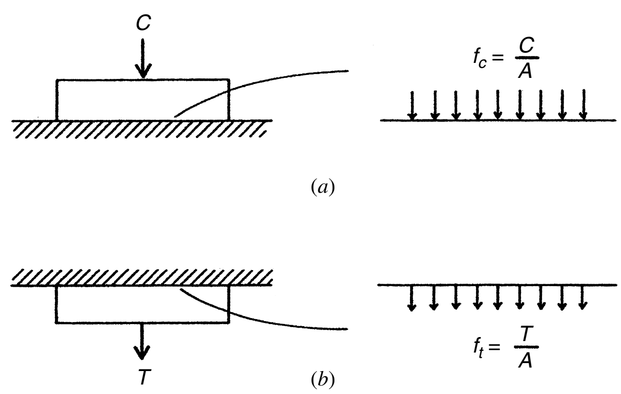


Figure 2.25 Development of direct stress.

If the block is adhered to the ceiling and operated on by a tension force, the determination of the tension stress is of the same form (see Figure 2.25*b*). In a structural member, such as the column or tension rod shown in Figure 2.24, development of internal direct force and the resulting direct stress are visualized in the same manner, the surface being that of a cut section at a right angle to the direct force.

The simple formula for direct stress can be used in three ways for different situations, as follows:

Given the crushing force and the area of contact (or the area of the cut section), find the magnitude of the unit stress. This computation takes the form presented in the illustration: $f = C/A$.

Given the area and some limit for the stress, find the limit for the direct force. For this computation we use the form $C = fA$.

Given the limit for the stress and the need to develop resistance to a specific amount of direct force, find the area of contact or cross section required. For this case we use the form $A = C/f$.

The following example problems demonstrate the use of the formula for direct stress.

Example 1. A 6-by-6 wood post (actually 5.5 in. on a side [1.40 mm]) sustains a compression load of 20,000 lb [89 kN]. Find the unit compressive stress in the post.

Solution. The cross-sectional area of the post can be obtained from Table A.8 in Appendix A or may simply be computed as $A = (5.5)^2 = 30.25\text{ in.}^2\ [0.0196\text{ m}^2]$. The stress is then found as

$$f = \frac{C}{A} = \frac{20,000}{30.25} = 661\text{ psi }[4.54\text{ MPa}]$$

Example 2. If the post in Example 1 has a maximum limit for compression stress of 1000 psi, what is the maximum total compression force that it can carry?

Solution. With the known values for f and A, the force is expressed as

$$C = fA = 1000(30.25) = 30,250\text{ lb }[135\text{ kN}]$$

Example 3. If the post in Example 2 must carry a load of 50,000 lb [222 kN] and the stress limit is 1000 psi [6.9 MPa], what area is required for the post cross section?

Solution. Using the given values for C and f, we obtain

$$A = \frac{C}{f} = \frac{50,000}{1000} = 50\text{ in.}^2\ [0.03226\text{ m}^2]$$

Strain due to direct stress is visualized as a linear change of shortening (due to compression) or lengthening (due to tension) in the direction of the force producing the stress. Strain is expressed in the form of a unitless quantity, indicated as a percentage or a decimal fraction. If the stress is constant throughout the length of the member (as it approximately is in a column or tension rod), the strain will also be constant and will accumulate in the total length change (called *deformation*) of the member. For a tension member, using e to designate the deformation of elongation and L for the original member length, we express the unit strain as ε (lowercase Greek epsilon) and determine it as follows:

$$\varepsilon = \frac{e}{L}$$

Shear Stress

There are three different situations in which force actions result in the development of shear stress. These are as follows:

Stress produced by a direct shearing action (cutting, slicing effect), called *direct shear stress*.

Stress produced in the normal functioning of beams, called *beam shear stress*.

Stress produced by torsion (twisting), called *torsional shear stress*.

In this section we treat only the first case, that of direct shear stress. Shear in beams is discussed in Chapter 3.

Figure 2.26 shows two examples that involve the development of direct shear stress. In Figure 2.26*a* the lateral slip of the tongue-and-groove joint is resisted by the development of shear stress at the base of the tongue. The shear stress is assumed to be of a uniform value on the cross section of the area at the base of the tongue, with a magnitude of

$$f_v = \frac{V}{A}$$

in which V is the shear force and A is the area of the section at the base of the tongue.

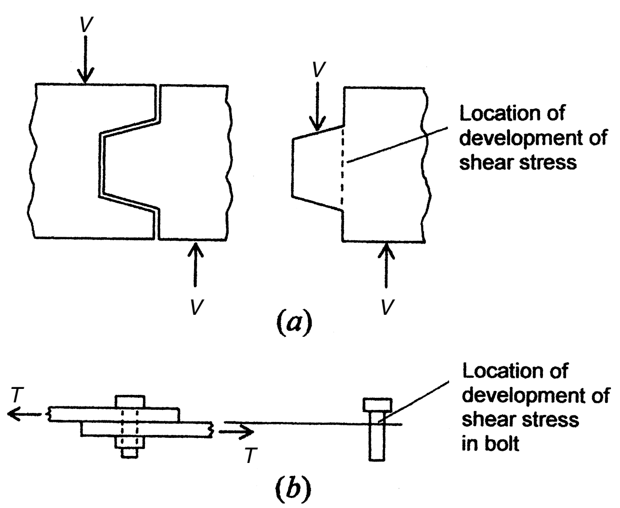


Figure 2.26 Development of shear stress.

In the example in Figure 2.26*b* the tension forces exerted on the two overlapping steel plates tend to produce a slippage which is resisted by the bolt that connects the plates, resulting in a slicing effect on the bolt. The expression for the shear stress on the cross section of the bolt is the same in form as that at the base of the tongue in Figure 2.26*a*.

When used in the manner shown in Figure 2.26*b*, the bolt is said to be in a condition of *single shear*, since the bolt needs to be sliced only once for the joint to fail. In the use of the bolt in Figure 2.27*a* the bolt must be sliced twice for the joint to fail; in this case the bolt is said to be in *double shear*. Extending this concept further, it may be observed that the hinge pin in Figure 2.27*b* must be sliced a total of eight places if the two halves of the hinge are to be separated by the tension forces. Actually, pins and bolts have other aspects of structural behavior in addition to shear. However, when considering only the shear effect, the area used in the stress formula is that of the cross section times the number of sliced sections.

The following example problems illustrate the use of these relationships for the determination of simple direct shear stresses.

Example 4. A wood tongue-and-groove joint is formed as shown in Figure 2.26*a*, with the width of the tongue at its base equal to 0.25 in. If the maximum stress is limited to 50 psi, what is the total shear load capacity of the joint, expressed in units of pounds per running foot of the joint length?

Solution. For a length of 1 ft (12 in.) of the joint the total area at the base of the tongue is $12(0.25) = 3.0\,\text{in.}^2$, and the limit for the load is

$$V = f_v A = 50(3.0) = 150\,\text{lb/ft}\ [2.19\,\text{kN/m}]$$

Example 5. A steel bolt is used as shown in Figure 2.27*a*. The tension force on the joint is 20 kips [89 kN] and the limit for shear stress is 14 ksi [100 MPa]. Find the minimum diameter required for the round bolt.

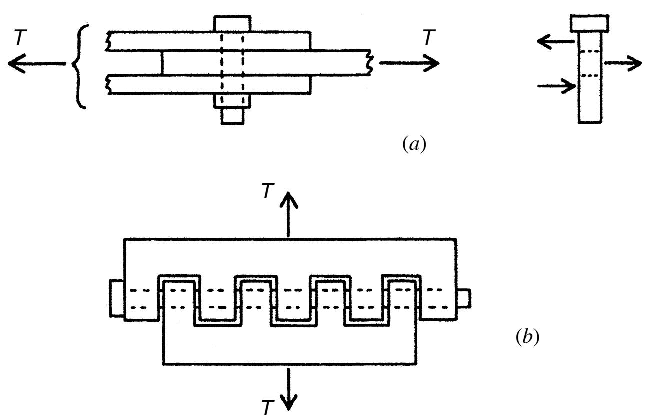


Figure 2.27 Development of multiple shear sections in bolted and pinned connections.

Solution. For the required total resisting area we find

$$A = \frac{T}{f_v} = \frac{20}{14} = 1.429\,\text{in.}^2\ [890\,\text{mm}^2]$$

Observing that the bolt is in double shear, we express the total resisting area of the two sliced cross sections as

$$A = 2\frac{\pi D^2}{4} = 0.5\pi D^2 = 1.571D^2$$

Then

$$1.571D^2 = 1.429$$

$$D = \sqrt{\frac{1.429}{1.571}} = \sqrt{0.906} = 0.954\,\text{in.}\ [23.8\,\text{mm}]$$

Example 6. A hinge of the form shown in Figure 2.27*b* is connected with a 0.25-in.- [6-mm-] diameter pin. Find the value of the unit shear stress in the pin if the force is 2000 lb [8.9 kN].

Solution. The total resisting area of the eight sliced sections is

$$A = 8\pi R^2 = 8\pi(0.125)^2 = 0.393\,\text{in.}^2\ [226\,\text{mm}^2]$$

and the shear stress is

$$f_v = \frac{2000}{0.393} = 5089\,\text{psi}\ [39.4\,\text{MPa}]$$

Bending Stress

When a member is subjected to moment, the effect is called *bending*. Such a situation is shown in Figure 2.28*a*; we observe the following regarding its actions:

The bending causes the member to curve.

The curving indicates that the material on one side of the member is being subjected to tension, while that on the opposite side is being subjected to compression.

Due to the reversal of stress across the member cross section, there will be some point at which the stress is zero.

Internal bending resistance in the member is developed by the opposition of the tension and compression stresses on the two sides of the bending member.

(*Note:* The reader should consult the material in Appendix A if the terms for some properties of cross sections used here are not familiar.)

If the member being bent has a symmetrical cross section, and the moment exists in a plane containing the axis of symmetry of the section (see Figure 2.28*b*), we may make the following observations:

The neutral stress point will lie on an axis of symmetry through the centroid of the section and perpendicular to the axis containing the bending moment. This axis where no stress occurs is called the *neutral axis*.

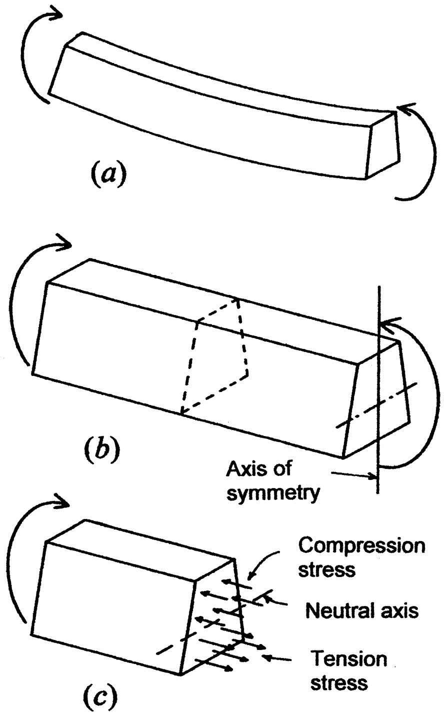


Figure 2.28 Development of bending stress.

If the member is composed of a single, elastic material, planar cross sections of the member at right angles to its longitudinal axis will retain their planar form during bending. As the beam curves, these plane sections will rotate about their neutral axes.

The radius of curvature of the bent member at any point along its length (assuming an original straight condition) will be inversely proportional to the bending moment at the point.

The observations just made can be verified by tests, and because of them, we can make the following derivations in terms of the stresses and strains in the member.

We first observe that due to the plane section observation the strain at any point is in direct proportion to its distance from the neutral axis. Thus, if the material is elastic—meaning that its stress is proportional to its strain—and the stress is below the material's failure limit, the stress at any point is proportional to the distance of the point from the neutral axis. Thus (see Figure 2.29*a*)

$$\frac{\varepsilon_y}{y} = K_1 \quad \text{and} \quad \frac{f_y}{y} = K_2$$

in which K_1 and K_2 are constants that derive from the properties of the material and the geometry of the member's cross section.

The total internal resisting moment at a section may be determined by a summation of the increments of moment resistance consisting of the effects of stresses on unit areas of the cross section. If we use a unit area that consists of a slice dy thick and y distance from the neutral axis (see Figure 2.29*b*), we can assume the stress to be a constant on the unit area. Thus the increment of internal force developed on the unit area is the product of the stress times the unit area, or

$$dF = f_y \, dA$$

and the unit moment developed by this unit force is

$$dM = dF(y) = (f_y \, dA)(y)$$

Using the calculus, the total resisting moment of the cross section can be expressed as

$$M = \int_{c_2}^{c_1} dM = \int_{c_2}^{c_1} (f_y \, dA)y = \int_{c_2}^{c_1} f_y y \, dA$$

Since f_y/y is a constant, we can make the following transformation:

$$M = \int_{c_2}^{c_1} \frac{f_y}{y} y^2 \, dA = \frac{f_y}{y} \int_{c_2}^{c_1} y^2 \, dA$$

The expression following the integral sign in this formula is the basic form for indication of the second moment of the area of the section (also called the *moment of inertia* and designated as I). The formula can thus be more simply written as

$$M = \frac{f_y}{y} I \quad \text{in which} \quad I = \int y^2 \, dA$$

or, transforming it into an expression for stress, we have

$$f_y = \frac{My}{I}$$

which is the general formula for bending stress.

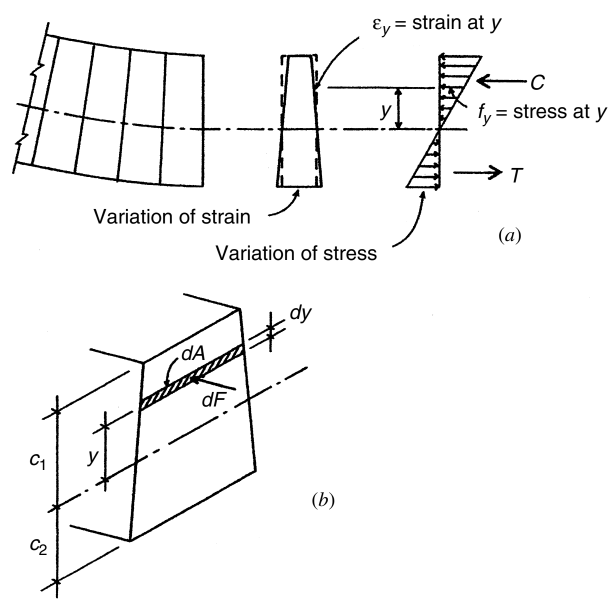


Figure 2.29 Bending stress on a beam cross section.

Referring to Figure 2.29, we note that the maximum bending stress will be obtained when $y = c$. Further, if the section is one with symmetry about its neutral axis, the two c distances shown will be equal. A commonly used expression for maximum bending stress is thus

$$f_{\max} = \frac{Mc}{I}$$

An additional simplification can be made by use of a single term for the combination of I and c. This term is called the *section modulus*, defined as

$$S = \frac{I}{c}$$

Substituting this in the stress formula reduces it to

$$f_{\max} = \frac{M}{S}$$

For beam design purposes, the formula is transposed to

$$S = \frac{M}{f}$$

When designing a beam by the allowable stress method, once the required resisting moment is determined and the material to be used is established (yielding a value for the limit of stress), a single property for the beam—S—can be found for the selection of the beam.

The following examples illustrate simple problems using the formula for bending stress.

Example 7. A beam has a T-shaped section, as shown in Figure 2.30. A bending moment occurs in the plane of the axis of symmetry of the section, producing compression in the upper portion and tension in the lower portion of the section. If the bending moment is 10 kip-ft [14 kN-m] and the moment of inertia is 30.71 in.^{4} [12.02×10^6 mm^4], find the values for the maximum tension and compression stresses due to bending.

Solution. For the compression at the top of the section we use the distance from the neutral axis of $y = 2.167$ in. and the stress formula in the form

$$f_y = \frac{My}{I} = \frac{[10(12)]2.167}{30.71} = 8.47\text{ ksi }[63.1\text{ MPa}]$$

Note that the moment in kip-feet must be changed to inch units, as the other values use inches.

For tension stress at the bottom we use the distance of $y = 3.833$ in.; thus

$$f_y = \frac{My}{I} = \frac{[10(12)]3.833}{30.71} = 14.98\text{ ksi }[111.6\text{ MPa}]$$

Example 8. For the beam section in Figure 2.30, find the value of S that indicates the maximum moment resistance based on a limited bending stress value.

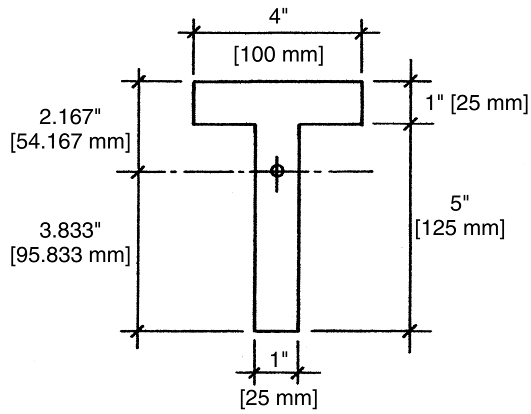


Figure 2.30 Reference for Examples 7 and 8.

Solution. If the section is limited by the maximum bending stress, we must use the greatest distance from the neutral axis to any edge, or 3.833 in. for the section given. Thus

$$S = \frac{I}{c} = \frac{30.71}{3.833} = 8.01\text{ in.}^3\ [125.4 \times 10^3\text{ mm}^3]$$

Strain and Deformation

Relationships between stress and strain permit the use of a highly effective technique in investigation of the behavior of structures. This consists of first visualizing the form of the structure as it is deformed by the loads acting on it. From the nature of the deformations it is usually possible to infer the particular strain conditions in the material whose gross accumulation produces the overall deformation. Finally, from the type of strain it is possible to determine the character of the stress and the type of internal action that relates to the stress development. Because it is more difficult to "see" stress and internal force, the direct interpretation of the geometric character of the structure's deformation becomes a practical analytic device.

The visualization of the deformed structure is usually not critical to the understanding of actions of simple tension or compression members, but it is often quite useful with members subjected to bending and torsion. It was used, for example, in the derivation of the bending stress formula (see Figure 2.29).

Deformation of the structure itself is sometimes a design issue. In building structures, deformations due to direct stress actions of tension and compression are usually so small in dimension as not to be critical, with the possible exception of very long tension members. One common deformation problem is that consisting of the deflection (sag) of beams. This may be critical for the loss of straightness (visible sag), the actual dimension of displacement from an original flat form, or the rotations of the beam ends, In structural design work, computations for deformations are usually limited to these determinations.

Stress–Strain Relationships

We have thus far discussed stress and strain as related to phenomena but have not dealt with their specific relations to each other. We will now consider some aspects of these relationships. A major factor in these relationships is the material of the structure.

The curves on the graph in Figure 2.31 indicate three basic types of stress–strain relationships. The curves are plotted with unit strain as the horizontal variable and unit stress as the vertical variable. The curves shown manifest relationships involving three types of material response as follows:

Elastic Behavior. This indicates a constant ratio of proportionality of stress to strain.

Inelastic Behavior. This is the general case when stress and strain do not remain in a constant relationship over the range of stress increase. The relationship may be predictable but is not one of constant proportionality.

Plastic Behavior. This is the case when increase in strain occurs at a relatively constant stress. This may occur because of the nature of a particular material but can also be due to heat, moisture, time, or chemical change.

Referring to Figure 2.31, curve 1 represents a material that is elastic virtually throughout its entire range of stress up to failure of the material. This is a characteristic response of ceramic materials such as glass. In this case stress and strain can be related by expressing the slope or angle of the line on the graph. This is ordinarily done by defining the slope in terms of the tangent of the angle, which is simply the ratio of stress to strain. This value is called the *elastic modulus*, or more commonly the *modulus of elasticity*, and is designated as E in standard notation. Thus, for curve 1, as shown in Figure 2.31,

$$E = \tan\theta_1 = \frac{\text{stress}}{\text{strain}} = \frac{f}{\varepsilon}$$

Typical values for E for common materials are given in Table 2.2. Note that a single value is obtained for all grades of structural steel and aluminum, while a range of values exists for different grades of wood and concrete.

Curve 2 in Figure 2.31 represents a material for which the relation of stress to strain varies continuously over the range of stress. This is the form of response of wood, concrete, and

Table 2.2 Values of Modulus of Elasticity (E) for Common Structural Materials

Material	E: ksi	E: GPa	Type of Stress
Steel	29,000	200	Tension, compression
Aluminum (structural alloys)	10,000	70	Tension, compression
Concrete (stone aggregate)	3–5000	20–35	Compression
Wood (Douglas fir)	1.5–2000	10–14	Tension, compression parallel to grain
Plastic			
Hard (acrylic)	1000	7	Tension, compression
Soft (vinyl)	350	2.4	Tension, compression

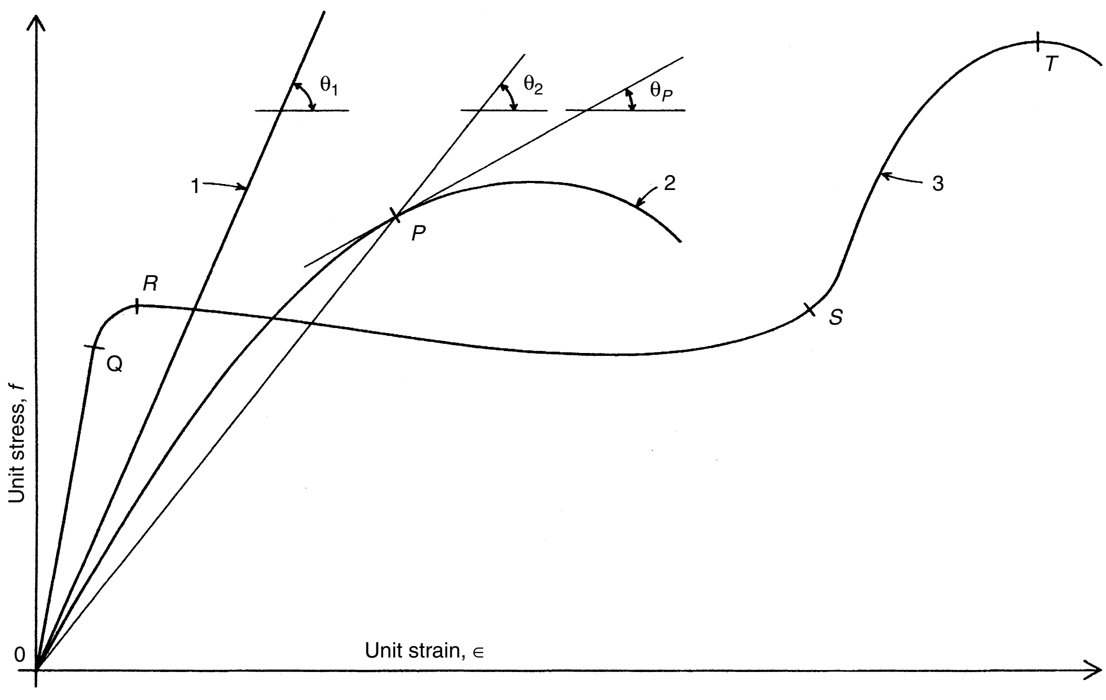


Figure 2.31 Forms of stress–strain behavior.

plastic. No single value of E can be defined for such a curve. Two values that are used for such a curve are the modulus at some specific point on the curve and an average modulus for some portion of the curve. On the curve in Figure 2.31, θ_p indicates the tangent modulus at point P and θ_2 is the average slope for a defined range of the curve, from zero to point P. Both of these values serve particular purposes in structural analysis. The average value is that given in Table 2.2 for concrete, wood, and plastic.

Curve 3 in Figure 2.31 represents the general form of response for ordinary structural-grade steel in direct stress. The curve is complex and must be discussed in detail. Referring to the letters on the curve, we note the following:

O to Q. The material is elastic in this range with an E of approximately 29,000 ksi [200 GPa]. Note that E has the same unit as stress since strain is dimensionless.

Q to R. At Q the curve begins to deviate from the straight line. Point Q establishes the *proportional limit* for the material, and as stress is increased, strain becomes increasingly inelastic. Finally, at some point (R), called the *yield point*, the material begins to deform plastically, with considerable increase in strain at an approximately constant stress level near that of Q and R. The maximum stress achieved before plastic deformation is called the *yield stress*.

R to S. If the material has strain as great as that shown in the range from R to S—as much as 15 to 20 times the strain at Q—it is said to be highly *ductile*. As the material nears the end of this range, it begins once again to develop additional stress resistance. The point at which the curve rises once again to the level of the stress at R establishes the end of the plastic deformation range relating to evaluation of ductility.

S to T. This range establishes the ultimate stress resistance of the material, which may be twice or more the value at R. Eventually, the material fails, usually in a brittle manner. This portion of the graph is not drawn to scale in the figure, as the strain at failure will be considerably greater.

The curves in Figure 2.31 are significantly different and indicate different requirements for evaluation of structural responses of different materials. This is indeed the case, as will be demonstrated in the discussions of design with the materials in Chapters 4 through 6.

Design Control of Stress and Strain

For design purposes the control of stress is a basic means for limiting the use of structures to situations that are within their capacities by some safe margin. With the allowable stress method of design this is accomplished by the establishment of limiting values for stresses that are some fraction of the demonstrated capacity of the materials to be used. This does not consist of determining a single stress value, as there are both different types of stress (tension, compression, shear, etc.) and special responses of materials (such as the effect of grain direction in wood).

Loading conditions may affect the considerations of stress response and stress limits. Considerations for loading include concerns for static versus dynamic effects, time duration of the load (wind versus gravity, etc.), and practical reliability of load determination.

Industry design standards and building codes are the usual sources for stress limits used in design work. These sources and the use of criteria and data from them are discussed in detail in Chapters 4 through 6.

Control of strain is not often a concern. Control of stress, in effect, results in control of strain. Accumulated strains in the form of gross deformations—such as the deflection of a beam—is one type of concern for strain. Cracking of concrete is related to a specific amount of strain under tension stress and its control must recognize that issue. Sharing of loads by materials that interact—such as concrete and steel in reinforced concrete—involves consideration of shared strain conditions.

2.4 SPECIAL TOPICS

This section contains discussions of a number of special topics pertaining to various aspects of structural behavior.

Thermal Effects

A special case of deformation is that which occurs when a material undergoes a change in temperature. In general, solid materials tend to expand when heated and contract when cooled. Although this is actually a volumetric change and does not occur at a constant rate for all ranges of temperature, it is possible to generalize with reasonably approximate accuracy for the case of a simple linear element at the temperate range of climates—from about -10 to $+120^\circ$F. Assuming a constant rate of expansion for this range (see Table 2.3), we may determine the accompanying linear (unit) deformation, such as inches per inch, by the following computation:

$$\varepsilon = C\,\Delta T$$

where

C = linear coefficient of expansion

ΔT = change in temperature

When thermal expansion or contraction is prevented, stresses are developed in the material proportional to those that would occur if the member was free to move and was acted on by a force producing the necessary stress to cause the same deformation or strain due to the thermal change. If elastic conditions are assumed, stress due to temperature change may be determined as follows:

$$f = \varepsilon E = C\,\Delta T\,E$$

in which E is the direct stress modulus of elasticity.

Table 2.3 Coefficient of Linear Expansion per Degree

Material	Coefficient °F	Coefficient °C
Aluminum	128×10^{-7}	231×10^{-7}
Copper	93×10^{-7}	168×10^{-7}
Steel	65×10^{-7}	117×10^{-7}
Concrete	55×10^{-7}	99×10^{-7}
Masonry (brick)	34×10^{-7}	61×10^{-7}
Wood (fir)	32×10^{-7}	58×10^{-7}

Example 9. A steel beam is subjected to a temperature change of 120°F from summer to winter. The beam is 60 ft long. Find (a) the length change if movement is not prevented and (b) the stress developed if movement is completely prevented.

Solution. From Table 2.3 we find the coefficient of expansion for steel to be 65×10^{-7}. Thus, for (a) the free-length change is

$$e = \varepsilon L = C\ \Delta T\ L = (65 \times 10^{-7})\ (120)\ (60 \times 12) = 0.562\ \text{in.}$$

For (b) we determine

$$f = \varepsilon E = C\ \Delta T\ E = (65 \times 10^{-7})(120)(29{,}000) = 22.6\ \text{ksi}$$

The force exerted by the beam on its constraints would be the product of this stress and the cross-sectional area of the beam.

Composite Elements

A special stress condition occurs when two or more different materials are assembled in a single element so that when a load is applied they strain together as a single mass. This type of structural member is called a *composite element*. An example of this is a reinforced concrete column. In an idealized condition we assume both materials to be elastic and make the following derivation for the distribution of stresses between the two materials.

If the two materials deform the same total amount (see Figure 2.32), we may express the total length change as

$$e = e_1 = e_2$$

where

e_1 = length change of material 1
e_2 = length change of material 2

Because the two materials have the same original length and the same total deformation, the unit strains in the two materials are the same; thus

$$\varepsilon_1 = \varepsilon_2$$

Assuming elastic conditions, these strains may be related to the stresses and the moduli of elasticity for the two materials; thus

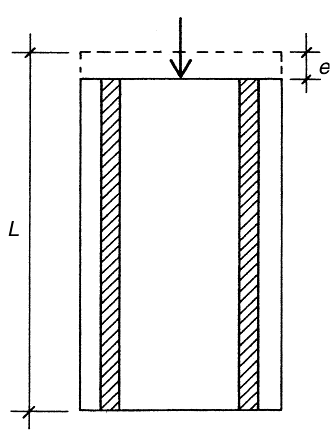


Figure 2.32 Deformation of the composite structure.

$$\varepsilon_1 = \frac{f_1}{E_1} = \varepsilon_2 = \frac{f_2}{E_2}$$

The relationships of the stresses in the two materials may thus be stated as

$$\frac{f_1}{f_2} = \frac{E_1}{E_2} \quad \text{or} \quad f_1 = f_2\frac{E_1}{E_2}$$

Expressed in various ways, the relationship is simply that the stresses in the two materials are proportional to their moduli of elasticity.

Example 10. A reinforced concrete column consists of a 12-in. square concrete section with four 0.75-in.-diameter round steel rods. The column sustains a compression load of 100 kips. Find the stresses in the two materials. (Assume an E of 4000 ksi for the concrete and 29,000 ksi for the steel.)

Solution. Consider the load to be resisted by two internal forces, P_s and P_c, that is, the load resisted by the steel and the load resisted by the concrete. Then

$$\text{Total } P = P_s + P_c = f_sA_s + f_cA_c$$

Using the previously derived relationship for the two stresses yields

$$f_s = \frac{E_s}{E_c}f_c = \frac{29{,}000}{4000}f_c = 7.25f_c$$

Substituting this in the expression for P yields

$$\begin{aligned} P = 100 &= f_sA_s + f_cA_c \\ &= 7.25f_cA_s + f_cA_c \\ &= f_c(7.25A_s + A_c) \end{aligned}$$

For the four steel bars

$$A = 4(\pi R^2) = 4[3.14(0.375)^2] = 1.77\ \text{in.}^2$$

Then the concrete area is

$$A_c = (12)^2 - 1.77 = 142.23\,\text{in.}^2$$

Substituting these values yields

$$100 = f_c[7.25(1.77) + 142.23] = 155.3f_c$$
$$f_c = \frac{100}{155.3} = 0.644\,\text{ksi}$$
$$f_s = 7.25f_c = 7.25(0.644) = 4.67\,\text{ksi}$$

Shear Effects

Horizontal and Vertical Shear

Consider the particle of material shown in Figure 2.33, having dimensions of a, b, and a unit dimension of 1 as the third dimension. The particle is subjected to a shear stress of f_1 on the top face, the total force effect of which is expressed as the product of the stress and the stressed area; thus

$$F_1 = f_1(a \times 1) = f_1 a$$

For equilibrium of the particle, there must be an equal and opposite force on the bottom face. This force is expressed as

$$F_2 = f_2(a \times 1) = f_2 a$$

By equating these two forces, we observe that $f_1 = f_2$.

The opposed efforts of F_1 and F_2 will induce a rotation of the particle which is resisted by the development of shear stresses f_3 and f_4 on the end faces of the particle. We may show that these two stresses will be equal; that is, in the same manner as was done for f_1 and f_2, $f_3 = f_4$. Thus the forces on the particles from all of these stresses will be in equilibrium if

$$(f_1 a)b = (f_3 b)a \quad \text{or} \quad f_1 = f_3$$

or, to summarize,

$$f_1 = f_2 = f_3 = f_4$$

which leads us to conclude that, if a shear stress exists in a plane in a material, there will be an equal shear stress in a mutually perpendicular plane.

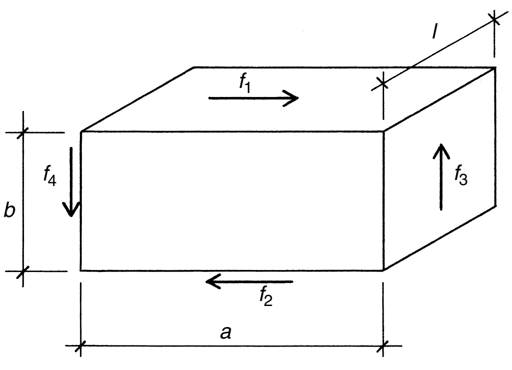


Figure 2.33 Three-dimensional development of shear.

Diagonal Stress Due to Shear

If we consider a particle that is subjected to shear to be cut along a diagonal (as shown in Figure 2.34), it may be observed that tension stress exists on one diagonal, while compression stress exists on the other diagonal. It may be shown that the magnitude of these diagonal stresses is equal to that of the shear stress that generates them.

Referring to the free-body diagram in Figure 2.34*b*, we note that the combined effects of the shear stresses on two adjacent sides of the particle must be resisted by the total effect of the diagonal tension stress acting on the diagonal cut area. Thus

$$T_1 = T_2 \quad \text{or} \quad f_v A\sqrt{2} = f_t \sqrt{2}A$$

from which $f_v = f_t$. These direct diagonal stresses will be additive to any other existing direct stresses, which is demonstrated later.

Stress on an Oblique Cross Section

Just as shear was shown to produce direct stresses, so we may show that direct force produces shear stresses. Consider the element shown in Figure 2.35 subjected to a tension force. If a section is cut that is not at a right angle to the force, there may be seen to exist two components of the internal force P. One component is at a right angle to the cut-section surface plane and the other is in the surface plane. These two components produce, respectively, tension stress and shear stress on the cut section. Referring to the angle of the oblique section, as shown in Figure 2.35*a*, we may express these stresses as

$$f_t = \frac{P\cos\theta}{A/\cos\theta} = \frac{P}{A}\cos^2\theta$$
$$f_v = \frac{P\sin\theta}{A/\cos\theta} = \frac{P}{A}\sin\theta\cos\theta$$

in which A is the area of the right-angle cross section of the member.

We note that when $\theta = 0, \cos\theta = 1$ and $\sin\theta = 0$, and therefore $f_t = P/A$ and $f_v = 0$. Note also that when the angle is 45°, $\cos\theta = \sin\theta = \sqrt{2}/2$, and the two component stresses will each be equal to one-half of P/A.

Example 11. The wood block shown in Figure 2.36 has its grain at an angle of 30° to the direction of force. Find the compression and shear stresses on a plane parallel to the grain.

Solution. Note that, as used in Figure 2.35, $\theta = 60°$. Then for the free-body diagram shown in Figure 2.36*b*

$$N = P\cos 60°, \quad V = P\sin 60°$$

The forces and the area of the oblique section can be used to find the two stresses. However, we can also use the

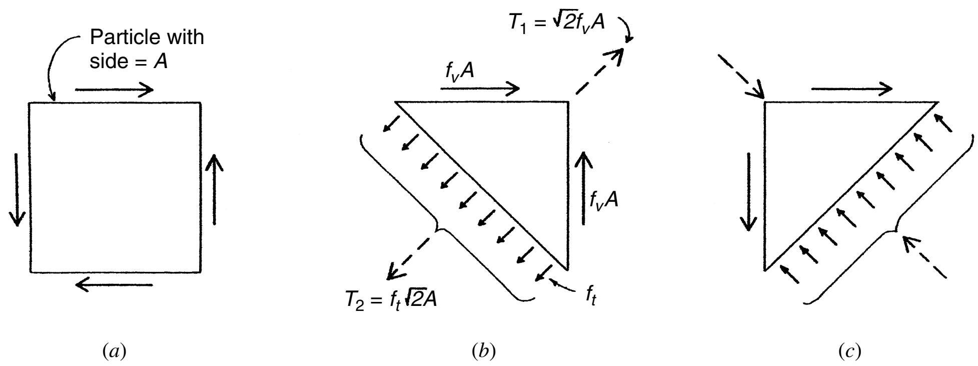


Figure 2.34 Direct diagonal stresses due to shear.

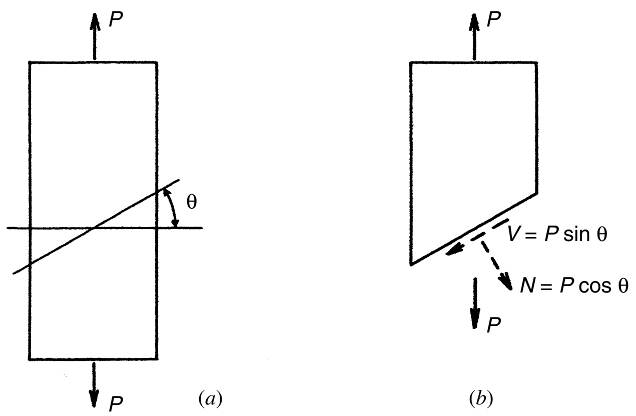


Figure 2.35 Development of stresses on an oblique section.

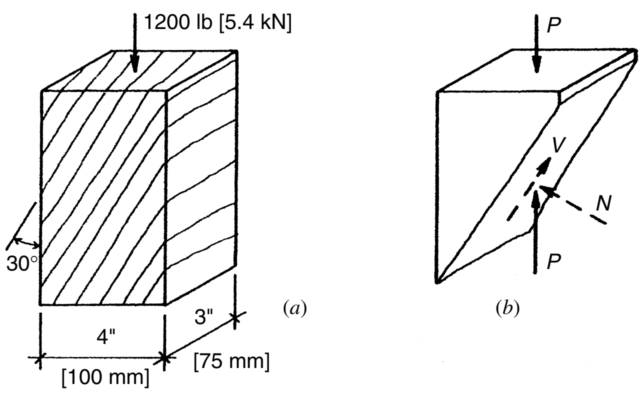


Figure 2.36 Reference for Example 11.

previously derived formulas. Thus

$$f_t = \frac{P}{A}\cos^2\theta = \frac{1200}{12}(0.5)^2 = 25\,\text{psi}\ [180\,\text{kPa}]$$

$$f_v = \frac{P}{A}\sin\theta\cos\theta = \frac{1200}{12}(0.5)(0.866) = 43.3\,\text{psi}\ [312\,\text{kPa}]$$

Shear Stress in Beams

Shear stress in a beam may be visualized by considering the beam to consist of stacked loose boards. Under the beam loading, the boards tend to slide over each other, taking the form shown in Figure 2.37. This type of deformation also tends to occur in a solid beam but is resisted by the development of horizontal shearing stresses, and, as we now know, there will also be vertical shearing stresses of equal magnitude.

Shear stresses in beams are not distributed evenly over cross sections of the beam as was assumed for the case of simple direct shear force. Investigation for shear stresses is done with formulas specific to the material of the beam. The following is a discussion of the general case for shear.

From observations of lab-tested beams and derivations considering the equilibrium of free bodies of beam segments under combined shear and moment actions, the following expression has been obtained for shear stress in a beam:

$$f_v = \frac{VQ}{Ib}$$

where

$V =$ shear force at beam section

$Q =$ moment about neutral axis of area of section between point of stress and edge of section (called *statical moment*)

$I =$ moment of inertia of section with respect to neutral axis

$b =$ width of section at point of stress

It may be observed from this formula that the maximum value for Q (and for shear stress) will be obtained at the neutral axis of the section. It may also be observed that the value

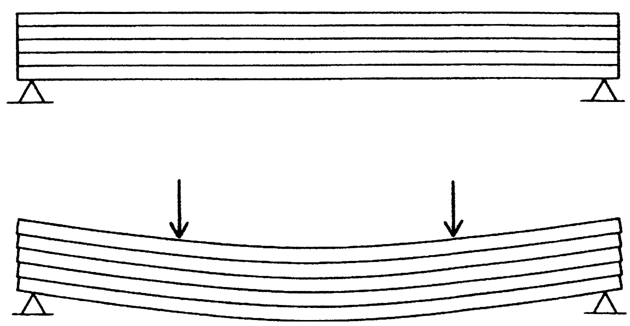


Figure 2.37 Visualization of horizontal shear stress.

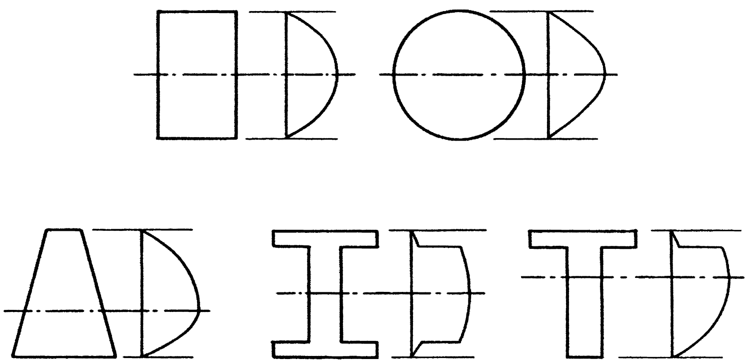


Figure 2.38 Distribution of shear stress in beams of various cross-sectional shapes.

of Q (and thus the shear stress) will be zero at the edges of the section. The form of shear stress distribution for various geometric shapes of beam sections is shown in Figure 2.38.

The following examples illustrate the use of the general formula for shear stress in a beam.

Example 12. A rectangular beam section with depth of 8 in. and width of 4 in. (Figure 2.39*a*) [200 and 100 mm] sustains a shear force of 4 kips [18 kN]. Find the maximum shear stress.

Solution. From Appendix A the value of the moment of inertia for a rectangular section about its neutral axis is $I = bd^3/12$; we thus determine

$$I = \frac{bd^3}{12} = \frac{4(8)^3}{12} = 170.7\ \text{in.}^4\ [67 \times 10^6\ \text{mm}^4]$$

The statical moment (Q) is the product of the area a' and its centroidal distance from the axis of the section ($\bar{y}$), as shown in Figure 2.39*b*. We thus compute Q as

$$Q = a'\bar{y} = [4(4)]2 = 32\ \text{in.}^3\ [500 \times 10^3\ \text{mm}^3]$$

The maximum shear stress at the neutral axis is thus

$$f_v = \frac{VQ}{Ib} = \frac{4000(32)}{170.7(4)} = 187.5\ \text{psi}\ [1.34\ \text{MPa}]$$

and the stress distribution is as shown in Figure 2.39*c*.

Example 13. A beam with the T-shape section shown in Figure 2.39*d* is subjected to a shear of 8 kips [36 kN]. Find the maximum shear stress and the shear stress at the location of the juncture of the web and the flange of the T.

Solution. Because this section is not symmetrical with respect to its horizontal centroidal axis, the first steps for this problem consist of locating the neutral axis and determining the moment of inertia for the section with respect to the neutral axis. To save space, this work is not shown here, although it is performed as Examples 1 and 8 in Appendix A. For determination of the maximum shear stress at the neutral (centroidal) axis, as shown in Figure 2.39*f*, we find Q using the bottom portion of the web. Thus

$$Q = a\bar{y} = [6.5(6)]3.25 = 126.75\ \text{in.}^3$$

and the maximum stress at the neutral axis is thus

$$f_v = \frac{VQ}{Ib} = \frac{8000(126.75)}{1046.7(6)} = 161.5\ \text{psi}$$

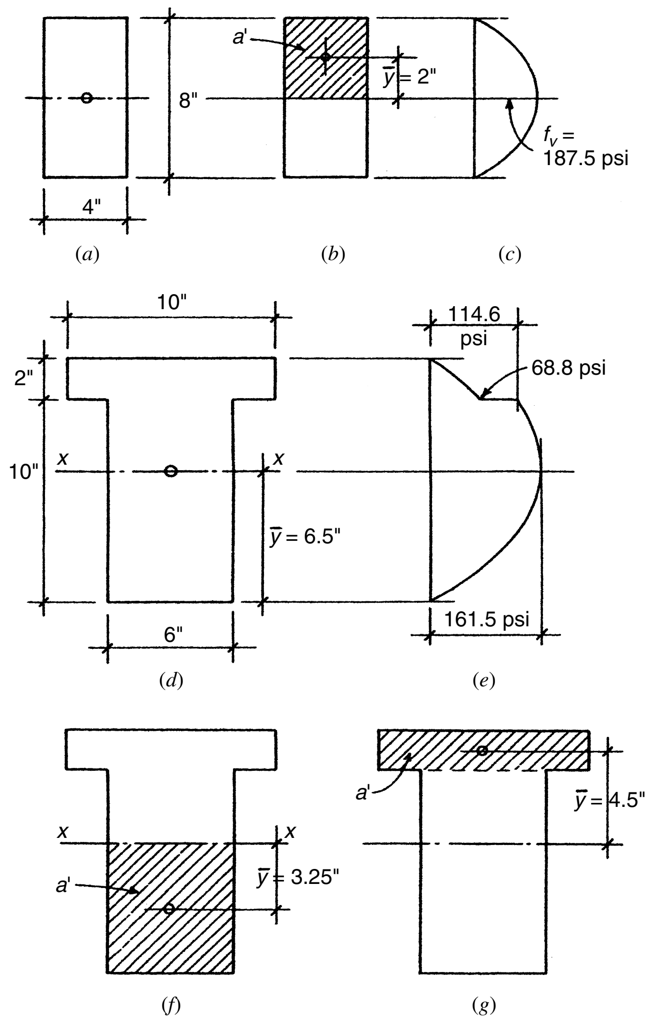


Figure 2.39 Reference for Example 12 and 13.

For the stress at the juncture of the web and flange we use the area of the flange, as shown in Figure 2.39*g* for Q. Thus

$$Q = [2(10)]4.5 = 90\,\text{in.}^3$$

and the two shear stresses in this location, as shown in Figure 2.39*e*, are

$$f_{v1} = \frac{8000(90)}{1046.7(6)} = 114.6\,\text{psi}$$

$$f_{v2} = \frac{8000(90)}{1046.7(10)} = 68.8\,\text{psi}$$

In most design situations it is not necessary to use the complex form of the general expression for beam shear stress. In wood structures the beam sections are mostly simple rectangular shapes, for which we can make the following simplification:

$$I = \frac{bd^3}{12}, \quad Q = (b)\,\frac{d}{2}\,\frac{d}{4} = \frac{bd^2}{8}$$

$$f_v = \frac{VQ}{Ib} = \frac{V(bd^2/8)}{(bd^3/12)b} = \frac{3}{2}\,\frac{V}{bd}$$

For steel beams—which are mostly I-shaped cross sections—the shear is taken almost entirely by the beam web (see shear distribution for the I shape in Figure 2.38). Since the stress distribution in the web is almost uniform, it is considered adequate to use a simplified computation of the form

$$f_v = \frac{V}{dt_w}$$

in which d is the overall beam depth and t_w is the thickness of the beam web.

Although shear stress distribution is quite complex in beams of reinforced concrete, the level of stress in the concrete tends to be the most critical concern. This stress can be generally evaluated by a simple formula, although actual design of the reinforced beam is complex. This process is discussed in Chapter 6.

Combined Direct and Shear Stresses

The stress actions shown in Figure 2.34 represent the conditions that occur when shear alone is considered. When shear occurs simultaneously with other effects, the various resulting stress conditions must be combined to produce the net effect. Figure 2.40 shows the result of combining a shear stress effect with a direct tension stress effect. For shear alone, the critical tension stress plane is at 45°, as shown in Figure 2.40*a*. For tension alone, the critical tension stress plane is at 90°, as shown in Figure 2.40*b*. For the combined stress condition, the critical unit stress will be some magnitude higher than either the shear or direct tension stress, and the critical tension stress plane will be at an angle somewhere between 45° and 90° (see Figure 2.40*c*).

Consider the beam shown in Figure 2.41. Various combinations of direct and shear stresses may be visualized in terms of the conditions of the cross section labeled $S\!-\!S$ in the figure. With reference to the points on the section labeled 1 through 5, we observe the following:

At point 1 the beam shear stress is zero, and the dominant stress is the compressive stress due to bending, which is oriented in a horizontal direction.

At point 3 the shear stress is a maximum, bending stress is zero, and the diagonal tension and compression stresses are in a 45° direction.

At point 5 the shear stress is zero; bending stress in tension predominates, acting in a horizontal direction.

At point 4 the net tension stress due to the combination of shear and bending will operate in a direction somewhere between the horizontal and 45°.

At point 2 the net tension will operate at an angle somewhat larger than 45°.

At point 1 the tension goes to zero as its direction approaches 90°.

The direction of the net tension stress is indicated for various points in the beam by short dark bars on the beam elevation at the bottom of Figure 2.41. The lighter dashed lines indicate the flow of tension force within the beam. If this figure were inverted, it would indicate the action of the net compressive stresses in the beam.

Tension force flow is especially critical to the behavior of concrete beams, as the concrete is very tension weak, and the flow of tension indicates logical placement of steel reinforcing bars.

Shear Center

Loadings on beams may produce other actions in addition to normal shear and bending. Potential lateral or torsional (twisting) buckling is one such type of action. Another type

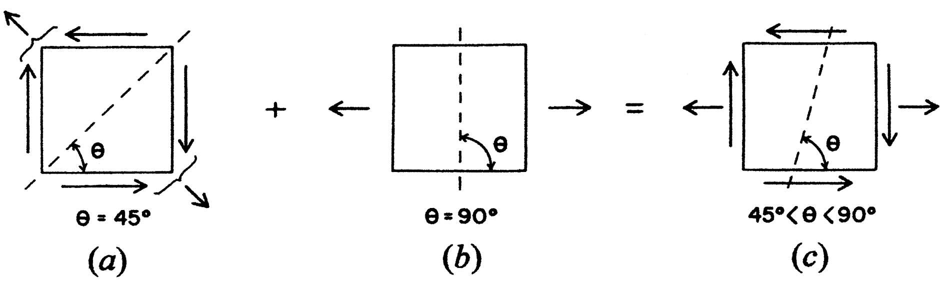


Figure 2.40 Combined effect of shear stress and direct stress.

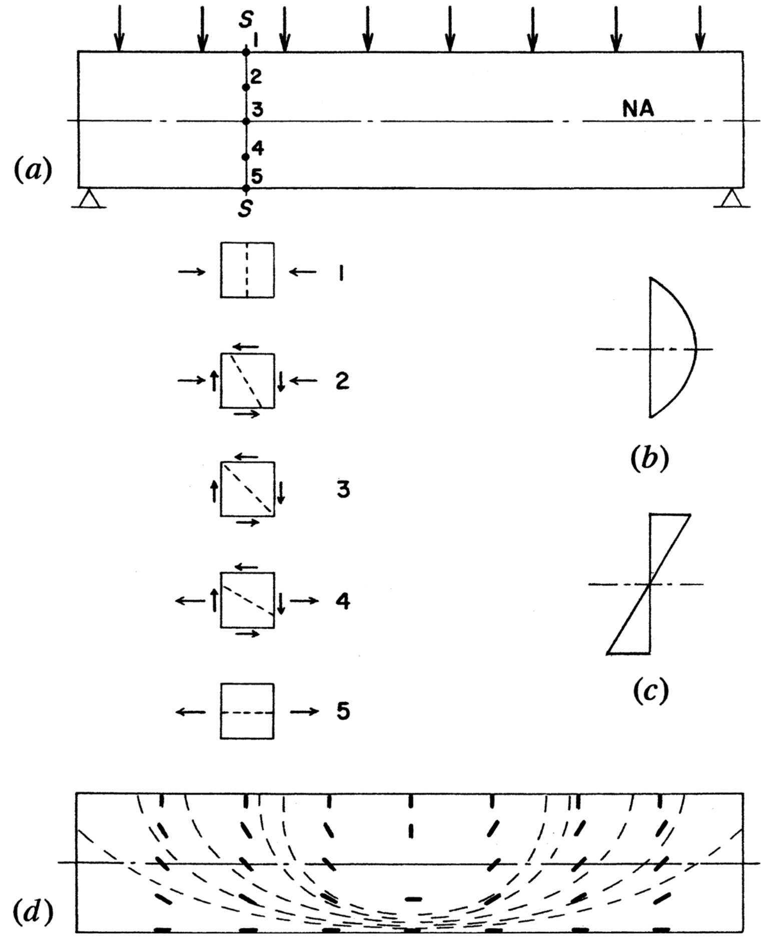


Figure 2.41 Direction of tension flow in a beam.

of action is a torsional effect which can occur if the plane of the bending moment does not coincide with the *shear center* of the beam cross section.

In Figure 2.42*a* a concentrated load is shown on the end of a cantilevered beam. The position of the load results in a moment that coincides with the vertical centroidal axis of the beam's rectangular section. This loading will produce the beam deformation shown in Figure 2.42*c* with a simple distribution of bending stress, the neutral axis coinciding with the horizontal centroidal axis of the beam section. If, however, the load is moved off center, as shown in Figure 2.42*b*, the beam is also subjected to a torsional twist and assumes the deformed shape shown in Figure 2.42*d*.

For beams with biaxial symmetry (symmetrical about both centroidal axes) the shear center will coincide with the centroid of the section. Thus for the beam in Figure 2.42 the twisting effect is avoided if the plane of the bending moment coincides with the centroidal axis. This relationship holds true for other doubly symmetrical sections, such as that of the steel I-shaped section shown in Figure 2.43*a*.

For sections that have no axis of symmetry parallel to the plane of bending, such as the C shape in Figure 2.43*c*, the shear center is at a location separate from the centroid of the section. However, it is always on any other symmetrical axis that may exist (such as the horizontal axis for the C shape).

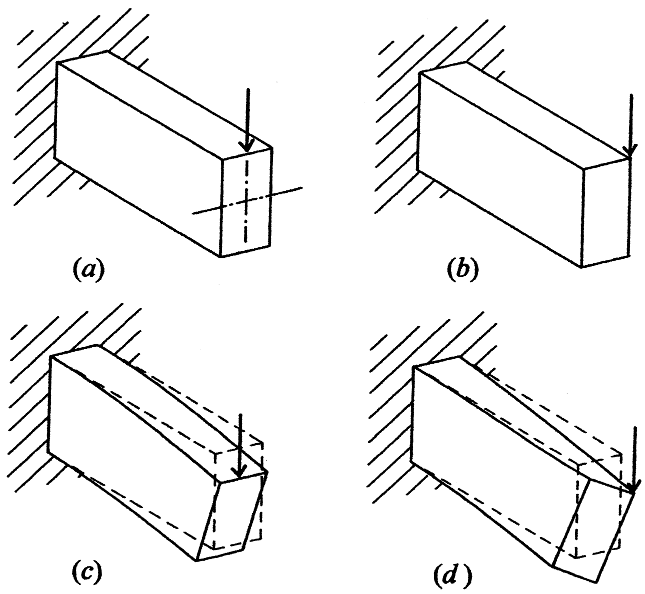


Figure 2.42 Effects of load placement on a beam.

For the C-shaped section, loading and bending moment that coincides with the vertical centroidal axis (Figure 2.43*c*) will produce a torsional twist. If used as a beam, a C-shape member must be braced against the torsional rotation or

have the loading coincide with the shear center, as shown in Figure 2.43*d*, if torsional stress and a twisting deformation are to be avoided.

Figure 2.44 shows the relation between the centroid and the shear center for various sections consisting of rolled steel shapes. For the I shape and the pipe (Figure 2.44*a*) the centroid and shear center coincide. For the C, T, and L shapes (Figure 2.44*b*) the two are at different locations. It is possible, however, to use the C, T, or L shape in ways that permit loading on a centroidal axis, as shown in Figures 2.44*c* and *d*, by orienting the loading on the one axis of symmetry or by producing a compound section with a symmetrical axis.

It may be possible, of course, to analyze for the torsional effect and to design a member to handle both the bending and twisting actions. However, the most common solution in many cases is to brace the member against the twisting action, if possible.

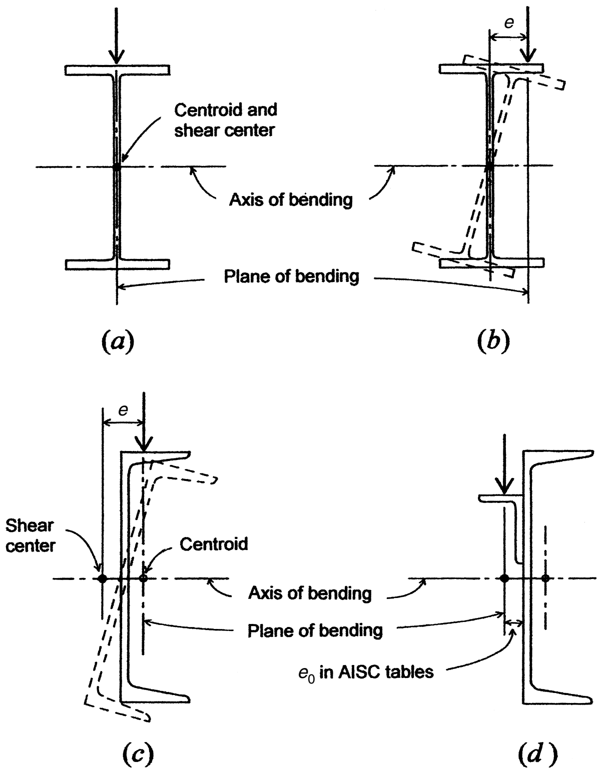


Figure 2.43 Torsional effect of a load not acting through the shear center of a beam.

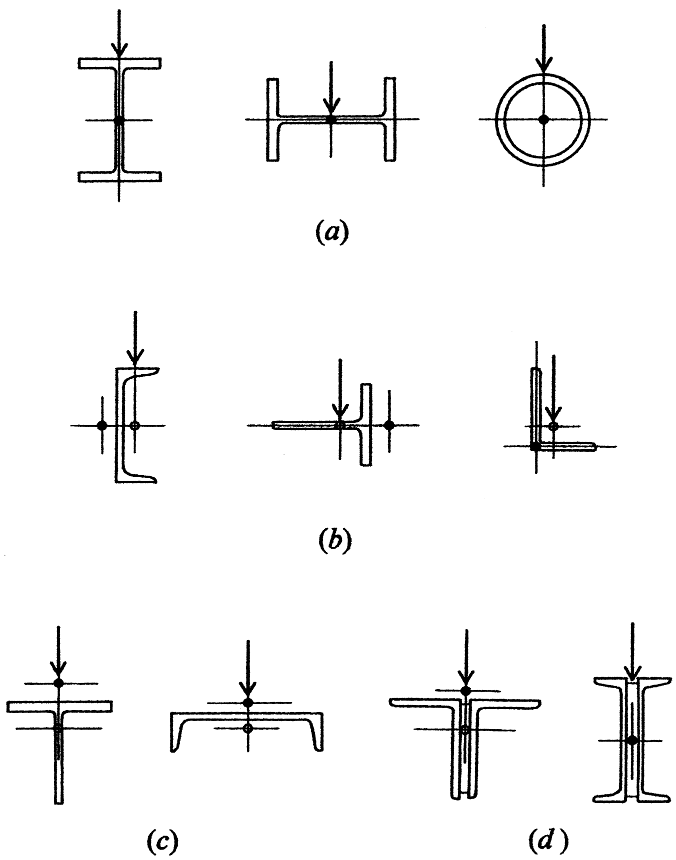


Figure 2.44 Shear centers and centroids of various sections.

Unsymmetrical Bending

There are various situations in which a structural member is subjected to bending in a manner that results in simultaneous bending about more than one axis. If the member is braced against torsion, the result may simply be a case of what is called *biaxial bending* or *unsymmetrical bending*. Figure 2.45 shows a common situation in which a roof beam is used for a sloping roof spanning between trusses or other beams that generate the sloped profile. With respect to vertical gravity

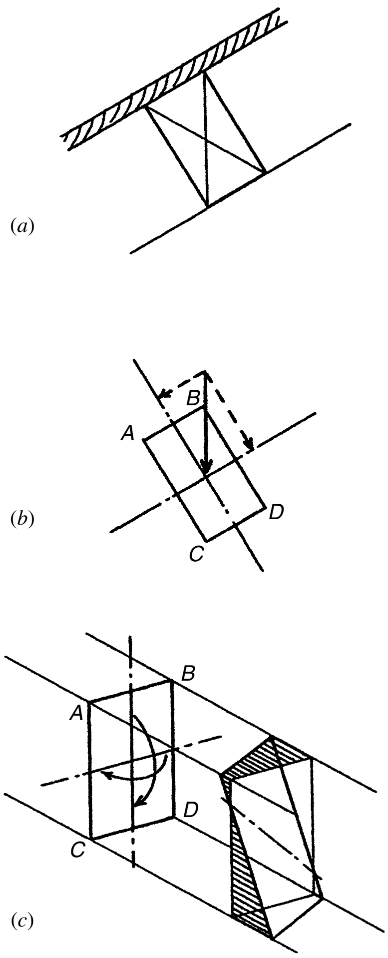


Figure 2.45 Development of biaxial bending.

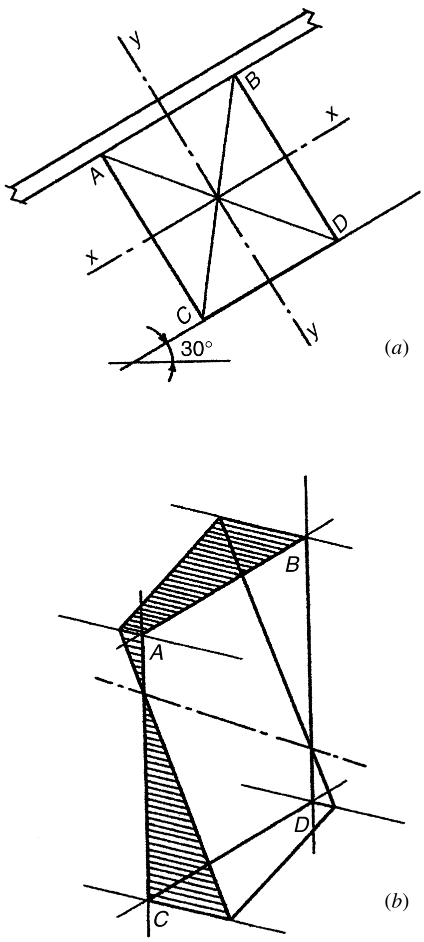


Figure 2.46 Reference for Example 14.

loads, the beam will experience bending in a plane that is rotated with respect to its major axes, resulting in components of bending about both its axes, as shown in Figure 2.45*b*. The bending moments about the two axes of the beam cross section will produce the following stresses:

$$f_x = \frac{M_x}{S_x} \quad \text{and} \quad f_y = \frac{M_y}{S_y}$$

These are maximum stresses that occur at the edges of the section.

The distribution of the combined stresses is of a form such as that shown in Figure 2.45*c*, and it can be described by determining the stresses at the four corners of the section. Noting the sense of the moments with respect to the two axes and using plus for compression and minus for tension, the net stress conditions at the four corners are as follows:

$$\text{At } A: -f_x + f_y$$

$$\text{At } B: -f_x - f_y$$

$$\text{At } C: +f_x - f_y$$

$$\text{At } D: +f_x + f_y$$

If a beam is used in this situation, it should be one with low susceptibility to torsion or lateral buckling, such as the almost square one shown. Otherwise the construction should be developed to provide adequate bracing.

Example 14. Figure 2.46 shows the use of an 8 × 10-in. wood beam in a sloping roof, with the beam rotated 30° to correspond to the roof slope. Find the net bending stress condition if the gravity load generates a moment of 10 kip-ft in a vertical pane.

Solution. From Table A.8 we find the properties of the beam to be $S_x = 112.8\,\text{in.}^3$ and $S_y = 89.1\,\text{in.}^3$ The components of the moment with respect to the major and minor axes of the section are

$$M_x = 10\cos 30^\circ = 10(0.866) = 8.66\,\text{kip-ft}$$

$$M_y = 10\sin 30^\circ = 10(0.5) = 5\,\text{kip-ft}$$

The corresponding maximum bending stresses are

$$f_x = \frac{M_x}{S_x} = \frac{8.66(12)}{112.8} = 0.921\,\text{ksi}$$

$$f_y = \frac{M_y}{S_y} = \frac{5(12)}{89.1} = 0.673\,\text{ksi}$$

The net stress conditions at the four corners of the section are determined as follows using plus for compression and minus for tension.

$$\text{At } A: +0.921 - 0.673 = +0.248 \text{ ksi}$$
$$\text{At } B: +0.921 + 0.673 = +1.594 \text{ ksi}$$
$$\text{At } C: -0.921 + 0.673 = -0.248 \text{ ksi}$$
$$\text{At } D: -0.921 - 0.673 = -1.594 \text{ ksi}$$

The form of the distribution of bending stress is as shown in Figure 2.46*b*. The following example illustrates the situation that occurs when a column is subjected to biaxial bending together with the compression load.

Example 15. Figure 2.47 shows a masonry column subjected to a compression load that is placed with eccentricities from both centroidal axes of the column. Find the distribution of net stresses due to this loading.

Solution. The load will generate bending equal to the product of the load times the eccentricity from the centroidal axes. The general expression for stress at a corner of the section is

$$f = \frac{N}{A} \pm \frac{Ne_x c_x}{I_x} \pm \frac{Ne_y c_y}{I_y}$$

where

N = compression load
A = area of section
e_x = eccentricity from x axis
c_x = extreme distance from x axis
I_x = moment of inertia about x axis
e_y, c_y, I_y = corresponding values for y axis

Using data from Figure 2.47*a*, we determine

$$I_x = \frac{bd^3}{12} = \frac{30(20)^3}{12} = 20{,}000 \text{ in.}^4$$
$$I_y = \frac{bd^3}{12} = \frac{20(30)^3}{12} = 45{,}000 \text{ in.}^4$$
$$A = 20(30) = 600 \text{ in.}^2$$

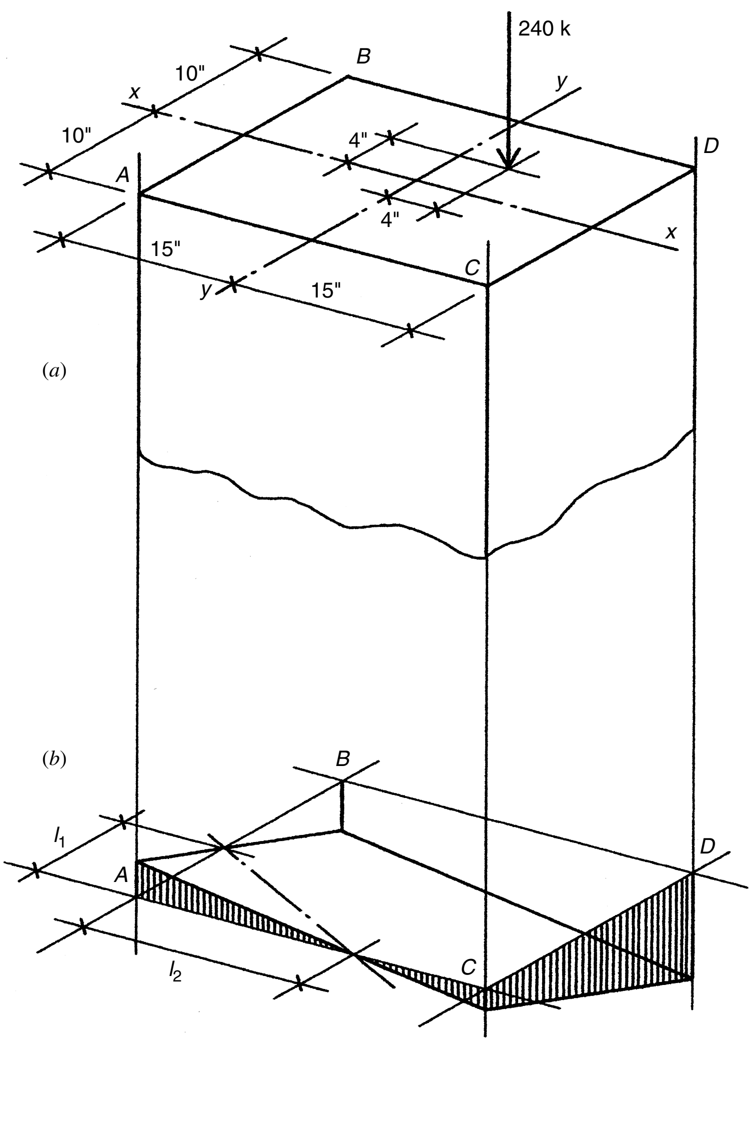


Figure 2.47 Reference for Example 15.

$$\frac{N}{A} = \frac{240{,}000}{600} = 400\,\text{psi}$$
$$\frac{Ne_x c_x}{I_x} = \frac{240{,}000(4)(10)}{20{,}000} = 480\,\text{psi}$$
$$\frac{Ne_y c_y}{I_y} = \frac{240{,}000(4)(15)}{45{,}000} = 320\,\text{psi}$$

The stresses at the four corners of the section, as shown in Figure 2.47*b*, are

$$\text{At } A: +400 - 480 - 320 = -400\,\text{psi}$$
$$\text{At } B: +400 + 480 - 320 = +560\,\text{psi}$$
$$\text{At } C: +400 - 480 + 320 = +240\,\text{psi}$$
$$\text{At } D: +400 + 480 + 320 = +1200\,\text{psi}$$

Location of the neutral axis can be determined by finding the values for distances l_1 and l_2, as shown on Figure 2.47*b*, by use of proportional triangles.

Inelastic Behavior

There are various situations in which the structural behavior of elements deviates sufficiently from the idealized form of response visualized for pure elastic behavior that some more accurate investigation is required. There are actually very few structural materials that conform to a straight stress–strain relationship all the way to ultimate failure of the material. The need to acknowledge a more realistic response depends on a number of considerations.

If the allowable stress method is used, the maximum level of stress permitted to occur for service conditions will typically be well below the ultimate failure range of the material. Thus, for the full range of working (service) load conditions, stress–strain responses may be close to those for an idealized material. Simplified stress and deformation computations may thus be reasonably adequate within the usual safety margins. What may be critical in these situations is the establishment of the allowable stress limits, since these must be based on some realistic evaluation of the structure's true limiting capacity at failure. It is common, therefore, to use ultimate limit evaluation—with usually inelastic behaviors—in establishing limiting stresses and some realistic safety factor.

Although ultimate, inelastic responses must be used to establish limits for the allowable stress method, these investigations are the central theme of the strength design method. For load capacity in terms of strength, this method does not concern itself with responses at the service load level. Service loads are used only to derive necessary safety factors, which is done by multiplying service loads by some number. This *factored load* (increased load) is used for design. The design is thus a limit design—literally a design for failure. If the failure at the factored load is accurately predicted, the response at the service load should be well below the limiting capacity of the structure. Use of this method requires a clear understanding of the full range of material response from zero to failure.

Ultimate responses are best dealt with in terms of the responses of specific materials and their common situations of usage. Discussion of these issues is therefore presented in Chapters 4 through 6 for the three most common structural materials: wood, steel, and concrete. For all situations, the classic, idealized form of stress and strain development is used for a reference, and its understanding is essential as a starting point for the structural designer.

Material behaviors can reliably be tested in testing laboratories, and, indeed, extensive testing of this sort is done. Establishment of design guidelines, procedures, and supporting data is heavily dependent on published results from structural tests of materials.

Friction

When forces act on objects in such a way as to tend to cause one object to slide on the surface of another object, a resistance to the sliding motion is developed at the contact face between the objects. This resistance is called *friction*, and it constitutes a special kind of force.

For the object shown in Figure 2.48*a*, being acted on by its own weight and the inclined force F, we may observe that the motion that is impending is one of sliding of the block toward the right along the surface of the supporting plane. The force that tends to cause this motion is the component of F that is parallel to the plane, $F \cos\theta$. The component of F that is vertical, $F \sin\theta$, works with the weight of the block to press the block against the plane. The sum of the weight of the block and the vertical component of F is called the *pressure* on the plane or the force *normal* (perpendicular) to the plane.

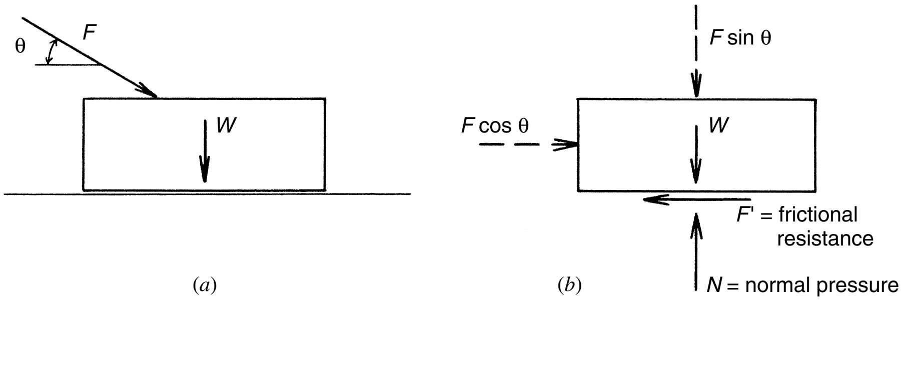


Figure 2.48 Development of friction.

A free-body diagram of the block is shown in Figure 2.48*b*. For equilibrium of the block, the two components of F must be resisted. For equilibrium in a vertical direction, normal to the plane, the reactive force N is required, its magnitude being $W + F\sin\theta$. For equilibrium in a horizontal direction, along the surface of the plane, a friction resistance must be developed with a magnitude equal to $F\cos\theta$.

The situation just described may result in one of three possibilities, as follows:

The block does not move because the potential friction resistance is more than adequate, that is,

$$\text{Frictional resistance } F' > F\cos\theta$$

The block moves because the friction is not sufficient, that is,

$$\text{Frictional resistance } F' < F\cos\theta$$

The block is just on the verge of moving because the potential friction is exactly equal to the force tending to induce motion, that is,

$$\text{Frictional resistance } F' = F\cos\theta$$

From observations and experimentation the following deductions have been made about friction:

The frictional resisting force always opposes motion; that is, it acts opposite to the slide-inducing force.

For dry, smooth surfaces, the friction resistance developed up to the point of motion is directly proportional to the normal pressure between the surfaces in contact. The maximum value for the friction resistance is

$$F' = \mu N$$

in which μ (lowercase Greek mu) is called the *coefficient of friction.*

The frictional resistance is independent of the area of contact.

The coefficient of static friction (before motion occurs) is greater than the coefficient of kinetic friction (during actual sliding). That is, for the same amount of normal pressure, the friction resistance is reduced once motion occurs.

Frictional resistance is ordinarily expressed in terms of its maximum potential value. Coefficients for static friction are determined by finding the ratio between the sliding force and the normal pressure at the moment motion just occurs. Simple experiments consist of using a block on an inclined plane with the angle of the plane's slope slowly increased until sliding occurs (see Figure 2.49*a*). Referring to the free-body diagram for the block (Figure 2.49*b*), we note that

$$F' = \mu N = W\sin\phi, \quad N = W\cos\phi$$

and, as noted previously, the coefficient of friction is expressed as the ratio of F' to N, or

$$\mu = \frac{F'}{N} = \frac{W\sin\phi}{W\cos\phi} = \tan\phi$$

Approximate values of the coefficient of static friction for various combinations of joined objects are given in Table 2.4.

Problems involving friction are usually one of two types. The first involves situations in which friction is one of the forces in a system, and the problem is to determine whether frictional resistance is sufficient to maintain the equilibrium of the system. For this type of problem the solution consists of writing the equations for equilibrium, including the maximum potential friction, and interpreting the results. If the frictional resistance is not large enough, sliding will occur; if it is just large enough or excessive, sliding will not occur.

The second type of problem involves situations in which the force required to overcome friction is to be found. In this case the slide-inducing force is simply equated to the frictional resistance, and the required force is determined.

Table 2.4 Range of Values for Coefficient of Static Friction

Contact Surfaces	Coefficient, μ
Wood on wood	0.40–0.70
Metal on wood	0.20–0.65
Metal on metal	0.15–0.30
Metal on stone, masonry, concrete	0.30–0.70

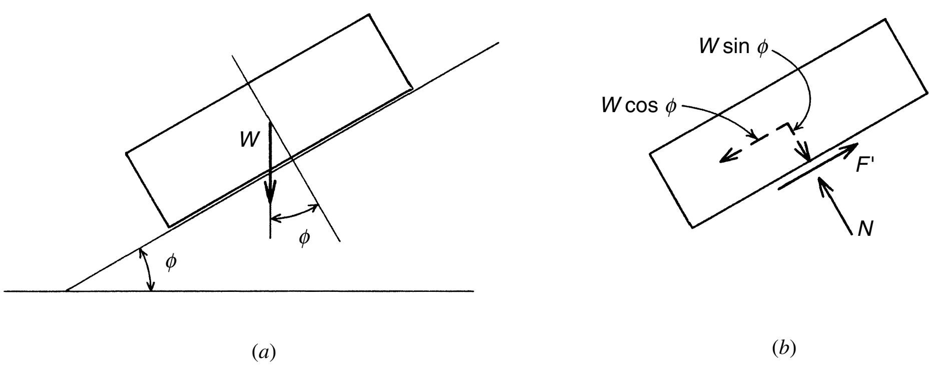


Figure 2.49 Development of sliding friction.

Note that in these problems sliding motions are usually of the form of pure translation (no rotation), and we may therefore treat the force systems as simple concurrent ones, ignoring moment effects. An exception is the problem in which tipping is possible, shown in Example 17.

Example 16. A block is placed on an inclined plane whose angle is slowly increased until sliding occurs. If the angle of the plane with the horizontal is 35° when sliding occurs, what is the coefficient of static friction between the block and the plane?

Solution. As previously derived, the coefficient of friction may simply be stated as the tangent of the angle of the plane. Thus

$$\mu = \tan\phi = \tan 35^\circ = 0.70$$

Example 17. Find the horizontal force P required to slide a block weighing 100 lb if the coefficient of friction is 0.30. (See Figure 2.50.)

Solution. For sliding to occur, the slide-inducing force P must be slightly larger than the frictional resistance F'. Equating these two yields

$$P = F' = \mu N = 0.30(100) = 30\,\text{lb}$$

The force must be slightly larger than 30 lb.

Example 18. Will the block shown in Figure 2.51*a* tip or slide?

Solution. We first determine whether the block will slide by computing the maximum frictional resistance and comparing it to the slide-inducing force:

$$F' = 0.60(8) = 4.8\,\text{lb}$$

and therefore the block will not slide.

We then evaluate the tipping potential by considering the block to be restrained horizontally at the base by the frictional resistance. Tipping may be visualized in terms of overturning about the lower right corner (*a* in Figure 2.51*b*). The overturning moment is

$$M = 4(6) = 24\,\text{in.-lb}$$

Resisting this is the moment of the block's weight, which is

$$M = 8(2) = 16\,\text{in.-lb}$$

Since the stabilizing moment offered by the weight is not sufficient, the block will tip.

While the illustration in Example 18 is somewhat academic, the situation has real implications, examples being the stability of cantilevered retaining walls and building shear walls as well as entire buildings.

2.5 DYNAMIC BEHAVIOR

A good laboratory course in physics should provide a reasonable understanding of the basic ideas and relationships involved in dynamic behavior. A better preparation is a

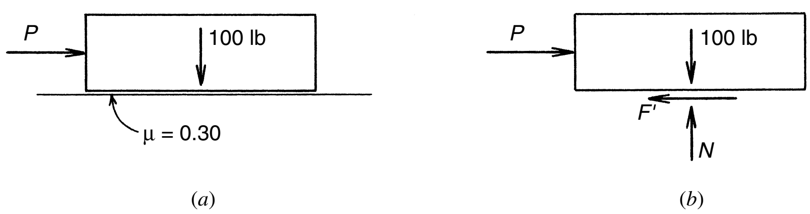


Figure 2.50 Reference for Example 17.

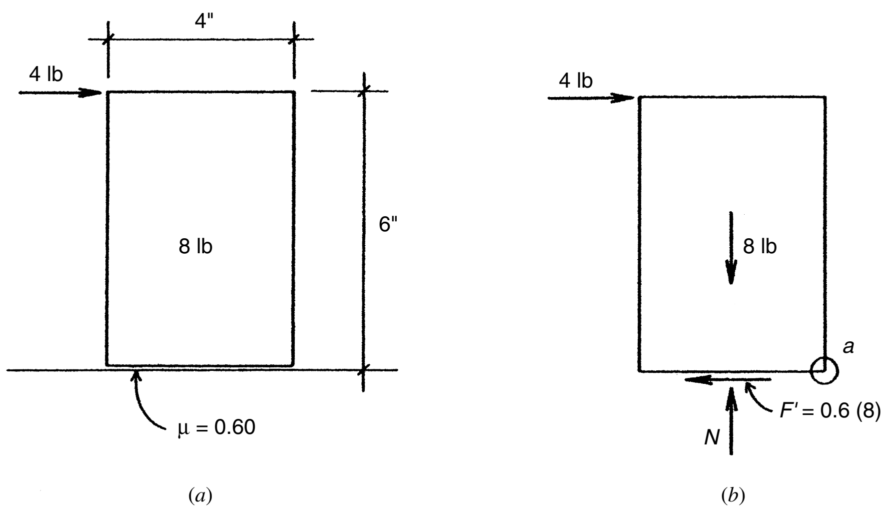


Figure 2.51 Reference for Example 18.

course in engineering dynamics that focuses on the topics in an applied fashion, dealing with their applications in various engineering problems. The material in this section consists of a brief summary of basic concepts in dynamics that will be useful to those with a limited background and that will serve as a refresher for those who have studied the topic before.

The general field of dynamics may be divided into the areas of *kinetics* and *kinematics.* Kinematics deals exclusively with motion, that is, with time–displacement relationships and the geometry of movements. Kinetics adds the consideration of the forces that produce or resist motion.

Kinematics

Motion can be visualized in terms of a moving point or in terms of the motion of a related set of points that constitute a body. The motion can be qualified geometrically and quantified dimensionally. In Figure 2.52 the point is seen to move along a path (its geometric character) a particular distance. The distance traveled by the point between any two separate locations on its path is called *displacement* (s). The idea of motion is that this displacement occurs over time, and the general mathematical expression for the time–displacement function is

$$s = f(t)$$

Velocity (v) is defined as the rate of change of the displacement with respect to time. As an instantaneous value, the velocity is expressed as the ratio of an increment of displacement (ds) divided by the increment of time (dt) elapsed during its displacement. Using the calculus, the velocity is thus defined as

$$v = \frac{ds}{dt}$$

That is, the velocity is the first derivative of the displacement.

If the displacement occurs at a constant rate with respect to time, it is said to have constant velocity. In this case the velocity may be expressed more simply without the calculus as

$$v = \frac{\text{total displacement}}{\text{total elapsed time}}$$

When velocity changes over time, its rate of change is called *acceleration* (a). Thus, as an instantaneous change,

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

That is, the acceleration is the first derivative of the velocity or the second derivative of the displacement with respect to time.

Motion

A major aspect of consideration in dynamics is the nature of motion. While building structures are not really supposed to move (as opposed to machine parts), their responses to force actions involve consideration of motions. These motions may actually occur in the form of very small deformations or may be a failure response that the designer must visualize. The following are some basic forms of motion:

Translation. This occurs when an object moves in simple linear displacement, with the displacement measured as a simple change of distance from some reference point.

Rotation. This occurs when the motion can be measured in the form of angular displacement, that is, in the form of revolving about a fixed reference point.

Rigid-Body Motion. A rigid body is one in which no internal deformation occurs and all particles of the body remain in fixed relation to each other. Three types of motion of such a body are possible. Translation occurs when all the particles of the body move in the same direction at the same time. Rotation occurs when all points in the body describe circular paths about some common fixed line in space, called the *axis of rotation.* Plane motion occurs when all the points in the body move in planes that are parallel. Motion within the planes may be any combination of translation or rotation.

Motion of Deformable Bodies. In this case motion occurs for the body as a whole as well as for the particles of the body with respect to each other. This is generally of more complex form than rigid-body motion, although it may be broken down into simpler component motions in many cases. This is the nature of motion

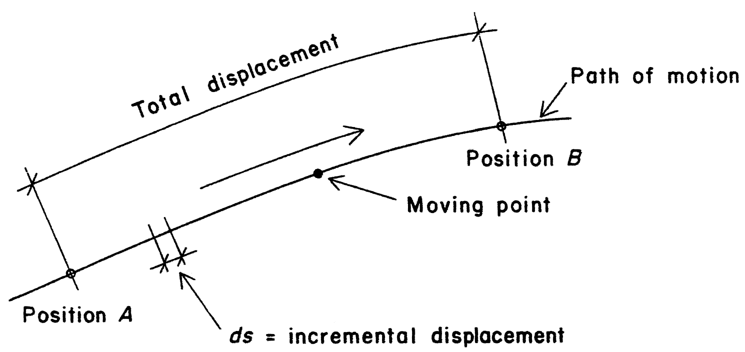


Figure 2.52 Motion of a point.

of fluids and of elastic solids. The deformation of elastic structures under load is of this form, involving both the movement of elements from their original positions and changes in their shapes.

Kinetics

As stated previously, kinetics includes the additional consideration of the forces that cause motion. This means that added to the variables of displacement and time is the consideration of the mass of the moving objects. From Newtonian physics the simple definition of mechanical force is

$$F = ma \text{ (mass times acceleration)}$$

Mass is the measure of the property of inertia, which is what causes an object to resist change in its state of motion. The more common term for dealing with mass is *weight,* which is a force defined as

$$W = mg$$

In which g is the constant acceleration of gravity (32.3 ft/sec^2).

Weight is literally a dynamic force, although it is the standard means of measurement of force in statics, when the velocity is assumed to be zero. Thus in static analysis force is simply expressed as

$$F = W$$

and in dynamic analysis, when using weight as the measure of mass, force is expressed as

$$F = ma = \frac{W}{g}a$$

Work, Power, Energy, and Momentum

If a force moves an object, work is done. *Work* is defined as the product of the force multiplied by the displacement (distance traveled). If the force is constant during the displacement, work may be simply expressed as

$$w = Fs = \text{(force)} \times \text{(total distance traveled)}$$

Energy may be defined as the capacity to do work. Energy exists in various forms: heat, mechanical, chemical, and so on. For structural analysis the concern is with mechanical energy, which occurs in one of two forms. *Potential energy* is stored energy, such as that in a compressed spring or an elevated weight. Work is done when the spring is released or the weight is dropped. *Kinetic energy* is possessed by bodies in motion; work is required to change their state of motion, that is, to slow them down or speed them up.

In structural analysis energy is considered to be indestructible, that is, it cannot be destroyed, although it can be transferred or transformed. The potential energy in the compressed spring can be transferred into kinetic energy if the spring is used to propel an object. In a steam engine the chemical energy in the fuel is transformed into heat and then into pressure of the steam and finally into mechanical energy delivered as the output of the engine.

An essential idea is that of the conservation of energy, which is a statement of its indestructibility in terms of input and output. This idea can be stated in terms of work by saying that the work done on an object is totally used and that it should therefore be equal to the work accomplished plus any losses due to heat, air friction, and so on. In structural analysis this concept yields a "work equilibrium" relationship similar to the static force equilibrium relationship. Just as all the forces must be in balance for static equilibrium, so the work input must equal the work output (plus losses) for work equilibrium.

Harmonic Motion

A special problem of major concern in structural analysis for dynamic effects is that of *harmonic motion.* The two elements generally used to illustrate this type of motion are the swinging pendulum and the bouncing spring. Both the pendulum and the spring have a neutral position where they will remain at rest in static equilibrium. If either of them is displaced from this neutral position, by pulling the pendulum sideways or compressing or stretching the spring, they will tend to move back to the neutral position when released. Instead of stopping at the neutral position, however, they will be carried past it by their momentum to a position of displacement in the opposite direction. This sets up a cyclic form of motion (swinging of the pendulum; bouncing of the spring) that has some basic characteristics.

Figure 2.53 illustrates the typical motion of a bouncing spring. Using the calculus and the basic motion and force equations, the displacement−time relationship may be derived as

$$s = A \cos Bt$$

The cosine function produces the basic form of the displacement−time graph, as shown in Fig. 2.53*b*. The maximum displacement from the neutral position is called the *amplitude.* The time elapsed for one full cycle is called the *period*. The number of full cycles in a given unit of time is called the *frequency* (usually expressed in cycles per second) and is equal to the inverse of the period

Every object subject to harmonic motion has a fundamental period (also called natural period), which is determined by its weight, stiffness, size, and so on.

Any influence that tends to reduce the amplitude in successive cycles is called a *damping effect.* Heat loss in friction, air resistance, and so on, are natural damping effects. Shock absorbers, counterbalances, cushioning materials, and other devices can also be used to damp the amplitude. Figure 2.53*c* shows the form of a damped harmonic motion, which is the normal form of most such motions, because perpetual motion is not possible without a continuous reapplication of the original displacing force.

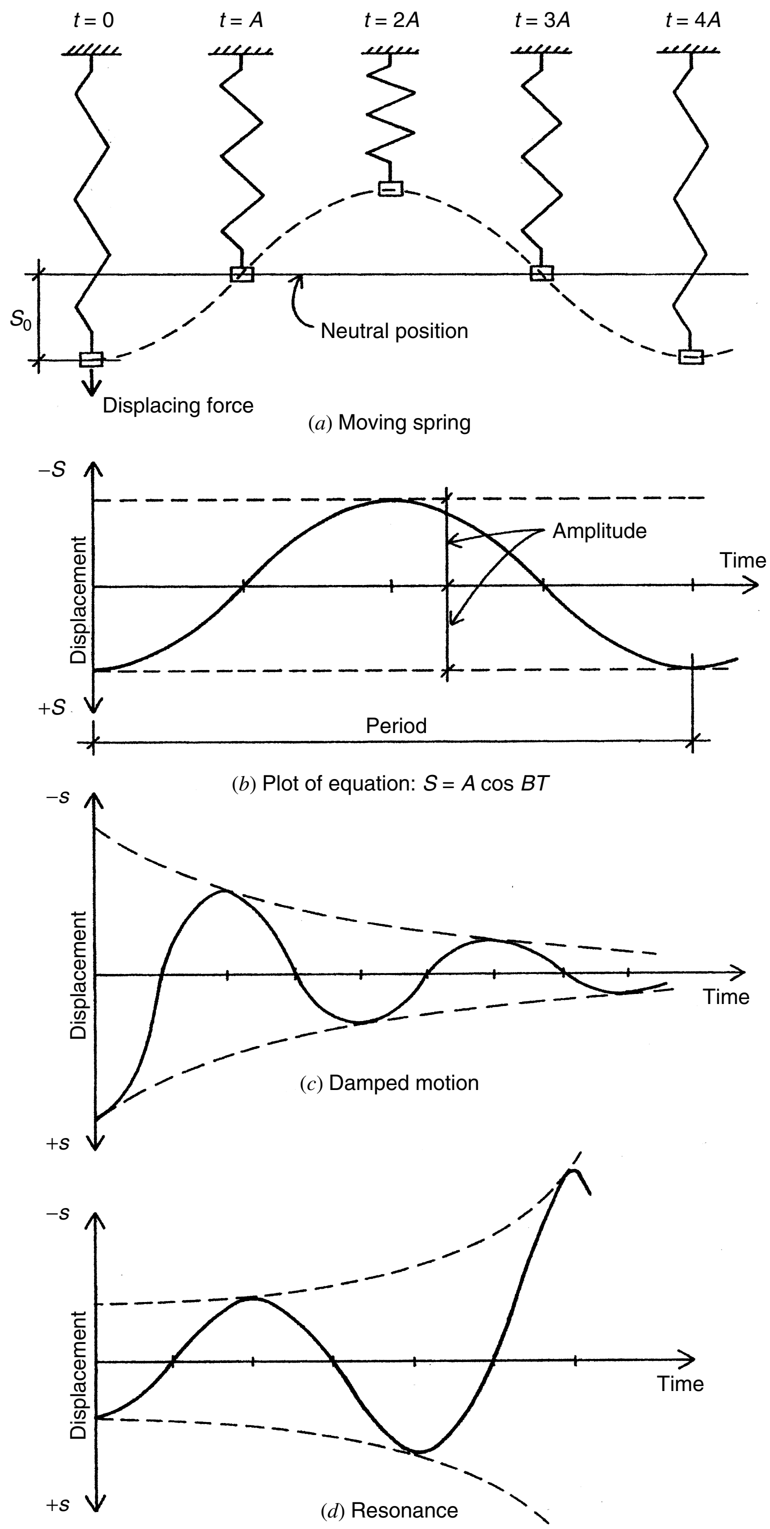


Figure 2.53 Considerations of harmonic motion.

Resonance is the effect produced when the displacing effort is itself harmonic with a cyclic nature that corresponds with the period of the impelled object. An example is someone bouncing on a diving board in rhythm with the fundamental period of the board, thus causing a reinforcement, or amplification, of the free motion of the board. This form of motion is illustrated in Figure 2.53*d*. Unrestrained resonant effects can result in intolerable amplitudes, producing destruction or damage of the moving object or its supports. A balance of damping and resonant effects can sometimes produce a constant motion with a flat profile of the amplitude peaks.

Loaded structures tend to act like springs. Within the elastic stress range of the materials, they can be displaced from a neutral (unloaded) position and, when released, will go into a form of harmonic motion. The fundamental period of the structure as a whole, as well as the periods of its parts, are major properties that affect responses to dynamic loads.

Equivalent Static Effects

Use of equivalent static effects essentially permits simpler analysis and design by eliminating the complex procedures of dynamic analysis. To make this possible, the load effects and the responses of the structure must be translated into static terms.

For wind load the primary translation consists of converting the kinetic energy of the wind into an equivalent static pressure, which is then treated in a manner similar to that for a distributed gravity load. Additional considerations are made for various aerodynamic effects, such as ground surface drag, building shape, and suction, but these do not change the basic static nature of the work.

For earthquake effects the primary translation consists of establishing a hypothetical horizontal static force that is applied to the structure to simulate the effects of sideward motions during ground movements. This force is calculated as some percentage of the dead weight of the building, which is the actual source of the kinetic energy loading once the building is in motion—just as the weight of the pendulum and the spring keeps them moving after the initial displacement and release. The specific percentage used is determined by a number of factors, including some of the dynamic response characteristics of the structure.

An apparently lower safety factor is used when designing for the effects of wind and earthquake because an increase is permitted in allowable stresses. This is actually not a matter of a less-safe design but merely a way of compensating for the fact that one is actually adding static (gravity) effects and *equivalent* static effects. The total stresses thus calculated are really quite hypothetical because in reality one is adding static strength effects to dynamic strength effects, in which case 2 + 2 does not necessarily make 4.

Regardless of the number of modifying factors and translations, there are some limits to the ability of an equivalent static analysis to account for dynamic behavior. Many effects of damping and resonance cannot be accounted for. The true energy capacity of the structure cannot be accurately measured in terms of the magnitudes of stresses and strains. There are some situations, therefore, in which a true dynamic analysis is desirable, whether it is performed by mathematics or by physical testing. These situations are actually quite rare, however. The vast majority of building designs present situations for which a great deal of experience exists. This experience permits generalizations on most occasions that the potential dynamic effects are really insignificant or that they will be adequately accounted for by design for gravity alone or with use of the equivalent static techniques.

CHAPTER

3

Structural Elements

This chapter treats the considerations for investigation of the behavior of various basic structural elements. These are the building blocks from which structural systems are developed.

3.1 BEAMS

The generic name for a structural member (element) that is used for spanning, sustains lateral (perpendicular) loading, and develops internal resisting force actions of bending and shear is a *beam.* Depending on its particular task in a structural system, a beam may be further described as a *joist*, *rafter, purlin*, *girder*, *header*, or *lintel*; however, for its fundamental behavior, it is classified as a beam.

Types of Beams

The most frequently used beam is the *simple beam.* As shown in Figure 3.1*a*, this consists of a single-span beam with supports at each end, offering only vertical force resistance. Because the supports do not offer restraint to the rotation of the beam ends, the beam takes the simple curved form of deformation as shown in the figure.

Supports that do not restrain rotation are called *free*, *pinned*, or *simple* supports. Thus the beam in Figure 3.1*a* is actually a simply supported beam, although it is more commonly called a simple beam.

A *cantilever beam* consists of a single-span beam with only one end support, as shown in Figure 3.1*b*. For stability of the beam, this support must be a rotation-resisting support, called a *fixed support* or a *moment-resisting support*.

Cantilevers exist less often as shown in Figure 9.2*b* than as extensions of beam ends over their supports, as shown at the right end of the beam in Figure 3.1*c*. The beam with an extended end is called an *overhanging beam*.

While the simple beam and single cantilever have deformed shapes with simple single curvature, the overhanging beam has multiple, or double, curvature (S shaped when the beam has a single extended end). This form of curvature is also found in beams that are continuous through more than one span, as shown in Figure 3.1*d*.

Figure 3.1*e* shows a single-span beam with both ends fully fixed against rotation. This is called a *restrained beam* or a *fixed-end beam* and it takes the profile of the doubly inflected curve shown.

Visualization of the deformed shape of a beam is a useful tool in investigation. It helps to establish the character of support reactions as well as the nature of distribution of internal force effects in the beam.

It is possible for beam support conditions to approach the true situation of fully fixed or complete freedom of moment restraint. Many supports, however, tend to offer partial restraint, being somewhere between the extreme conditions illustrated in Figure 3.1. Details of the connections at the beam supports as well as the nature of the supporting structures will qualify these conditions. For initial investigation, however, it is common to assume either simple or fully fixed supports, reserving judgment as to any need for adjustment until more is determined about the final form and details of the structure.

Load and Support Conditions

Members that serve as beams exist in a variety of situations and sustain many types of loads. The most common types of loading conditions are the following (see Figure 3.2):

Uniformly Distributed Load. The dead weight of the beam itself is constituted as a load that is distributed evenly along the beam length, as shown in Figure 3.2*a*. This is a common loading and is called a uniformly

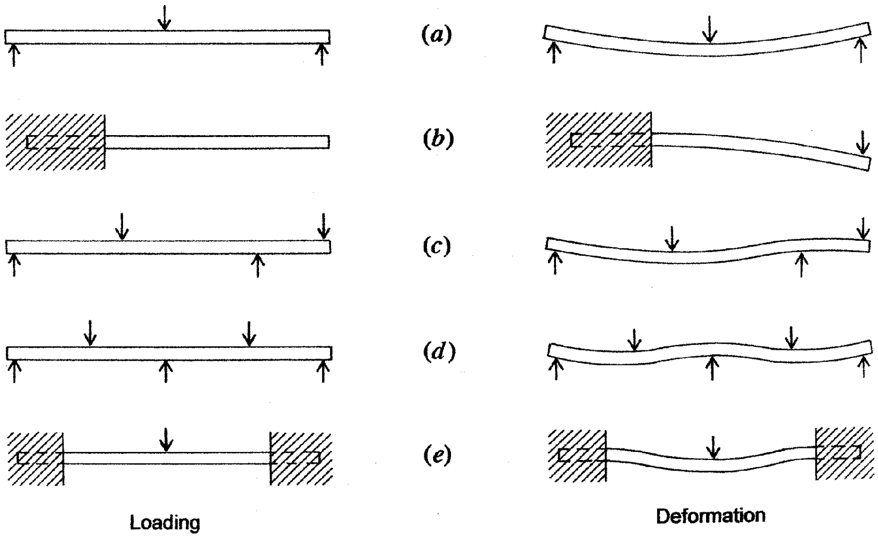


Figure 3.1 Types of beams.

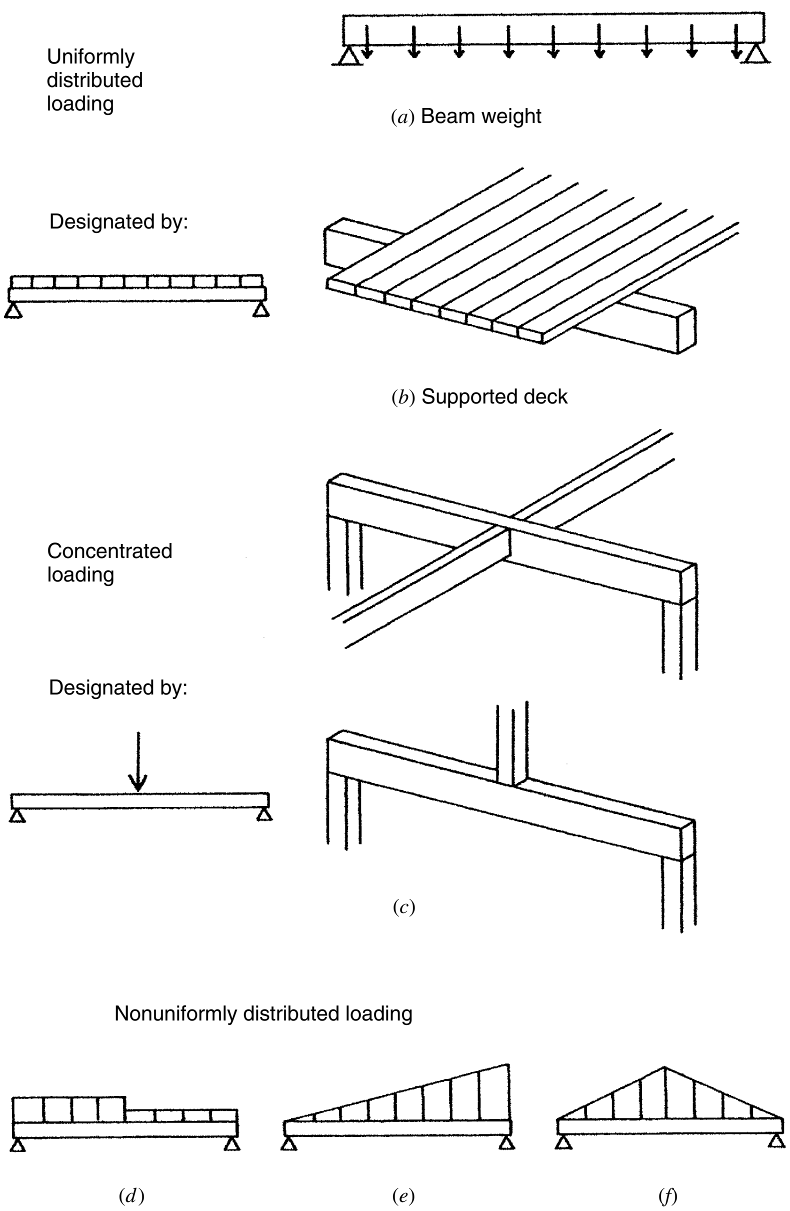


Figure 3.2 Beam loading conditions.

distributed load, or simply a uniform load. If the beam supports a roof or floor deck directly, as shown in Figure 3.2*b*, the weight of the deck and any loads carried by the deck will usually also be uniformly distributed on the beam.

Concentrated Load. The second most common load is one in which the force is delivered to the beam at a single location, effectively as though it were concentrated at a point. In framing systems, beams that support the ends of other beams (Figure 3.2*c*) sustain concentrated loads consisting of the end reactions of the supported beams.

Nonuniformly Distributed Load. Complexities of the building form or construction sometimes result in distributed loads on beams that are not uniform in magnitude along the beam length. The loading shown in Figure 3.2*d* indicates a change in the magnitude of the distributed loading over part of the beam length. In Figure 3.2*e* the distributed load varies continuously in magnitude from zero at one end to a maximum value at the other end. Another load that varies in magnitude is shown in Figure 3.2*f*; this is the form of load commonly assumed for a beam that serves as a lintel over an opening in a masonry wall.

As with other structural elements, beams often sustain combinations of loads rather than a single loading. For a given design situation, it may be necessary to investigate the behavior of the beam for several loading combinations.

In many cases the design of the supports is an extension of the design of the beam. Actually, few structural members can be designed completely as entities; each is a part of a system, and the entire system must be considered at some time in the design of the structure. Thus in a real design situation it is necessary to realize that while the beam behavior depends on support conditions, the requirements for the supports depend on the beam actions.

The loads and force actions generated by the supports constitute an external force system acting on the beam. The character of the beam and the nature of this external force system will determine whether the beam is stable or unstable and whether it is statically determinate or indeterminate.

Stability has to do with the general capability of a structure to resolve the forces it sustains, regardless of their magnitude. Determinacy refers to the capability of using the simple conditions of statics to solve for the behavior of the beam. The illustrations in Figure 3.3 show the range of possibilities in these regards.

In Figure 3.3*a* the single-span beam is supported at one end with a free support capable of vertical force resistance only. The beam is obviously unstable. It can be made stable by the provision of an additional reaction component in the form of a second vertical support or a moment resistance at the presently supported end, as shown in Figure 3.3*b*.

If two additional support components are added to the beam in Figure 3.3*a*, it constitutes an overkill of the unstable condition, producing a situation that is stable but statically redundant. This is not necessarily bad, but it means that we cannot solve for the reactions at the supports by the sole use of equations for static equilibrium. The situation is therefore described as being statically indeterminate.

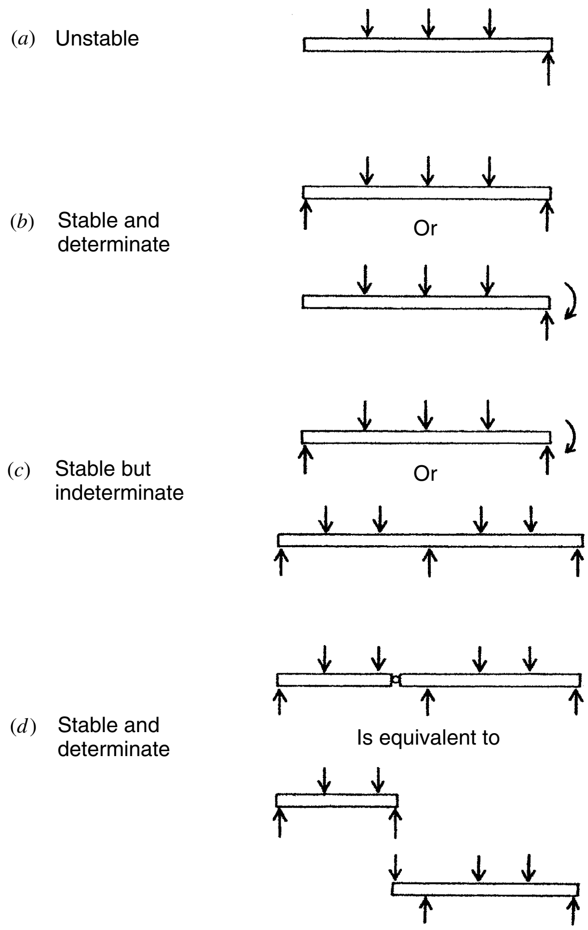


Figure 3.3 Stability and determinacy of beams.

A manner in which determinacy can be restored to the two-span beam in Figure 3.3*c* is shown in Figure 3.3*d*. This consists of producing the beam in two segments that are connected by a pin-type connection within one span. This permits the investigation of the beam in sequence as two statically determinate beams.

The work in this section deals primarily with statically determinate beams. General treatment of the topic of indeterminate structures is beyond the scope of this book, although some approximate investigations are discussed at various points in the book. Most wood and steel beams are constituted as statically determinate, whereas concrete beams are typically multispan and thus indeterminate.

Beam Actions

The behavior of beams involves various actions that may need consideration in structural investigation for design. The following are the major considerations:

Flexure, or Bending. Bending is a primary beam function involving the need for some resistance to internal

moment at most cross sections of the beam. It requires the development of stresses that vary in magnitude and switch from compression to tension across the beam section, as discussed in Section 2.3. For most loadings, the internal bending moment will vary across the beam length. Of critical concern is the greatest magnitude of internal moment, producing the requirement for maximum bending resistance by the beam. Flexure unavoidably produces deformations of rotation and deflection of the beam.

Shear. Shear as a result of the direct load effects is the other primary beam function. Shear stress development in the beam is not as simple as in the case of direct shear. As discussed in Section 2.3, the vertical shear on a horizontal beam produces an equal reactive horizontal shear and diagonal tension and compression, any of which may be critical for the beam, depending on its material and form.

Rotation and Deflection. Beam deformation is manifested as angular change and deflection. The angular change (rotation) may be visualized as the movement of vertical plane sections or of tangents to the curved beam profile. Deflection is the distance of dislocation of points in the beam from their previous, unloaded positions.

Lateral Buckling. If a beam lacks lateral stiffness and is not braced, it may buckle sideways due to the column-like action of the compression side (top) of the beam. Chief determinants of this action are the lateral bending stiffness of the beam section and the stiffness of the beam material. The usual solution is bracing; if not, moment resistance is reduced.

Torsional (Rotational) Buckling. If the beam lacks torsional resistance and is not adequately braced, it may be rolled over by the loads or at its end by the support force. As with lateral buckling, bracing is the best solution or else load capacity must be reduced.

Torsional Moment. Beams may experience a twisting effect (called torsion) due to direct effects of loading. This may be caused by loading which is not aligned with the beam's vertical axis (called *eccentric loading*) or by moment transfer from attached framing. This is not the same as torsional buckling, although the effect is similar, consisting of a rolling over of the beam.

Bearing. If a beam is supported by direct bearing on a support, the support force must be developed as a vertical compression on the beam end and as a contact-bearing pressure on the support.

Reactions

For statically determinate beams the first step in the investigation of beam behavior is the determination of the effects of the supports on the beam—called the *reactions.* For the simplest case the reactions are a set of vertical forces that respond directly to the vertical loads on the beam, constituting with the loads a system of coplanar, parallel forces. This system yields to a solution by consideration of static equilibrium if there are not more than two unknowns (corresponding to the number of conditions for equilibrium of a parallel force system; see discussion in Section 2.2).

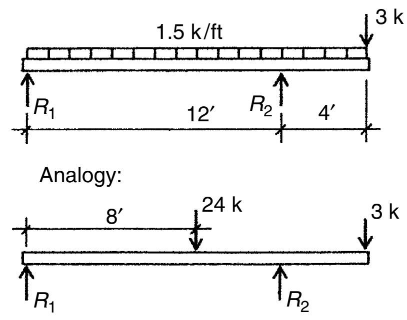


Figure 3.4 Reference for Example 1.

Using the beam shown in Figure 3.4, the following example demonstrates the usual procedure for finding the reactions for a single-span beam with one extended end and two vertical reactions.

Example 1. Find the reaction forces at the supports, R_1 and R_2, for the beam in Figure 3.4.

Solution. The general solution is to write two equations involving the two unknowns and then to solve them simultaneously. This procedure is simplest if one equation can be written that involves only one of the unknowns. For this procedure, consider a summation of moments of the forces about a point on the line of action of one of the reactions; thus, using the location of R_1 as the center of moments,

$$\sum M = (24 \times 8) + (3 \times 16) - (R_2 \times 12) = 0$$

from which

$$R_2 = \frac{192 + 48}{12} = 20 \text{ kips}$$

For a second step write any equilibrium equation that includes the action of R_1 and it will be an equation with one unknown. One choice for this is a simple summation of vertical forces; thus

$$\sum F_v = 0 = +R_1 + 20 - 24 - 3$$

from which

$$R_1 = 7 \text{ kips}$$

For a check write another equation for R_1 and it will test the system for equilibrium; for a summation of moments about R_2,

$$\sum M = (7 \times 12) + (3 \times 4) - (24 \times 4) = 0$$

$$84 + 12 - 96 = 0$$

which verifies the answer for R_1.

This is the usual form of solution for a beam with two supports loaded only with loads perpendicular to the beam.

Shear

Internal shear in a beam is the direct force effort required of the beam for the equilibrium of the loads on the beam. Because the shear itself is a force, it is possible to use a simple summation of forces to establish equilibrium.

Consider the beam shown in Figure 3.5, which is the same beam used for the solution for reactions in Example 1. With the reactions known, to find the internal shear at some point in the beam—say at 3 ft from the left end—cut a section of the beam at that point, remove the portion of the beam to the right, and consider the equilibrium for the remaining portion to the left. This free body is acted on by the loads and reactions that are directly applied to it plus the actions of the removed portion, the latter representing the internal forces at the section. To find the internal shear consider an equilibrium of vertical forces; thus

$$\sum F_v = 0 = 7 - 4.5 + V, \qquad V = -2.5 \text{ kips}$$

The minus sign for the internal shear indicates that it acts downward on the free body, as shown in Figure 3.4*b*.

Now consider the internal shear at a point 10 ft from the left end of the beam using a similar load summary on the free body shown in Figure 3.5*c*,

$$\sum F_v = 0 = +7 - 15 + V, \qquad V = +8 \text{ kips}$$

as shown in Figure 3.5*c*.

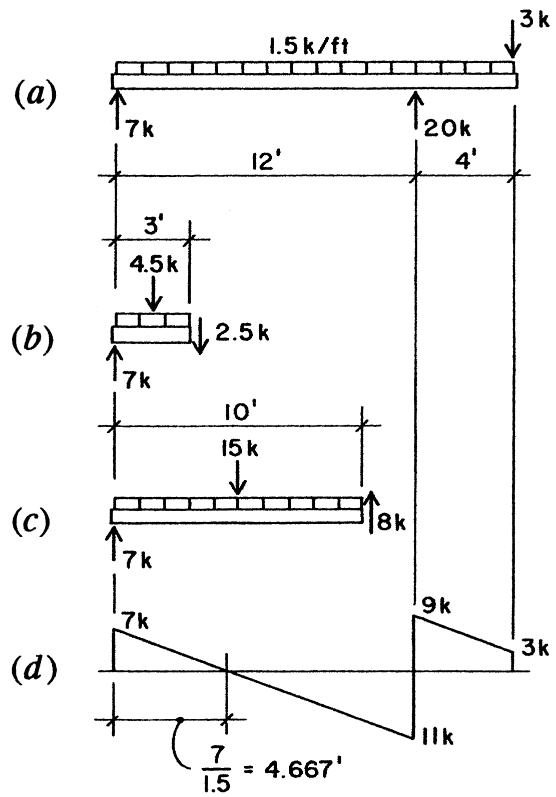


Figure 3.5 Determination of internal shear in a beam.

This process may be continued to find internal shear values at any point in the beam. A technique generally used is that of displaying the internal shear values and the form of their variation by using a simple graph, called the *shear diagram*. For this beam the shear diagram takes the form shown in Figure 3.5*d*. The convention used to create this diagram is to proceed from left to right along the beam and simply plot the values of the reactions and the loads as they are encountered.

Since there are no loads beyond either end of the beam, the shear diagram begins and ends with values of zero. Starting at the left end in the example, the first load encountered is the upward force of 7 kips at the reaction. As we proceed to the right, downward load is steadily encountered at the rate of loading shown: 1.5 kips/ft. A second point to note is where the graph crosses the zero baseline, which will occur at $7/1.5 = 4.667$ ft from the left end. The graph then continues to decline at the same rate, but the sign of the shear changes from there to the other support reaction.

The shear diagram descends steadily between the supports, reaching a value of −11 kips at the right support. It then rises at the value of the reaction force to a value of +9 kips, from which it continues to decline, reaching a value of 3 kips just before encountering the load at the end of the beam. Incorporating that final load returns the graph to the base value of zero at the beam end.

Points at which the shear diagram values switch signs are significant as they indicate points of maximum bending moment in the beam. This will be demonstrated in the following discussion.

Bending Moment

Internal bending moment in a beam is the effort required by the beam for consideration of complete equilibrium at all points in the beam. Consider the free body shown in Figure 3.5*b*, for which a summation of vertical forces was considered to determine the internal shear at the cut section. This is not actually a complete resolution of equilibrium for the free body, as equilibrium of moments must also be considered. We thus proceed with investigation of the beam in Example 1 by consideration of the development of internal bending moments.

Figure 3.6 once again presents the beam from Example 1, with the solutions for the reactions and the distribution of internal shear shown at the top of the illustration. The free body for the cut section at 3 ft from the left end is shown in Figure 3.6*a*. We will now consider the conditions required for equilibrium of moments for this free body; thus, referring to the figure, a summation of moments at the cut section is

$$\sum M = (7 \times 3) - (4.5 \times 1.5) = 21 - 6.75 = 14.25 \text{ kip-ft}$$

This indicates that the forces tend to rotate the free body with a clockwise moment of 14.25 kip-ft at the section. For equilibrium, this must be resisted by an internal bending moment of the same magnitude and opposite sense of

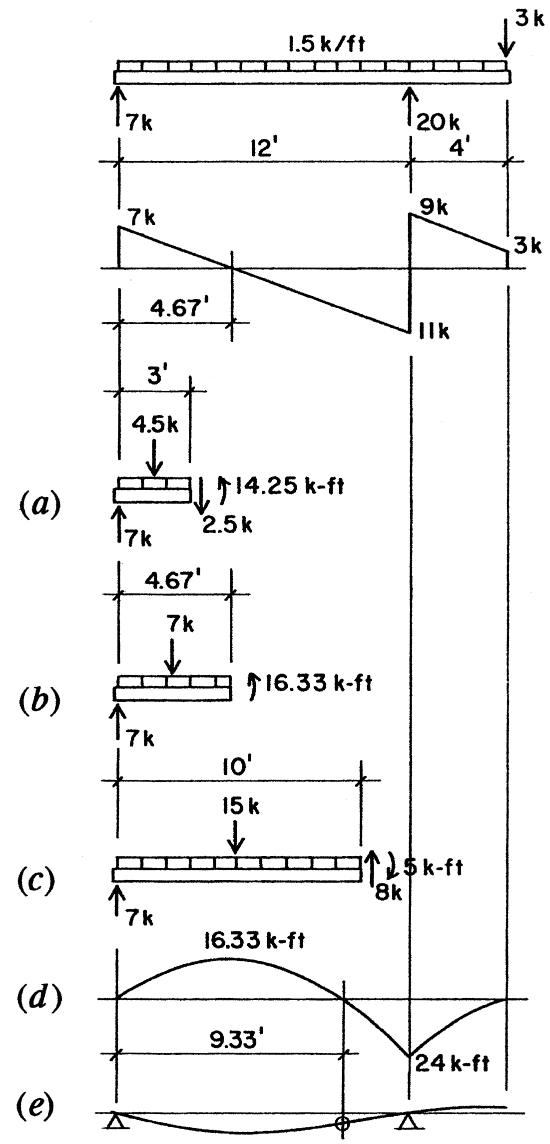


Figure 3.6 Determination of internal bending moment in a beam.

rotation. This is indicated by the externalized moment on the free body. It represents what the right portion of the beam was doing before it was cut away.

The internal moment just determined could also be found by use of a free-body portion to the right of the cut section. The reader can verify that a summation of moments on this free body will produce the same answer for the internal moment at the cut section.

Consideration of the rotational sign of the internal moment shown in Figure 3.6*a* will indicate that the moment tends to cause compression in the top and tension in the bottom of the beam. For the horizontal beam this sense of the moment is usually described as positive, while a moment that develops the opposite rotational effect—and thus tension in the top of the beam—is described as negative.

Consider now a cut section at the previously determined location of zero shear at 4.667 ft from the left support (see Figure 3.6*b*). A moment summation at this point will reveal an internal moment of 16.33 kip-ft. Further investigation of cut sections will determine that this is the point of maximum positive moment in the beam.

Finally, consider a cut section at 10 ft from the left end of the beam, at which point a moment summation will determine an internal bending moment of −5 kip-ft. This switch in the sign of the moment indicates that compression is in the bottom and tension in the top of the beam. Thus, it may be observed that the sign of moment changes from positive to negative somewhere between the cut section at 4.667 ft and that at 10 ft.

If we continue to determine moments at cut sections, we will eventually establish the overall form of the moment variation in the beam, which takes the form of the graph in Figure 3.6*d*, called the *moment diagram*. The curved form of the diagram is a result of the distributed loading, which produces a graph consisting of segments of parabolas. For the positive moment portion of the graph, the diagram takes the form of a symmetrical parabola with its apex at the point of zero shear.

Figure 3.6*e* shows the form of the profile of the beam as deformed from a straight horizontal line by the loads. This diagram is called the *deflected shape*, or simply the *deflection diagram*. Visualization of the deflected shape is a very useful device in the investigation of members subjected to bending. In most cases this visualization can be made on the basis of consideration of the loads, the supports, and the beam form before other investigation is done. If so, it provides immediate clues to the character of the moment in the beam.

Referring to Figure 3.6, we can make the following statements with regard to investigation of beams:

The internal shear at any point along a beam is the sum of the loads and reactions on one side of that point.

The internal moment at any point along a beam is the sum of the moments of the loads and reactions on one side of the point.

The change of the moment between any two points along the beam is equal to the area of the shear diagram between the two points.

Points of maximum value on the moment diagram correspond to points where the shear diagram goes to or passes through zero.

The sign of the internal bending moment is related to the form of curvature of the deflected shape.

Points of zero moment along a beam indicate points of change of the curvature of the deflected shape—called *inflection* or *contraflexure*.

When performing an investigation of beam behavior, it is important to follow the process demonstrated in the preceding examples. Where more than a single method is available for determining critical values, a second method can be used to verify the computations.

It is also useful to recognize the relationships between elements of the investigation as described above. These

are also useful in verifying the results of the mathematical computations. Mathematics is often abstract and having other simple observations to verify results is important.

Stresses in Beams

Stresses in beams vary along the beam length and across individual cross sections. The variation of bending stress, as discussed in Section 2.3, occurs as shown in Figure 3.7*a* on a simple rectangular cross section. On the same cross section, shear stress, as also discussed in Section 2.3, varies as shown in Figure 3.7*b*. As stated, these stresses vary along the beam length, so that the stress condition in a beam is not a single situation, but rather a complex series of situations. Typically, however, only a few isolated situations are critical for design.

Of critical concern for design are the maximum stresses. The most obvious of these are the maximum tension and compression stresses due to bending and the maximum shear stress. For the beam shown in Figure 3.7, consisting of a simple beam with a single, uniformly distributed load, we observe that these maximum stresses will occur as follows:

Maximum shear will occur at the neutral axis of the cross section at the ends of the beam.

Maximum tension will occur at the bottom of the beam at midspan.

Maximum compression will occur at the top of the beam at midspan.

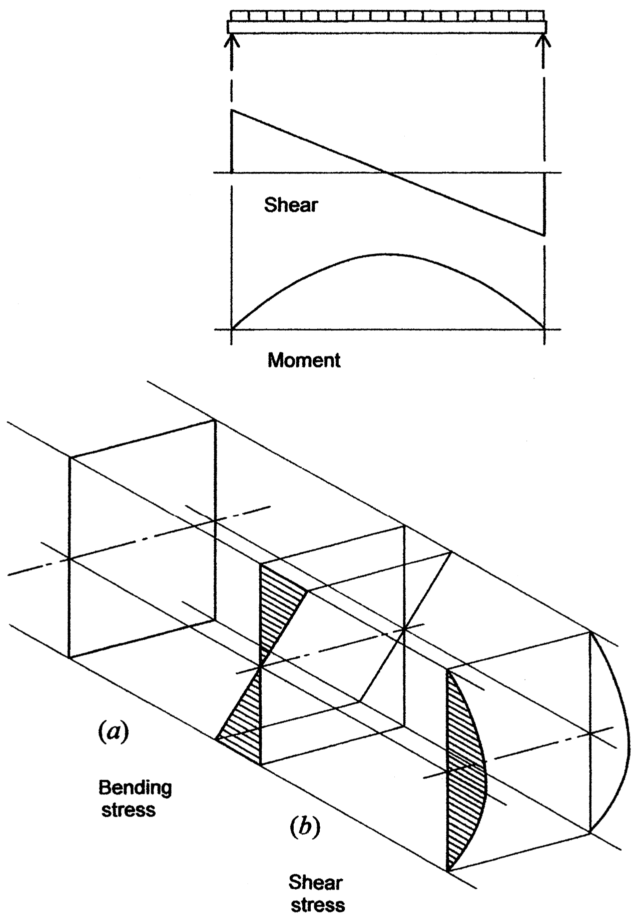


Figure 3.7 Distribution of stresses in a beam.

In this example it may be noted that the maximum shear stress occurs where bending stress is zero, and the maximum bending stresses occur where the shear stress is zero. This is frequently the case with simple beams. However, with overhanging or continuous beams negative moments at the supports may be the maximum moments in the beam. Thus in Figure 3.6, for example, the maximum value for shear and the maximum value for moment both occur at the right support; thus all the critical stresses will occur at this point.

Although we observe the stress conditions for shear and bending as separate phenomena, they do not truly occur as such. In some cases, therefore, we must consider their combinations and interactions. Some of these situations are discussed in the chapters that deal with specific structural materials.

In various situations beam actions may occur in combination with other actions, such as tension, compression, or torsion. While these situations involve combinations of stresses, their actual combined effects are often treated as interactive phenomena, rather than as combined stress concerns. Some of these situations are discussed in this chapter with regard to tension and compression members. They are also discussed in the chapters on individual materials.

Rotation and Deflection

There are various situations in structural design in which it is necessary to determine the actual deformation of a beam. Most often this has to do with deflection and usually with a single value of maximum deflection. For beams in ordinary situations deflections are usually determined through the use of derived formulas that incorporate the variables and represent the situation for a particular loading, span, and support condition.

Rotations are not often of interest in themselves, although problems in connection design or of the effects of a beam end on its supports may involve their consideration. Computations for rotations are more often done as part of the procedure for some other investigation, such as the deflection of a complex beam or of the sideways deflection of a framed structure.

For both deflections and rotations, the effects on the structure itself may be less of concern than the effects on other parts of the building construction.

Use of Tabulated Values for Beams

Some of the most common beam loadings are shown in Figure 3.8. In addition to the formulas for the reactions R, the maximum shear V, and the maximum bending moment M, expressions for maximum deflection are also given.

In Figure 3.8, if the loads P and W are in pounds or kips, the values for reactions and shear will be the same. If the span is in feet, values for bending will be in units of foot-pounds or kip-feet. The distributed load W used in Figure 3.8 is the total load on the beam, not the load per unit length of the beam, which is designated as w; thus $W = wL$.

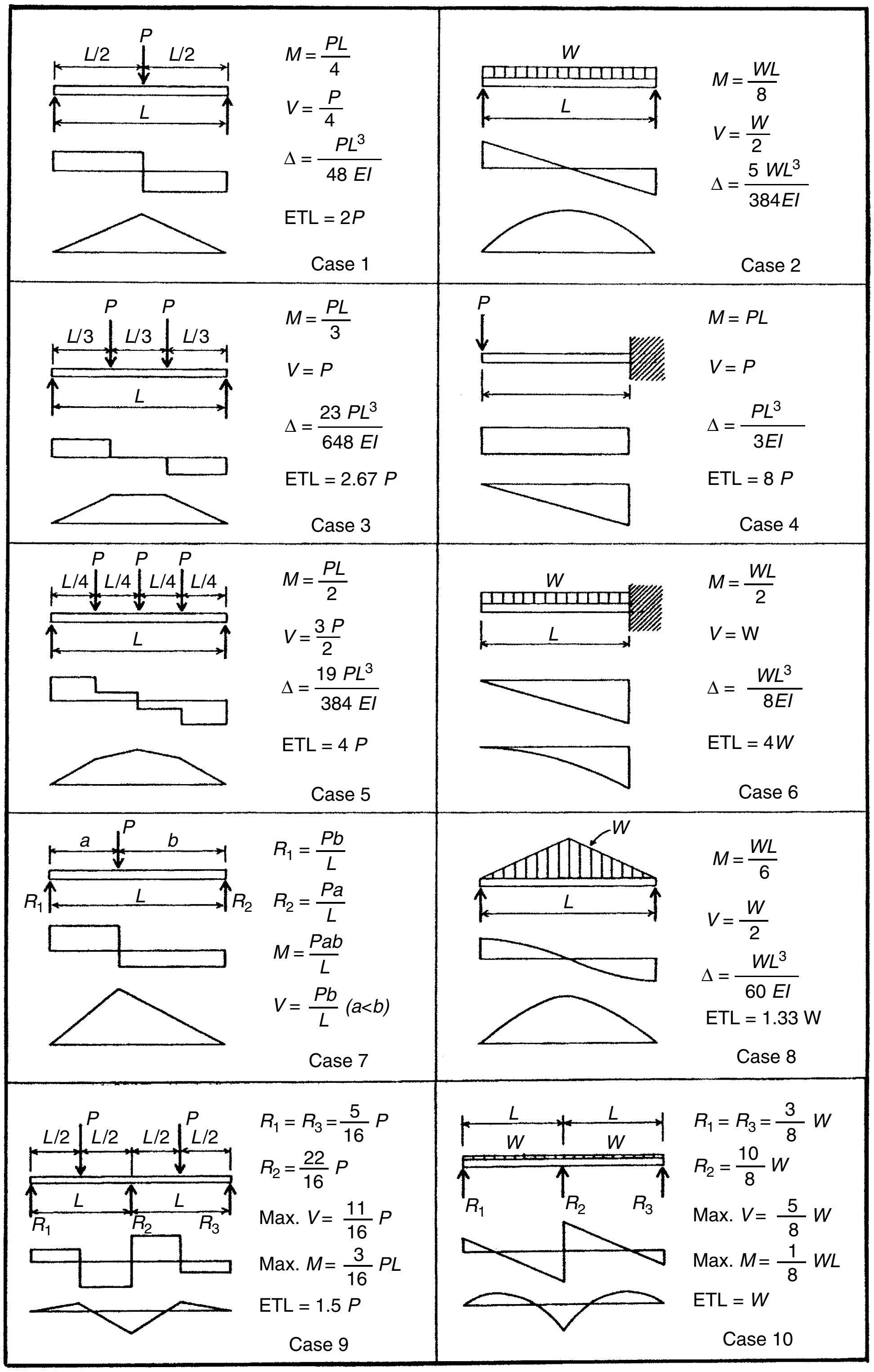


Figure 3.8 Values for typical beam loadings and support conditions.

Also given in Figure 3.8 are values designated ETL, which stands for *equivalent tabular load*. These factors may be used to derive a hypothetical uniformly distributed load that, when applied to a simply supported beam, will produce approximately the same magnitude of maximum bending moment as that for the given case of loading. Use of these factors is illustrated in later parts of the book.

Buckling

Buckling of beams—in one form or another—is mostly a problem with beams that are weak on their transverse axes, that is, the axis of the beam cross section at right angles to the axis of bending. This is not a frequent condition in concrete beams, but it is a common one with beams of wood or steel or with narrow trusses that perform beam functions. The

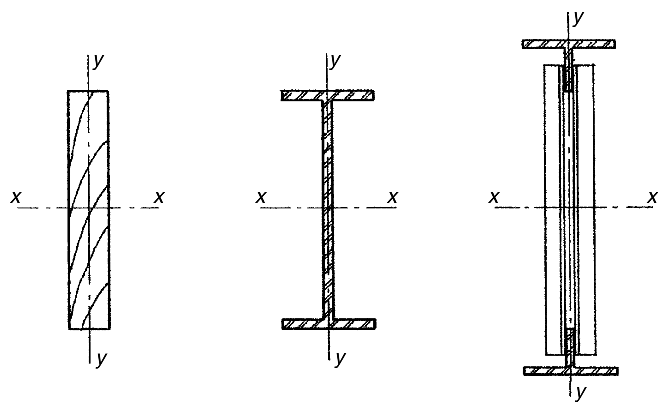


Figure 3.9 Sections of spanning elements that lack lateral strength.

cross sections shown in Figure 3.9 illustrate members that are relatively susceptible to buckling in beam actions.

Other than redesigning the beam section, there are two general approaches to solving a buckling problem. The first—and mostly preferred—approach is to brace the member so as to prevent the buckling response. To visualize where and how the bracing should be done, we must consider the various possibilities for buckling, the three major ones being those shown in Figure 3.10.

Figure 3.10*a* shows the response described as *lateral* (literally sideways) *buckling*. This is caused by the compression column action of the portion of the beam that develops compressive bending stress. For a simple span beam, the critical location for this is at the midspan where bending moment is maximum. If the beam is unbraced, the length over which buckling occurs is the full span of the beam. If lateral bracing is provided, the buckling length becomes the distance between bracing points.

The other type of buckling, called *torsional buckling*, commonly occurs in one of two ways. The first way is at the supports, as shown in Figure 3.10*c*, where the concentrated effect of the support force may cause a roll-over effect. The second way in which torsional buckling occurs is shown in Figure 3.10*d*, consisting of a twisting effect caused by the tension bending stress.

Torsional buckling can be controlled by selection of beams that have significant torsion-resisting cross sections. For the most part, this means sections like those of concrete or timber beams with wide dimensions in proportion to their depth. At the other extreme are sections like those in Figure 3.9, which are weak in both lateral bending and torsion.

Various other elements of the structure may serve to brace a beam against buckling effects. For beams that support decks directly, the attachment of the deck to the beam top is usually adequate to resist lateral buckling at this location. In addition, the bending resistance of the deck will help resist torsional buckling in the beam span. For beams that support the ends of other beams, the supported beams will usually serve adequately for bracing, reducing the length over which buckling occurs to the spacing of the supported beams.

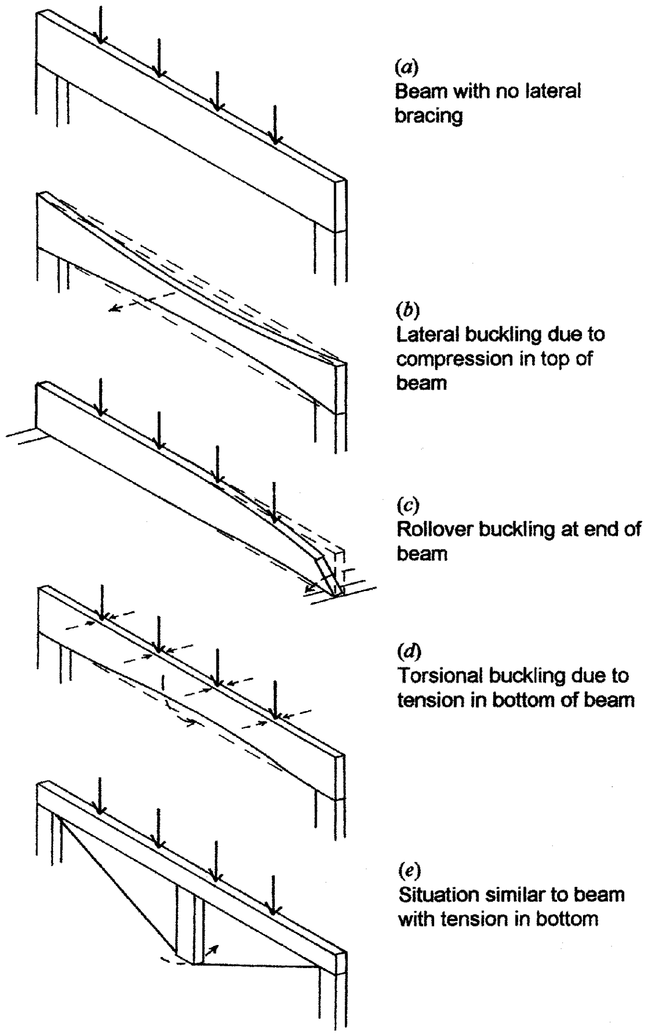


Figure 3.10 Buckling failure of beams.

3.2 TENSION ELEMENTS

Tension elements are used in a number of ways in building structures. Structural behavior may be simple, as in the case of a single hanger or tie rod, or extremely complex, as in the case of cable networks or restraining cables for tents and pneumatic structures. This section contains discussions of the nature and problems of various elements and examples of investigation of several simple elements.

Axially Loaded Elements

The simplest case of tension occurs when a linear element is subjected to tension and the tension force is aligned on an axis that coincides with the centroid of the member's cross section. A member that is loaded in this manner is said to sustain axial load. The internal tension stress is assumed to be evenly distributed on the cross section and is expressed as

$$f = \frac{T}{A}$$

In the usual case with an axially loaded member, the stress is also evenly distributed over the length of the member. If the material of the member is elastic, the strain may thus be expressed as

$$\varepsilon = \frac{f}{E}$$

and the elongation of a member L distance in length is expressed as

$$e = \varepsilon L = \frac{f}{E} L = \frac{TL}{AE}$$

A tension force limited by a specific magnitude of e may be expressed as

$$T = \frac{AEe}{L}$$

When a member is short, as in the case of a short hanger or a truss member, the usable tension capacity is determined by stress. However, for very long members, elongation may be critical and may limit the capacity to a value below that of the safe stress limit.

Example 2. An arch spans 100 ft [30 m] and is tied at its spring points by a 1-in.- [25-mm-]diameter round steel rod. Find the limit for the axial tension force in the rod if stress is limited to 22 ksi [150 MPa] and total elongation is limited to 1.0 in. [25 mm].

Solution. The cross-sectional area of the rod is

$$A = \pi R^2 = 3.14(0.5)^2 = 0.785 \text{ in.}^2 \ [491 \text{ mm}^2]$$

The maximum force based on stress is

$$T = fA = 22(0.785) = 17.27 \text{ kips } [73.65 \text{ kN}]$$

and the maximum force based on elongation is

$$T = \frac{AEe}{L} = \frac{0.785(29{,}000)(1.0)}{100(12)} = 18.97 \text{ kips } [82 \text{ kN}]$$

In this case the stress limit is critical. However, if the span were only a few feet longer, the elongation would be critical.

Net Section and Effective Area

The development of tension in a structural member involves connecting it to something. Achieving tension connections often involves situations that reduce the load-carrying effectiveness of the tension member. Two examples of this are the bolted connection and the threaded connection.

Bolts are commonly used with members of wood or steel. Insertion of the bolts requires drilling or punching of holes in the connected members. If a cross section for stress visualization is cut in the member at the location of a bolt hole (or a row of bolts), a reduced area is obtained, called a *net section*. If a stress computation is made for this area, the unit stress obtained will be higher than that at unreduced sections of the member.

The total behavior of a bolted connection is more complex and involves many considerations besides the tension on net sections. (See discussion of bolted connections in Chapters 4 and 5.) Nevertheless, it is often the case that considerations for the connection may be the limiting factors in establishing the tension capacity for bolted tension members.

A simple tension element frequently used is a round steel rod with spiral threads cut into its ends to facilitate a nut for a connection. The cutting of the ends reduces the cross section of the rod, producing a net section for critical tension stress. The cut thread is also a consideration for the limit of direct tension on a bolt. A special kind of rod is one that has forged enlargements at its ends—called *upset ends*—which have sufficient diameters so that the cutting of threads does not produce a net section smaller than that of the main part of the rod. These special rods are mostly used only when dynamic loading is a critical concern.

In steel structures it sometimes happens that practical considerations of achieving connections make it difficult to develop fully the potential of the connected member. Figure 3.11 shows a common connection involving a steel angle, in which one leg of the angle is fastened by welding to a supporting element. At the connection the tension force is developed only in the leg of the angle that is directly grabbed by the welds. At some distance along the angle, some tension will be developed in the other leg; however, for a conservative design it is not unreasonable to ignore the unconnected leg and consider the connected leg to function in the manner of a flat bar. The full angle will still be effective for stiffness or

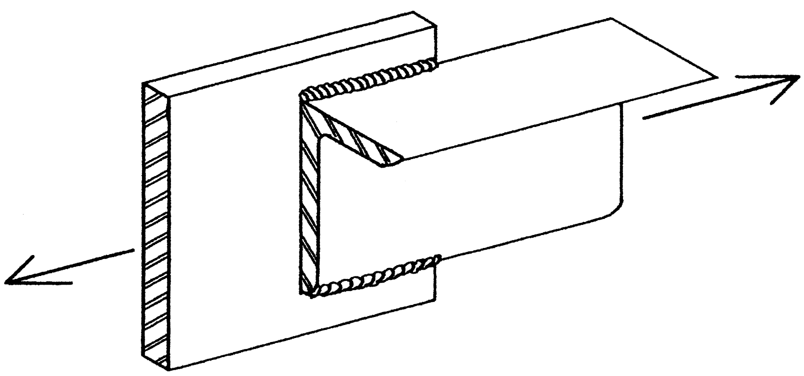


Figure 3.11 Development of effective area in tension members.

other considerations, but the tension resistance is limited by the form of the member and the layout of the connection.

Flexible Elements

Tension elements are unique in that there is little basis for limitations on slenderness or aspect ratio. By comparison, compression elements and beams have critical concerns for slenderness as it effects buckling. Thus, tension elements may often have considerable flexibility.

Flexibility can be high for tension rods, straps, or single angles, but the highest degree of flexibility occurs with elements whose materials and formation produce a truly flexible member, such as a rope, chain, wire, or cable. These highly flexible elements can only develop tension by following the natural path of the tension force.

When the superflexible tension element is used for spanning, it cannot assume the rigid, minutely deflected shape of a beam. It must, instead, assume a profile that permits it to act only in pure tension. This profile must be "honest," that is, not necessarily one that the designer may concoct, but one that the loads and the loaded structure can actually achieve. This is a critical issue with the design of structures using cables, and the overall form of such structures must be carefully developed for feasibility of the functioning of the cables.

Spanning Cables

Flexible elements may be used for spanning if they are properly supported and are permitted to assume the profile natural to resolution by pure internal tension. Consider the single-span cable shown in Figure 3.12*a*, spanning horizontally and supporting only its own dead weight. The natural form assumed by the cable is a catenary curve, which is a fourth-degree parabola. Except for cables that actually do carry only their own weight (such as electrical transmission lines) or which carry loads proportionally small with respect to their weights, this form is not particularly useful for spanning structures.

In this section we deal with problems in which the weight of the cable can be ignored without significant error. When this is assumed, the cable profile becomes a pure response to the static resolution of the loads. The cable will thus assume a simple parabolic form (Figure 3.12*b*) when carrying a uniformly distributed load with respect to the span and a form consisting of straight segments (Figure 3.12*c*) when loaded with individually concentrated loads.

Consider the cable shown in Figure 3.13*a*, supporting a single concentrated load W and having four exterior reaction components, H_1, V_1, H_2, and V_2, as shown in Figure 3.13*b*. Without consideration of the internal nature of the structure, this analysis is indeterminate with regard to the use of static equilibrium conditions alone. To analyze the structure, we must use the fact that the cable is incapable of developing bending or shear resistance, and therefore there can be no internal bending moment at any point along the cable. We may also note that individual segments of the cable operate as two-force elements; thus the direction of T_1, for example,

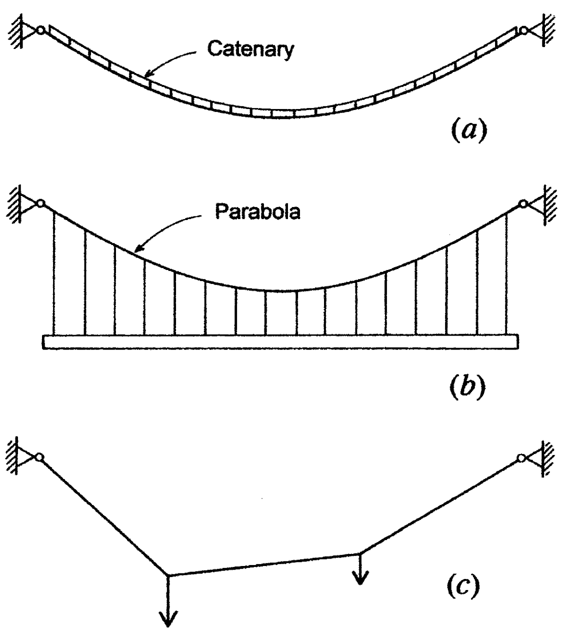


Figure 3.12 Cable response to loads.

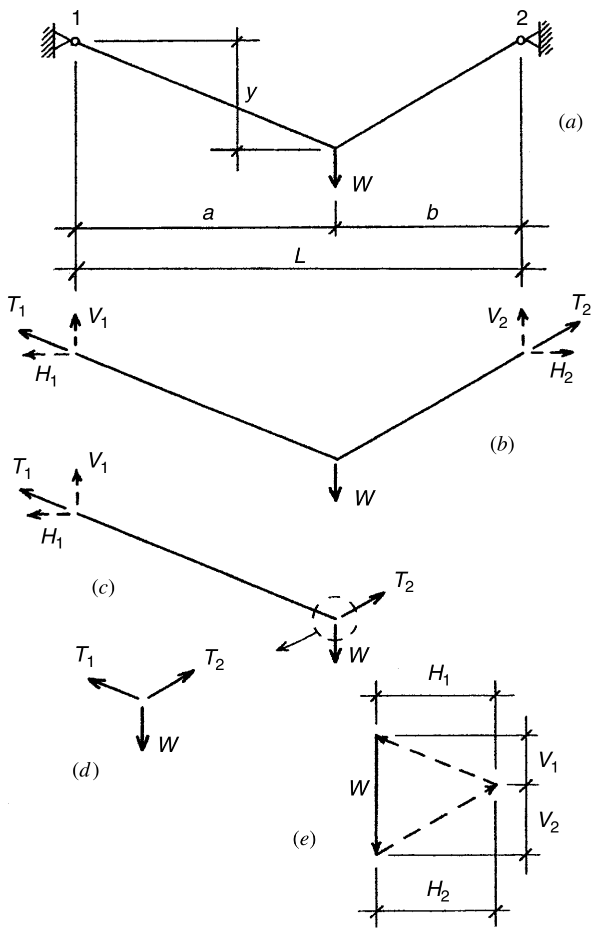


Figure 3.13 Cable with single concentrated load.

must be the same as the slope of the left segment of the cable. Similarly, the direction of T_2 must be the same as the slope of the right segment.

Referring to the free-body diagram of the whole cable in Figure 3.13*b*, if we take moments about support 2,

$$\sum M_2 = (W \times b) - (V_1 \times L) = 0$$

$$(W \times b) = (V_1 \times L)$$

$$V_1 = \frac{b}{L} W$$

Similarly, $V_2 = (a/L)W$.

For a check of the computations, the sum of the vertical forces must be zero; thus $V_1 + V_2 = W$.

With the vertical components of the reactions found, the horizontal components may be found by considering the relations between the components and the slopes of the cable segments. Thus, for H_1

$$\frac{H_1}{V_1} = \frac{a}{y} \quad \text{and} \quad H_1 = \frac{a}{y} V_1$$

Referring to the free-body diagram for the whole cable in Figure 3.13*b*, note that the two horizontal reaction components are the only horizontal forces external to the cable; therefore, they must be equal in magnitude and opposite in sign for horizontal equilibrium. The values for the two tension reactions can thus be found from the vector combinations of the components; thus

$$T_1 = \sqrt{H_1^2 + V_1^2} \quad \text{and} \quad T_2 = \sqrt{H_2^2 + V_2^2}$$

This problem can also be solved numerically with a graphic construction, as shown in Figure 3.13*e*. This figure is produced by drawing the two dashed lines that represent the two unknown tension forces at slopes corresponding to those in the cable system layout.

Example 3. Find the horizontal and vertical components of the reactions and the tension forces in the cable for the system shown in Figure 3.14*a*.

Solution. Using the relations just derived for the structure in Figure 3.13 yields

$$V_1 = \frac{b}{L} W = \frac{8}{13}(100) = 61.54 \text{ lb}$$

$$V_2 = \frac{a}{L} W = \frac{5}{13}(100) = 38.46 \text{ lb}$$

$$H_1 = H_2 = \frac{a}{y} V_1 = \frac{5}{6}(61.54) = 51.28 \text{ lb}$$

$$T_1 = \sqrt{V_1^2 + H_1^2} = \sqrt{(61.54)^2 + (51.28)^2} = 80.1 \text{ lb}$$

$$T_2 = \sqrt{V_2^2 + H_2^2} = \sqrt{(38.46)^2 + (51.28)^2} = 64.1 \text{ lb}$$

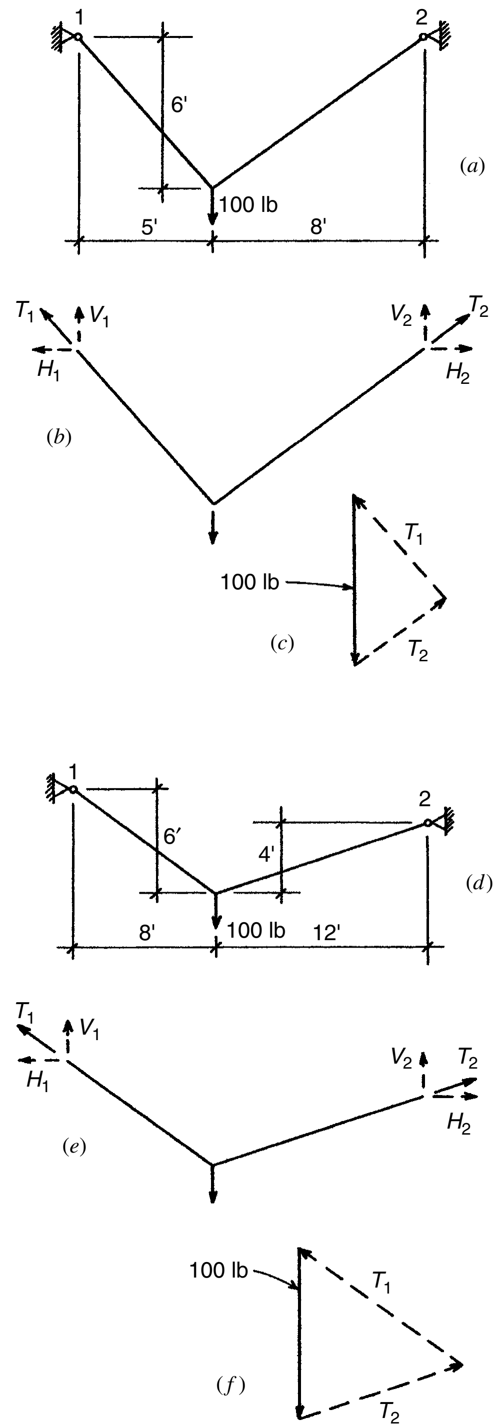


Figure 3.14 References for Examples 3 and 4.

When the two supports are not at the same elevation, the preceding problem becomes somewhat more complex. The solution is still determinate, however, and may be accomplished as follows.

Example 4. Find the horizontal and vertical components of the reactions and the tension forces in the cable for the system shown in Figure 3.14*d*.

Solution. Referring to the free-body diagram in Figure 3.14e, we note that

$$\sum F_v = 0$$

and thus $V_1 + V_2 = 100$;

$$\sum F_h = 0$$

and thus $H_1 + H_2 = 0$ and $H_1 = H_2$; and

$$\sum M_2 = 0 = (V_1 \times 20) - (H_1 \times 2) - (100 \times 12)$$

From the moment equation

$$10V_1 - H_1 = 600$$

From the geometry of T_1, we observe

$$H_1 = \tfrac{8}{6}V_1$$

and substituting gives

$$10V_1 - \tfrac{8}{6}V_1 = \tfrac{52}{6}V_1 = 600$$

and thus

$$V_1 = \tfrac{6}{52}(600) = 69.23 \text{ lb}$$

Then

$$H_1 = \tfrac{8}{6}V_1 = \tfrac{8}{6}(69.23) = 92.31 \text{ lb} = H_2$$

$$V_2 = 100 - V_1 = 100 - 69.23 = 30.77 \text{ lb}$$

and, from the vector additions, $T_1 = 115.4$ lb and $T_2 = 97.3$ lb.

When a cable is loaded by more than one concentrated load, the rest position of the loaded cable must be found. However, if the location of any single load point is known, the problem is statically determinate.

A cable loaded with a uniformly distributed load along a horizontal span (not along the cable itself), as shown in Figure 3.15*a*, assumes a simple second-degree parabolic form. Referring to the free-body diagram in Figure 3.15*b*, it may be observed that the horizontal component of the internal tension is the same for all points along the cable, due to the equilibrium of horizontal forces. The vertical component of the internal force varies as the slope of the cable changes, becoming a maximum value at the supports, and the minimum internal tension will occur at the center of the span. Referring to Figure 3.15*c*

$$V_1 = V_2 = \frac{wL}{2}$$

and a summation of moments about the left supports yields

$$\sum M = \left(\frac{wL}{2} \times \frac{L}{4}\right) - (H \times y) = 0$$

Thus

$$H = \frac{wL^2}{8y}$$

which is the general expression for H in the cable.

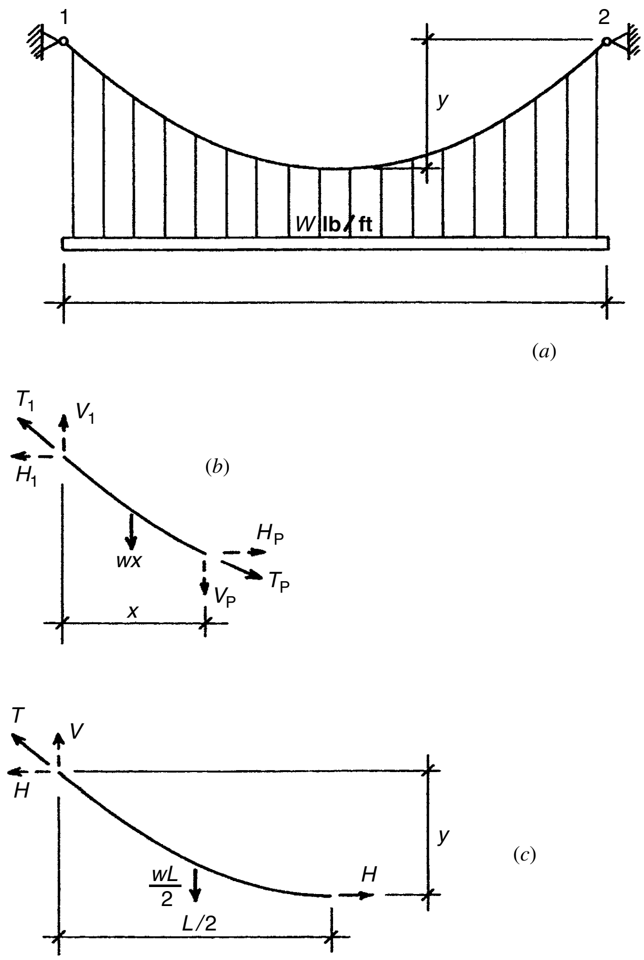


Figure 3.15 Behavior of the uniformly loaded cable.

Combined Action: Tension Plus Bending

Various situations occur in which both axial force of tension and a bending moment exist at the same cross section in a structural member. Consider the hanger shown in Figure 3.16, in which a 2-in.^{2} steel bar is welded to a plate which is bolted to the bottom of a steel beam. A short piece of steel with a hole in it is welded to the face of the bar, and a load is hung from the hole.

In this situation, the steel bar is subjected to a combination of tension and bending, both of which are created by the hung load. The bending occurs because the load is not applied axially to the bar; the bending moment thus produced has a magnitude 2(5000) = 10,000 in.-lb.

For this simple case, the stresses due to the two phenomena can be found separately and added as follows: For the tension effect (Figure 3.16*c*)

$$f_a = \frac{N}{A} = \frac{5000}{4} = 1250 \text{ psi}$$

For the bending we first find the section modulus of the cross section; thus

$$S = \frac{bd^2}{6} = \frac{2(2)^2}{6} = 1.333 \text{ in.}^3$$

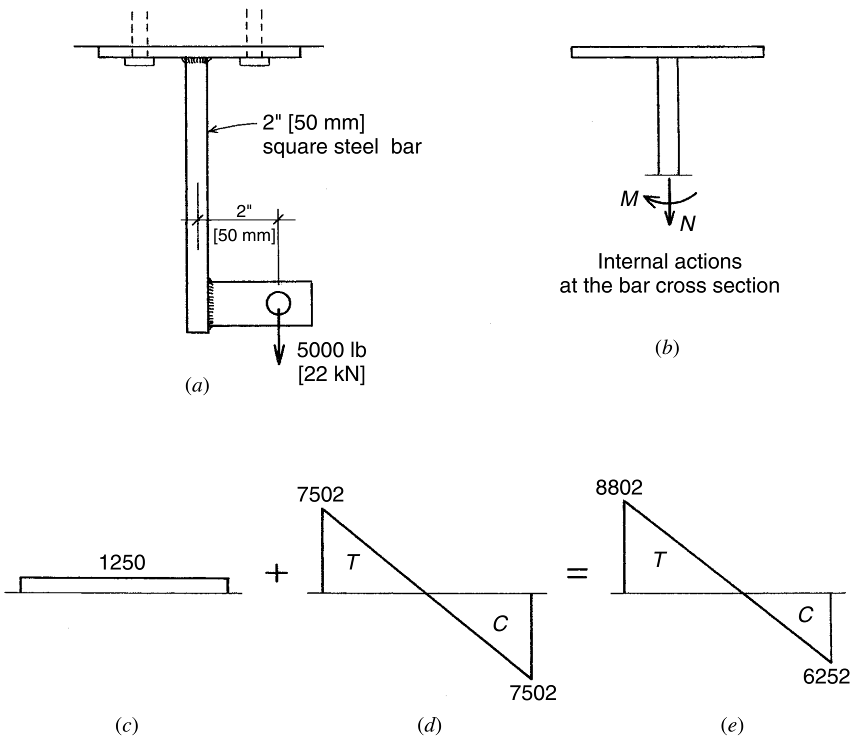


Figure 3.16 Development of the combined stress.

Then, for the bending stress (Figure 3.16*d*)

$$f_b = \frac{M}{S} = \frac{10{,}000}{1.333} = 7502 \text{ psi}$$

and the stress combinations are (Figure 3.16*e*)

$$\text{Maximum } f \text{ in tension: } \quad f = f_a + f_b = 1250 + 7502$$
$$= 8752 \text{ psi}$$
$$\text{Minimum } f \text{ in tension: } \quad f = f_a - f_b = 1250 - 7502$$
$$= -6252 \text{ psi}$$

with the minus sign indicating that this is actually a compression stress.

This simple analysis is probably adequate for this situation, with a relatively short hanger bar. However, when the tension member has other cross sections, the design codes may require a different investigation, described as an *interaction* condition. This type of investigation will be illustrated in the discussion of compression elements.

3.3 COMPRESSION ELEMENTS

Compression is developed in a number of ways in structures, including the compression component that accompanies the development of internal bending resistance and diagonal compression due to shear. In this section we consider elements whose primary structural purpose is the resistance of direct compression.

Types of Compression Elements

A number of types of primary compression elements are used in building structures. Major ones are the following:

Columns. These are usually linear vertical elements, used when supported loads are concentrated, or when a need for open space precludes the use of bearing walls. In various situations columns may also be called *posts*, *piers*, or *struts*.

Piers. This term generally refers to relatively stout columns. The term is also used to describe massive bridge supports, abutments for arches, deep foundation elements cast in excavated holes, and vertical masonry or concrete elements that are transitional between columns and walls.

Truss Members. Compression members in trusses function like columns. They may also be subjected to bending in addition to their primary truss functions.

Bearing Walls. When walls are used for supports, taking vertical compression, they function like very wide columns. Exterior walls, however, are usually also designed for bending due to horizontal forces of wind, earthquakes, or soil pressure.

Slenderness

The general case for axial compression capacity as related to slenderness is indicated in Figure 3.17. The limiting conditions are those of the very stout column (not slender, usually described as *short*) element that fails essentially in compressive stress and the very slender element that has its failure precipitated by buckling.

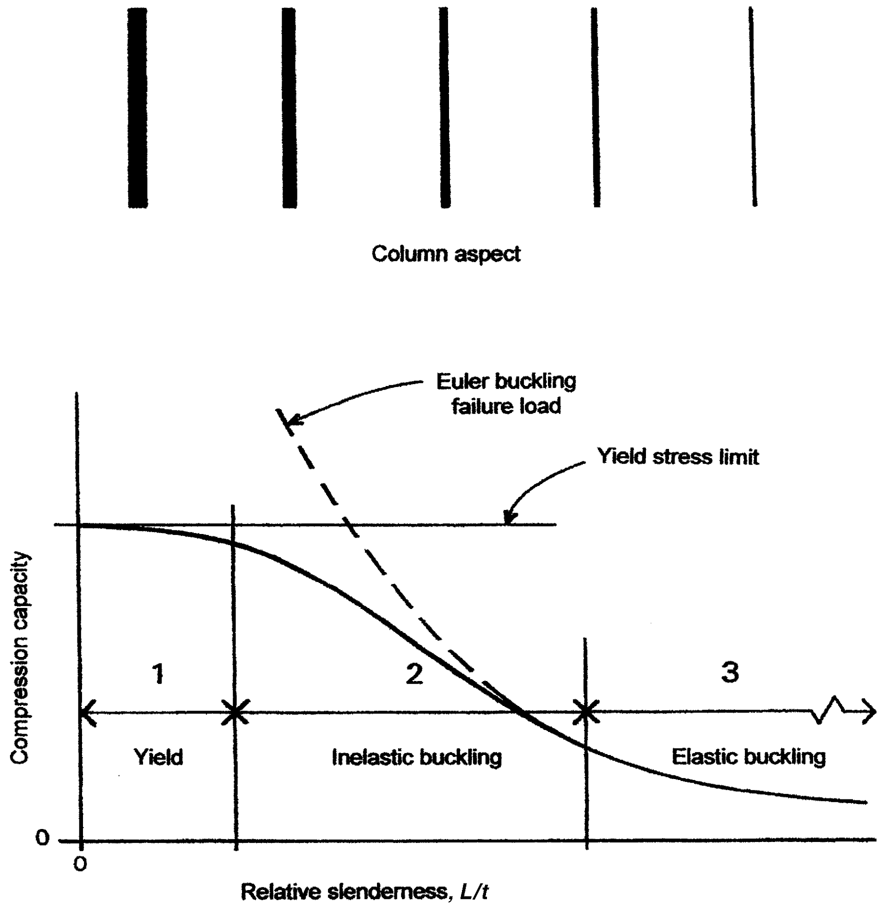


Figure 3.17 Compression capacity of columns, related to relative slenderness.

Short Compression Elements

When subjected primarily to axial compression force, the capacity of the short compression member is directly proportional to the mass of the material and its strength in resisting crushing stresses (Figure 3.17, zone 1). Depending on the origin of the compression load, there may also be some concentrated effects at the point of application of the load that may limit the total load applied, but the strength of the compression element is a function of its cross-sectional area and its crushing resistance. The limit for compression stress depends on the type of material. In Figure 3.17 this limit is indicated as the yield limit, which is appropriate to ductile structural steel.

Slender Compression Elements

Very slender elements that sustain compression tend to buckle (Figure 3.17, zone 3). Buckling is a sudden lateral deflection at right angles to the direction of the compression. If the member is held in position, the buckling may serve to relieve the member of the compressive effort and the member may spring or snap back into alignment if the compressive force is removed. If the force is not removed, the member will quickly fail—essentially by bending.

Compression Elements of Intermediate Slenderness

The two limiting response mechanisms—crushing and buckling—are distinctly different in nature, relating to different properties of the material and of the form of the element. At intermediate points on the response graph in Figure 3.17 the curve achieves a transition between these conditions, having some aspects of both responses. As it happens, this is where most building columns fall, being neither very stout nor very skinny.

The classic means for describing elastic bulking is the Euler curve, having form

$$P = \frac{\pi^2 EI}{L^2}$$

From the form of this equation, it may be observed that the pure elastic buckling response is:

Proportional to the stiffness of the material of the member (E)

Proportional to the bending stiffness of the member, indicated by the moment of inertia (I) of its cross section

Inversely proportional to the member length; or, actually, to the seond power of the length. The length in this case is an indication of potential slenderness.

Buckling of columns may be affected by constraints, such as lateral bracing that prevents the sideways deflection, or by end connections that restrain rotation. Figure 3.18*a* shows

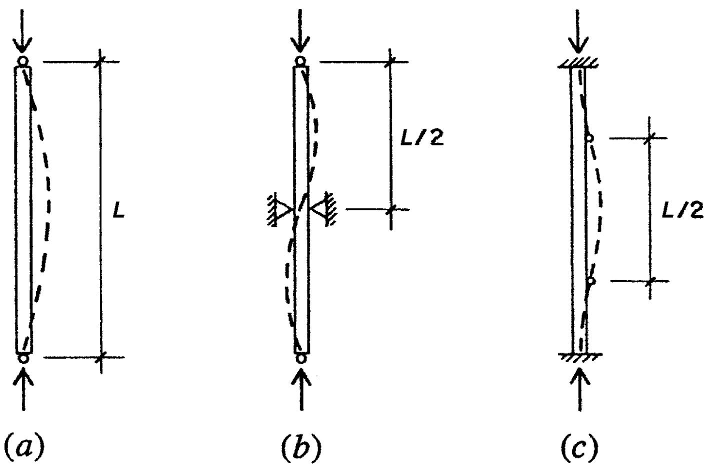


Figure 3.18 Effects of restraints on column buckling.

the case for the member that is the basis for the response indicated by the Euler formula. This form of response may be modified by lateral bracing, as shown in Figure 3.18*b*, in which case the effective buckling length is reduced. Rotational restraint at the column ends, as shown in Figure 3.18*c*, can also produce this effect.

Conditions such as all three cases in Figure 3.18 occur commonly in many building structures. Recommended modifications for these, as well as other restraints, are included in design standards. The usual form of modification is an adjustment of the buckling length to one noted as the *effective buckling length*.

Lack of straightness in an unloaded condition can be a critical factor for the slender column. This condition adds a bending effect to the column when loaded.

Interaction: Compression Plus Bending

There are a number of situations in which structural members are subjected to the combined effects of axial compression plus bending. Stresses developed by these two actions are both of the direct stress type and can be directly combined for the consideration of net stress conditions. This process was demonstrated earlier for the tension hanger shown in Figure 3.16. However, the actions of a column and a bending member are essentially different in character, and it is therefore customary to consider this combined activity by what is described as interaction.

For a given compression member consideration for interaction begins with a definition of the compression capacity without bending and the bending capacity without compression. From the start a basis is established for considerations of cases with both compression and bending.

The classic form of interaction is represented by the graph in Figure 3.19. Referring to the notation on the graph:

The maximum axial compression capacity of the member with no bending is P_o.

The maximum bending moment capacity of the member is M_o.

At a compression load below P_o, indicated as P_n, the member is assumed to have some tolerance for a bending moment, indicated as M_n, in combination with the compression load.

Combinations of P_n and M_n are assumed to fall on a line connecting P_o and M_o. The equation for this line has the form

$$\frac{P_n}{P_o} + \frac{M_n}{M_o} = 1$$

For various reasons, real structural members do not adhere to the classic straight-line form of response as shown in Figure 3.19. Figure 3.20 shows a form of response characteristic of reinforced concrete columns. There is some approximation of the pure interaction response in the middle range of this graph, but major deviation occurs at both ends of the range.

At the low-moment end, where almost pure compression occurs, the column is capable of developing only some percentage of its full, theoretical capacity. There are many reasons for this, mostly having to do with the composite form of the concrete plus steel member.

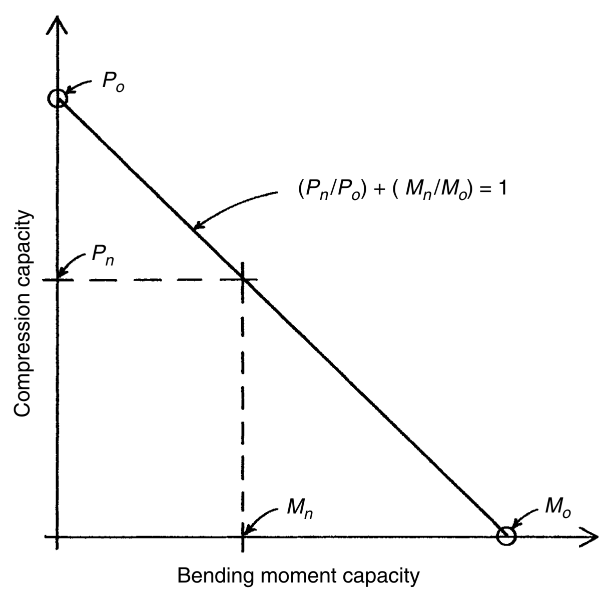


Figure 3.19 Column interaction: compression plus bending.

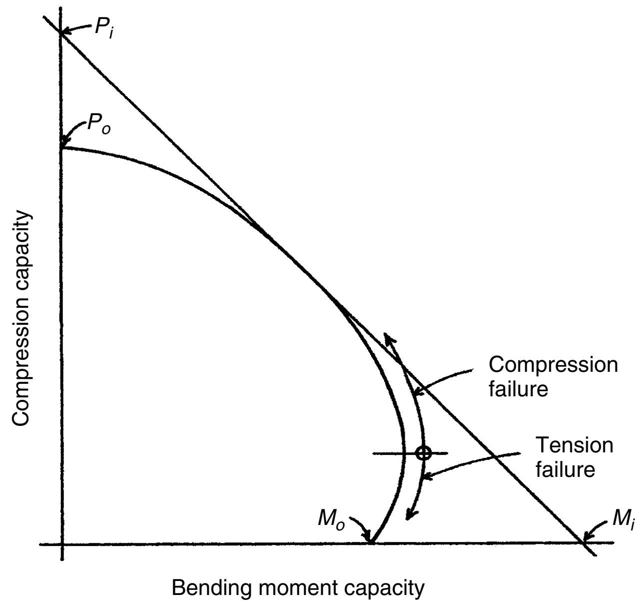


Figure 3.20 Interaction response of a concrete column.

At the high end of the moment range, where little compression occurs, the column tends to function primarily as a beam, with failure characterized as a tension failure due to yielding of the reinforcing steel. Adding compression here tends to increase the moment resistance, counteracting some of the tension. With more compression, however, the failure mode moves back to a compression failure.

There are also deviations from the norm with wood and steel columns, which are discussed in Chapters 4 and 5.

Combined Stress: Compression Plus Bending

Figure 3.21 illustrates an approach to the combined effect of compression and bending moment on a cross section. In this case the "section" is actually the contact face between the footing and the supporting soil. However the combined force and moment may originate, we make a transformation into an equivalent eccentric (nonaxial) force N with an eccentricity of e that produces the same effect on the cross section.

The value of e is established simply by dividing the moment by the force normal to the cross section, as shown in the figure. The net, or combined, stress distribution on the

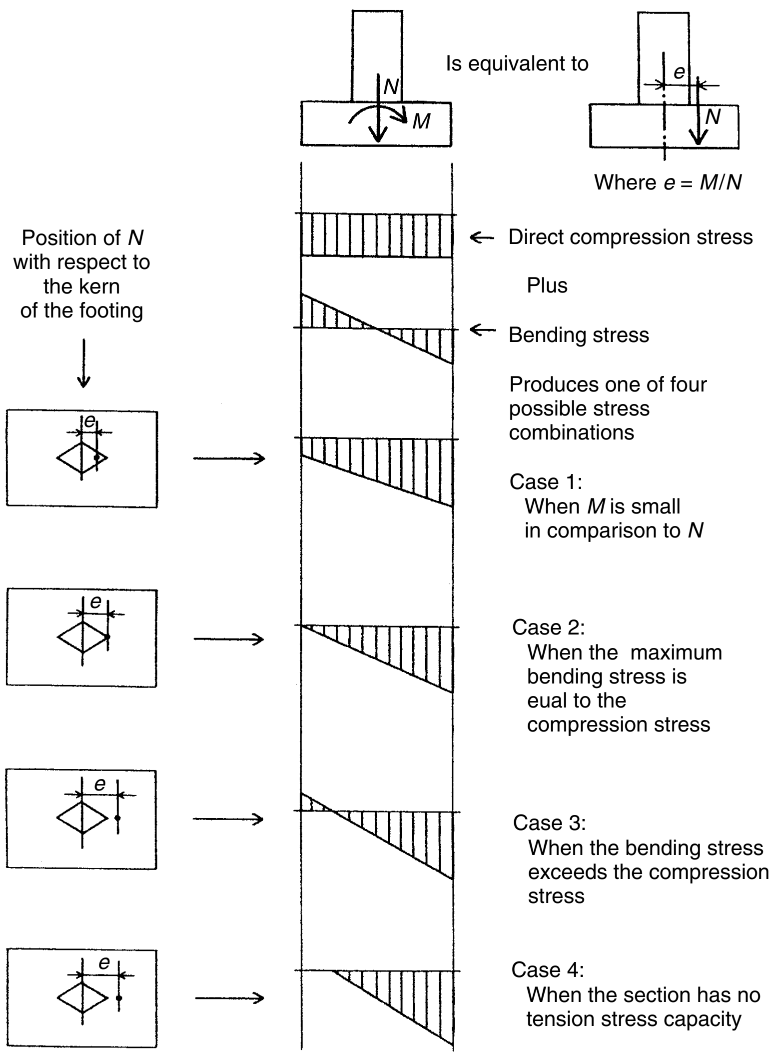


Figure 3.21 Combined stress due to compression plus bending.

section is visualized as the sum of the separate stresses due to the normal force and the moment. For the stresses on the two extreme edges of the footing the general formula for the combined stress is

$$p = \frac{N}{A} \pm \frac{Nec}{I}$$

As shown in Figure 3.21, this produces one of three possible situations for the net stress.

The first case occurs when e is small, resulting in very little bending stress. The section is thus subjected to all compressive stress, varying from a maximum value on one edge to a minimum value on the opposite edge.

The second case occurs when the two stress components are equal, so that the minimum edge stress becomes zero. This is the boundary condition between the first and third cases, since any increase in the eccentricity will tend to produce some net tension stress on the section. This is a significant limit for the footing since tension stress is not possible for the soil-to-footing contact face. Thus case 3 is only possible in a beam or column where tension can be developed. The value for e that corresponds to case 2 can be derived by equating the two components of the combined stress; thus

$$\frac{N}{A} = \frac{Nec}{I}, \qquad e = \frac{I}{Ac}$$

This value establishes what is called the *kern limit*, or simply the *kern* of the section.

The kern is a zone around the centroid of the section within which an eccentric force will not cause tension on the section. The form of this zone may be established for any shape of cross section by application of the formula derived for the kern limit. The forms of the kern limit zones for three common shapes of section are shown in Figure 3.22.

When tension stress is not possible, eccentricities beyond the kern limit will produce a *cracked section*, which is shown as case 4 in Figure 3.21. In this situation some portion of the section becomes unstressed (or cracked), and the compressive stress on the remainder of the section must develop the entire resistance to the compression and bending moment.

Figure 3.23 shows a technique for the analysis of the cracked section, called the *pressure wedge method*. The pressure wedge represents the total compressive force developed by the soil pressure. Analysis of the static equilibrium of this wedge and the force and moment on the section produces two relationships that may be utilized to establish the dimensions of the stress wedge. These relationships are as follows:

The total volume of the wedge is equal to the vertical force on the section. (Sum of the vertical force equals zero.)

The centroid of the wedge is located on a vertical line with the eccentric force on the section. (Sum of the moments on the section equals zero.)

Referring to Figure 3.23, the three dimensions of the stress wedge are w, the width of the footing; p, the maximum soil pressure; and x, the limit of the uncracked portion of the section. With w known, the solution of the wedge analysis consists of determining p and x. For the rectangular footing, the triangular stress wedge will have its centroid at the third point of the triangle. As shown in the figure, this means that x will be three times the dimension a. With the value for e determined, a may be found and the value for x established.

The volume of the stress wedge may be expressed in terms of its three dimensions as follows:

$$V = \tfrac{1}{2}wpx$$

Using the static equilibrium relationship stated previously, this volume may be equated to the force on the section. Then, with the values of w and x established, the value of p may be found as follows:

$$N = V = \tfrac{1}{2}wpx, \qquad p = \frac{2N}{wx}$$

Example 5. Find the maximum soil pressure for a square footing that sustains a compression force of 100 kips and a moment of 100 kip-ft. Find the pressure for footing widths of (a) 8 ft, (b) 6 ft, and (c) 5 ft.

Solution. The first step is to determine the equivalent eccentricity; thus

$$e = \frac{M}{N} = \frac{100}{100} = 1\text{ ft}$$

Then, for (a), the kern limit is $\frac{8}{6} = 1.33$ ft, which establishes case 1 in Figure 3.21, and the pressure is determined as

$$p = \frac{N}{A} + \frac{Mc}{I} = \frac{100}{64} + \frac{100 \times 4}{341.3} = 1.56 + 1.17 = 2.73\text{ ksf}$$

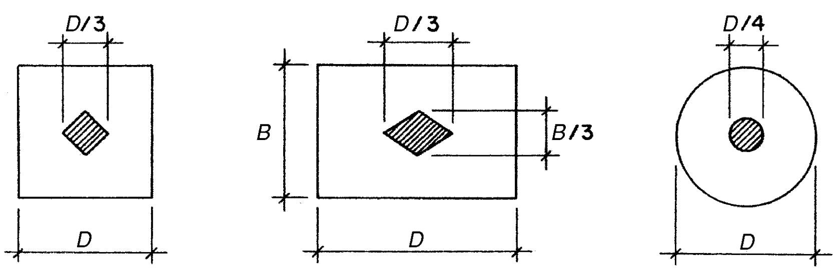


Figure 3.22 Kern limits for common shapes.

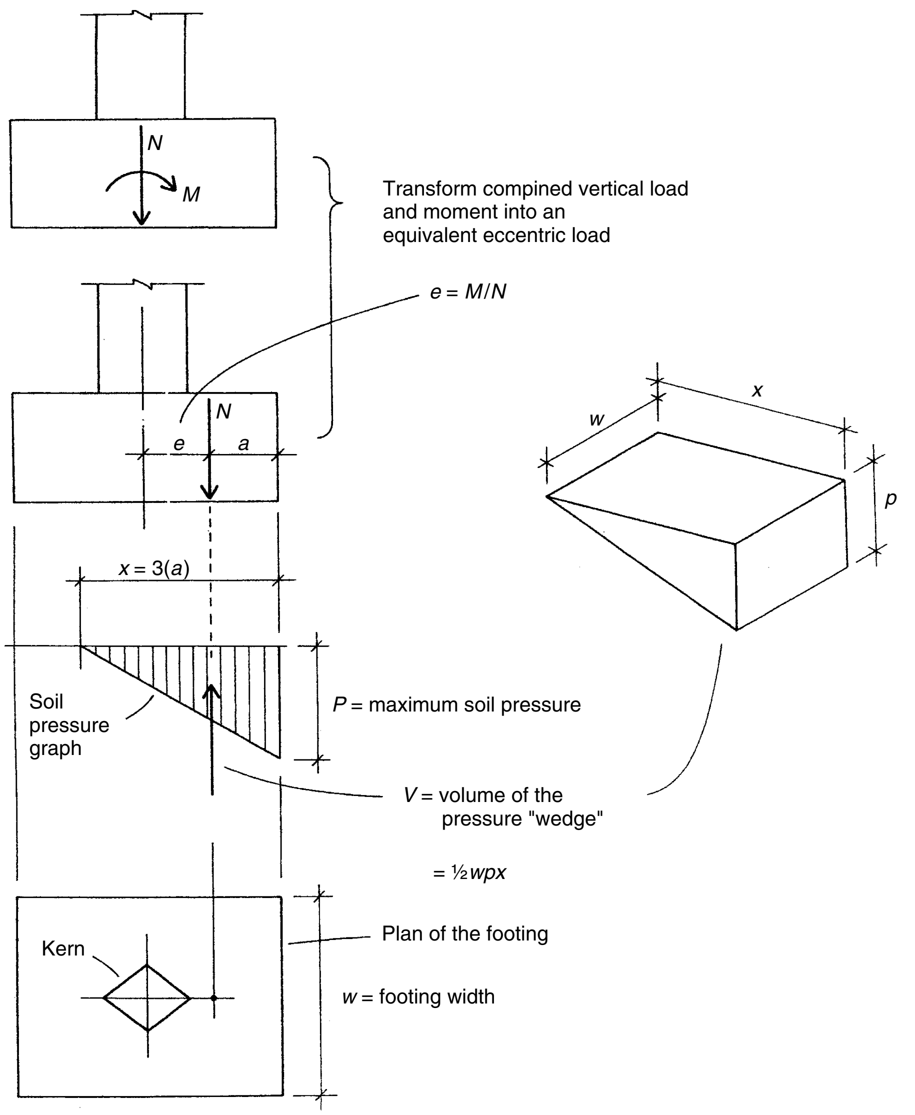


Figure 3.23 Analysis of the cracked section.

in which

$$A = (8)^2 = 64 \text{ ft}^2$$

$$I = \frac{bd^3}{12} = \frac{(8)^4}{12} = 341.3 \text{ ft}^4$$

$$c = \frac{w}{2} = 4 \text{ ft}$$

For (b) the kern limit is $\frac{6}{6} = 1$ ft, which is equal to e. Then from case 2 in Figure 3.21,

$$p = 2\left(\frac{N}{A}\right) = 2\left(\frac{100}{36}\right) = 5.56 \text{ ksf}$$

For (c) the eccentricity exceeds the kern limit, so the analysis must be done as shown in Figure 3.23. Thus

$$a = \frac{w}{2} - e = \frac{5}{2} - 1 = 1.5 \text{ ft}$$

$$x = 3a = 3(1.5) = 4.5 \text{ ft}$$

$$p = \frac{2N}{wx} = \frac{2(100)}{5(4.5)} = 8.89 \text{ ksf}$$

Bending in Columns

Bending moments can be developed in structural members in a number of ways. When the member is subjected to an axial compression force, there are various ways in which the bending effect and the compression effect can relate to each other. Figure 3.24*a* shows a common situation that occurs when an exterior wall functions as a bearing wall or contains a column. The combination of gravity load and lateral load due to wind or seismic action can result in the loading shown. If the member is quite flexible and the deflection is considerable, an additional moment is developed as the axis of the member deviates from the action line of the compression force. This moment is the simple product of the compression force and the lateral deflection of the member at any point; that is, P times Δ, as shown in Figure 3.24*d*. It is thus referred to as the P–delta effect. There are various other situations that can result in this effect.

Figure 3.24*b* shows an end column in a rigid-frame structure where bending is induced in the top of the column due to the moment in the end of the beam. Figure 3.24*c*

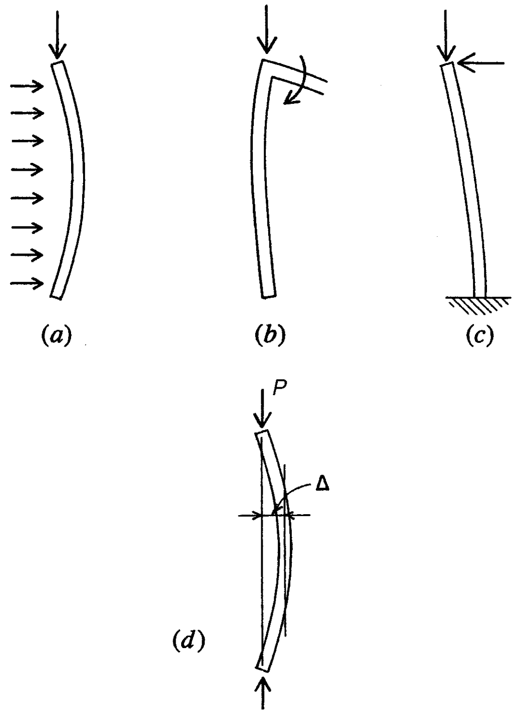


Figure 3.24 Bending in columns.

shows the effect of a combination of gravity load and lateral load applied at the top of a cantilevered column. The effects shown in Figures 3.24*b* and *c* will be combined when a rigid frame is subjected to combined lateral and gravity loads.

In all of these, as well as other situations, the P–delta effect may or may not be critical. The major factor that determines its seriousness is the relative flexibility of the structure in general, but particularly the stiffness of the member that directly sustains the effect. In a worst-case scenario, the P–delta effect causes an additional deflection, which in turn results in an additional P–delta effect, which in turn causes more deflection, and so on, resulting in a progressive failure.

In most cases, the P–delta effect will be combined with other conditions. For the slender element, it may work to precipitate a buckling failure. In other cases, the moment due to the P–delta effect is simply combined with the moment from other actions for an interaction or combined stress condition.

Compression of Confined Materials

Solid materials have the capability to resist linear compression effects. Fluids can resist compression only if they are in a confined situation, such as air in an auto tire or oil in a hydraulic jack. Compression of a confined material results in a three-dimensional compressive stress condition, visualized as a triaxial condition, as shown in Figure 3.25.

A major occurrence of the triaxial stress condition is that which occurs in loose soil materials. The supporting soils for foundations are typically buried below some amount of soil overburden. The upper soil mass—plus the general effect of

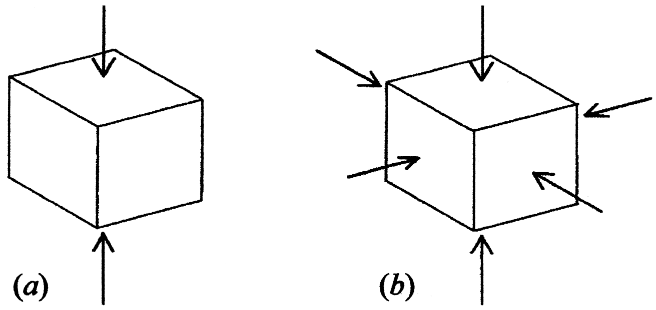


Figure 3.25 Effect of constraint on compression stress.

the surrounding soils—creates a considerable confinement. This confinement permits some otherwise compression-weak materials to accept some amount of compressive force. Loose sands and soft clays must have this confinement, although they are still not very desirable bearing materials, even with confinement. The continued presence is necessary for their stability and removal or reduction of confining soils can cause problems.

While confinement is mandatory for loose or fluid materials, it can also enhance the compression of solids. An example of this is the concrete at the center of a reinforced concrete column, typically constructed with a wrapping of steel ties or spirals. At loads near the ultimate strength of the concrete, the confinement can significantly increase the compression capacity of the column.

Confinement is a basic technique used for air-inflated structures. A simple case is the membrane-wrapped building with its surface held in position by holding a difference of air pressure between the inside and outside. Another possibility involves the inflation of an object to make it rigid (like an air mattress or life raft) and then using it to serve some structural function.

For relatively massive structures, such as large piers or very thick walls, confinement can enhance the strength with what may otherwise be relatively weak materials. Adobe (sun-dried mud) and other weak masonry may resist little compression when a single unit is crushed. Yet a thick construction is strong—mostly due to the confinement of the materials near the center of the mass.

3.4 TRUSSES

Trussing is essentially a means of stabilizing a framework of linear elements by arranging them in triangulated patterns. Figure 3.26*a* shows a simple frame of four members with connections achieved with joints characterized as *pinned*. This type of joint is common in wood and steel structures, being one that can transmit direct or shear forces, but has little or no resistance to moment.

Under the action of lateral force, the frame shown in Figure 3.26*a* will collapse sideways. The addition of the diagonal member, as shown in Figure 3.26*b*, is a means of stabilizing the frame, the device being to reduce the rectangle

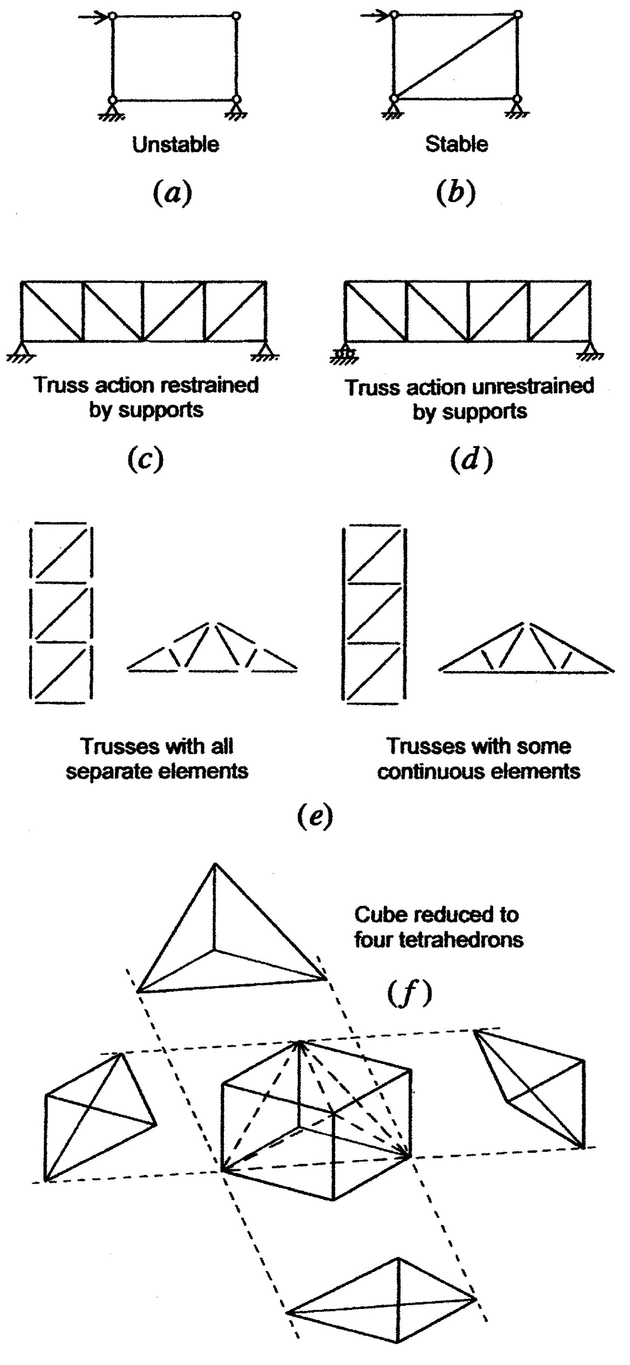


Figure 3.26 Aspects of trusses.

to two triangles. The frame is thus converted to a vertically cantilevered truss.

In a pure truss the members are all considered to be connected by pins, thus being subject only to axial tension or compression. In a truss of slender elements, with loads applied only at the joints, this condition is mostly true. However, in some situations joints may actually be quite rigid, members may be short and stout, and loads may be applied directly to members. For any of the latter conditions, the truss members may have significant additional structural actions, which must be added to those developed by truss functions.

Interior triangulation is a basic necessity for a truss. In addition, the external supports must have certain qualifications. There must be sufficient reaction components for the stability of the truss, but the truss must also be allowed to deform freely under actions of the loads. Consider the truss in Figure 3.26*c*, which is supported at its ends. Vertical reactions are sufficient for resistance to vertical loads. Horizontal reaction components are zero for vertical loading but are potentially useful for resisting any horizontal loads. With vertical loads, the bottom chord is subjected to tension and tends to elongate. This is not possible, however, if both of the supports resist horizontal motion; thus the truss as supported is not able to function properly. If one of the supports has its horizontal resistance removed, as shown in Figure 3.26*d*, the truss is still stable and is capable of the necessary deformation.

A simplification often used for small trusses is that of having some members continuous through the joints (see Figure 3.26*e*). If members remain reasonably flexible, this does not usually affect the basic truss action, except to reduce slightly the truss deflections.

Trussing may also be used in three dimensions. Whereas the triangle is the basic unit of the planar truss, the tetradedron (four-sided solid) is the basic unit of the spatial truss. Although other geometries are possible, three-dimensional trussing is often achieved by trussing of the three basic, mutually perpendicular planes of the orthogonal system (x–y–z); each plane being separately developed as a planar trussed system, as shown in Figure 3.26*f*.

Truss forms derive from many considerations of usage. Figure 3.27 shows a number of common truss forms for both simple flat-spanning elements and planar bents. Some traditional truss forms have been named for the designers who first developed them. As the size of a truss increases, the amount of interior triangulation increases in order to keep individual members relatively short.

Truss Loading

For truss behavior—involving only tension or compression in the truss members—we assume all loads to be applied only at joints of the truss. This may be literally true for some loads when they are applied through framing that occurs at the location of the truss joints. However, loads are frequently applied directly to the truss members; thus the members affected have two functions: as truss members (taking tension or compression) and as directly loaded members (taking bending and shear).

For investigation of the whole truss, we consider only joint loadings. Loads that are distributed otherwise are collected as joint loads for the truss analysis. Then in the design of the individual members the true loading is considered and the combined actions required of individual members are recognized.

Design Forces for Truss Members

The primary concern in analysis of trusses is the determination of the critical forces for which each member of the truss must

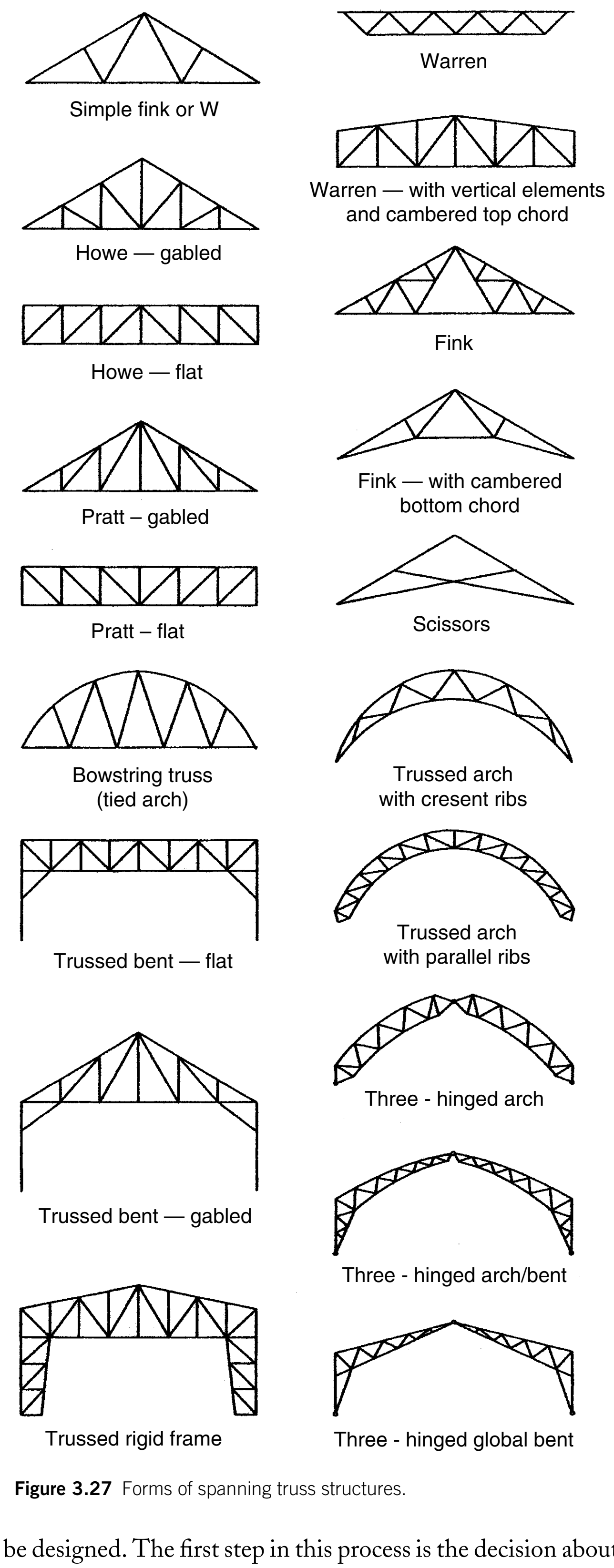


Figure 3.27 Forms of spanning truss structures.

be designed. The first step in this process is the decision about which combinations of loading must be considered. In some cases the potential combinations may be quite numerous. Where both wind and seismic actions are potentially critical, and more than one type of live load occurs (such as roof loads plus hanging loads), the theoretical combinations of loadings can be overwhelming. However, designers are usually able to exercise judgment in reducing the sensible combinations to a reasonable number.

Once the required design loading conditions are established, the usual procedure is to perform separate analyses for each of the loadings. The values obtained can then be combined at will for each individual member to ascertain the particular combination that produces the critical result for the member. This sometimes means that certain members will be designed for one combination and others for different combinations.

Individual connections at joints must also be designed for the various load combinations that affect them. Selection of the connection form and the details of the connections will relate to the form of the truss members as well as to the specific loads applied to the joints. There is an unavoidable interaction in the truss design in this regard which requires considerable judgment by designers.

Figure 3.28 shows a typical roof truss in which the actual gravity loading consists of the roof load distributed continuously along the top chords and a supported ceiling load distributed continuously along the bottom chords. The top chords are thus designed for a combination of compression plus bending and the bottom chords for a combination of tension plus bending. These load combinations will somewhat increase the sizes of members over what they might be with loads only at the joints.

Methods of Investigation

Simple, statically determinate, planar trusses are quite easily analyzed for the effects of ordinary loadings. Depending on the complexity of the truss form, its lack of symmetry, and the diversity of loadings, the analysis may be done by one of several methods. These are as follows:

Graphical Analysis. This method is explained in Section 2.2; it consists of successive applications of the use of force polygons for the joints. If done at a sufficient size and with a reasonable degree of accuracy, it can produce data adequate for most design work.

Algebraic Method of Joints. This is the algebraic correlative of the graphical method, consisting of successive solutions of the individual concurrent force systems at each truss joint.

Method of Sections. This consists of cutting sections through the truss and considering the free body on either side of the cut section.

Beam Analogy Method. For flat-spanning, parallel-chord trusses, this is a shortcut method using the method of sections, in which the variations observable on the shear and moment diagrams for the span are used for rapid determination of member forces.

Computer-Aided Methods. Programs are available for determination of member forces, once the truss layout is established and load conditions are defined.

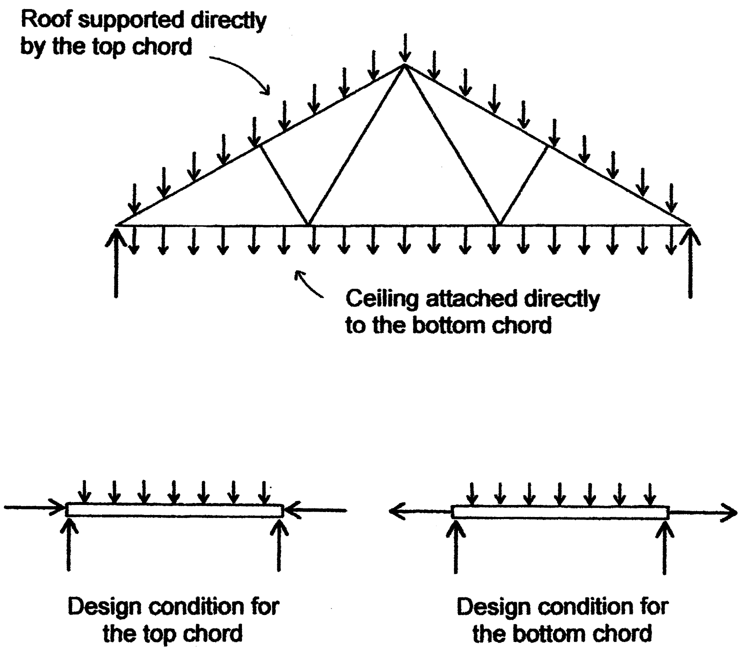


Figure 3.28 Effects of loads applied directly to truss chords.

The method to be used in a particular situation may depend on many factors. Computer-aided methods offer many advantages to the professional designer. However, truss design is not usually an everyday occurrence and may not justify an investment in computer software or the time to learn how to use it. Unless an exceptionally complex truss form is required, or a highly optimized design is desired, simpler hand methods may be preferred. For a simple, modest-sized truss with ordinary static loads, hand analysis will probably be quick and adequate.

Truss Deflection

When used in situations where they are most capable of being utilized to the best of their potential, trusses will seldom experience critical deflections. In general, trusses possess great stiffness in proportion to their mass of materials. When the deflection of a truss is significant, it is usually the result of one of two causes. The first of these is the ratio of the truss span to the depth. For efficiency of the truss, this ratio is usually quite low when compared to the normal ratio for beams. However, when the ratio approaches that for a beam, deflection may well become an issue.

The second—and potentially major—cause for truss deflection is excessive deformation of truss joints. A particular problem is that experienced with trusses that are assembled with ordinary bolts. Because bolt holes must be somewhat larger than the bolts to facilitate assembly, considerable slippage is accumulated when the many joints are loaded. This is a reason for favoring of joints with welds or high-strength bolts for steel trusses and split-ring connectors or other enhancers for wood trusses.

General Considerations for Trusses

A historically common use of the truss is to achieve the simple, double-slope, gabled roof form. This is typically done by use of sloping top chords and horizontal bottom chords, as shown in Figure 3.29. Depending on the size of the span, the interior of the simple triangle formed by the chords may be filled by various arrangements of triangulated members. Some of the terminology used for the components of such a truss, as indicated in Figure 3.29, are as follows:

Chord Members. These are the top and bottom boundary members of the truss, analogous to the top and bottom flanges of a steel I-shaped beam. For trusses of modest size, these members are often made of a single element that is continuous though several joints, with a total length limited only by the maximum-length piece ordinarily obtainable from suppliers.

Web Members. The interior members of the truss are called web members. Unless the truss is very tall, there are usually no interior joints, so web members are ordinarily a single piece between chord joints.

Panels. Most trusses have a pattern that consists of some repetitive modular unit, ordinarily referred to as the panel of the truss. Joints are sometimes referred to as panel points.

A critical dimension for a truss is its overall height, which is referred to as its *rise* or its *depth*. For the truss in Figure 3.29, this dimension relates to the establishment of the roof pitch and also determines the lengths of interior

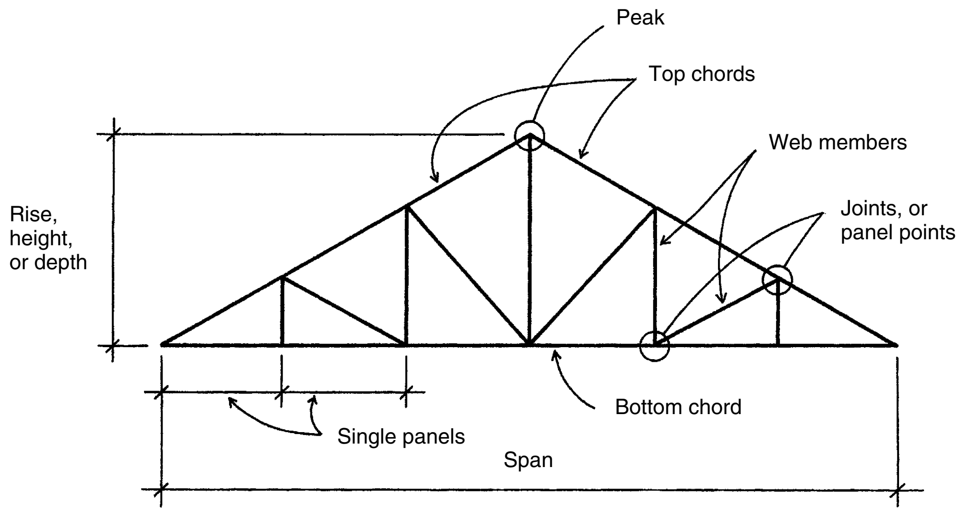


Figure 3.29 Truss terminology.

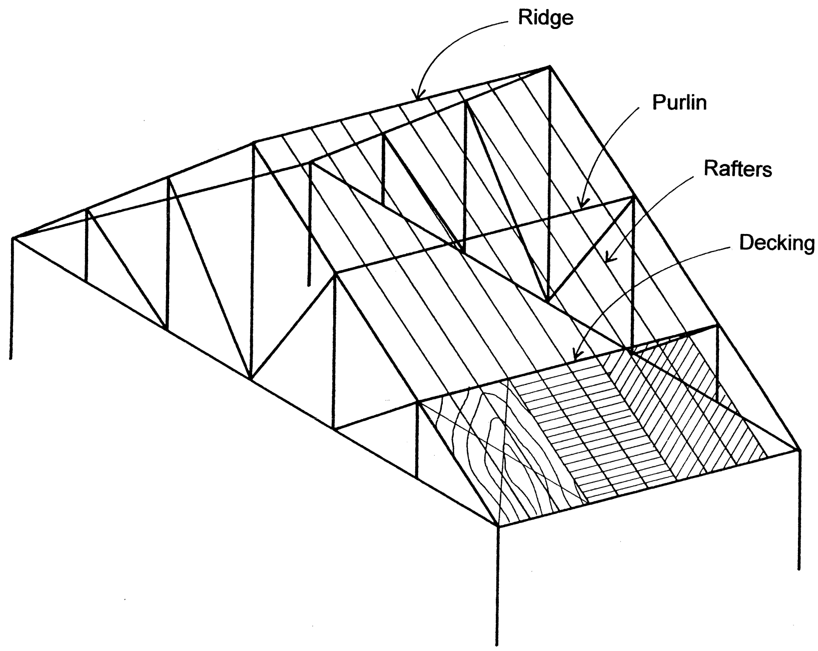


Figure 3.30 A roof structure with trusses.

members. A critical concern with regard to the efficiency of the truss as a spanning structure is the ratio of the span of the truss to its height. Although beams and joists may have span/height ratios of 20 or more, trusses generally require much lower ratios.

Trusses may be used in a number of ways as part of the total structural system for a building. Figure 3.30 shows a series of single-span, planar trusses in the form shown in Figure 3.29, with other elements of the building structure that develop the roof system and provide support for the trusses. In this example, the trusses are spaced a considerable distance apart. For this situation, it is common to use beams (called purlins) to span between the trusses, usually supported at the top chord joints of the trusses to avoid bending in the chords. The purlins, in turn, support a series of closely spaced rafters that are parallel to the trusses. The roof surface is developed with a deck that is attached to the rafters. Thus the roof surface actually floats above the level of the top of the trusses.

Figure 3.31 shows a structure for a flat roof using parallel-chorded trusses. This system may also be used for the flat surface of a floor.

When the trusses are slightly closer together, it may be more practical to eliminate the purlins and to increase the size of the truss top chords to accommodate the additional bending due to the support of the rafters. As an extension of this idea, if the trusses are quite closely spaced, it may be possible to eliminate the rafters and have a slightly heavier deck that is directly supported by the truss top chords. Choices between these options will be influenced mostly by

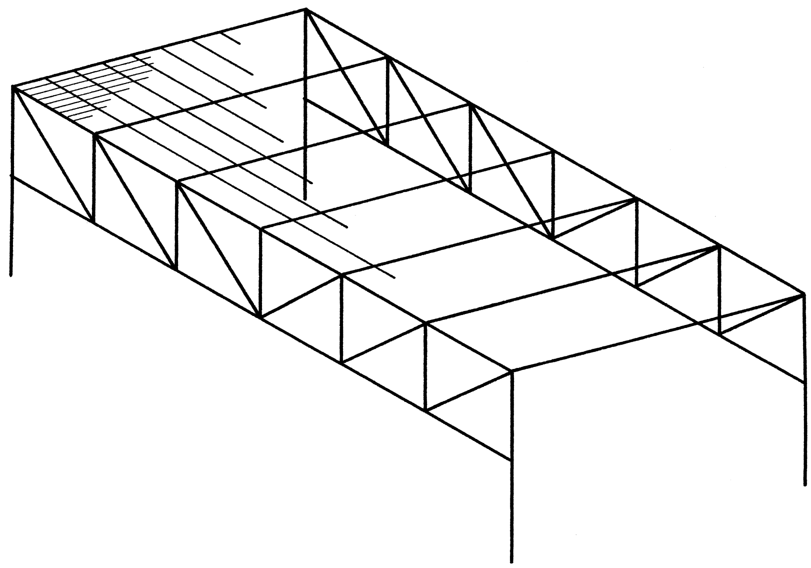


Figure 3.31 Flat-spanning, parallel-chorded trusses.

the materials of the structure (wood or steel) and the size of the truss span.

For various situations, additional elements may be required for the complete structural system. If a ceiling is required, another framing system must be developed at or below the level of the truss bottom chords. Also, if the roof framing and ceiling framing do not provide adequate lateral bracing, other elements must be used for this purpose. Extra structural elements may also be required for the support of heavy equipment for various services, such as heating, ventilation, and air conditioning (HVAC), lighting, audio systems, and so on.

Bracing for Trusses

Single planar trusses are very thin structures that require some form of lateral bracing. The compression chord of the truss must be designed for its laterally unbraced length. In the plane of the truss, the chords are braced by the other truss members at the truss joints. Thus, within the truss plane, the laterally unbraced lengths for all truss members are their own full lengths. However, if there is no lateral bracing system for the trusses, the unbraced lengths of the chords become the full span length—obviously not a feasible situation.

In most buildings, other elements of the construction provide some or all of the lateral bracing for the trusses. In the structural system in Figure 3.32*a*, the top chord of the truss is braced at each truss joint by the purlins. If the roof deck is a reasonably rigid planar structural element and is adequately attached to the purlins, this constitutes a very adequate bracing for the compression chord members, which is a major bracing problem for the truss.

However, it is also necessary to brace the truss generally for out-of-plane movement throughout its height. In Figure 3.32*a* this is done by providing a vertical plane of X-bracing at every other panel point of the trusses. This bracing serves primarily to brace the bottom chords—not for compression buckling, but for general lateral movement involving a roll-over of the trusses. The purlin does an additional service by serving as part of this vertical plane of trussed bracing.

The lateral X-bracing in a single bay, together with two adjacent trusses, forms a structural unit that is fully braced. It would be possible, therefore, to place the X-bracing only in every other bay. However, the X-bracing may also be used for a general bracing of the building, in which case it would need to be continuous.

Light trusses that directly support a deck, as shown in Figure 3.32*b*, are usually adequately braced at the top chord level by the deck that is attached to them. This constitutes continuous bracing, so that the unbraced length of the top chord in this case is virtually zero. Additional bracing in this situation may consist of spaced X-bracing, or simply of continuous ties between trusses at the level of the bottom chords.

Another form of bracing is shown in Figure 3.32*c*, consisting of a horizontal plane of X-bracing at the level of the bottom chords. As in the case of the vertical X-bracing, only alternate bays may be braced, unless the bracing is part of the lateral bracing system for the building.

3.5 RIGID FRAMES

Frames in which members are connected in a manner that permits the transfer of end moments from member to member are commonly called *rigid frames*. Rigid in this case refers to the character of the joints, not necessarily to the deformation character of the whole frame. In fact, many rigid frames have critical deflection problems, and the control of movements—especially sideways movements due to lateral loads—is often an important design factor. Most rigid frames are indeterminate; however, for common situations it is often acceptable to use approximate methods for analysis and design.

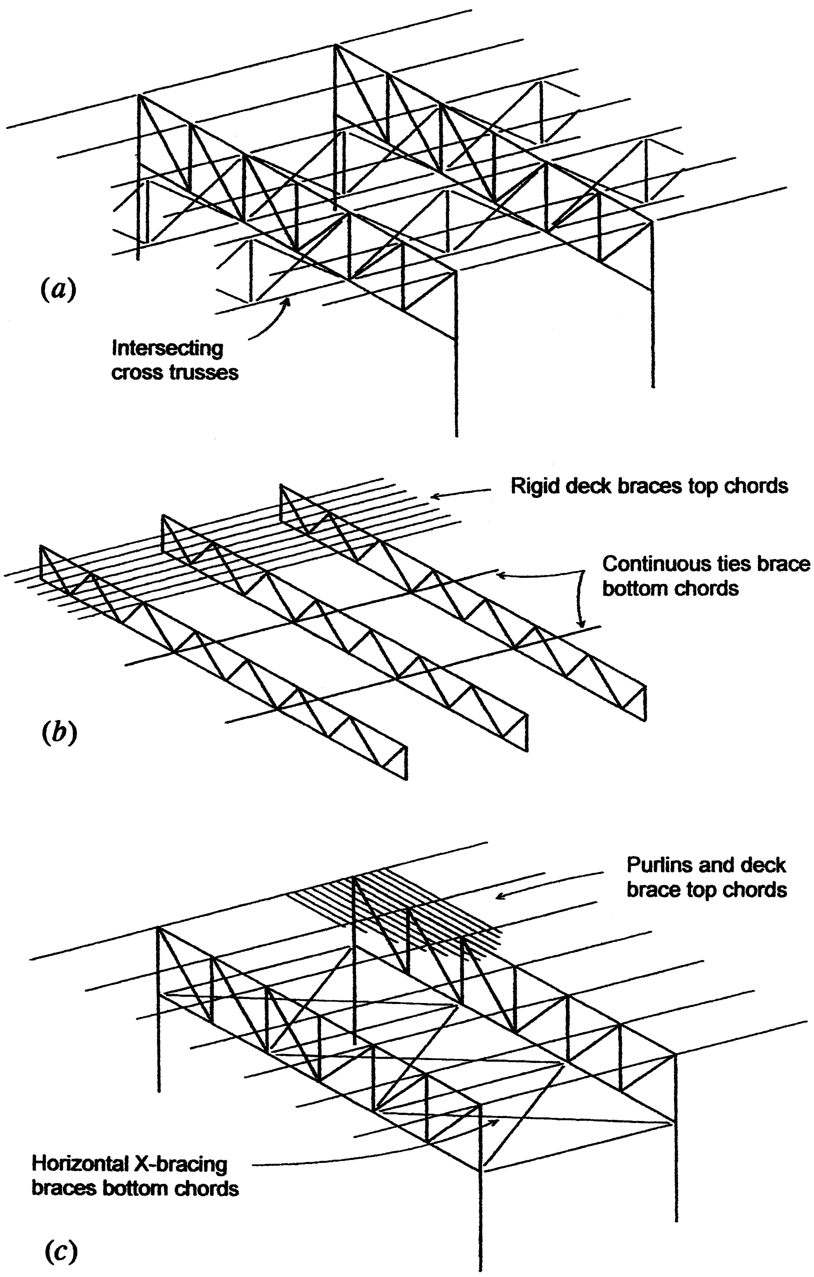


Figure 3.32 Forms of lateral bracing for trusses.

Aspects of Rigid Frames

When members are connected to each other by joints that act essentially as pinned (moment-free) connections, the members are free to deflect and to rotate at the joints without affecting the deformation of the other connected members. When members are rigidly connected, they tend to offer restraint to each other's movements. This can be a positive effect, producing stability of the frame and reducing deflections of spanning members. It can also cause problems in some situations, such as the following:

Unbalanced Loading. When live load is high in comparison to dead load, random loading can cause some members to deform excessively, resulting in transfer of major effects to attached members.

Mismatched Member Sizes. Long-span beams attached to small columns will transfer considerable twist to the columns. Alternate short and long spans of continuous beams will result in serious deformation of the short-span members. Frame layout and lengths and sizes of members must be matched in ways that control these effects.

Restrained Deformations. Member deflections and joint rotations are natural and necessary to the functioning of the rigid frame. If infilling construction (especially walls) restricts the frame deformations, loads will be transferred to the stiffer restraining construction. In this event, damage can occur if the restraining construction does not have adequate structural capacity.

Because of the interactions of members in rigid frames, each frame member is usually subjected to a combination of internal actions. This is made more complex when the frame is subjected to a number of different load combinations, is three dimensional, has a large number of members, or lacks symmetry. When all of these conditions are present, the investigation of frame behavior and the determination of critical design values for the individual members become arduous chores.

Simple Determinate Frames

Consider the frame shown in Figure 3.33*a*, consisting of two members rigidly joined at their intersection. The vertical member is fixed at its base, providing the necessary support conditions for stability of the frame. The horizontal member is loaded with a uniformly distributed loading and functions as a simple cantilever beam. The frame is described as a *cantilever frame* because of the single fixed support. The five sets of figures shown in Figures 3.33*b* through *f* are useful elements for the investigation of the behavior of the frame:

Free-Body Diagram of the Whole Frame (Figure 3.33*b*). This shows the layout and the reaction components at the support.

Free-Body Diagrams of the Members (Figure 3.33*c*). These are of great value in visualizing the equilibrium of the individual members and the interactions of the members. They also help to visualize the nature of internal forces in the members.

Shear Diagrams of Members (Figure 3.33*d*). These help mostly to visualize how moment is developed in the members.

Moment Diagrams for the Individual Members (Figure 3.33*e*). These are very useful in determining the deformation of the frame. The sign convention used is to plot the moment on the compression side of the member.

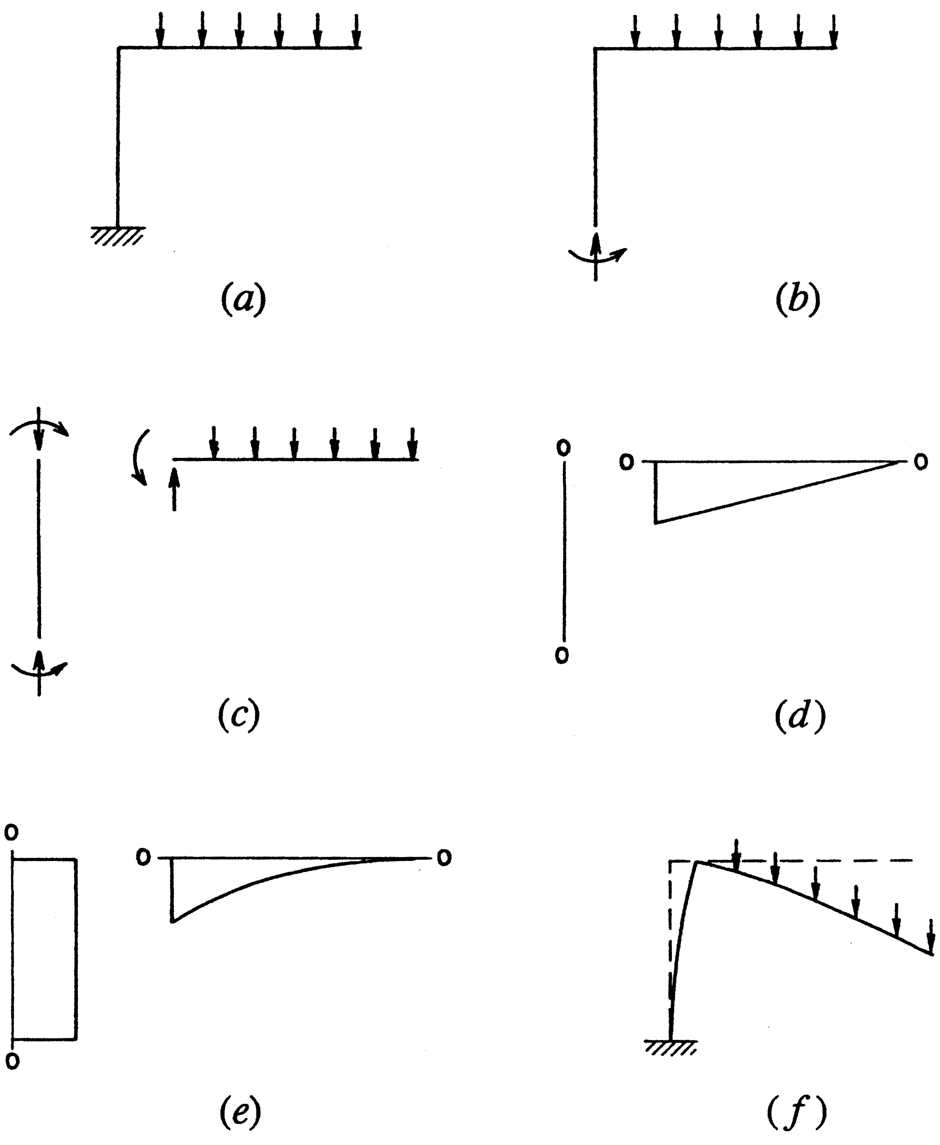


Figure 3.33 Investigation of the cantilever rigid frame.

Deformed Shape of the Loaded Frame (Figure 3.33*f*). This is an exaggerated profile of the bent frame, superimposed on an outline of the unloaded frame for reference. It is very useful for visualization of the frame behavior.

The following examples illustrate the process of investigation for simple cantilevered frames.

Example 6. Find the components of the reactions and draw the free-body diagrams, shear and moment diagrams, and the deformed shape of the frame shown in Figure 3.34*a*.

Solution. The first step is to determine the reactions. From the free body of the whole frame in Figure 3.34*b*,

$$\sum F_v = 0 = -8 + R_v, \qquad R_v = 8 \text{ kips} \quad \text{(up)}$$

and, with respect to the support,

$$\sum M = 0 = M_R - (8 \times 4)$$

$$M_R = 32 \text{ kip-ft} \quad \text{(clockwise)}$$

Note that the sense, or sign, of the reaction components is determined from the logical development of the free-body diagram of the frame.

Considerations of the free-body diagrams of the members will yield the actions of the joints in transmitting moments to other members. Each member free body is treated separately for its equilibrium.

In this frame there is no shear in the vertical member of the frame; thus there is no variation in moment from the top to the bottom of the member.

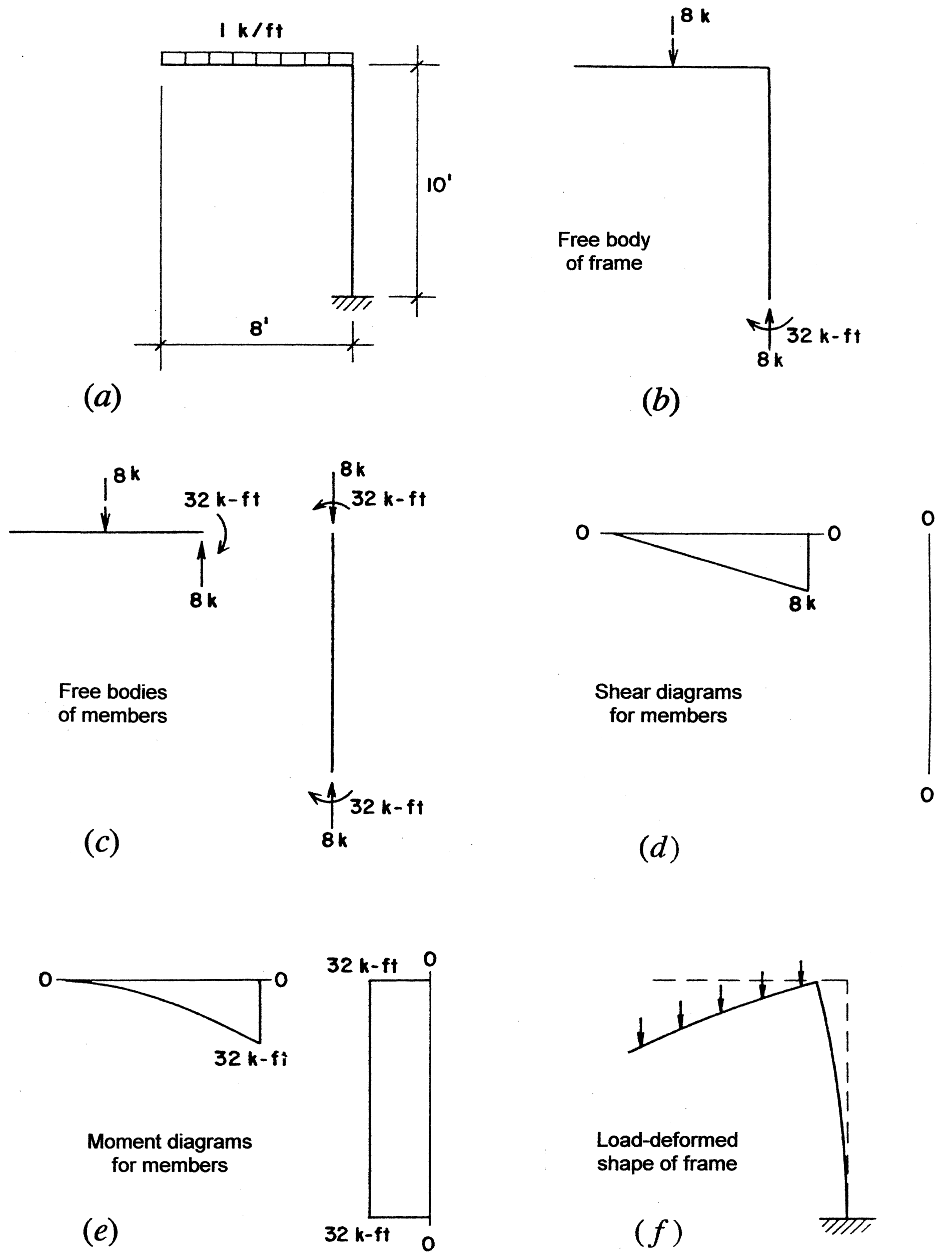


Figure 3.34 Example 6.

Example 7. Find the components of the reactions and draw the free-body diagrams, shear and moment diagrams, and the deformed shape of the frame shown in Figure 3.35*a*.

Solution. In this frame there are three reaction components required for stability, since the loads and reactions form a general coplanar force system. Using the free-body diagram of the whole frame, the three equilibrium conditions for a coplanar system are used to find reaction moment and force components.

Note that the inflection point occurs in the larger vertical member because the moment of the horizontal load about the support is greater than that of the vertical load. In this case this computation must be done before the deformed shape can be accurately drawn.

The reader should verify the equilibrium of the member free bodies and the correlation of all the diagrams.

Most spanning rigid frames are statically indeterminate and their investigation for behavior requires more than analysis for simple static equilibrium. The following example presents a special case in which the limited capability of one support reduces the problem to permit static analysis. We offer it as a chance to see some aspects of the spanning bent.

Example 8. Find the components of the reactions and draw the free-body diagrams, shear and moment diagrams, and the deformed shape of the frame shown in Figure 3.36*a*.

Solution. Note that the right-hand support allows for an upward vertical reaction only, whereas the left-hand support

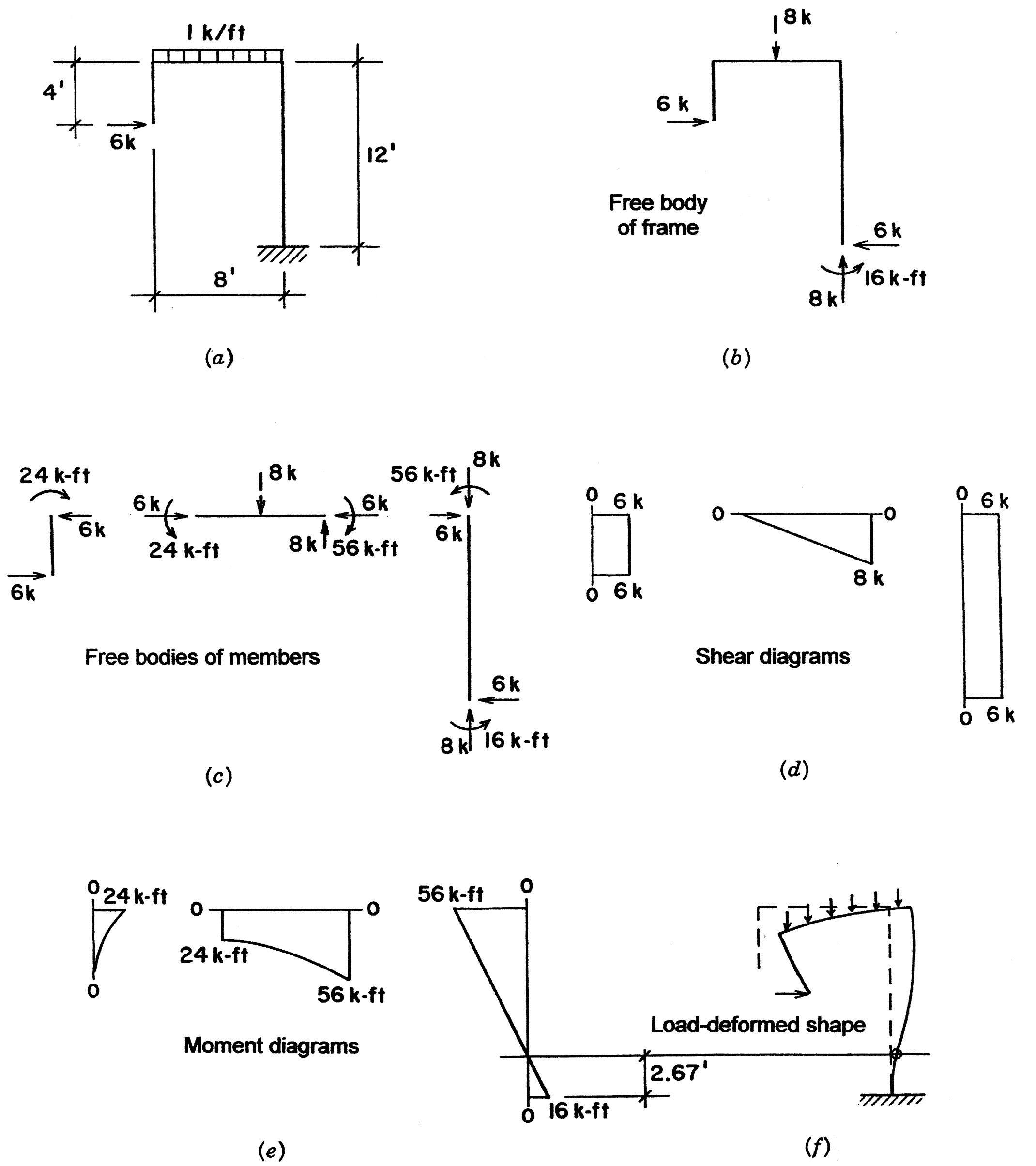


Figure 3.35 Example 7.

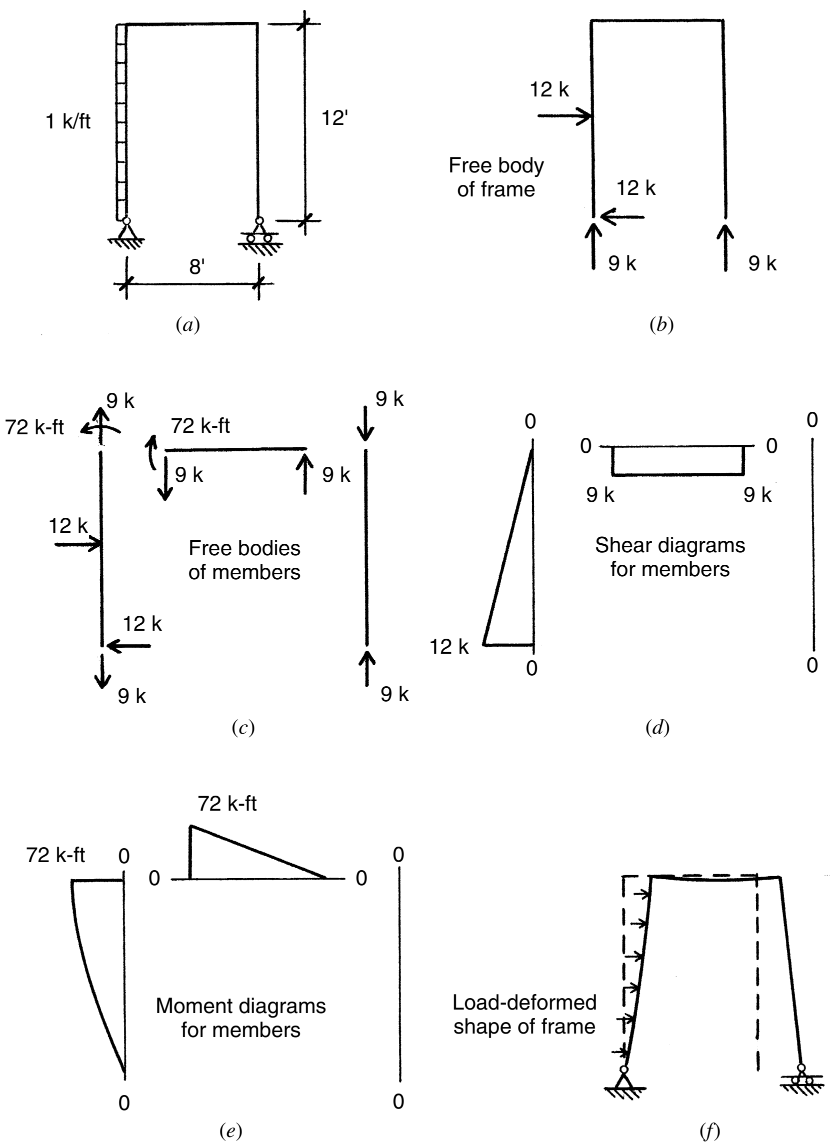


Figure 3.36 Example 8.

allows for both vertical and horizontal components. Neither support provides for moment resistance.

The typical elements of investigation, as illustrated for the previous two examples, are shown in the figure. The suggested procedure for the work is as follows:

Sketch the deflected shape—a little tricky in this case, but a good exercise.
Consider equilibrium of the whole frame to find the reactions.
Consider equilibrium of the left-hand vertical member to find the internal action at its top.
Consider equilibrium of the horizontal member.
Consider equilibrium of the right-hand vertical member.
Draw the diagrams and check for correlation.

Indeterminate Rigid Frames

There are many possibilities for the development of rigid frames for building structures. Two common types of frames are the single-span bent and the vertical, planar bent, consisting of the multistory columns and multispan beams in a multistory building.

As with other complex structures, the highly indeterminate rigid frame presents a good case for the use of a computer-aided process. Programs for this are available and are used routinely by most professional engineering offices.

Rigid-frame behavior is much simplified when the joints of the frame are not displaced; that is, when they move only by rotating, This is usually only true for the case of gravity loading on a symmetrical frame. If the frame is not symmetrical, or the load is not uniformly distributed, or lateral loads are applied, frame joints will likely move sideways (called *sidesway* of the frame) and additional internal forces will be generated by the joint displacements. If joint displacement is considerable, there may be significant increases of forces in various members of the frame.

Lateral deflection of a rigid frame is related to the general stiffness of the frame. When several frames share a loading,

such as in the case of a multistory building, the relative stiffness of individual frames must be determined to find their share of applied loads.

The Single-Span Bent

One commonly used rigid frame is the single-span bent used to create a single, column-free space. It is one of many options for this situation.

Figure 3.37 shows two possibilities for a rigid frame for a single-span bent. In Figure 3.37*a* the frame has pinned bases for the columns, resulting in the load-deformed shape shown in Figure 3.37*c*, and the reaction components are shown in Figure 3.37*e*. The frame in Figure 3.37*b* has fixed bases for the columns, resulting in the modified behavior indicated. These are common situations—the base condition depending on the supporting structure as well as the connections of the columns to the supports.

The frames in Figure 3.37 are both technically statically indeterminate and require analysis by something more than statics. However, if the frame is symmetrical and the loading is uniform, the upper joints will not move sideways and the behavior is of a classic form. Values for the reactions and the internal forces in members can be obtained from tabulations in various references.

Figure 3.38 shows the single-span bent under a lateral load applied at the upper joint. In this case the upper joints move sideways, and the frames take the deformed shape indicated with reaction components as shown. Technically, both of these frames are statically indeterminate, although some reasonable assumptions can reduce the pin-based frame to one that is actually statically determinate.

For the pin-based frame, a moment equation can be written about one column base that will cancel out the moments of the vertical component at that location plus the moments of both horizontal components. That leaves an equation with only one unknown force: the vertical reaction at the opposite base. With no vertical load, the vertical component at the opposite base will have equal magnitude and opposite sign.

If the construction of the column bases is symmetrical, it is reasonable to assume the two horizontal reaction components to be equal; thus they will each be equal to one-half the

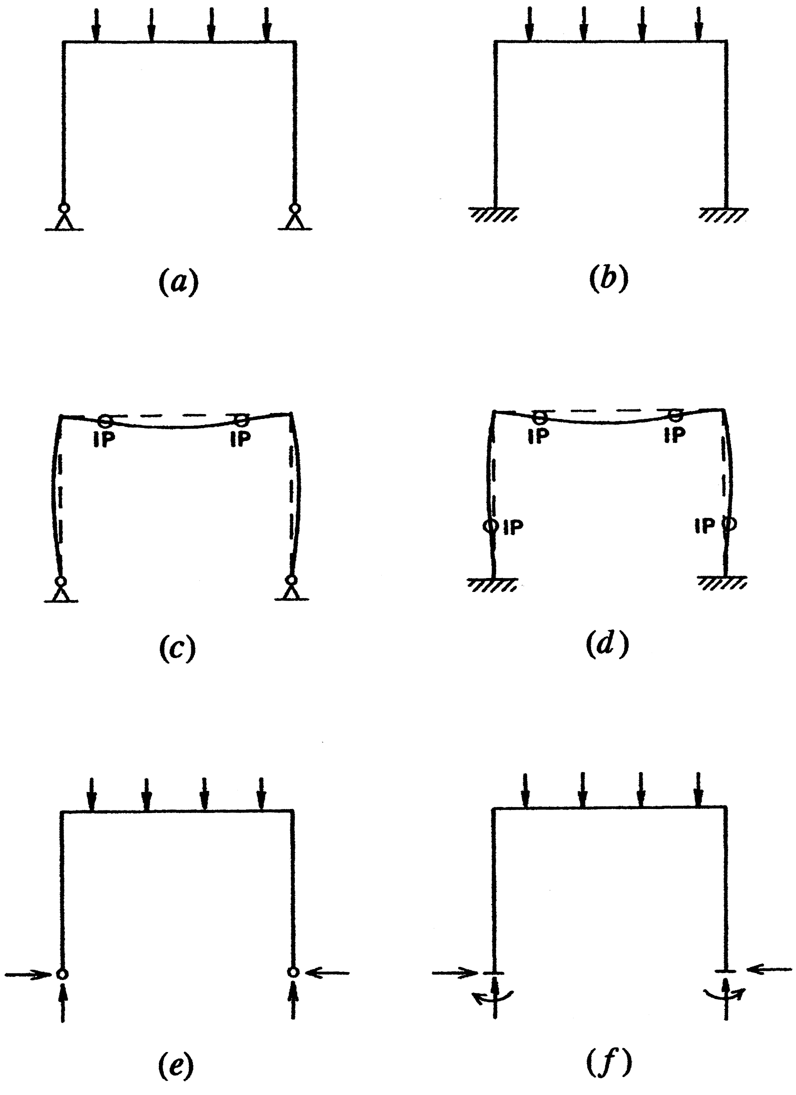


Figure 3.37 Single-span bents with gravity load.

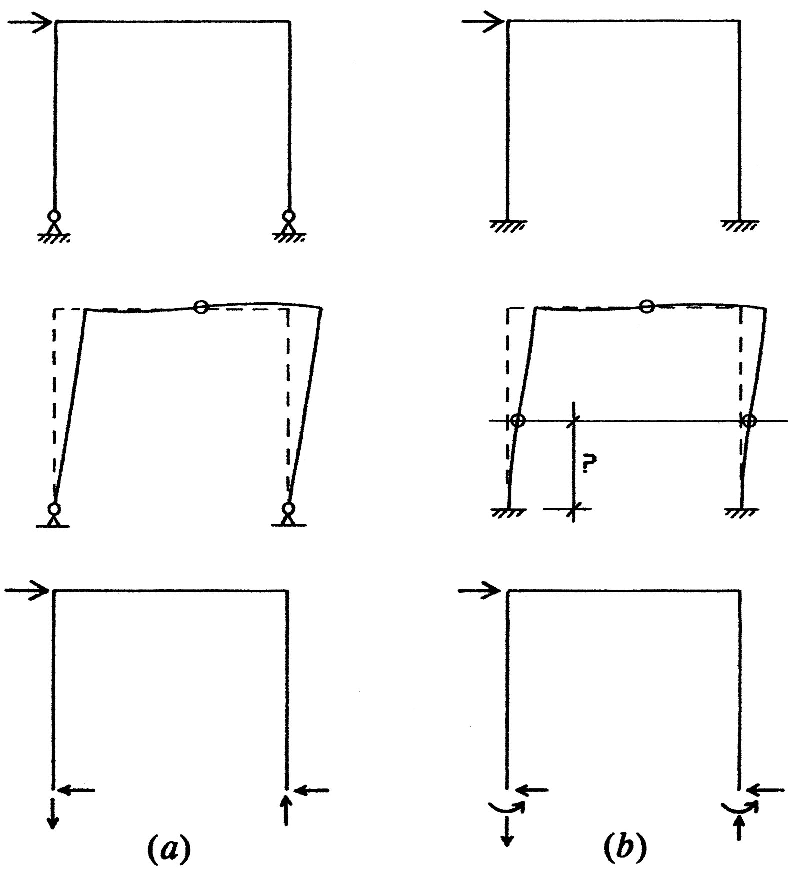


Figure 3.38 Single-span bents with lateral load.

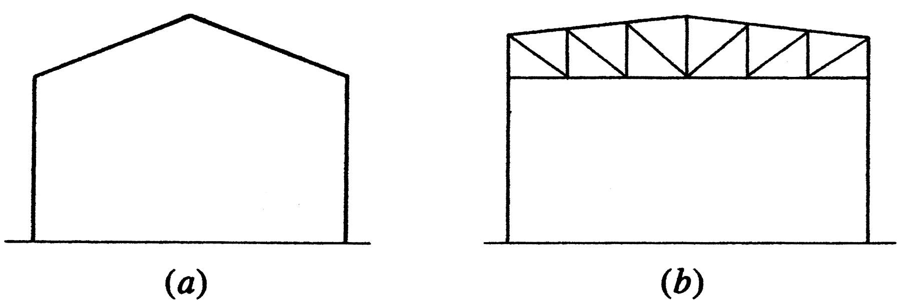


Figure 3.39 The gabled, single-span bent.

load. With the reactions determined, free-body diagrams of the members can be analyzed and the complete behavior of the frame determined as shown in the previous examples. However, the response of the frame to gravity loading remains indeterminate.

The same analysis can be used to find the force components of the reactions for the frame with fixed bases in Figure 3.38*b*. However, analysis for the base moments is still indeterminate.

A popular variation of the single-span bent is the gabled form shown in Figure 3.39*a*. The significant angle of the slope provides both additional interior height and the possibility of using a fast-draining roof covering, which is usually much less expensive and more reliable than roofing for a flat surface. Although the bent may be as shown, with a moment-resistive joint at the peak, it can also be constructed with a pin joint at this location, forming a so-called *three-hinged* structure.

Another usage of the rigid bent is in the mixed-type structure shown in Figure 3.39*b*, called a trussed bent. In this case the horizontal member is a truss and the column continues upward to become a part of the truss end. Because of the typical high stiffness of the truss, it is usually assumed that the column is held fixed at the bottom of the truss. With this assumption, the column is assumed to be for vertical axial compression only for gravity loads (due to very little rotation at the truss end). For lateral load, the column is considered to flex as shown for the example in Figure 3.38*c*.

Approximate Analysis of Multistory Bents

The multistory rigid bent is usually quite indeterminate and its investigation complex when it includes both vertical and lateral loads. Except for very early stage design approximations, the analysis is sure to be done with a computer-aided process. The software for such analyses is readily available.

For preliminary design, however, it is possible to use approximation methods to obtain member sizes with reasonable accuracy. This permits early determination of approximate column sizes—a critical concern for early stages of architectural planning. An example of such a design is given for the building in Section 10.8.

A general discussion of the use of the rigid frame for lateral bracing is given in Chapter 9.

3.6 SPECIAL STRUCTURES

Two-Way Spanning Structures

Spanning structures often consist of assemblages of single linear elements (trusses, beams, decks) that interact to work together, with individual structural behavior that is quite simple. However, the spanning system may also be defined by a system that has a more complex behavior, with the structural functioning of individual elements essentially interdependent on that of other elements. Such a structure is the two-way spanning system.

For a two-way system it is not possible to consider the functioning of a single element without acknowledging the behavior of the whole system. Some structures that have this behavior are the following:

Two-Way Spanning Trusses. Trusses may act individually when they span in a single direction and provide only simple support for connecting elements. However, a system of intersecting trusses may also be developed as a two-way spanning structure if the truss configurations, jointing, and support conditions are developed to achieve this action.

Cable Nets. Spanning cables may function individually or in simple tandem sets, such as the cables of a suspension bridge. However, they may also be used to define a surface in the form of laced sets of intersecting cables, draped in suspended form or arched to restrain a pneumatically developed surface.

Concrete Systems. The essentially nonlinear nature of cast concrete can be exploited to produce two-way spanning slabs or intersecting beam systems. Continuity of both sets of intersecting beams is a natural feature of the cast system, although it is not generally feasible in assemblages of separate linear elements of wood or steel.

The two-way spanning structure is in general quite indeterminate and its investigation for behavior is very complex. Investigation methods and computer-aided design techniques exist and are frequently used, but the work is considerably more laborious than that for simpler systems. Approximation techniques may be used for early stages of design in order to produce a simulation model that can be useful in the more exact stages of investigation.

An advantage of two-way spanning structures is the increased efficiency resulting from the mutually supporting nature of the member interactions. If the problems of more costly and time-consuming investigation and design and usually more complex construction can be tolerated, these systems may actually result in reduced cost for the structure.

Two-way spanning systems of sitecast concrete have been used for many years. One popular system for multistory apartments is the flat-plate system that uses only a single-thickness solid slab with supporting columns. The elimination of beams allows the underside of the slab to directly define a ceiling surface and the top of the slab to directly define a floor surface. The result of this is the production of the shortest floor-to-floor story height with the savings in all vertical elements of the building construction.

In planning of structures that have two-way spanning actions, if optimal utilization is to occur, attention must be given to the nature of the system. A major consideration is the planning of the supports for the system. If the system is used to define a single square unit in plan, there are several options for the layout of the supports. Architectural design considerations must be made, but the effects on the structure must be considered if the spanning structure is to capitalize on its inherent potential for high efficiency.

In Figure 3.40*a* a square system is shown with supports at the four corners. While the interior portion of the spanning system will function as a two-way structure, the edges must function essentially as linear elements spanning between the supports and providing support for the interior system. The edges will therefore be quite heavy.

Figures 3.40*b* and *c* show supports for a square system that eliminate the heavy spanning edge structure, replacing it with bearing walls or closely spaced columns.

Figure 3.40*d* is a variation on the corner-supported scheme in Figure 3.40*a*, resulting in a reduction of the span effect of the interior portion but requiring relatively heavy edge cantilevers.

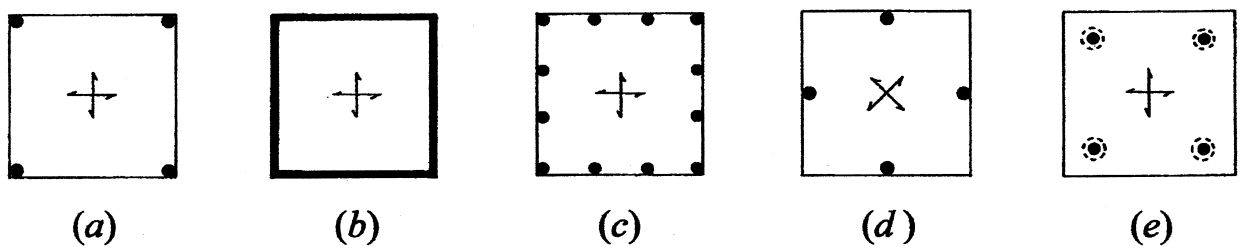
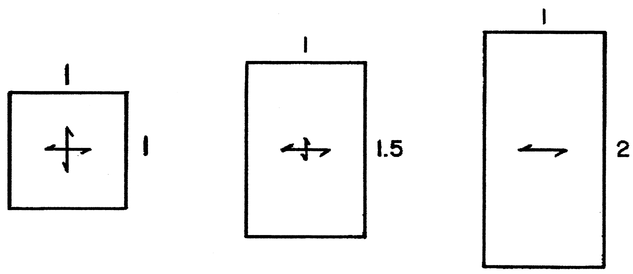


Figure 3.40 Alternative supports for a single two-way spanning structure.

An optimal situation for the spanning structure is shown in Figure 3.40*e*, in which the four corner columns are pulled in to allow the spanning structure to project as a cantilever on all sides. This substantially reduces the spanning requirement for the interior structure. Widening of the column tops can further reduce demand by spreading the punching effect of the columns.

For two-way spanning elements, the plan should be as square as possible. As the plan becomes oblong rather than square (see Figure 3.41), the increased stiffness of the shorter span tends to reduce the spanning contribution of the longer span. At a ratio of 1.5 : 1,80% of the load is carried by the shorter span, and at 2 : 1 the two-way effect pretty much disappears.



(*a*) (*b*) (*c*)

Figure 3.41 Effect of span ratio on a two-way spanning element.

CHAPTER

4

Wood Structures

This chapter treats the use of wood from trees as a basic material for building structures. While an understanding of the nature of the basic material is important, it is equally important to be aware of the products developed from the basic material and of the various structural systems that can be developed with the products.

4.1 GENERAL CONCERNS FOR WOOD

Wood from trees has a long history of usage for structural purposes, most notably in regions where large stands of trees exist (see Figure 4.1). At the time of the early colonization of the United States, vast areas of the country were covered with forests. It was, indeed, a major problem for early settlers of the eastern, southeastern, and midwestern areas. Travel was difficult because of the dense growth and, up to the middle of the nineteenth century, was mostly accomplished by using the many navigable rivers. As in many countries today, land for cultivation of crops or grazing of animals was claimed by burning off or otherwise destroying forest lands.

While much of that early dense forest was lost—most notably vast stands of hardwood trees—a considerable amount of timber was used for construction. Thus a heritage of wood construction was developed and an extensive industry was established. This industry extends to today, with wood remaining as a major source for building construction uses.

We no longer build extensively with construction that directly utilizes the source. Log cabins, roughly hacked boards, and pole construction with peeled logs do not account for the majority of buildings. Today, wood as a building material is treated as an industrialized product, receiving considerable processing on the way to the construction site. Still, a major use—and one treated extensively in this book—is that of the lightly processed pieces of wood that are cut directly from the logs, smoothed up a bit, and used as quickly as possible in their solid-sawn form. This product is what we generally refer to as *lumber*, and the lumberyard is still a major business in almost every large community in the United States. Wood is indeed the all-American building material and will be found somewhere on just about every building site.

This section deals with some of the basic issues concerning the use of wood, with concentration on the direct usage for structural lumber.

Sources of Wood

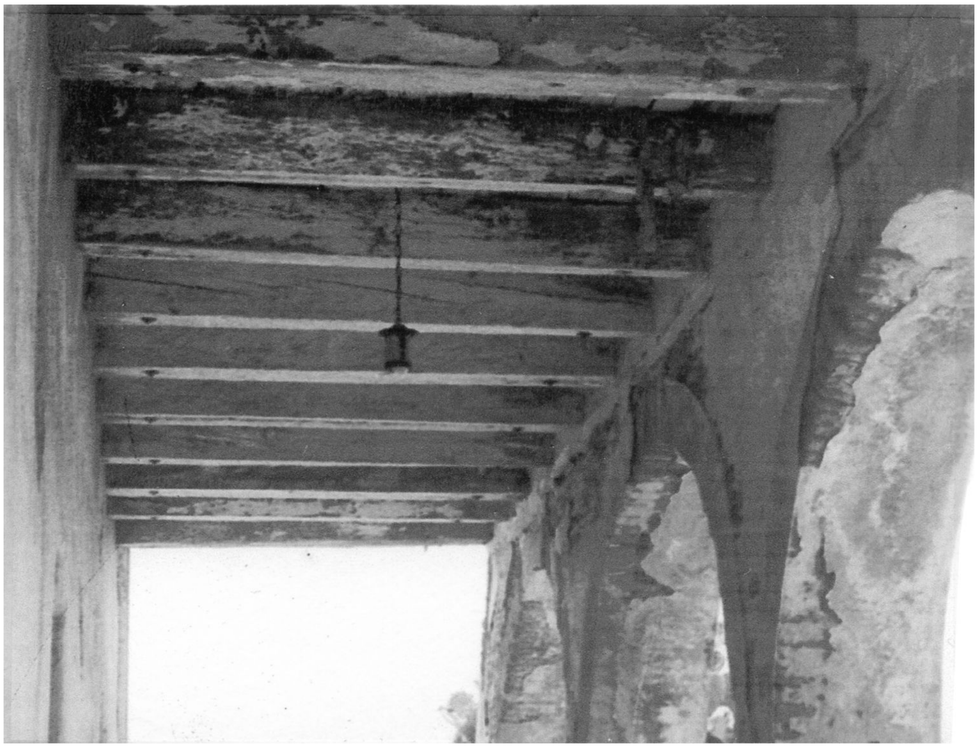
The particular type of tree from which wood comes is called the *species*. Although there are thousand of species of trees, most structural wood used in the United States comes from a few dozen species that are selected for commercial processing.

The two groups of trees used for building purposes are the *softwoods* and *hardwoods*. Most softwood trees like pine and spruce are coniferous, or cone bearing, whereas hardwood trees have broad leaves exemplified by oak and maple. Softwoods are indeed mostly softer than hardwoods, although there are other properties that define the types.

The two species of trees used most extensively for structural wood in the United States are Douglas fir and southern pine. However, several other species are also used, depending partly on regional availability. Although the terms *timber* and *lumber* are often used interchangeably, current industry usage tends to reserve timber for structural wood members of large cross-sectional area.

Tree Growth

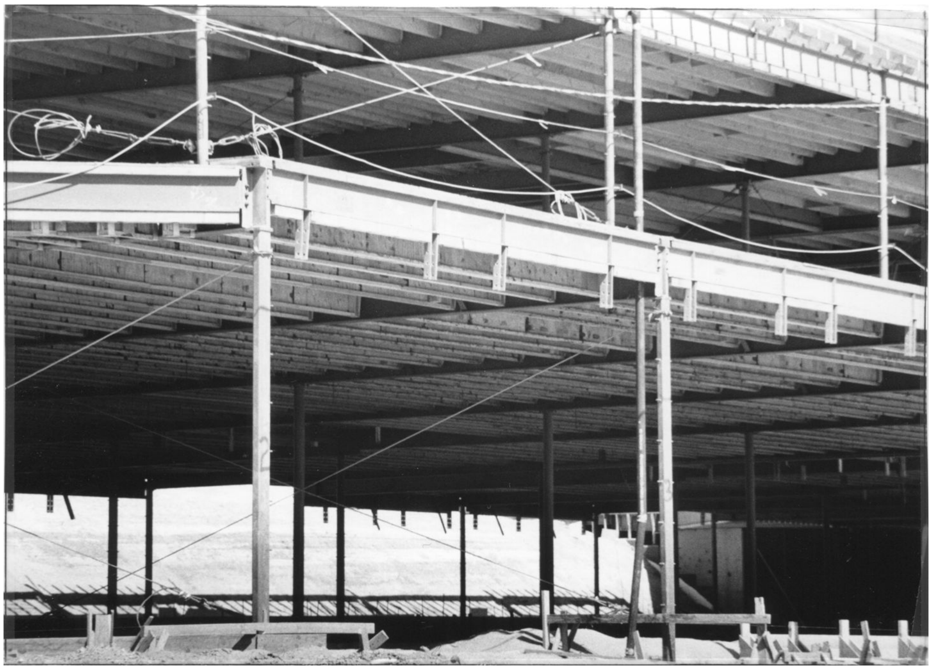
The trees used for lumber in the United States are exogenous; that is, they increase in size by growth of new wood on the outer surface under the bark. The cross section of a tree trunk reveals the layers of new wood that are formed annually. These layers, called *annual rings*, are typically composed of



(*a*)

(*b*)

Figure 4.1 Utilization of wood for building structures through the ages. Left: Hand-hewn timbers create a roof for a mission church, a craft with roots extending thousands of years. Right, top: Solid-sawn lumber, cleanly articulated in a simple post-and-beam structure. Although the example shown here is a contemporary construction, it also has long roots. Right, bottom: Present usage of wood products in a mixed steel and wood structure. Wood is used here for the plywood deck and fabricated I-beams with particleboard webs and sawn lumber top and bottom flanges.



(*c*)

Figure 4.1 (*continued*)

alternating light and dark material. In most areas, the lighter, more porous layers are grown in the warmer months of the year (spring and summer in the northern hemisphere) and the denser, darker layers are grown in the colder months.

The number of layers of annual rings at the base of the tree trunk indicates the age of a tree. To build up a trunk large enough to be able to saw structural lumber requires several years of growth, the number of years depending on the climate and the type of tree. In a real sense, however, no matter how many years it takes, wood is a renewable source of building materials.

The youngest band of annual rings at the outer edge of the tree is called the *sapwood*. This is usually lighter in color than wood at the center of the log, which is called the *heartwood*. For specific purposes either the sapwood or the heartwood may be the more desirable material. However, how an individual piece of lumber is cut from the log with respect to orientation to the general pattern of the annual rings is often of greater concern. The structure of wood is composed primarily of long and slender cells called *fibers*. These cells have a hollow, tubular form with an orientation of their lengths in the longitudinal direction of the log (up the tree for transporting of water and nutrients during growth). This gives cut pieces of wood a character described as its *grain*, with the grain being directed along the length of cut pieces of lumber. This in turn provides a reference for viewing various structural actions as relating to the grain, that is, as being *parallel to the grain*, *perpendicular to the grain*, or at some *angle to the grain*.

The fibrous, tubular cells of the wood are composed primarily of *cellulose* and the material that binds the cells is called *lignin*. These two materials are the main chemical components of wood.

Density of Wood

The difference in the arrangement and size of the cell cavities and the thickness of the cell walls determine the specific gravity, or relative *density*, of various species of wood. The strength of wood is closely related to its density. The term *close grained* refers to wood with narrow, closely spaced annual rings. Certain woods, such as Douglas fir and southern yellow pine, show a distinct contrast between the springwood and summerwood, and the proportion of summerwood affords a visual basis for approximating strength and density. The solid material in wood is about 1.53 times the weight of water, but the wood cells contain open spaces in varying degrees. These spaces are typically filled partly with air and partly with water. The weight of wood varies with regard to the amount of open cell space and the amount of trapped water. For purposes of computation in this book, the average weight of structural softwood is taken as 35 pcf (pounds per cubic feet).

Density of wood is generally linked closely with strength of the wood; thus, the strongest woods—and the strongest grades for a single species—will usually be quite heavy pieces.

Defects in Lumber

Any irregularity in wood that affects its strength or durability is called a *defect*. Because of the natural characteristics of the material, several common defects are inherent in wood. The most common are described here.

A *knot* is a portion of a branch or limb that has been surrounded by subsequent growth of the tree. There are several types of knots, and the strength of a structural member is affected by the size and location of those it may contain. Grading rules for structural lumber are specific concerning the number, size, and position of knots and their presence

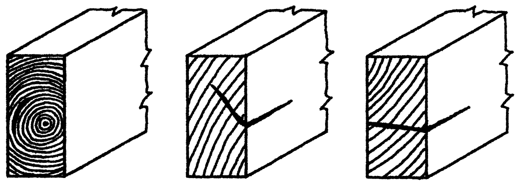


Figure 4.2 Defects in structural lumber.

is considered when establishing design values for structural response.

A *shake* is a separation along the grain, principally between annual rings. The cross section of a shake is shown in Figure 4.2*a*. Shakes reduce the resistance to shear, and consequently members subjected to bending are directly affected by their presence. The strength of members in longitudinal compression (directed parallel to the wood grain), such as columns and posts, is not greatly affected by shakes.

A *check* is a separation along the grain, the greater part of which occurs across the annual rings (Figure 4.2*b*). Checks generally develop during the process of seasoning (drying out from the green condition). Like shakes, checks also reduce the resistance to shear.

A *split* is shown in Figure 4.2*c*. It is a lengthwise separation of the wood that extends through the piece from one surface to another. Splits obviously have a major effect in reducing shear resistance.

A *pitch pocket* is an opening parallel to the annual rings that contains pitch, either solid or liquid.

Logs are typically tapered in form, and when a long piece of lumber is sawn from a relatively short tree trunk, or from a log that is not held straight during sawing, a condition may occur that is described as having a piece of lumber with a *slope of grain*. This has some direct effects on certain structural uses and is one of many properties of an individual piece of lumber that is noted when the piece is evaluated for structural applications.

A major concern for wood for construction is the general problem of *decay* of the wood. This is actually a natural process for the organic (once-living) material, and preserving the wood is literally a nature-defying effort. Some decay occurs within the tree even during its growth period and pockets of decay are another form of defect of the sawn lumber pieces. Old decay may be arrested or suspended by treatment of the wood or simply be eliminated by cutting out the decayed portions. Often of greater concern is continuing or new decay, which is a major problem for the general development of the construction.

Numerous treatments of wood are possible, including the impregnation of the wood mass with chemicals to arrest future decay. Wood generally untreated and exposed to the weather is especially vulnerable in this regard.

Seasoning of Wood

All wood contains moisture, and the serviceability of wood for construction is generally improved by reduction of the amount of moisture below the content in the *freshly cut* pieces, referred to as *green* wood. The process of removing moisture from green wood is known as *seasoning*; it is accomplished by exposing the wood to relatively drier air for an extended period of time or by heating it in kilns to drive out the moisture. Whether *air dried* or *kiln dried*, seasoned wood is generally stiffer, stronger, and less subject to shape changes.

Drying out of the wood results in shrinkage of the cellular structure of the material. This occurs differently in the three primary directions: along the grain, parallel to the annual rings, and perpendicular to the annual rings. This is where the orientation of the grain, as well as its relative uniformity and absence of large defects, becomes quite important. Shape and dimensional changes of some degree are to be expected, affecting a property described as the *dimensional stability* of the wood. It is quite important that as much of this change as is possible should occur before installation of the wood in the construction assemblage.

The *moisture content* of wood is the ratio of the weight of contained water in a piece of wood to the weight of an oven-dried (zero-moist) sample, expressed as a percentage. Specific limits are set for this value for structural applications.

Wood freshly cut from a log usually has a relatively high moisture content—and is sometimes described as a *green wood*. For construction, it is desirable to have the cut lumber in a low-moisture-content condition to avoid most of the shape changes that occur. This is usually possible for thin wood pieces (2 by and thinner) but is mostly not feasible for thick timbers.

4.2 WOOD PRODUCTS AND SYSTEMS

Present forms of wood construction derive their heritage from the use of solid-sawn wood pieces. Today the pieces we use may be somewhat more refined, but the basic forms of the structures are essentially the same. Figure 4.3 shows the basic form of *light wood frame construction*, a system that utilizes primarily 2-by lumber elements (2-by-4 studs, 2-by-10 floor joists, etc.). Most of the single-family houses, as well as other small buildings, in the United States have been built with this form of construction for the past 150 years.

Larger pieces of solid-sawn wood are also used for construction of wood structures. Where the heavier members (called mostly *timber*) predominate in the system, the system is usually described as *timber construction*.

Structural Lumber

The general group of solid-sawn pieces is referred to as *lumber*; or more specifically as *structural lumber*. The following discussion treats various considerations for the use of structural lumber.

Nominal and Dressed Sizes

A piece of structural lumber is designated by its nominal cross-sectional dimensions. As an example, a designation of 6 by 12 (also written 6×12) indicates a member with

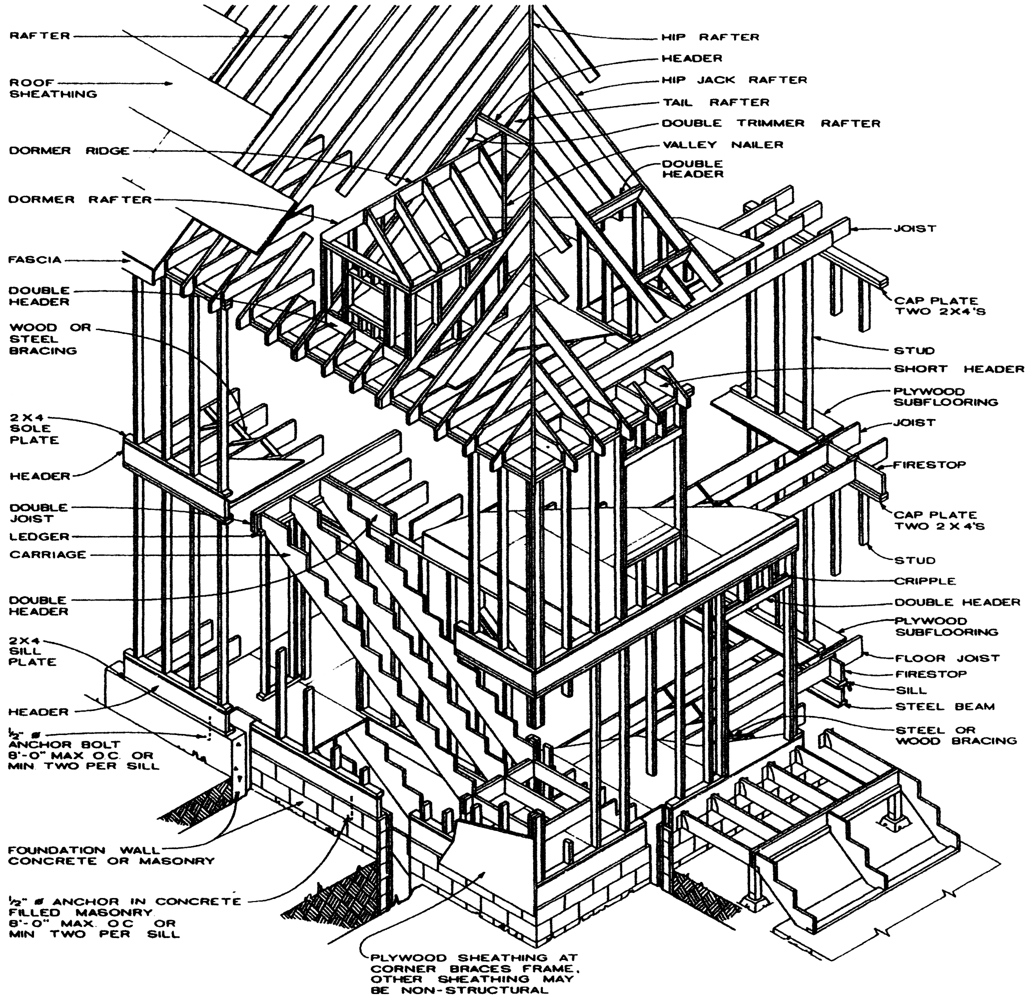


Figure 4.3 Light wood frame construction with 2-in. nominal size solid-sawn lumber. Reproduced from *Architectural Graphic Standards,* 7th ed., with permission of the publisher, John Wiley & Sons, Hoboken, NJ.

a width of 6 in. and a depth of 12 in. However, these are actually *nominal dimensions* which roughly describe the shape as sawn from the log. When the member faces are smoothed by planing and/or sanding, the member is said to be *dressed* or *surfaced*. The 6 by 12 when dressed will have actual dimensions of 5.5 by 11.5 in. Standard lumber sizes produced by the lumber industry are listed in Table A.8, which yields both the nominal and actual dimensions of the pieces.

Use Classification of Structural Lumber

Because the effects of natural defects on the strength of lumber vary with the type of loading, and specific types of defects relate to structural usage of members, structural lumber is classified according to both *size* and *use*. All dimensions used in classification are nominal dimensions and not actual dimensions. The four principal classifications are as follows:

Dimension Lumber. Rectangular cross sections with nominal dimensions that range from 2 in. to 4 in. in thickness and 2 in. or more in width. A further distinction is made between *light framing* 2 to 4 in. wide and *joists* and *planks* 5 in. and wider.

Beams and Stringers. Rectangular cross sections 5 in. or more thick and with a width at least 2 in. greater than the thickness, graded for strength in bending when loaded on the narrow face.

Posts and Timbers. Square or nearly square cross sections with nominal dimensions 5 by 5 in. and larger and with width not more than 2 in. greater than thickness are graded primarily for use as posts or columns or other uses where bending strength is not a major concern.

Decking. This consists of lumber 2 to 4 in. thick and 4 in. or more wide. Jointing of members is provided by tongue-and-groove edges or edges splined (slotted) for interlocking on the narrow face. Decking is graded for use with the wide face placed flatwise in contact with supporting members.

Grading of Structural Lumber

Grading is necessary to establish the quality of lumber. Structural grades are identified in relation to strength properties and use classifications. Structural grading may

be done by mechanical testing of wood samples but is most often accomplished by visual inspection. Once graded, a piece of structural lumber is stamped with the grade and the grading authority or standard used. Use of specific grades is established by the structural designer for each type of structural member and is ordinarily designated for use on the structural construction documents.

Miscellaneous Wood Products and Elements

While structural lumber is still in wide use, there is an ever-increasing array of other structural products that use wood as a primary material. In the development of complete structural systems it is common to mix a variety of products. Figure 4.4 shows the roof structure for a warehouse in which lumber elements are used for the short spanning rafters and the purlins that support the rafters. However, plywood is used for the roof deck, spanning between the rafters, and large glued-laminated girders are used to span between columns. The mix is further extended here by use of steel pipe columns for support of the girders. And, of course, a large number of steel elements are used for the assembly of the system.

The general term *engineered wood products* is now used to describe manufactured structural wood products other than lumber. Many of these products are developed from wood that is reconstituted in some form, such as by slicing it into thin plies, shredding it into fibers, or forming it into chips or strands.

These reconstituted materials are then subjected to some forming and bonding processes to produce laminated veneer lumber, plywood, oriented strand board, or composite panels. Combinations of sawn lumber and engineered products can also be assembled to produce glued-laminated lumber and I-joists. The following sections describe these products and their applications in building structures.

Glued-Laminated Structural Members

The gluing together of multiple laminations of standard 2-in.-nominal-thickness lumber has been used for many years to produce large beams and girders. This is really the only option for using sawn wood for large members that are beyond the feasible range of size for single sawn pieces. However, there are other reasons for using the laminated beam that include the following:

Higher Strength. Lumber used for laminating consists of a moisture content described as kiln dried. This is the opposite end of the quality range from the green wood condition ordinarily assumed for solid-sawn members. This, plus the minimizing effect of flaws due to lamination, permits use of stresses for flexure and shear that are much higher than those allowed for single-piece members. The result is that much smaller sections can often be used, which helps to offset the usually higher cost of the laminated products.

Better Dimensional Stability. This refers to the tendency for wood to warp, split, shrink, and so on. Both the use of the kiln-dried materials and the laminating process itself tend to create a very stable product. This is often a major consideration where shape change can adversely affect the building construction.

Shape Variability. Lamination permits the production of curved, tapered, and other special profile forms for beams, as shown in Figure 4.5. Cambering as compensation for service load deflection, sloping for roof drainage, and other useful custom profiling can be done with relative ease. This is otherwise possible only with a truss or a built-up section.

Laminated beams have seen wide use for many years and industrywide standards are well established. Cross-sectional

Figure 4.4 Roof structure for a warehouse utilizing a variety of structural products.

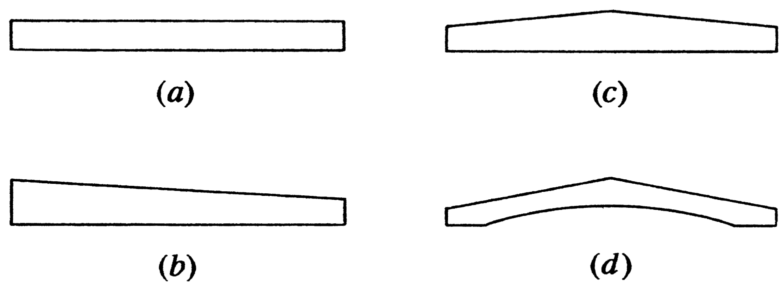


Figure 4.5 Alternative profiles for glued-laminated beams: (a) straight; (b) single sloped; (c) double sloped; double sloped with arched bottom.

sizes are derived from the number of laminations and the size of the individual pieces used. Thus depths are multiples of 1.5 in. and widths are slightly less than the lumber size as a result of the finishing of the product. Minor misalignments and the unavoidable sloppiness of the gluing process result in an unattractive surface. Finishing of the sides of beams may simply consist of smoothing them off, although various special surface textures can also be created.

Investigation and design of glued-laminated timber members are done primarily with the procedures explained for solid-sawn beams as described in Section 4.4. Criteria for design is provided in most building codes, in the *National Design Specification* (NDS) (Ref. 3), and in the literature provided by manufacturers and suppliers of the products. These elements are manufactured products and are mostly not able to be transported great distances, so information about them should be obtained from local suppliers.

Individual elements of glued-laminated timber can be custom profiled to produce a wide variety of shapes for structures. Two forms commonly used are the three-hinged arch (Figure 4.6*a*) and the gabled bent (Figure 4.6*b*). A critical consideration is that of the radius of curvature of the member, which must be limited to what the wood species and the laminate thickness can tolerate. For very large elements this is not a problem, but for smaller structures the curvature limits of 2-in.-nominal-thickness lumber may be critical.

Manufacturers of laminated products usually produce the arch and gabled elements as standard forms. Structural design of the products is usually done by the manufacturer's engineers. Form limits, size range, connection details, and other considerations for these products should be investigated with individual manufacturers.

Custom shapes can be produced, such as those with double curvature, as shown in Figure 4.6*c*. Many imaginative structures have been designed using the form variation potential of this process.

Columns may be produced with 1.5-in. laminations, presenting the same advantages as those described for beams: higher strength and dimensional stability being most critical. It is also possible to produce glued-laminated columns of greater length than that obtainable with solid-sawn members. In general, laminated columns are used less frequently than beams or girders and are mostly chosen only when a special shape is desired or when some of the inherent limitations of other options are restrictive.

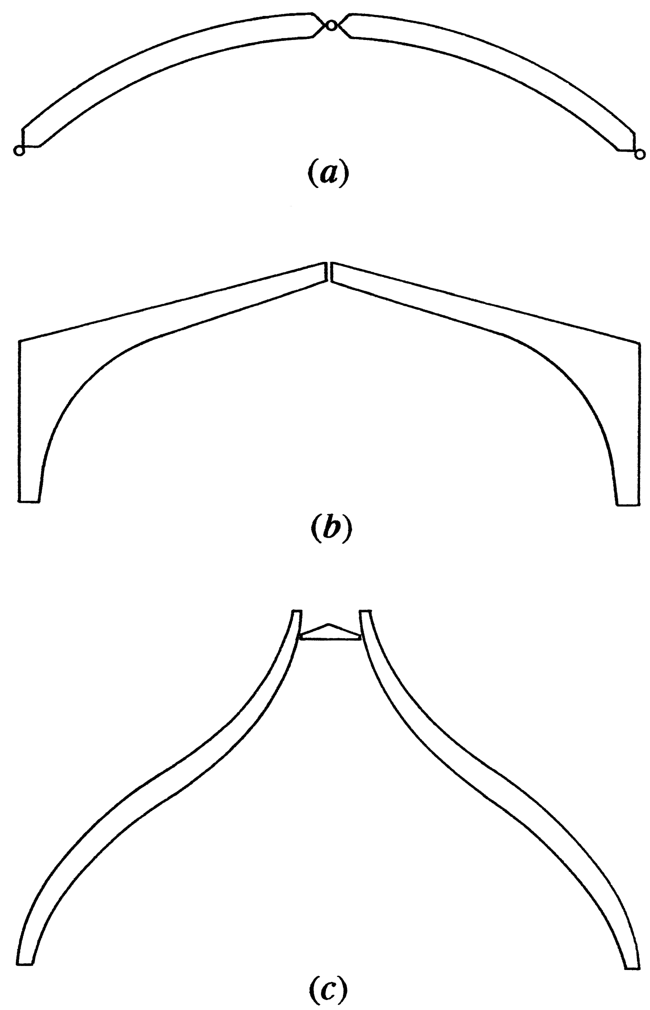


Figure 4.6 Nonstraight laminated elements: (a) arch segments; (b) gabled frame; (c) doubly curved elements.

Structural Composite Lumber

Structural composite members are manufactured using wood elements that are bonded together by gluing. Although this classification theoretically includes glued-laminated timber members, this term is used primarily to describe other elements, such as:

Laminated Veneer Lumber (LVL). These consist of laminated members using thin veneers similar to those used for plywood, except that here the grain of the

laminations are all parallel to the length of the finished members. See Figure 4.7*a*.

Parallel Strand Lumber (PSL). These consist of long thin strands of wood placed with their orientation parallel to the length of members and bonded together by a combination of resin and compression. See Figure 4.7*b*.

Both of these processes are used to produce linear lumber pieces similar in form and intended function to those produced as solid-sawn elements. Advantages include higher strength, dimensional stability, and virtually unlimited length of pieces. In addition, the raw materials used—especially for PSL—can be obtained from smaller size, faster growth trees. The forest resource is thus similar to that for paper products, which is a vast cultivated system.

A major use for both of these products is for chord members of trusses and I-joists. All of the advantages described are significant for this application.

PSL members were originally introduced mostly as replacements for small lumber pieces such as the 2 by 4. However, they are now produced in a considerable size range and are generally competitive with sawn products in many situations. Only glued-laminated timbers can presently achieve larger cross-sectional sizes and member lengths, but time will tell.

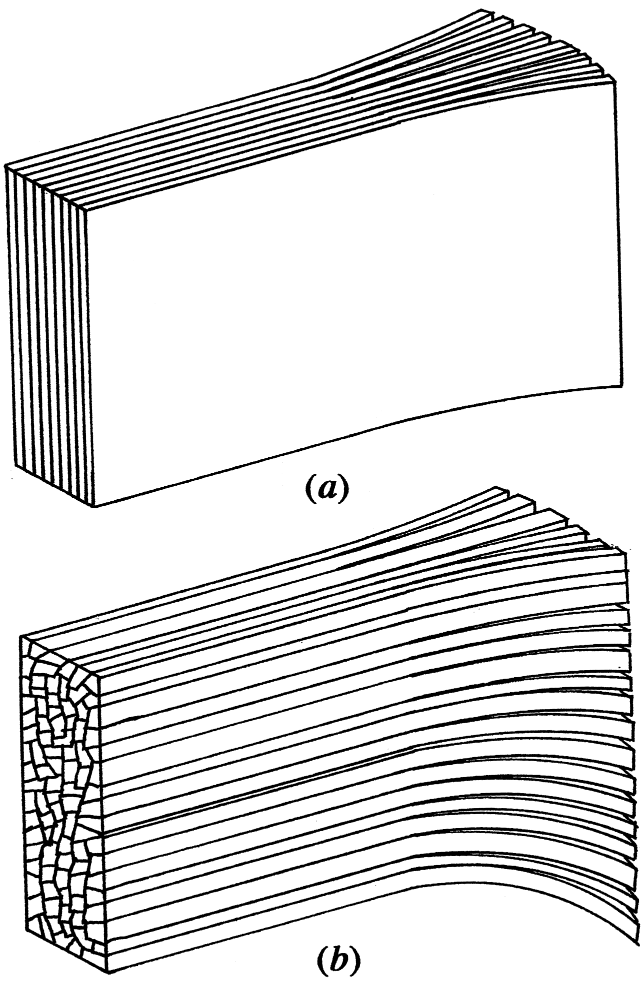


Figure 4.7 Structural composite elements: (*a*) laminated veneer; (*b*) parallel strand.

Wood Structural Panels

Plywood. In earlier times, surfacing of walls, roofs, and floors was mostly achieved with solid-sawn wood boards. By the mid-twentieth century, this surfacing was mostly achieved with plywood panels. At present, plywood is still mostly used for floor decks and for heavily loaded shear walls, but other panel materials are increasingly used for walls and roofs. Plywood is discussed in Section 4.4.

The following discussion deals with the other two types of panels currently used for structural applications.

Composite Panels. Composite panels are produced in a manner similar to that used for plywood. Thin sheet veneers are glued together in layers, called plies, to build up the panel thickness. As with plywood, the two outside veneers are usually of wood with the grain of both veneers in the same direction. The primary difference here is the use of wood fiber materials for the inner plies. Use of fiber materials for the major portion of the bulk of the panel is a cost-saving feature as well as a utilization of less critical resources. A principal feature of these panels is their close resemblance to plywood as far as the viewed surface. Where the viewed surface is a concern, the cost reduction may be significant.

Oriented Strand Board (OSB). OSB is produced with thin chips or wafers of wood and a bonding resin. The wafers are placed in layers with a switch in direction of the grain of the wafers in alternating layers, producing a structural similarity to plywood in terms of two-way bending resistance. This product is now widely used for wall and roof sheathing in place of the more expensive and resource-depleting plywood. However, *all* of its properties should be considered for each application, including structural properties but also moisture resistance, thermal expansion, and fastener holding.

Prefabricated Wood I-Joists

Wood I-shaped joists are formed with chords (top and bottom elements) of sawn lumber or laminated wood veneers. (See Figure 4.8.) Grooves are cut in the chord elements and a web of plywood or OSB is glued into the grooves. Criteria for design of these structural products are now included in codes and industry standards, but these are essentially the products of individual manufacturers and specific design data should be obtained from the producers.

These products were originally developed to extend the span range slightly beyond the limit represented by 2-by-12 lumber joists and rafters. While this is still the case, smaller

Figure 4.8 Wood I-joists.

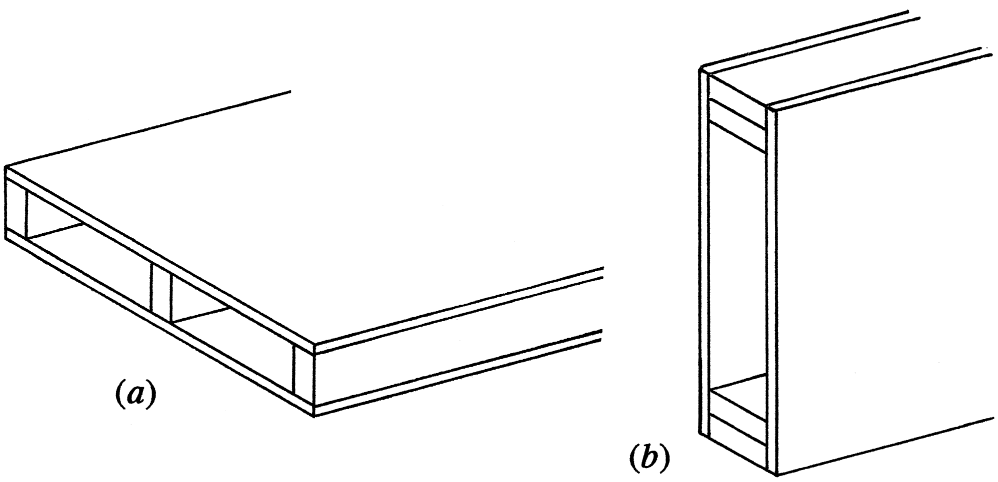


Figure 4.9 Built-up wood structural units.

I-joists are now competitive with sawn lumber in smaller sizes as well. Advantages include more dimensional stability (resistance to warping, curling, etc.) and better resistance to lateral and torsional buckling. Elimination of bridging and blocking may be significant.

Built-Up Panel and Lumber Beams

Structural wood panels and sawn wood lumber can be combined in various ways. The I-joist is one such product. Another form is the so-called *sandwich panel* or *stressed-skin panel*, which consists of two large panels attached to and separated by a frame of sawn wood elements. (See Figure 4.9*a*.) In this assembly the panels and sawn wood switch roles from that in the I-joist. Here the panels serve as bending resisters at the top and bottom and the sawn elements serve as the shear-resisting web. Sandwich panels can be assembled to form prefabricated wall or deck units.

Another structural product of this type is the built-up beam formed with top and bottom elements of sawn lumber and panels that are nailed to the sides of the lumber. (See Figure 4.9*b*.) One application of this structure is formed as part of an ordinary stud wall formation. The most frequent such use is for headers over wide openings in walls. For the header the wall top plate (usually two 2-in.- nominal-thickness elements) and a similar doubled member at the top edge of the opening are used as the top and bottom elements of the beam. The built-up header is then completed by using wall sheathing as the web of the panel/lumber assemblage.

Pole Structures

A type of construction used extensively in ancient times and still used today in some regions is that which employs wood poles as vertical structural elements. Although processed poles cut to have a constant diameter are obtainable, most poles

are simply tree trunks with branches and soft outer layers of material stripped away. There are generally three ways in which such poles may be used: as timber piles driven into the ground, as vertical building columns in a frame structure, or as buried-end poles extending partly below grade and partly above grade (such as fence posts or electric wire transmission poles). The following discussion is limited to consideration of buried-end poles.

As a foundation element, the buried-end pole is typically used to raise a building above the ground. Driven timber piles may be used in this manner, but control of the precise location of the pole top makes them less usable for extended members of the building framework. With regard to the building construction, the two chief means of using poles are for *pole-frame buildings* and *pole-platform buildings*, as shown in Figure 4.10. For a pole-frame building (Figure 4.10*a*), the buried poles are extended above grade to function as building columns. For a pole platform (Figure 4.10*b*), the poles are cut off at some level above grade and a flat structure (the platform) is built on top of them, possibly providing support for a conventional wood frame structure.

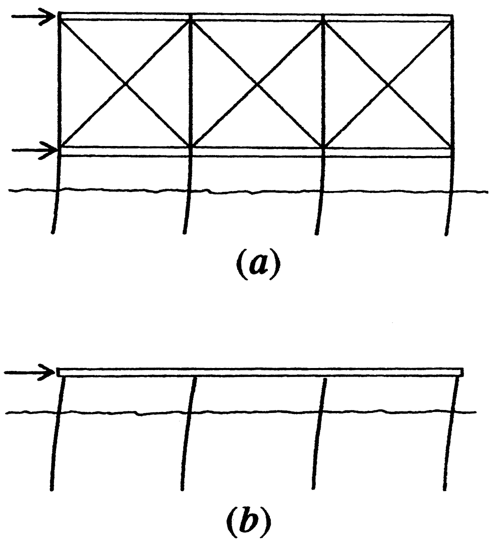


Figure 4.10 Pole construction: (*a*) pole frame; (*b*) pole platform.

Pole foundations must usually provide both vertical and lateral support for a building. For vertical loads, the pole end simply transfers vertical load by direct bearing. The three common forms for buried-end pole foundations are shown in Figure 4.11. In Figure 4.11*a* the bottom of the hole is filled with concrete to provide a footing, a preferred method when the soil at the bottom of the hole is very compressible. In Figure 4.11*b* the pole bears directly on the bottom of the hole and a stabilizing method sometimes used is to pour a concrete collar near the top to help with lateral loads. If lag bolts are used to anchor the collar, the collar will also assist with vertical support. Of course, both the footing and collar can be used for extra strong support. In Figure 4.11*c* the hole is completely backfilled with concrete, which is the most positive means of support for both vertical and lateral loads.

Pole construction is frequently used for utility buildings in regions where poles are readily available. Local codes and common practices—based mostly on experience—often determine details of the construction. Need for depth of embedment is an issue of particular concern and is often based more on experience than science.

For lateral loads, the situation is usually quite different for pole-frame and pole-platform buildings. For the pole-frame construction, lateral bracing is usually provided in a conventional manner as for other wood frame construction. The buried pole ends must resist lateral force, but there is less concern for rotation of the pole at grade level. For the pole platform, while the structure built on top of the platform may be conventionally braced, the platform itself depends heavily on the resistance to lateral movement of the poles at grade level. For the platform there should be either some constraint of the pole at ground level or some additional depth of penetration into the ground—or both.

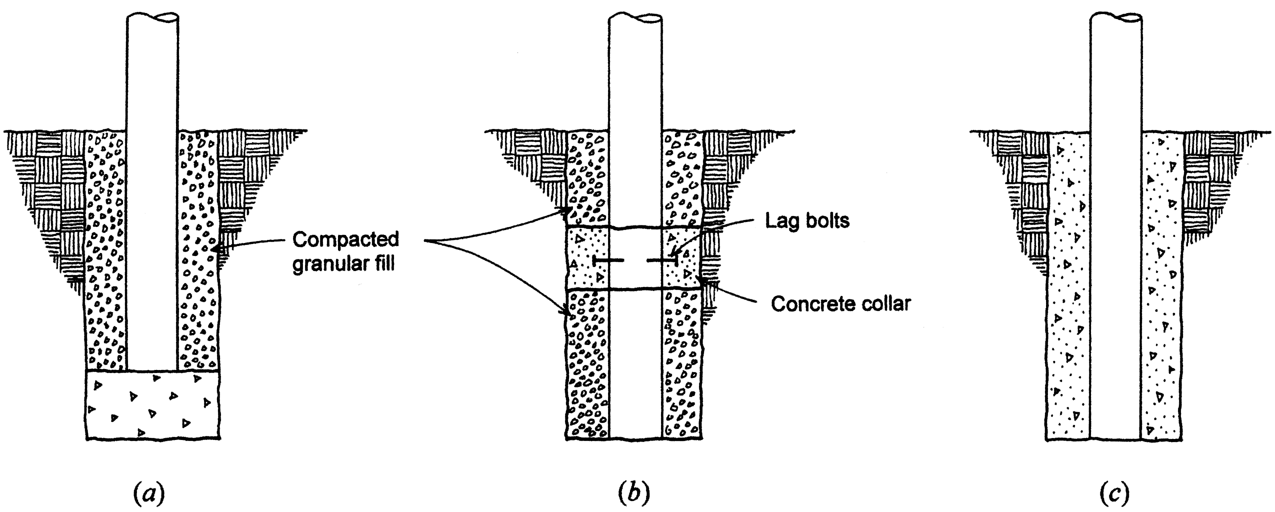


Figure 4.11 Foundations for poles.

4.3 DESIGN DATA FOR STRUCTURAL LUMBER

There are many factors to be considered in determining the unit stresses to be used for design of wood structures. Extensive testing has produced values known as *reference design values*. To obtain values for design work, the base values are modified for consideration, such as loss of strength from defects, the size and position of knots, the size of standard dimension members, the degree of density of the wood, and the condition of seasoning or specific value of moisture content of the wood at the time of use. For specific uses of the structure, modifications may be made for considerations such as load duration, direction of stress with respect to the wood grain, and type of structural element. Separate groups of design values are established for decks, closely spaced rafters and joists, large-dimension beams, and posts (columns).

Table 4.1 gives reference design values to be used for ordinary allowable stress design. It is adapted from the NDS (Ref. 3) and gives data for one popular wood species: Douglas fir-larch. To obtain values from the table the following is determined:

Species. The NDS publication lists values for several species, only one of which is included in Table 4.1.

Moisture Condition at Time of Use. The specific moisture condition corresponding to the table values is given with the species designation in the table. Adjustments for other conditions are described in the table footnotes or in various specifications in the NDS.

Grade. This is indicated in the first column of the table and is based on visual grading standards.

Size and Use. The second column of the table gives size ranges or usages of the lumber.

Structural Function. Individual columns in the table give values for various stress conditions. The last two columns give the material moduli of elasticity.

In the reference document there are extensive footnotes for this table. Data from Table 4.1 are used in various example computations in this book and some issues treated in the document footnotes are explained. In many situations there are modifications (or adjustments, as they are called in the NDS) to the design values, as will be explained later. Referred to in a footnote to Table 4.1, Table 4.2 yields

Table 4.1 Reference Design Values for Visually Graded Lumber of Douglas Fir–Larch[a] (Values in psi)

Species and Commercial Grade	Member Size and Use Classification	Bending, F_b	Tension Parallel to Grain, F_t	Shear Parallel to Grain, F_v	Compression Perpendicular to Grain, $F_{c\perp}$	Compression Parallel to Grain, F_c	Modulus of Elasticity E	Modulus of Elasticity E_{min}
Dimension Lumber 2–4 in. thick								
Select structural	2 in. & wider	1500	1000	180	625	1700	1,900,000	690,000
No. 1 and better		1200	800	180	625	1550	1,800,000	660,000
No. 1		1000	675	180	625	1500	1,700,000	620,000
No. 2		900	575	180	625	1350	1,600,000	580,000
No. 3		525	325	180	625	775	1,400,000	510,000
Stud		700	450	180	625	850	1,400,000	510,000
Timbers								
Dense select structural	Beams and stringers	1900	1100	170	730	1300	1,700,000	620,000
Select structural		1600	950	170	625	1100	1,600,000	580,000
Dense No. 1		1550	775	170	730	1100	1,700,000	620,000
No. 1		1350	675	170	625	925	1,600,000	580,000
No. 2		875	425	170	625	600	1,300,000	470,000
Dense select structural	Posts and timbers	1750	1150	170	730	1350	1,700,000	620,000
Select structural		1500	1000	170	625	1150	1,600,000	580,000
Dense No. 1		1400	950	170	730	1200	1.700,000	620,000
No. 1		1200	825	170	625	1000	1,600,000	580,000
No. 2		750	475	170	625	700	1,300,000	470,000
Decking								
Select Dex	2–4 in. thick, 6–8 in. wide	1750	—	—	625	—	1,800,000	660,000
Commercial Dex		1450	—	—	625	—	1,700,000	620,000

Source: Data adapted from *National Design Specification*® (NDS®) *for Wood Construction*, 2005 edition (Ref. 3), with permission of the publisher, American Forest and Paper Association. The tables in the reference document have data for many other species and also have extensive footnotes.

[a]Values listed are for reference in determining design use values subject to various modifications. See Table 4.2 for size adjustment factors for dimension lumber (2–4 in. thick). For design purposes, values are subject to various modifications.

Table 4.2 Size Adjustment Factors (C_F) for Dimension Lumber, Decking, and Timber

Dimension Lumber					
		Thickness (breadth), F_b			
Grades	Width (depth)	2 and 3 in.	4 in.	F_t	F_c
Select structural,	2, 3, 4 in.	1.5	1.5	1.5	1.15
No. 1 and better,	5 in.	1.4	1.4	1.4	1.1
No. 1, No. 2, No. 3	6 in.	1.3	1.3	1.3	1.1
	8 in.	1.2	1.3	1.2	1.05
	10 in.	1.1	1.2	1.1	1.0
	12 in.	1.0	1.1	1.0	1.0
	14 in. and wider	0.9	1.0	0.9	0.9
Stud	2, 3, and 4 in.	1.1	1.1	1.1	1.05
	5 and 6 in.	1.0	1.0	1.0	1.0
	8 in. and wider	Use No. 3 grade values and size factors			

Decking	
2 in.	3 in.
1.10	1.04

Beams and Stringers—Loads Applied to Wide Face			
Grade	F_b	E and E_{min}	Other Properties
Select structural	0.86	1.0	1.0
No. 1	0.74	0.9	1.0
No. 2	1.0	1.0	1.0

Timbers, $d > 12$ in., $C_F = (12/d)^{1/9}$ for F_b only

Source: Data adapted from *National Design Specification*® (NDS®) *for Wood Construction*, 2005 edition (Ref. 3), with permission of the publisher, American Forest and Paper Association.

adjustment factors for dimension lumber and decking based on the dimensions of the piece.

Bearing Stress

There are various situations in which a wood member may develop a contact bearing stress, essentially a surface compression stress. Some examples are the following:

At the base of a wood column supported in direct bearing. This is a case of bearing stress that is in a direction parallel to the grain.

At the end of a beam that is supported by bearing on a support. This is bearing stress perpendicular to the grain.

Within a bolted connection, at the contact surface between the bolt and the wood at the edge of the bolt hole.

In a timber truss where a compression force is developed by direct bearing between the two members. This is frequently a situation involving bearing stress that is at some angle to the grain other than parallel or perpendicular. Common in the past, this form of joint is seldom used today.

For connections, the bearing condition is usually incorporated into the general assessment of the unit value of connecting devices. The situation of stress at an angle to the grain requires the determination of a compromise value somewhere between the allowable values for the two limiting stress conditions for stresses parallel and perpendicular to the wood grain.

Adjustment of Design Values

The values given in Table 4.1 are basic references for establishing the allowable values to be used for design. The table values are based on some defined norms, and in many cases the design values will be adjusted for actual use in structural computations. In some cases the form of the modification is a simple increase or decrease achieved by a percentage factor. Table 4.3 lists the various types of adjustment and indicates their applicability to various reference design values. The types of adjustment factors are described in the following discussions:

Load Duration Factor, C_D. The Table 4.1 values are based on so-called *normal* duration loading, which is actually somewhat meaningless. Increases are permitted for very short duration loading, such as wind and earthquakes. A decrease is required when the critical design loading is a long time in duration (such as a major dead load). Table 4.4 gives a summary of the NDS requirements for modifications for load duration.

Table 4.3 Applicability of Adjustment Factors for Sawn Lumber, ASD

	F_b	F_t	F_v	$F_{c\perp}$	F_c	E	E_{min}
ASD only							
Load duration	C_D	C_D	C_D	—	C_D	—	—
ASD and LRFD							
Wet service	C_M	C_M	C_M	C_M	C_M	C_M	C_M
Temperature	C_t	C_t	C_t	C_t	C_t	C_t	C_t
Beam stability	C_L	—	—	—	—	—	—
Size	C_F	C_F	—	—	C_F	—	—
Flat use	C_{fu}	—	—	—	—	—	—
Incising	C_i	C_i	C_i	C_i	C_i	C_i	C_i
Repetitive member	C_r	—	—	—	—	—	—
Column stability	—	—	—	—	C_P	—	—
Buckling stiffness	—	—	—	—	—	—	C_T
Bearing area	—	—	—	C_b	—	—	—
LRFD only							
Format conversion	K_F	K_F	K_F	K_F	K_F	—	K_F
Resistance	ϕ_b	ϕ_t	ϕ_v	ϕ_c	ϕ_c	—	ϕ_s
Time effect	λ	λ	λ	λ	λ	—	—

Table 4.4 Adjustment Factors for Design Values for Structural Lumber due to Load Duration, C_D[a]

Load Duration	Multiply Design Values by:	Typical Design Loads
Permanent	0.9	Dead load
Ten years	1.0	Occupancy live load
Two months	1.15	Snow load
Seven days	1.25	Construction load
Ten minutes	1.6	Wind or earthquake load
Impact[b]	2.0	Impact load

Source: Data adapted from *National Design Specification*® (NDS®) *for Wood Construction*, 2005 edition (Ref. 3), with permission of the publisher, American Forest and Paper Association.

[a]These factors shall not apply to reference modulus of elasticity E, to reference modulus of elasticity for beam and column stability $E_{\min}$, or to compression perpendicular to the grain reference design values $F_{c\zeta}$ based on a deformation limit.

[b]Load duration factors greater than 1.6 shall not apply to structural members pressure treated with water-borne preservatives or fire-retardant chemicals. The impact load duration factor shall not apply to connections.

Wet-Service Factor, C_M. The NDS document on which Table 4.1 is based defines a specific assumed moisture content on which the table values are based. Increases may be allowed for wood that is specially cured to a lower moisture content. If exposed to weather or other high-moisture conditions, a reduction of some design values may be required.

Temperature Factor, C_t. Where prolonged exposure to temperatures over 150° F exists, design values must be reduced. The adjustment factor varies for different reference values and includes consideration for moisture condition and exposure of the wood.

Beam Stability Factor, C_L. Design flexural stress must be adjusted for conditions of potential buckling. The general situation of buckling and the remedies for its prevention are discussed in Section 3.1.

Size Factor, C_F. For dimension lumber, adjustments for size are made for design stresses of bending, shear, and tension as described in Table 4.2. For beams 5 in. or thicker, with depth exceeding 12 in., adjustment of bending stress is made as described in Section 4.4. Other adjustments may be required for columns and for beams loaded for bending on their wide face.

Flat-Use Factor, C_{fu}. When sawn lumber 2 to 4 in. thick is loaded on the wide face (as with a plank), some adjustments are required as described in the NDS references for Table 4.1.

Incising Factor, C_i. Incising refers to small indentation-form cuts made on the surface of lumber that is treated by impregnation of chemicals for enhancement of resistance to fire or rot. A reduction of all reference values is required for this condition.

Repetitive-Member Factor, C_r. When wood beams of dimension lumber (mostly joists and rafters) are closely spaced and share a load, they may be eligible for an increase of 15% in reference design values; this condition is described as *repetitive-member use*. To qualify, the members must be not less than three in number, must support a continuous deck, must not be over 24 in. on center, and must be joined by construction that makes them share deflections (usually bridging or blocking). This increase is also permitted for built-up beams formed by direct attachment of multiple dimension lumber elements.

Column Stability Factor, C_P. This adjustment is performed in the typical processes of investigation and design of wood columns, which is discussed in Section 4.6. Most often, an adjustment consisting of a reduction of permissible compression stress parallel to the grain is required for relatively slender columns.

Buckling Stiffness Factor, C_T. This adjustment is made only for the modified modulus of elasticity, $E_{\min}$, in certain situations involving wood members subjected to combined compression and bending. Its principal application is in the design of the top chords of wood trusses.

Bearing Area Factor, C_b. This factor is provided for the case of bearing perpendicular to the grain when the length of bearing is small. It applies primarily to situations where bearing is transferred from a wood member to a steel plate or washer. The NDS provides a formula for determination of an adjusted design stress for these situations.

Modulus of Elasticity

The modulus of elasticity is a measure of the relative stiffness of a material. For wood, two reference values are used for the modulus of elasticity. The basic reference value is designated E and is the value used for ordinary deformations, primarily the deflection of beams. The other value is designated $E_{\min}$ and it is used for stability computations involving the buckling of beams and columns. Values for the stability modulus of elasticity are given in the last column in Table 4.1. Applications for determination of buckling effects in columns are presented in Section 4.6.

4.4 WOOD-SPANNING SYSTEMS

Many wood elements can be used for the purpose of creating horizontally spanning building structures. This section deals with some of the elements and the requirements for them that are utilized for design.

Beams

Beams may consist of solid-sawn lumber, glued-laminated timbers, or various engineered materials. The following discussion treats the common case of the solid-sawn section.

Design for Bending

The design of a wood beam for strength in bending is accomplished by use of the flexure formula (Section 2.3). The

form of this equation used in design is

$$S = \frac{M}{F_b}$$

where

M = maximum bending moment
F_b = allowable extreme fiber (bending) stress
S = required beam section modulus

Beams must be considered for shear, deflection, end bearing, and lateral buckling, as well as for bending stress. However, a common procedure is to first find the beam size required for bending and then to investigate for other conditions. Such a procedure is as follows:

1. Determine the maximum bending moment.
2. Select the wood species to be used.
3. From Table 4.1, determine the value for bending stress.
4. Consider appropriate modifications for the design stress to be used.
5. Using the flexural formula, find the required section modulus.
6. Select a beam size from Table A.8.

Example 1. A simple beam has a span of 16 ft [4.88 m] and supports a total uniformly distributed load, including its own weight, of 6500 lb [28.9 kN]. Using Douglas fir-larch, select structural grade, determine the size of the beam with the least cross-sectional area on the basis of limiting bending stress.

Solution. The maximum bending moment for this condition is

$$M = \frac{WL}{8} = \frac{6500 \times 16}{8} = 13{,}000 \text{ ft-lb} \quad [17.63 \text{ kN-m}]$$

The next step is to use the flexure formula with the allowable stress to determine the required section modulus. A problem with this is that there are two different size/use groups in Table 4.1, yielding two different values for the allowable bending stress. Assuming single-member use, the part listed under "Dimension Lumber" yields a stress of 1500 psi for the chosen grade, while the part under "Timbers, Beams and Stringers," yields a stress of 1600 psi. Using the latter category, the required value for the section modulus is

$$S = \frac{M}{F_b} = \frac{13{,}000 \times 12}{1600} = 97.5 \text{ in.}^3 \quad [1.60 \times 10^6 \text{ mm}^3]$$

while the value for $F_b = 1500$ psi may be determined by proportion as

$$S = \frac{1600}{1500} \times 97.5 = 104 \text{ in.}^3$$

From Table A.8, the smallest members in these two size categories are: 4 by 16 ($S = 135.661$ in.^{3}, $A = 53.375$ in.^{2}) and 6 by 12 ($S = 121.229$ in.^{3}, $A = 63.25$ in.^{2}). For the 4 by 16 the allowable stress is not changed by Table 4.2. as the factor from the table is 1.0. Thus the 4 by 16 is the choice for the least cross-sectional area in spite of having the lower value for bending stress.

Size Factors for Beams

Beams greater than 12 in. in depth with thickness of 5 in. or more have reduced values for the maximum allowable bending stress. This reduction is achieved with a reduction factor determined as

$$C_F = \left(\frac{12}{d}\right)^{1/9}$$

Values for this factor for standard lumber sizes are given in Table 4.5. For the preceding example, neither section qualifies for size reduction modification.

Lateral Bracing

Design specifications provide for the adjustment of bending capacity or allowable bending stress when a member is vulnerable to a buckling failure. To reduce this effect, thin beams (mostly joists and rafters) are often provided with bracing that is adequate to prevent both lateral (sideways) buckling and torsional (roll-over) buckling. The NDS requirements for bracing are given in Table 4.6. If bracing is not provided, a reduced bending capacity must be determined from rules given in the specifications.

Common forms of bracing consist of bridging and blocking. Bridging consists of crisscrossed wood or metal members in rows. Blocking consists of solid, short pieces of lumber the same size as the framing; these are fit tightly between the members in rows.

Beam Shear

As discussed in Section 3.7, the maximum beam shear stress for the rectangular sections ordinarily used for wood beams is expressed as

$$f_v = \frac{1.5V}{A}$$

where

f_v = maximum unit horizontal shear stress, psi
V = total vertical shear force at the section, lb
A = cross-sectional area of the beam

Table 4.5 Size Factors for Solid-Sawn Beams, C_F

Actual Beam Depth		
in.	mm	C_F
13.5	343	0.987
15.5	394	0.972
17.5	445	0.959
19.5	495	0.947
21.5	546	0.937
23.5	597	0.928

Table 4.6 Lateral Support Requirements for Rectangular Sawn-Wood Beams

Ratio of Depth to Breadth $(d/b)^a$	Required Conditions to Avoid Reduction of Bending Stress
$d/b \leq 2$	No lateral support required.
$2 < d/b \leq 4$	Ends held in position to prevent rotation or lateral displacement.
$4 < d/b \leq 5$	Compression edge held in position for entire span, and ends held in position to prevent rotation or lateral displacement.
$5 < d/b \leq 6$	Compression edge held in position for entire span, ends held in position to prevent rotation or lateral displacement, and bridging or blocking at intervals not exceeding 8 ft.
$6 < d/b \leq 7$	Both edges held in position for entire span, and ends held in position to prevent rotation or lateral displacement.

Source: Adapted from data in *National Design Specification*® (NDS®) *for Wood Construction* (Ref. 3), with permission of the publisher, American Forest and Paper Association.
[a]Ratio of nominal dimensions for standard sections.

Wood is relatively weak in shear resistance, with the typical failure producing a horizontal splitting of the beam ends. This is most frequently only a problem with heavily loaded beams of short span for which bending may be low but the shear force is high. Because the failure is one of horizontal splitting, it is common in wood design to describe this stress as horizontal shear, which is how the allowable shear stress is labeled in Table 4.1.

Example 2. A 6-by-10 beam of Douglas fir-larch, No. 2 dense grade, has a total uniformly distributed load of 6000 lb [26.7 kN]. Investigate for shear stress.

Solution. For this loading condition the maximum shear at the beam end is one half of the total load, or 3000 lb. Using the true dimensions of the section from Table A.8, the maximum stress is

$$f_v = \frac{1.5V}{A} = \frac{1.5 \times 3000}{52.25} = 86.1\ \text{psi} \quad [0.594\ \text{MPa}]$$

Referring to Table 4.1, under the classification "Beams and Stringers," the allowable stress is 170 psi. The beam is therefore adequate for this loading condition.

For uniformly loaded beams that are supported by end bearing, the code permits a reduction in the design shear force to that which occurs at a distance from the support equal to the depth of the beam.

Bearing

Bearing occurs at beam ends when a beam sits on a support or when a concentrated load is placed on top of a beam within the span. The stress developed at the bearing contact area is compression perpendicular to the grain, for which an allowable value ($F_{c\perp}$) is given in Table 4.1.

Although the design values given in the table may be safely used, when the bearing length is quite short, the maximum permitted level of stress may produce some indentation in the edge of the wood member. If the appearance of such a condition is objectionable, a reduced stress is recommended. Excessive deformation may also produce some significant vertical movement, which may be a problem for the construction.

Example 3. An 8-by-14 beam of Douglas fir-larch, No. 1 grade, has an end bearing length of 6 in. [152 mm]. If the end reaction is 7400 lb [32.9 kN], is the beam safe for bearing?

Solution. The developed bearing stress is equal to the end reaction divided by the product of the beam width and the length of bearing. Thus

$$f_c = \frac{\text{bearing force}}{\text{contact area}} = \frac{7400}{7.5 \times 6} = 164\ \text{psi} \quad [1.13\ \text{MPa}]$$

This is compared to the allowable stress of 625 psi from Table 4.1, which shows the beam to be quite safe.

Example 4. A 2-by-10 rafter cantilevers over and is supported by the 2-by-4 top plate of a stud wall. The load from the rafter is 800 lb [3.56 kN]. If both the rafter and the plate are No. 2 grade, is the situation adequate for bearing?

Solution. The bearing stress is determined as

$$f = \frac{800}{1.5 \times 3.5} = 152\ \text{psi} \quad [1.05\ \text{MPa}]$$

This is considerably less than the allowable stress of 625 psi, so the bearing is safe.

Example 5. A two-span 3-by-12 beam of Douglas fir-larch, No. 1 grade, bears on a 3-by-14 beam at its center support. If the reaction force is 4200 lb [18.7 kN], is this safe for bearing?

Solution. Assuming the bearing to be at right angles, the stress is

$$f = \frac{4200}{2.5 \times 2.5} = 672\ \text{psi} \quad [4.63\ \text{MPa}]$$

This is slightly in excess of the allowable stress of 625 psi.

Deflection

Deflections in wood structures tend to be most critical for rafters and joists, where span-to-depth ratios are often pushed to the limit. However, long-term high levels of bending stress can also produce sag, which may be visually objectionable or cause problems with the construction. In general, it is wise to be conservative with deflections of wood structures. Push the limits and you will surely get sagging floors and roofs and possibly very bouncy floors. This may in some cases make a strong argument for use of glued-laminated beams or even steel beams.

For the common uniformly loaded beam, the deflection takes the form of the equation

$$\Delta = \frac{5WL^3}{384EI}$$

Substitutions of relations between W, M, and flexural stress in this equation can result in the form

$$\Delta = \frac{5L^2 f_b}{24Ed}$$

Using average values of 1500 psi for f_b and 1500 ksi for E, the expression reduces to

$$\Delta = \frac{0.03L^2}{d}$$

where

Δ = deflection, in.

L = span, ft.

d = beam depth, in.

Figure 4.12 is a plot of this expression with curves for nominal dimensions of depth for standard lumber. For reference the lines on the graph corresponding to ratios of deflection of $L/180$, $L/240$, and $L/360$ are shown. These are commonly used design limitations for total-load and live-load deflections, respectively. Also shown for reference is the limiting span-to-depth ratio of 25 to 1, which is commonly considered to be a practical span limit for general purposes.

For beams with other values for bending stress and modulus of elasticity, true deflections can be obtained as

$$\text{True } \Delta = \frac{\text{true } f_b}{1500} \times \frac{1{,}500{,}000}{\text{true } E} \times \Delta \text{ from graph}$$

The following examples illustrate problems involving deflection. Douglas fir-larch is used for these examples and for the problems that follow them.

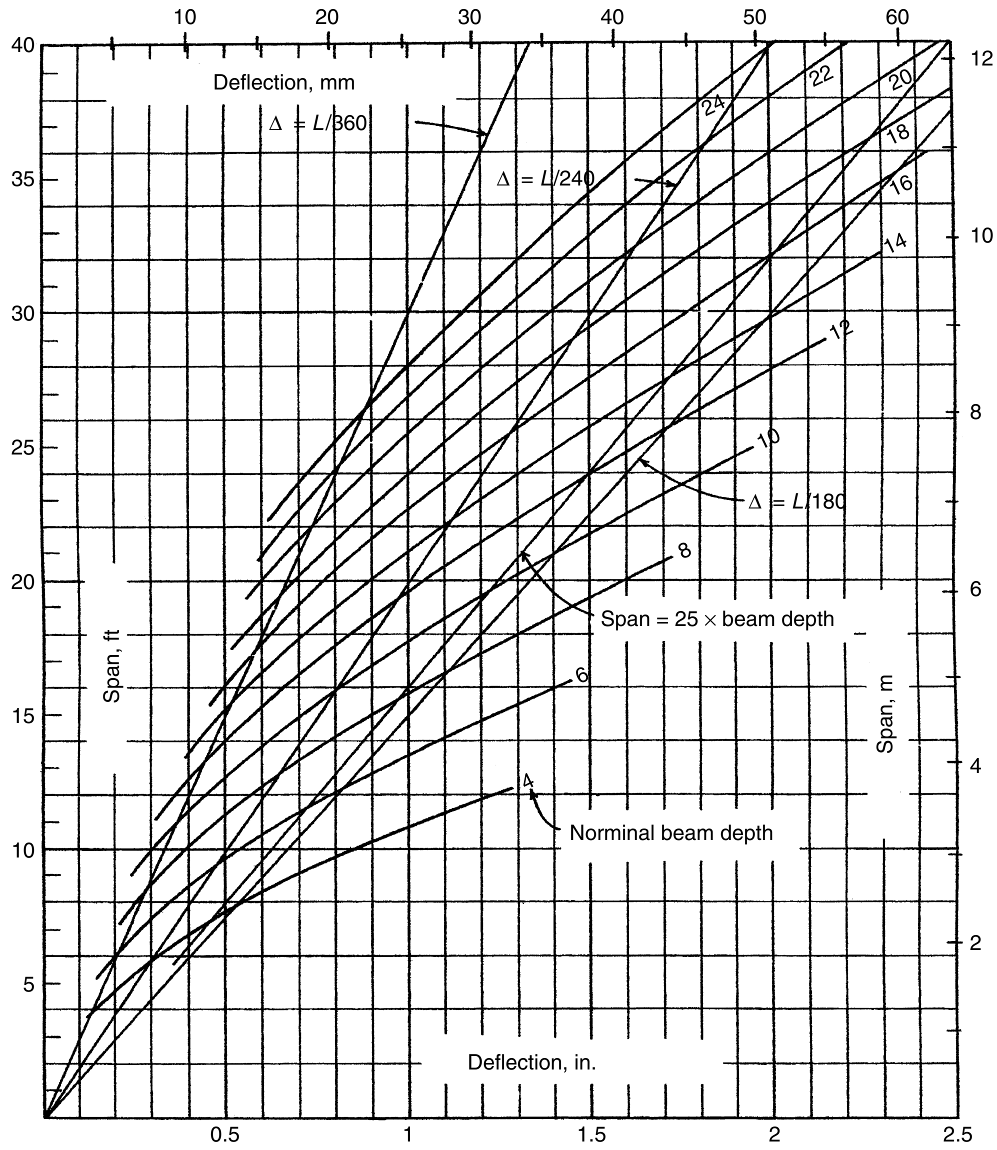


Figure 4.12 Approximate deflection of wood beams. Assumed conditions: maximum bending stress of 1500 psi, modulus of elasticity of 1,500,000 psi.

Example 6. An 8-by-12 wood beam with $E = 1{,}600{,}000$ psi is used to carry a total uniformly distributed load of 10 kips on a simple span of 16 ft. Find the maximum deflection of the beam.

Solution. From Table A.8 find the value of $I = 950$ in.^{4} for the 8-by-12 section. Then, using the deflection formula for this loading,

$$\Delta = \frac{5WL^3}{384EI} = \frac{5 \times 10{,}000 \times (16 \times 12)^3}{384 \times 1{,}600{,}000 \times 950} = 0.61 \text{ in.}$$

or, using the graph in Figure 4.12,

$$M = \frac{WL}{8} = \frac{10{,}000 \times 16}{8} = 20{,}000 \text{ ft-lb}$$

$$f_b = \frac{M}{S} = \frac{20{,}000 \times 12}{165} = 1455 \text{ psi}$$

From Figure 4.12, $\Delta = {\sim}0.66$ in. Then

$$\text{True } \Delta = \frac{1455}{1500} \times \frac{1{,}500{,}000}{1{,}600{,}000} \times 0.66 = 0.60 \text{ in.}$$

which shows reasonable agreement with the computed value.

Example 7. A beam consisting of a 6-by-10 section with $E = 1{,}400{,}000$ psi spans 18 ft and carries two concentrated loads. One load is 1800 lb and is placed at 3 ft from one end of the beam, and the other load is 1200 lb, placed at 6 ft from the opposite end of the beam. Find the maximum deflection due only to the concentrated loads.

Solution. For an approximate computation, use the equivalent uniform load method, consisting of finding the hypothetical total uniform load that will produce a moment equal to the actual maximum moment in the beam. Then the deflection for uniformly distributed load may be used with this hypothetical (equivalent uniform) load. Thus

$$\text{If:} \quad M = \frac{WL}{8}$$

$$\text{Then:} \quad W = \frac{8M}{L}$$

For this loading the maximum bending moment is 6600 ft-lb (the reader should verify this by the usual procedures) and the equivalent uniform load is thus

$$W = \frac{8M}{L} = \frac{8 \times 6600}{18} = 2933 \text{ lb}$$

and the approximate deflection is

$$\Delta = \frac{5WL^3}{384EI} = \frac{5 \times 2933 \times (18 \times 12)^3}{384 \times 1{,}400{,}000 \times 393} = 0.70 \text{ in.}$$

As in the previous example, the deflection could also be found by using Figure 4.12, with adjustments made for the true maximum bending stress and the true modulus of elasticity.

Joists and Rafters

Floor joists and roof rafters are closely spaced beams that support floor or roof decks. They are common elements of the structural system described as the *light wood frame* (see Figure 4.3). These may consist of sawn lumber, light trusses, laminated pieces, or composite elements achieved with combinations of sawn lumber, laminated pieces, plywood, or particleboard. The discussion in this chapter deals only with sawn lumber, typically in the class called *dimension lumber* having nominal thickness of 2 to 4 in. Although the strength of the structural deck is a factor, spacing of joists and rafters is typically related to the dimensions of the panels used for decking. The most used panel size is 48 by 96 in., from which are derived spacings of 12, 16, 19.2, 24, and 32 in.

Floor Joists

A common form of floor construction is shown in Figure 4.13. The structural deck shown in the figure is plywood, which produces a top surface not generally usable as a finished surface. Thus, some finish must be used, such as the hardwood flooring shown here. More common now for most interiors is carpet or thin tile, both of which require some smoother surface than the structural plywood panels, resulting in the use of *underlayment* typically consisting of wood fiber panels.

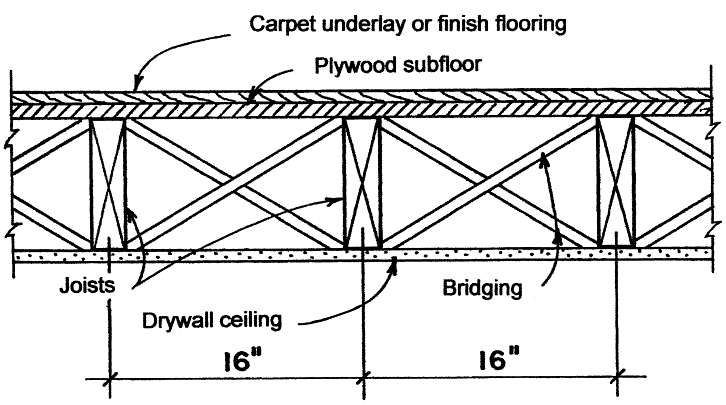


Figure 4.13 Typical floor construction, with solid-sawn joists, plywood structural deck, bridging, and directly applied drywall panel ceiling.

A drywall panel finish (paper-faced gypsum plaster board) is shown here for the ceiling directly attached to the underside of the joists. Since the floor surface above is usually required to be horizontally flat, the same surface can thus be developed for the ceiling. Of course, a separate *suspended* structure may also be provided for the ceiling support.

Lateral bracing for joists is usually provided by the attached deck. If additional bracing is required (see Table 4.6), it may consist of bridging as shown in Figure 4.13 or of solid blocking consisting of short pieces of the joist elements aligned in rows between the joists. If blocking is used, it will normally be located so as to provide for edge nailing of the deck panels. This nailing of all the edges of panels is especially critical when the joist and deck construction is required to serve as a horizontal diaphragm for wind or seismic forces. (See discussion in Chapter 9.)

Solid blocking is also used under any supported walls perpendicular to the joists or under walls parallel to the joists but not directly above a joist. Any loading on the joist construction other than the usual assumed dispersed load on the deck should be considered for reinforcement of the regular joist system. One way to give extra local strength to the system is to double up joists, a common practice at the edges of large openings in the floor.

With the continuous, multiple-span effect of decking and possible inclusion of bridging or blocking, there is a potential for load sharing by adjacent joists. This is the basis for classification as a *repetitive member*, permitting an increase of 15% in the allowable bending stress.

Floor joists may be designed as beams; however, the most frequent use of joists in light wood framing systems is in situations that are well defined in a short range of conditions. Spans are usually quite short, both dead and live loads are predictable, and a relatively few wood species and grades are most commonly used. This allows for the development of tabulated lists of joist sizes from which appropriate choices can be made. Table 4.7 is an abbreviated sample of such a table.

Example 8. Using Table 4.7, select joists to carry a live load of 40 psf and a dead load of 10 psf on a span of 15 ft, 6 in. Wood is Douglas fir-larch, No. 2 grade.

Solution. From Table 4.7, possible choices are 2 by 10 at 12 in., 2 by 12 at 16 in., or 2 by 12 at 19.2 in.

Table 4.7 Maximum Spans for Floor Joists (ft-in.)[a]

	Joist Size			
Spacing (in.)	2 × 6	2 × 8	2 × 10	2 × 12
Live load = 40 psf, Dead load = 10 psf, Maximum live-load deflection = $L/360$				
12	10-9	14-2	17-9	20-7
16	9-9	12-7	15-5	17-10
19.2	9-1	11-6	14-1	16-3
24	8-1	10-3	12-7	14-7
Live load = 40 psf, Dead load = 20 psf, Maximum live-load deflection = $L/360$				
12	10-6	13-3	16-3	18-10
16	9-1	11-6	14-1	16-3
19.2	8-3	10-6	12-10	14-10
24	7-5	9-50	11-6	13-4

Source: Compiled from data in the *International Building Code* (Ref. 4), with permission of the publisher, International Code Council.
[a]Joists are Douglas fir–larch, No. 2 grade. Assumed maximum available length of single piece is 26 ft.

Note that the values in Table 4.7 are based on a maximum deflection of 1/360 of the span under live load. In using the table for the example, it is also assumed that there is no modification of the reference stress values. If true conditions are significantly different from those assumed for Table 4.7, the full design procedure for a beam is required.

Rafters

Rafters are used for roof decks in a manner similar to floor joists. While floor joists are typically installed dead flat, rafters are commonly sloped to achieve roof drainage. For structural design it is common to consider the rafter span to be the horizontal projection, as indicated in Figure 4.14.

As with floor joists, rafter design is frequently accomplished with the use of safe load tables. Table 4.8 is representative of such tables and has been reproduced from the *International Building Code* (IBC) (Ref. 2). Organization of the table is similar to that for Table 4.7. The following example illustrates the use of the data in Table 4.8.

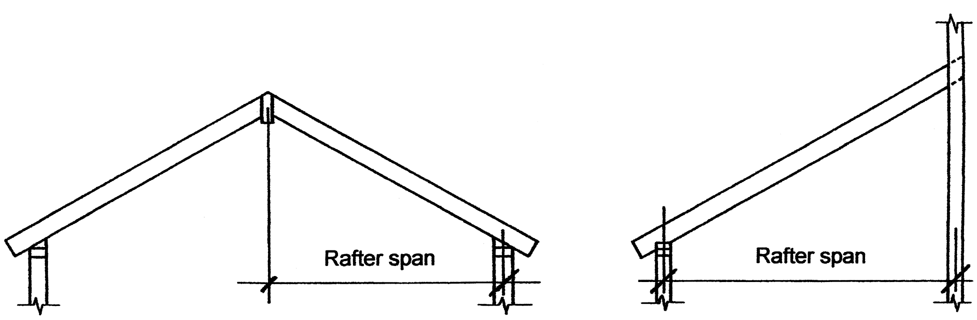


Figure 4.14 Definition of rafter span.

Table 4.8 Maximum Spans for Rafters (ft-in.)[a]

	Rafter Size				
Spacing (in.)	2 × 4	2 × 6	2 × 8	2 × 10	2 × 12
Live load = 20 psf, Dead load = 10 psf, Maximum live-load deflection = $L/240$					
12	9-10	15-6	20-5	25-8	26-0
16	8-11	14-1	18-2	22-3	25-9
19.2	8-5	13-1	16-7	20-3	23-6
24	7-10	11-9	14-10	18-2	21-0
Live load = 20 psf, Dead load = 20 psf, Maximum live-load deflection = $L/240$					
12	9-10	14-4	18-2	22-3	25-9
16	8-6	12-5	15-9	19-3	22-4
19.2	7-9	11-4	14-4	17-7	20-4
24	6-11	10-2	12-10	15-8	18-3

Source: Compiled from data in the *International Building Code* (Ref. 4), with permission of the publisher, International Code Council.
[a]Rafters are Douglas fir–larch, No. 2 grade. Ceiling is not attached to rafters. Assumed maximum available length of single piece is 26 ft.

Example 9. Rafters are to be used for a roof span of 16 ft. Live load is 20 psf; total dead load is 10 psf; live load deflection is limited to 1/240 of the span. Find the rafter size required for Douglas fir-larch of No. 2 grade.

Solution. (1) From Table 4.8, possible choices are for 2 by 8 at 16 in., 2 by 8 at 19.2 in., or 2 by 10 at 24 in.

Decking for Roofs and Floors

Materials used to produce roof and floor surfaces include the following:

Boards of nominal 1-in.-thickness solid-sawn lumber, typically with tongue-and-groove edges

Solid-sawn wood elements thicker than 1-in. nominal dimension (usually called planks or planking) with tongue-and-groove or other edge development to prevent vertical slipping between adjacent units

Plywood of appropriate thickness for the span and the construction

Other panel materials, including those of compressed wood fibers or particles

Proprietary products, used mostly for roof decks

Plank deck is especially popular for roof decks which are exposed to view from below. A variety of forms of products used for this construction are shown in Figure 4.15. Widely used is a nominal 2-in.-thick unit which may be of solid-sawn form (Figure 4.15*a*) but is now more likely to be of glued-laminated form (Figure 4.15*c*). Thicker units can be obtained for considerable spans between supporting members, but the thinner plank units are most popular.

Plank decks and other special decks are fabricated products produced by individual manufacturers. Information about their properties should be obtained from suppliers or the manufacturers. Plywood decks are widely used where their structural properties are critical. Plywood is an immensely variable material, although a few selected types are commonly used for structural purposes.

Plywood

Plywood is the term used to designate structural wood panels made by gluing together multiple layers of thin wood veneer (called *plies*) with alternate layers having their grain direction at right angles. The outside layers are called the *faces* and the others *inner plies*. Inner plies with the grain perpendicular to the faces are called *crossbands*. There is usually an odd number of plies so that the faces have the grain in the same direction. For structural applications in building construction, the common range of panel thickness is from $\frac{5}{16}$ to $1\frac{1}{8}$ in.

The alternating grain direction of the plies gives the panels considerable resistance to splitting, and as the number

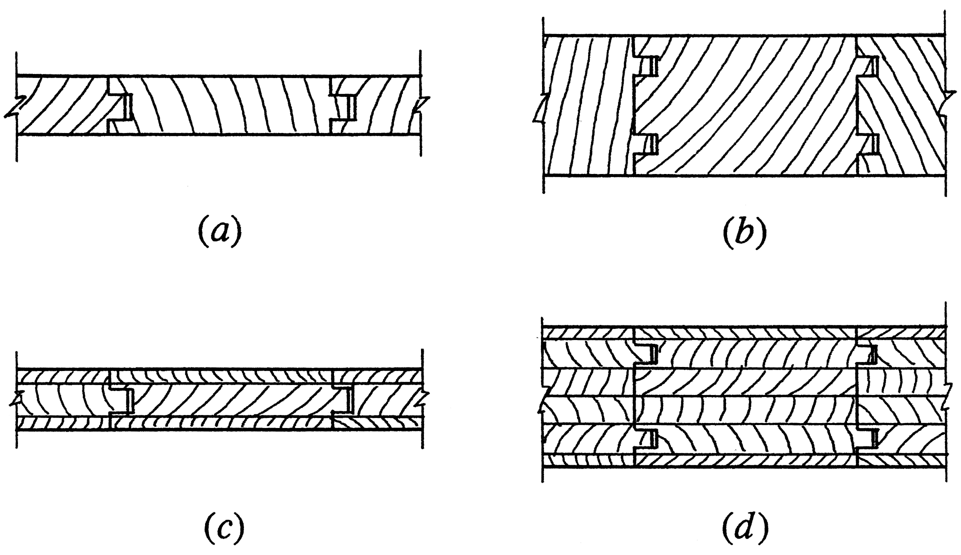


Figure 4.15 Units for board and plank decks.

of plies increases, the panels become approximately equal in strength in both directions. Thin panels may have only three plies, but for most structural applications plies will number from five to nine.

Types and Grades of Plywood

Many different kinds of plywood panels are produced. For structural applications, the principal distinctions other than panel thickness are the following:

Exposure Classification. Panels described as being *exterior* are used where high moisture conditions are enduring, such as outdoor uses and bathrooms, laundry rooms, and other high-moisture interior spaces. A classification of *exterior* 1 is for panels where the usage is for interior conditions, but the panels may be exposed to weather conditions during construction.

Structural Rating. Code-approved sheathing is identified as to class for purpose of establishing reference design values. Identification is achieved by marking of panels with an indelible stamp. Information in the stamp designates several properties of the panel, including basic structural capabilities.

Design Usage Data for Plywood

Data for structural design of plywood may be obtained from industry publications or from individual plywood manufacturers. Data are also provided in most building codes. Tables 4.9 and 4.10 are reproductions of tables in the IBC (Ref. 2). These provide data for the loading and span capabilities of rated plywood panels. Table 4.9 treats panels with the panel face grain perpendicular to the supports and Table 4.10 treats panels with the face grain parallel to the supports. Footnotes to these tables present various qualifications, including some of the loading and deflection criteria.

Plywood Diaphragms

Plywood deck-and-wall sheathing is frequently utilized to develop diaphragm actions for resistance to lateral loads from wind or earthquakes. Considerations for design of both horizontal diaphragms and vertical diaphragms (shear walls) are discussed in Chapter 9. Where both gravity loading and lateral loading must be considered, choices for the construction must relate to both problems. In the past most shear walls in light wood frame construction were achieved with plywood; however, recently, the material of choice is usually *oriented strand board*, except where exceptional strength is required.

Usage Considerations for Plywood

The following are some of the principal usage considerations for ordinary applications of structural plywood panels.

Choice of Thickness and Grade. This is largely a matter of common usage and local code acceptability. In most cases, for economy, the thinnest, lowest grade panels will usually be used unless various concerns require otherwise.

Modular Supports. With the usually common panel size of 48 by 96 in., logical spacing for studs, rafters, and

Table 4.9 Data for Plywood Roof and Floor Deck, Face Grain Perpendicular to Supports

Panel Span Rating	Panel Thickness (in.)	Maximum Span (in.) × 25.4 for mm		Load[5] (lb/ft²) × 0.0479 for kN/m²		Maximum Span (in.)
Roof/Floor Span	× 25 4 for mm	With Edge Support[6]	Without Edge Support	Total Load	Live Load	× 25.4 for mm
12/0	5/16	12	12	40	30	0
16/0	5/16, 3/8	16	16	40	30	0
20/0	5/16, 3/8	20	20	40	30	0
24/0	3/8, 7/16, 1/2	24	20[7]	40	30	0
24/16	7/16, 1/2	24	24	50	40	16
32/16	15/32, 1/2, 5/8	32	28	40	30	16[8]
40/20	19/32, 5/8, 3/4, 7/8	40	32	40	30	20[8,9]
48/24	23/32, 3/4, 7/8	48	36	45	35	24
54/32	7/8, 1	54	40	45	35	32
60/48	7/8, 1, 1 1/8	60	48	45	35	48
SINGLE-FLOOR GRADES		ROOF[3]				FLOOR[4]
Panel Span Rating (in.)	Panel Thickness (in.)	Maximum Span (in.) × 25.4 for mm		Load[5] (lb/ft²) × 0.0479 for kN/m²		Maximum Span (in.)
× 25.4 for mm		With Edge Support[6]	Without Edge Support	Total Load	Live Load	× 25.4 for mm
16 oc	1/2, 19/32, 5/8	24	24	50	40	16[8]
20 oc	19/32, 5/8, 3/4	32	32	40	30	20[8,9]
24 oc	23/32, 3/4	48	36	35	25	24
32 oc	7/8, 1	48	40	50	40	32
48 oc	1 3/32, 1 1/8	60	48	50	50	48

[1]Applies to panels 24 in. (610 mm) or wider.
[2]Floor and roof sheathing conforming with this table shall be deemed to meet the design criteria of Section 2312.
[3]Uniform load deflection limitations 1/180 of span under live load plus dead load, 1/240 under live load only.
[4]Panel edges shall have approved tongue-and-groove joints or shall be supported with blocking unless 1/4-in. (6.4 mm) minimum thickness underlayment or 1 1/2 in. (38 mm) of approved cellular or lightweight concrete is placed over the subfloor, or finish floor is 3/4-in. (19 mm) wood strip. Allowable uniform load based on deflection of 1/360 of span is 100 pounds per square foot (psf) (4.79 kN/m²) except the span rating of 48 in. on center is based on a total load of 65 psf (3.11 kN/m).
[5]Allowable load at maximum span.
[6]Tongue-and-groove edges, panel edge clips [one midway between each support, except two equally spaced between supports 48 in. (1219 mm) on center], lumber blocking, or other. Only lumber blocking shall satisfy blocked diaphgrams requirements.
[7]For 1/2-in. (12.7 mm) panel, maximum span shall be 24 in. (610 mm).
[8]May be 24 in. (610 mm) on center where 3/4-in. (19 mm) wood strip flooring is installed at right angles to joist.
[9]May be 24 in. (610 mm) on center for floors where 1 1/2 in. (38 mm) of cellular or lightweight concrete is applied over the panels.

Source: Reproduced from *International Building Code* (Ref. 2), with permission of the publisher, International Code Council, Inc.

Table 4.10 Data for Plywood Roof Deck, Face Grain Parallel to Supports

ALLOWABLE LOAD (PSF) FOR WOOD STRUCTURAL PANEL ROOF SHEATHING CONTINUOUS OVER TWO OR MORE SPANS AND STRENGTH AXIS PARALLEL TO SUPPORTS (Plywood Structural Panels Are Five-Ply, Five-Layer Unless Otherwise Noted)[a,b]

PANEL GRADE	THICKNESS (in.)	MAXIMUM SPAN (in.)	LOAD AT MAXIMUM SPAN (psf)	
			Live	Total
Structural I sheathing	$^7/_{16}$	24	20	30
	$^{15}/_{32}$	24	35[c]	45[c]
	$^1/_2$	24	40[c]	50[c]
	$^{19}/_{32}$, $^5/_8$	24	70	80
	$^{23}/_{32}$, $^3/_4$	24	90	100
Sheathing, other grades covered in DOC PS 1 or DOC PS 2	$^7/_{16}$	16	40	50
	$^{15}/_{32}$	24	20	25
	$^1/_2$	24	25	30
	$^{19}/_{32}$	24	40[c]	50[c]
	$^5/_8$	24	45[c]	55[c]
	$^{23}/_{32}$, $^3/_4$	24	60[c]	65[c]

For SI: 1 in. = 25.4 mm, 1 psf = 0.0479 kN/m^2.

a. Roof sheathing conforming with this table shall be deemed to meet the design criteria of Section 2304.7.

b. Uniform load deflection limitations $^1/_{180}$ of span under live load plus dead load, $^1/_{240}$ under live load only. Edges shall be blocked with lumber or other approved type of edge supports.

c. For composite and four-ply plywood structural panel, load shall be reduced by 15 psf.

Source: Reproduced from *International Building Code* (Ref. 2), with permission of the publisher, International Code Council, Inc.

joists become number divisions of the 48- or 96-in. dimensions: 12, 16, 19.2, 24, 32, or 48 in. However, spacing of framing must also relate to what is attached to the other side of a wall or as a directly applied ceiling as well as to the structural requirements for the framing.

Panel Edge Supports. Panel edges not falling on a support may need some provision for nailing, especially for roof and floor decks. Solid blocking is the usual answer, although some thick panels may have tongue-and-groove edges.

Attachment to Supports. For reference design values for shear loading in diaphragms, attachment is usually considered to be achieved with common wire nails. Required nail size and spacing relate to panel thickness and code minimums as well as to required shear capacities for diaphragms. Attachment is now mostly achieved with mechanically driven fasteners rather than by old fashioned pounding with a hand-held hammer. This means of attachment and the fasteners used are usually rated for capacity in terms of equivalency to ordinary nailing.

4.5 WOOD TRUSSES

Wood trusses are used for a wide range of applications. They are used mostly for roof structures owing to the lighter loads, more frequent use of longer spans, generally reduced requirements for fire resistance, and the ability of trusses to accommodate many roof profile forms. This section presents a general discussion of issues relating to the use of trusses that utilize wood elements. Design of trusses for various situations is presented in the examples in Chapter 10. Trussing is also used for lateral bracing for wind and earthquakes, which is discussed in Chapter 9.

Types of Trusses

Figure 4.16 shows some of the common forms for roof trusses. While the bottom chord of a truss is most often horizontal, the top chords most often need to accommodate water drainage, and the common truss profiles reflect this need. The height or *rise* of a truss divided by the span is called the *pitch*: the rise of a symmetrical gable-form truss (of triangular profile) divided by half the span is the *slope*, which is the tangent of the angle of slope of the top chord.

Unfortunately, these two terms are often used interchangeably to define the slope. Another way of expressing the slope is to give the amount of rise per foot of the span. A roof that rises 6 in. in a horizontal distance of 12 in. has a slope of "6 in 12." Reference to Table 4.11 should clarify this terminology.

Loads on Trusses

The first step in the design of a roof truss consists of computing the loads the truss will be required to support. These are dead and live loads. The former includes the weight of all construction materials supported by the truss; the latter includes loads resulting from snow and wind and, on flat roofs, occupancy loads and an allowance for the possible ponding of water due to impaired drainage.

Table 4.11 Roof Pitches and Slopes

Pitch rise/span	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{4}$	1/3.46	$\frac{1}{3}$	$\frac{1}{2}$
Slope degrees	14° 3′	18° 26′	21° 48′	26° 34′	30° 0′	33° 0′	45° 0′
Slope ratio	3 in 12	4 in 12	4.8 in 12	6 in 12	6.92 in 12	8 in 12	12 in 12

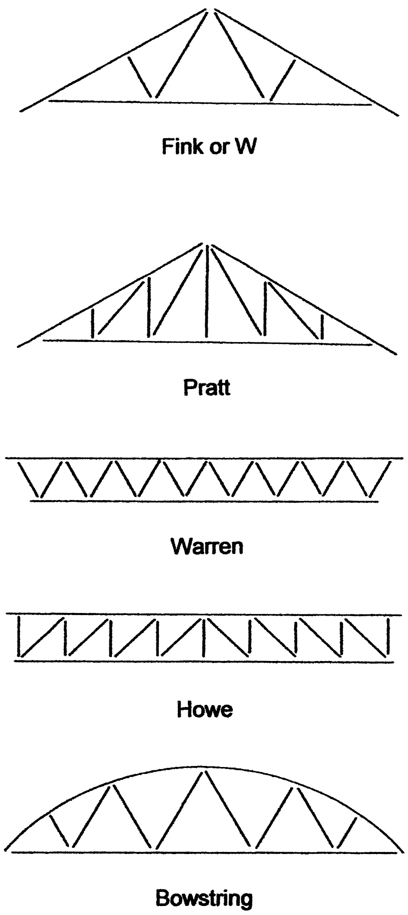


Figure 4.16 Common forms for wood roof trusses.

Table 4.12 Approximate Weight of Wood Trusses in Pounds per Square Foot of Supported Roof Surface

Span		Slope of Roof			
ft	m	45°	30°	20°	Flat
Up to 40	Up to 12	5	6	7	8
40–50	12–15	6	7	7	8
50–65	15–20	7	8	9	10
65–80	20–25	9	9	10	11

Table 4.12 provides estimated weights of wood trusses for various spans and pitches. With respect to the latter, one procedure is to establish an estimate in pounds per square foot of roof surface and consider this load as acting at the panel points of the upper chord. After the truss has been designed, its actual weight may be computed and compared with the estimated weight.

Truss Members and Joints

The three common forms of truss construction are those shown in Figure 4.17. The single-member type, with all members in one plane as shown in Figure 4.17*a*, is that used most often to produce the simple W-form truss (Figure 4.17*a*), with members usually of 2 in. nominal thickness. Joints may use plywood gussets as shown in Figure 4.17*a* but are more often made with metal-connecting devices when trusses are produced as products by a manufacturer.

For larger trusses the form shown in Figure 4.17*b* may be used, with members consisting of multiples of standard lumber elements. Joints are usually made with bolts and some form of shear developers such as split rings. However, large trusses can also still be made with single timber pieces, thick steel gussets, and bolts.

In the so-called *heavy-timber truss* individual members are of large timber or glued-laminated elements, usually occurring in a single plane as shown in Figure 4.17*c*. A common type of joint in this case is one using steel plates attached by lag screws or through-bolts. Depending on the truss pattern and loading, it may be possible to make some joints without gussets, as shown in Figure 4.18*a* for the diagonal compression member and the bottom chord connection. This was common in the past, but it requires carpentry work not easy to obtain today.

Although wood members have considerable capacity to resist tension, achieving tension connections is not so easy. Thus in some trusses tension members are made of steel, as shown for the vertical member in Figure 4.18*a*. A common form for manufactured trusses is that shown in Figure 4.18*b* where a flat-chorded truss has chords of solid wood and all interior members of steel (see Figure 4.19).

The selection of truss members and jointing methods depends on the size of the truss and the loading conditions. Unless trusses are exposed to view, the specific choices of members and details of the fabrication are often left to the discretion of the fabricators of the trusses.

Timber Trusses

Years ago, single members of timber were often used for large trusses. Cast iron and steel elements were used for various tasks in this construction. Figure 4.20 shows an example of this type of structure as it was developed in the early nineteenth century. This form of construction is seldom seen today except in restoration or reproduction of historic buildings. The special parts, such as the cast iron support shoe and the highly crafted notched joints, are generally unattainable.

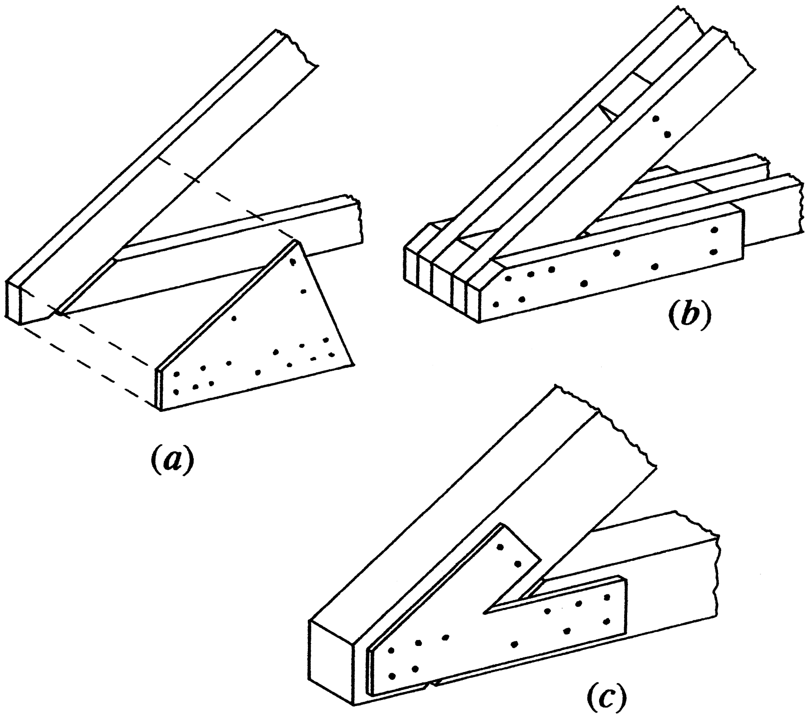


Figure 4.17 Forms of construction for wood trusses.

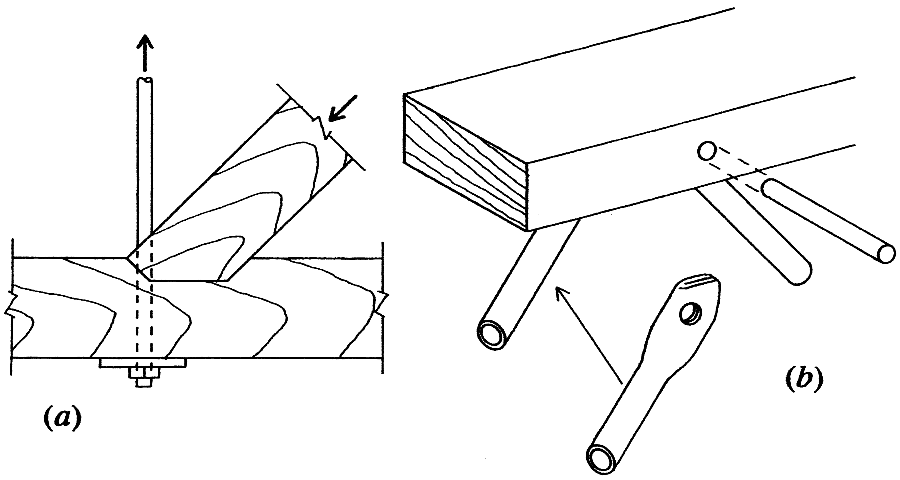


Figure 4.18 Forms of construction with both wood and steel members.

Trusses of the size and general form of that shown in Figure 4.20 are now mostly made with multiple-element members and bolted joints, as shown in Figure 4.17*b*. Although steel elements may be used for some tension members, this is done less frequently today because joints using shear developers are capable of sufficient load resistance to permit use of wood tension members. It was chiefly the connection problem that inspired the use of steel rods in the timber truss. The one vestige of this composite construction today is the use of steel members in the manufactured trusses of the form shown in Figures 4.18*b* and 4.19

Heavy timber trusses are sometimes still used with general timber construction that is exposed to view. These trusses are usually achieved with members of solid-sawn or glued-laminated timbers and with joints employing steel gusset plates and bolts. See Figure 4.21. Although these trusses are made to take the appearance of relatively rough, hand-crafted construction, they are mostly produced by the manufacturers of general truss systems.

Manufactured Trusses

The majority of trusses used for building structures today are produced as manufactured products, and the detailed engineering design is done largely by the engineers in the employ of the manufacturers. Since shipping of trusses over great distances is not generally feasible, the use of these products is chiefly limited to a region within a reasonable distance from a particular manufacturing facility.

There are three principal types of manufactured truss. Manufacturers vary in size of operation; some produce only

Figure 4.19 Manufactured trusses with wood and steel members.

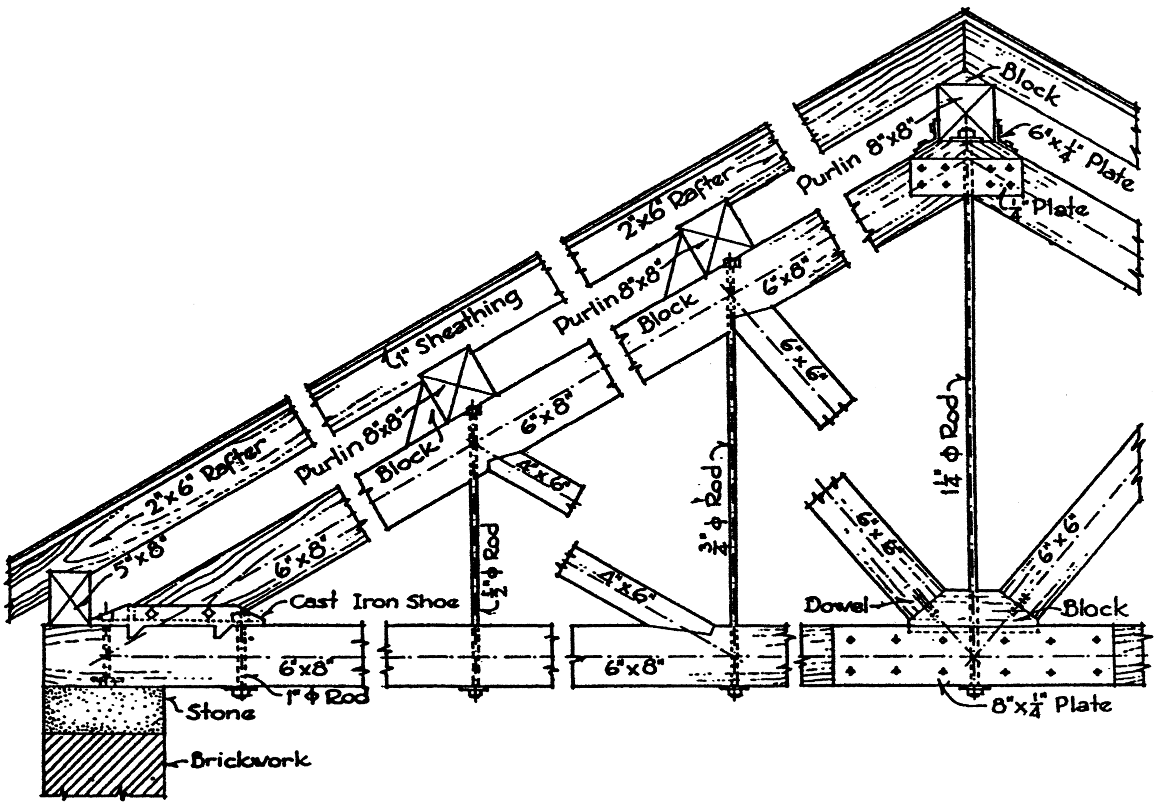


Figure 4.20 Early combination wood and steel truss. Reproduced from *Materials and Methods of Construction*, 1932, with permission of the publisher, John Wiley & Sons, New York.

(*a*)

(*b*)

Figure 4.21 Contemporary heavy timber truss, achieved with solid-sawn members, bolts, and thick steel gussets.

one type, while others have a range of products. The three common forms are as follows:

A simple gable-form, W-pattern truss (Figure 4.16*a*) with truss members of single-piece, 2-in.-nominal-thickness lumber. These are quite easy to produce and can be turned out in small quantities by small local suppliers in many cases.

A composite truss, usually consisting of a combination of wood and steel elements, as shown in Figures 4.18*b* and 4.19. These are more sophisticated in fabrication detail than the simple W-trusses, and they are usually produced as proprietary products by larger companies. These are available in a wide range of sizes for uses ranging from simple floor joists to long-spanning roof trusses. They usually compete with open-web steel joists in regions where they are readily available.

A large, long-span truss, usually using multiple-element members, as shown in Figure 4.17*b*. These may be produced with some standardization by a specific manufacturer but are often customized to some degree for a particular building. One form is the bowstring truss, which in reality is a tied arch (see Figure 4.22).

Suppliers of manufactured trusses usually have some standardized models, but they always have some degree of variability to accommodate the specific usage conditions for a particular building.

Figure 4.22 Roof structure with long-span manufactured bowstring trusses using multiple lumber members and joints with steel bolts and split-ring shear connectors.

Today, almost all building materials and products are industrially produced. There is hardly any stigma attached any more to objects that are manufactured, rather than hand made. The illusion of being hand made can be created, but the real processes must be acknowledged.

4.6 WOOD COLUMNS

The type of wood column used most frequently is the *simple solid column*, which consists of a single sawn piece of wood that is square or close to square in cross section. Solid columns of circular cross section are also considered simple solid columns and typically consist of trimmed, but not sawn, tree trunks which are called *poles*. A *spaced column* is a bolted assembly of two or more sawn pieces with their longitudinal axes parallel and separated at their ends and at the middle point of their length by blocking. Two other types are *built-up columns,* consisting of bundled multiple sawn pieces, and *glued-laminated columns.* The *studs* in light wood framing are also columns.

Slenderness Ratio for Columns

In wood construction the slenderness ratio used for design of a freestanding solid column is the ratio of its unbraced length to the dimension of its shortest side, expressed as L/d. See Figure 4.23.

When members are braced so that the laterally unsupported length with respect to one face is less than that to the other, L is the distance between lateral supports that prevent movement in the direction along which the corresponding d value is measured. This is illustrated in Figure 4.24. If

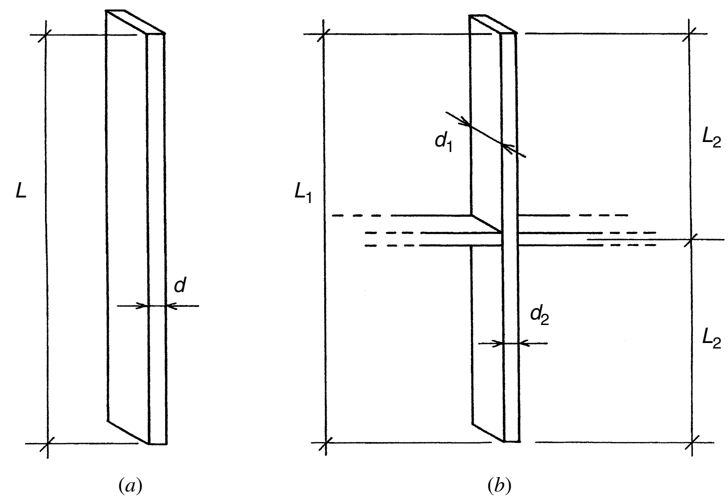


Figure 4.23 Determination of unbraced height of a column, as related to the critical column thickness dimension.

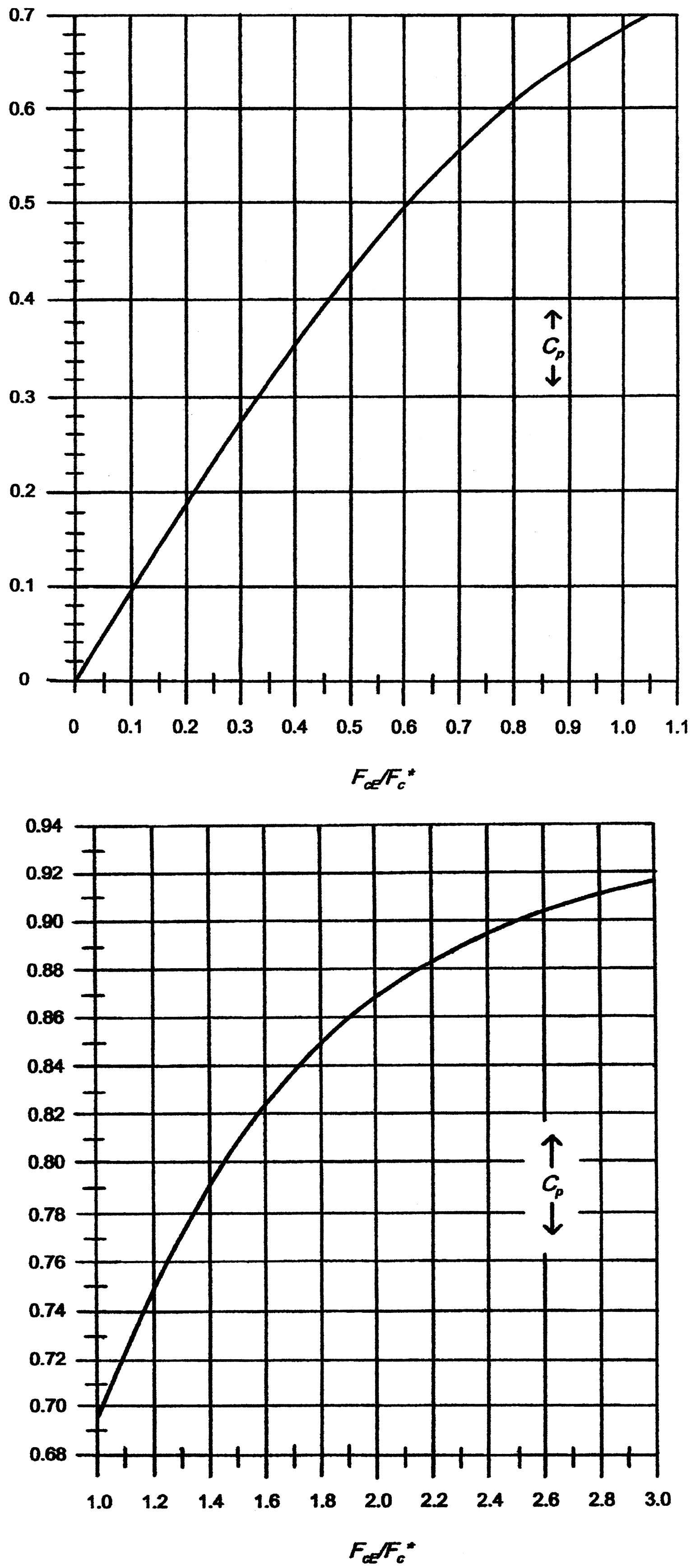


Figure 4.24 Column stability factor C_p as a function of F_{cE}/F_c^*.

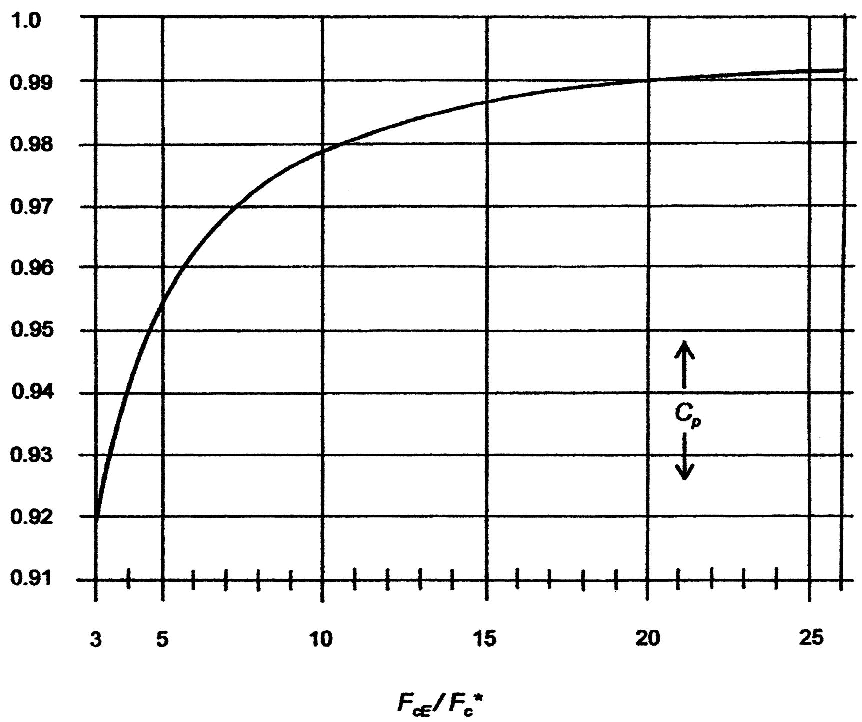


Figure 4.24 (*continued*)

the section is not square or round, it may be necessary to investigate two *L/d* conditions for such a column to determine which is the limiting case.

The slenderness ratio for simple solid columns is limited to $L/d = 50$, although this is quite slender for a freestanding column. During construction, before all bracing members are established, a temporary slenderness ratio of 75 is permitted by codes.

Compression Capacity of Simple Solid Columns

Compression capacity of columns is discussed in Section 3.3, with the form of the relationship between axial compression capacity and column slenderness displayed in Figure 3.17. The limiting conditions of this relationship are the very short column and the very slender column. The short member—such as a block of wood—fails in crushing (zone 1 in Figure 3.17), which is limited by the mass of the material (cross-sectional area of the column) and the stress limit for compression parallel to the grain of the wood. The very slender member—such as a yardstick—fails in elastic sideways buckling (zone 3 in Figure 3.17), which is determined by the stiffness of the material (its E value) and the bending stiffness of the member cross section (its modulus of elasticity, I). Between these two extremes (zone 2 in Figure 3.17), which is where most wood columns fall, the behavior is determined by a transition between the two distinctly different forms of response. Thus the form of the double-inflected curve in Figure 3.17 is determined.

For investigation, a single formula is used effectively covering the whole range of slenderness. The formula and its various factors are complex, and its use involves considerable computation; nevertheless, the basic process is essentially simplified through the use of a single defined relationship.

The work that follows demonstrates the use of the basic column formula and will show why the use of various shortcuts are commonly used for design work. Otherwise, design necessarily consists of a laborious trial-and-error method.

Investigation for Column Load Capacity

The basic formula for determination of the capacity of a wood column based on the allowable stress design method is

$$P = (F_c^*)(C_p)(A)$$

where

A = area of column cross section
F_c^* = design value for compression, modified
C_p = column stability factor
P = allowable column axial compression load

The column stability factor is determined as

$$C_p = \frac{1 + (F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1 + (F_{cE}/F_c^*)}{2c}\right]^2 - \frac{F_{cE}/F_c^*}{c}}$$

where

F_{cE} = Euler buckling stress as determined by formula below
$c = 0.8$ for sawn lumber

For the buckling stress

$$F_{cE} = \frac{0.822E'_{\min}}{(L_e/d)^2}$$

where

$E'_{\min}$ = modulus of elasticity for stability
L_e = effective buckling length
d = column cross-sectional width

As used here these formulas relate to the case of a simple solid-sawn column with no modification for stiffness.

For solid-sawn columns, the formula for C_p is simply a function of the value of F_{cE}/F_c^* with the value of c being a constant of 0.8. It is therefore possible to plot a graph of the value for C_p as a function of the value of F_{cE}/F_c^*, as is done in three parts in Figure 4.24. Accuracy of values obtained from Figure 4.25 is low but is usually acceptable for column design work. Of course, greater accuracy can always be obtained with the use of the formula.

The following examples illustrate the use of the NDS formulas for columns.

Example 10. A wood column consists of a 6 by 6 of Douglas fir-larch, No. 1 grade. Find the safe axial compression load for unbraced lengths of (1) 2 ft, (2) 8 ft, and (3) 16 ft.

Solution. From Table 4.1 find $F_c = 1000$ psi, $E_{\min} = 580{,}000$ psi. With no basis for adjustment given, the F_c value is used as the F_c^* value in the formulas.

For length 1, $L/d = 2(12)/5.5 = 4.36$. Then

$$F_{cE} = \frac{0.822E_{\min}}{(L/d)^2} = \frac{0.822(580{,}000)}{(4.36)^2} = 25{,}080 \text{ psi}$$

$$\frac{F_{cE}}{F_c^*} = \frac{25{,}080}{1000} = 25.08$$

$$C_p = \frac{1+25.08}{1.6} - \sqrt{\left[\frac{1+25.08}{1.6}\right]^2 - \frac{25.08}{0.8}} = 0.992$$

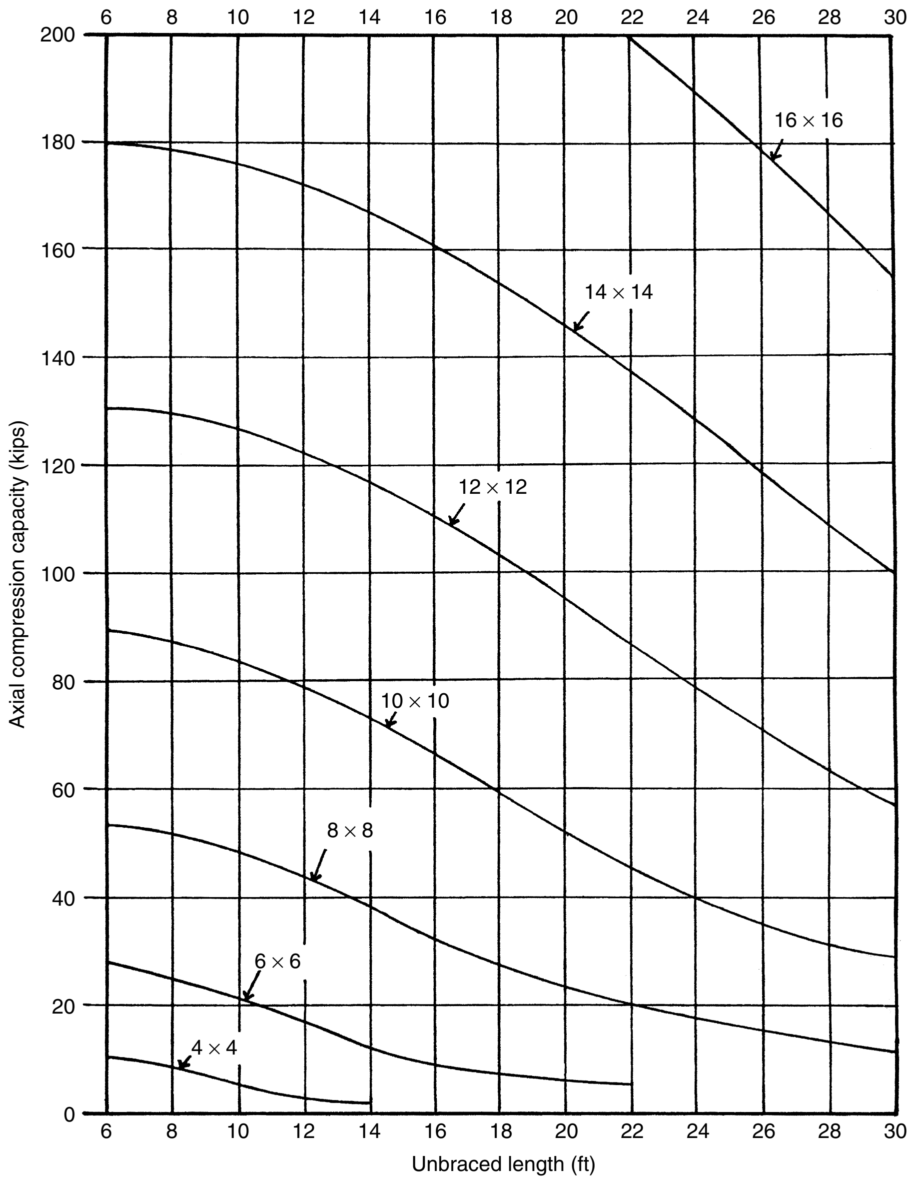


Figure 4.25 Load capacity in kips for wood columns of Douglas fir-larch, No. 1 grade.

and the allowable compression load is

$$P = F_c^* C_p A = (1000)(0.992)(5.5)^2 = 30{,}008 \text{ lb}$$

As mentioned previously, Figure 4.24 may be used to simplify the computation.

For length (2), $L/d = 8(12)/5.5 = 17.45$, for which $F_{cE} = 3916$ psi, $F_{cE}/F_c^* = 0.391$, and from Figure 4.24, $C_p = 0.82$; thus

$$P = (1000)(0.82)(5.5)^2 = 24{,}800 \text{ lb}$$

For length (3), $L/d = 16(12)/5.5 = 34.9$, for which $F_{cE} = 1566$ psi, $F_{cE}/F_c^* = 1.566$, and from Figure 4.24, $C_p = 0.35$; thus

$$P = (1000)(0.35)(5.5)^2 = 10{,}590 \text{ lb}$$

Example 11. Wood 2 by 4s are to be used for a wall with wood grade of Douglas fir-larch, stud grade. If the wall is 8.5 ft high, what is the compression capacity of a single stud?

Solution. It is assumed that the wall construction braces the studs on their weak axis (1.5 in.) and the critical d is thus 3.5 in. Then $L/d = 8.5(12)/3.5 = 29.14$. From Table 4.1, $F_c = 850$ psi, $E_{\min} = 510{,}000$ psi. From Table 4.2, an adjustment of 1.05 modifies F_c to $1.05(850) = 892.5$ psi. Then

$$F_{cE} = \frac{0.822(510{,}000)}{(29.14)^2} = 494 \text{ psi}$$

$$\frac{F_{cE}}{F_c^*} = \frac{494}{892.5} = 0.554$$

From Figure 4.24, $C_p = 0.47$; thus

$$P = (892.5)(0.47)(1.5 \times 3.5) = 2202 \text{ lb}$$

Design of Wood Columns

The design of columns is complicated by the relationships in the column formulas. The allowable stress for the column is dependent upon the actual column shape and dimensions, which are not known at the beginning of the design process. This does not allow for simply inverting the column formulas to derive required properties for the column. A trial-and-error process is therefore indicated. For this reason, designers typically use various design aids: graphs, tables, or computer-aided processes.

Because of the large number of wood species and grades, resulting in many different values for allowable stress and modulus of elasticity, precisely tabulated capacities become impractical. Nevertheless, aids using average values are available and simple to use for design. Figure 4.25 is a graph on which the axial compression load capacity of some square column sections of a single species and grade are plotted. The other variable on the graph is the lateral unbraced height of the column. Table 4.13 yields capacities in a similar range of variables. Note that for both Figure 4.25 and Table 4.13 the smaller size column sections fall into the classification of "dimension lumber" rather than for "timbers," as defined for Table 4.1. This makes for one more complication in the design process.

Round Columns

Solid wood columns of round cross section are not used extensively in building construction. As for load capacity, round and square columns of the same cross-sectional area will support the same axial loads and have the same degree of stiffness.

When designing a wood column of circular cross section, a simple procedure is to design a square column first and

Table 4.13 Safe Service Loads for Wood Columns[a]

Column Section		Unbraced Length (ft)										
Nominal Size	Area (in.^{2})	6	8	10	12	14	16	18	20	22	24	26
4 × 4	12.25	11.1	7.28	4.94	3.50	2.63						
4 × 6	19.25	17.4	11.4	7.76	5.51	4.14						
4 × 8	25.375	22.9	15.1	10.2	7.26	6.46						
6 × 6	30.25	27.6	24.8	20.9	16.9	13.4	10.7	8.71	7.17	6.53		
6 × 8	41.25	37.6	33.9	28.5	23.1	18.3	14.6	11.9	9.78	8.91		
8 × 8	56.25	54.0	51.5	48.1	43.5	38.0	32.3	27.4	23.1	19.7	16.9	14.6
8 × 10	71.25	68.4	65.3	61.0	55.1	48.1	41.0	34.7	29.3	24.9	21.4	18.4
10 × 10	90.25	88.4	85.9	83.0	79.0	73.6	67.0	60.0	52.9	46.4	40.4	35.5
10 × 12	109.25	107	104	100	95.6	89.1	81.2	72.6	64.0	56.1	48.9	42.9
12 × 12	132.25	130	128	125	122	117	111	104	95.6	86.9	78.3	70.2
14 × 14	182.25	180	178	176	172	168	163	156	148	139	129	119
16 × 16	240.25	238	236	234	230	226	222	216	208	200	190	179

[a]Load capacity in kips for solid-sawn sections of No. 1 grade Douglas fir–larch with no adjustment for moisture or load duration conditions.

then select a round column with an equivalent cross-sectional area. To find the required diameter of the equivalent round column, the side dimension of the square column is multiplied by 1.128.

Poles

Poles are round timbers consisting of the peeled logs of coniferous trees. When long they are tapered in form, which is the natural form of the tree trunk. As columns, poles are designed with the same basic criteria used for rectangular sawn sections.

For a tapered column, a conservative assumption for design is that the critical column diameter is the least diameter at the small end. If the column is very short, this is reasonable. However, for a tall column, with buckling occurring near the midheight of the column, this is very conservative, and the code provides for adjustment. Because of the typical lack of straightness and presence of many flaws, however, most designers prefer to use the unadjusted small-end diameter for design computations. In regions where poles are readily available, pole construction is extensively used for utilitarian buildings.

Stud Wall Construction

Studs are the vertical elements used for wall framing in light wood construction. Studs serve utilitarian purposes of providing for attachment of wall surfacing but also serve as columns when the wall provides bearing wall functions. The most common studs are 2 by 4s spaced at intervals of 12, 16, or 24 in., the spacing deriving from the common 4-by-8-ft panels used for wall coverings. Attachment of covering is one criterion for selection of stud size.

Studs of nominal 2 in. thickness must be braced on the weak axis when used for story-high walls, a simple requirement deriving from the limiting ratio of L/d of 50 for columns. If the wall is surfaced on both sides, the studs are usually considered to be adequately braced by the surfacing. If the wall is not surfaced or is surfaced on only one side, horizontal blocking between studs must be provided, as shown in Figure 4.26. The number of rows of blocking and the spacing of the blocking will depend on the wall height and the need for column action by the studs.

Studs may also serve other functions, as in the case of an exterior wall subjected to wind forces. For this situation the studs must be designed for the combined actions of compression plus bending.

In colder climates it is now common to use wider studs to accommodate more insulation within the wall. This may produce some redundancy of strength for one-story walls but may actually be required for tall or heavily loaded walls. Design for direct wind pressure or for shear wall functions may also affect the choice of stud size.

If vertical loads are high or bending is great, it may be necessary to strengthen a stud wall. This can be done in a number of ways, such as:

Decreasing stud spacing; 16 in. is most common, 12 in. is possible.
Increasing the stud thickness from 2 in. nominal to 3 in. nominal.
Increasing the stud width from 4 in. nominal to 6 in. nominal or greater.
Using doubled studs or large timbers as posts at locations of heavy concentrated loads, such as reactions of beams.

In general, studs are columns and must comply with the various requirements for design of solid-sawn sections. Any appropriate grade may be used, although special stud grades are commonly used for ordinary 2-by-4 studs.

Table 4.14, which is adapted from a table in the IBC (Ref. 2), provides data for the selection of studs for both bearing and nonbearing walls. The code stipulates that these data must be used in lieu of any engineering design for the studs, which means that other possibilities may be considered if computations can support a case for them.

Stud wall construction is often used as part of a general light wood construction system described as *light wood frame construction*. This system has been highly refined over many years of usage in the United States, with its present most common form as shown in Figure 4.3. The joist-and-rafter construction discussed in Section 4.4 and the stud wall construction discussed here are the primary elements of this system. In most applications, the system is almost entirely composed of 2-in.-nominal-dimension lumber.

Built-Up Columns

In various situations single columns may consist of multiple elements of solid-sawn sections. Although the description

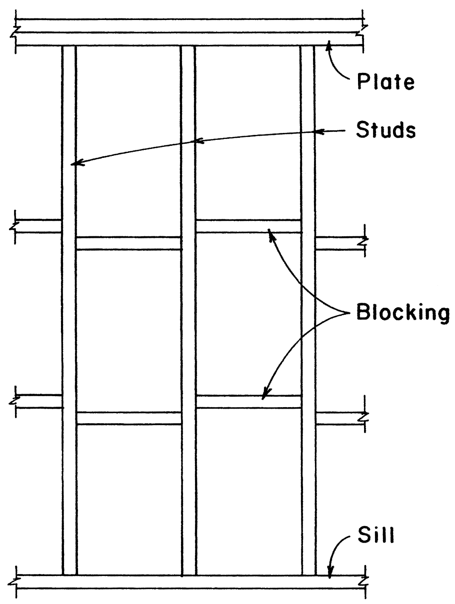


Figure 4.26 Stud wall construction with blocking.

Table 4.14 Requirements for Stud Wall Construction

		Bearing Walls			Nonbearing Walls	
Stud Size (in.)	Laterally Unsupported Stud Height[a] (ft)	Supporting Roof and Ceiling Only	Supporting One Floor, Roof, and Ceiling	Supporting Two Floors, Roof, and Ceiling	Laterally Unsupported Stud Height[a] (ft)	Spacing (in.)
		Spacing (in.)				
2 × 3[b]	—	—	—	—	10	16
2 × 4	10	24	16	—	14	24
3 × 4	10	24	24	16	14	24
2 × 5	10	24	24	—	16	24
2 × 6	10	24	24	16	20	24

Source: Compiled from data in the *International Building Code* (Ref. 2), with permission of the publisher, International Code Council.
[a]Listed heights are distances between points of lateral support placed perpendicular to the plane of the wall. Increases in unsupported height are permitted where justified by analysis.
[b]Shall not be used in exterior walls.

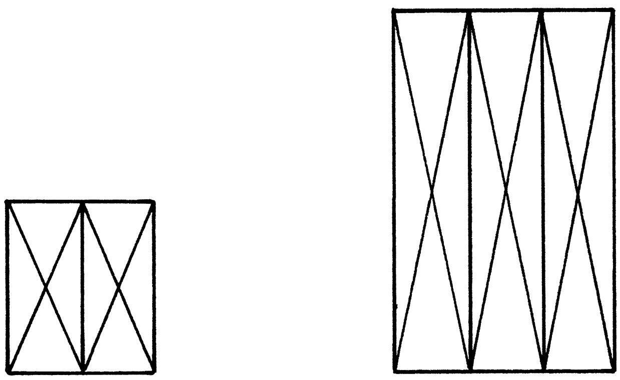


Figure 4.27 Cross sections of built-up columns with multiple lumber elements.

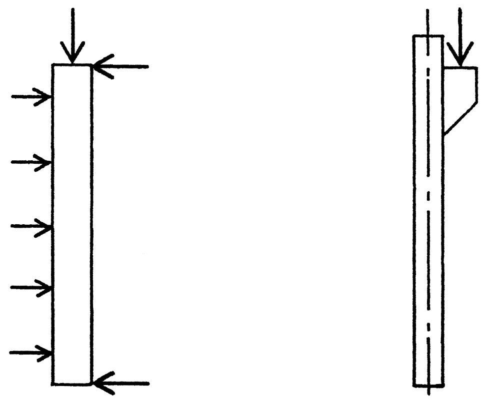


Figure 4.28 Common cases involving combined compression and bending in columns.

includes glued-laminated and spaced columns, the term *built-up column* is generally used for multiple-element columns such as those shown in Figure 4.27. Built-up columns usually have the elements attached to each other by mechanical devices such as nails, spikes, lag screws, or machine bolts. The NDS (Ref. 3) has data and procedures for design of built-up columns.

Columns with Bending

In wood structures columns with bending occur most frequently as shown in Figure 4.28. Studs in exterior walls represent the situation shown in Figure 4.28*a*, with a loading consisting of vertical gravity load plus horizontal wind load. Due to the use of common construction details, columns carrying only vertical loads may sometimes be loaded eccentrically, as shown in Figure 4.28*b*.

The basic actions of a column and a bending member are essentially different in character, and it is therefore customary to consider this combined activity by what is called *interaction*. The basic nature of interaction is discussed in Section 3.3 and the relationships are displayed on a graph, as shown in Figure 3.20. The form of the graph is derived from the formula

$$\frac{P_n}{P_0} + \frac{M_n}{M_0} = 1$$

A graph similar to that in Figure 3.20 can be produced using stresses rather than loads and moments, since stresses are directly proportional to the loads and moments that produce them. This is the procedure generally used in wood and steel design, with the graph taking a form expressed as

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$$

where

f_a = computed stress due to load
F_a = allowable column action stress
f_b = computed stress due to bending
F_b = allowable bending stress

Various effects cause deviation from the pure, straight-line form of interaction, including inelastic behavior, effects of lateral stability or torsion, general effects of member cross-sectional form, and flaws in and lack of straightness of the column. While having a general form similar to the pure interaction formula, the formulas prescribed by design codes are more complex in response to the various qualifying conditions.

Investigation of Columns with Bending

For solid-sawn columns the NDS provides the following formula for investigation.

$$\left(\frac{f_c}{F'_c}\right)^2 + \frac{f_b}{F_b\left(1 - f_c/F_{cE}\right)} \leq 1$$

where

f_c = computed compressive stress
F'_c = adjusted reference design value for compressive stress
f_b = computed bending stress
F_b = reference design stress for bending
F_{cE} = value determined for investigation of buckling

This equation assumes that bending is in one direction only. The NDS provides additional equations to be used for bending on two axes (biaxial bending), although we will not address the problem here.

As with other column design problems, this formula does not lend itself to direct design determinations. Thus, the usual trial-and-error process is required.

The following examples demonstrate applications for the investigation of columns with bending.

Example 12. An exterior wall stud of Douglas fir-larch, stud grade, is loaded as shown in Figure 4.29*a*. Investigate the stud for the combined loading. (*Note:* This is the wall stud from the building example in Section 10.2.)

Solution. From Table 4.1, $F_c = 850$ psi, $F_b = 700$ psi, and $E_{\min} = 510{,}000$ psi. Note that these values are not changed by Table 4.2, as the table factors are all 1.0. With inclusion of the wind loading, the stress values (but not E) may be increased by a factor of 1.6 (see Table 4.4).

Assume that wall surfacing braces the 2-by-6 studs adequately on their weak axis ($d = 1.5$ in.), so the critical value for d is 3.5 in. Thus

$$\frac{L}{d} = \frac{11 \times 12}{5.5} = 24$$

$$F_{cE} = \frac{0.822E_{\min}}{(L/d)^2} = \frac{0.822 \times 510{,}000}{(24)^2} = 728 \text{ psi}$$

The first investigation involves the gravity load without wind, for which the stress increase factor is omitted. Thus

$$F_c^* = 850 \text{ psi}, \qquad \frac{F_{cE}}{F_c^*} = \frac{728}{850} = 0.856$$

From Figure 4.24, $C_p = 0.63$, and the stud compression capacity is

$$P = F_c^* C_p A = (850)(0.63)(8.25) = 4418 \text{ lb}$$

This is compared to the given load for the 16-in. stud spacing, which is

$$P = (16/12)(1720) = 2293 \text{ lb}$$

which demonstrates that the gravity-only loading is not a critical concern.

Proceeding with consideration for the combined loading, we determine that

$$F_c^* = 1.6F_c = 1.6(850) = 1360 \text{ psi}$$

$$\frac{F_{cE}}{F_c^*} = \frac{728}{1360} = 0.535$$

From Table 4.24, $C_p = 0.45$. For the load combination with wind, the adjusted vertical load is

$$P = \tfrac{16}{12}[\text{dead load} + 0.75(\text{live load})]$$
$$= \tfrac{16}{12}(600 + 840) = 1920 \text{ lb}$$

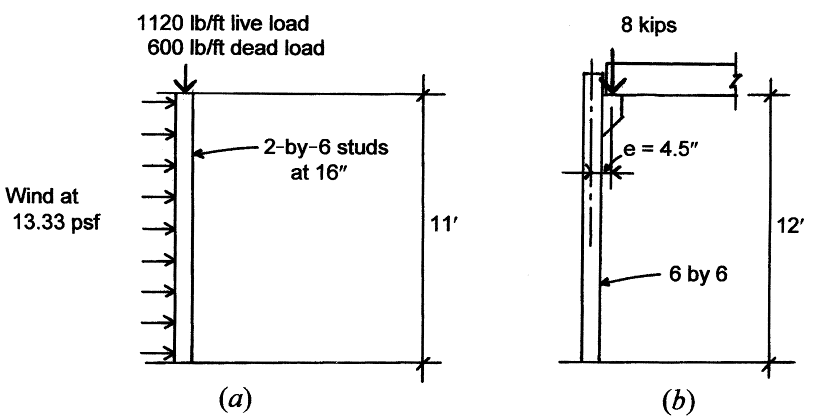


Figure 4.29 Reference for Examples 12 and 13.

Then

$$F'_c = C_p F^*_c = 0.45 \times 1360 = 612 \text{ psi}$$

$$f_c = \frac{P}{A} = \frac{1920}{8.25} = 233 \text{ psi}$$

For the wind load use 0.75(13.33) = 10 psf. Then

$$M = \frac{16}{12}\frac{wL^2}{8} = \frac{16}{12}\frac{10(11)^2}{8} = 202 \text{ lb-ft}$$

$$f_b = \frac{M}{S} = \frac{202 \times 12}{7.563} = 320 \text{ psi}$$

$$\frac{f_c}{F_{cE}} = \frac{233}{728} = 0.320$$

Then, using the code interaction formula,

$$\left(\frac{f_c}{F'_c}\right)^2 + \frac{f_b}{F_b\left(1 - f_c/F_{cE}\right)} \leq 1$$

$$\left(\frac{233}{612}\right)^2 + \frac{320}{1.6 \times 700(1 - 0.320)} = 0.139 + 0.420 = 0.599$$

As the result is less than 1, the stud is adequate.

Since the 2-by-6 stud has significant redundant strength for both loading conditions in the preceding example, it may be advisable to try a 2 by 4, unless there are other compelling reasons for having the 2 by 6.

Example 13. The column shown in Figure 4.29*b* is of Douglas fir-larch, dense No. 1 grade. Investigate the column for combined compression and bending.

Solution. From Table 4.1, $F_c = 1200$ psi, $F_b = 1400$ psi, and $E_{\min} = 620{,}000$ psi. From Table A.8, $A = 30.25$ in.^{2} and $S = 27.7$ in.^{3} Then

$$\frac{L}{d} = \frac{12 \times 12}{5.5} = 26.18$$

$$F_{cE} = \frac{0.822 \times 620{,}000}{(26.18)^2} = 744 \text{ psi}$$

$$\frac{F_{cE}}{F_c} = \frac{744}{1200} = 0.62$$

From Figure 4.24, $C_p = 0.51$ and

$$f_c = \frac{8000}{30.25} = 264 \text{ psi}$$

$$F'_c = C_p F_c = (0.51)(1200) = 612 \text{ psi}$$

$$\frac{f_c}{F_{cE}} = \frac{264}{744} = 0.355$$

$$f_b = \frac{M}{S} = \frac{8000 \times 4.5}{27.7} = 1300 \text{ psi}$$

and for the column interaction

$$\left(\frac{f_c}{F'_c}\right)^2 + \frac{f_b}{F_b\left(1 - f_c/F_{cE}\right)} \leq 1$$

$$\left(\frac{264}{612}\right)^2 + \frac{1300}{1400(1 - 0.355)} = 0.186 + 1.440 = 1.626$$

As this exceeds 1, the column is inadequate. Since bending is the main problem, a second try might be for a 6 by 8 or a 6 by 10, or for an 8 by 8 if a square section is required.

4.7 FASTENERS AND CONNECTIONS FOR WOOD

Structures of wood typically consist of large numbers of separate pieces that must be joined together. For assemblage of building construction, fastening is most often achieved by using some steel device, common ones being nails, screws, bolts, and sheet metal fasteners. This section deals with some aspects of connections achieved with bolts and nails.

Bolted Joints

When steel bolts are used to connect wood members, there are several design concerns. Some of the principal concerns are the following:

Net Cross Section in Member. Holes made for the placing of bolts reduce the wood member cross section. For this investigation, the hole diameter is assumed to be $\frac{1}{16}$ in. larger than that of the bolt. Common situations are shown in Figure 4.30. When bolts in multiple rows are staggered, it may be necessary to make two investigations, as shown in the illustration.

Bearing of the Bolt on the Wood. This compressive stress limit varies with the angle of wood grain to the load direction.

Bending of the Bolt. Long, thin bolts in thick wood members will bend considerably, causing a concentration of bearing stress at the edge of the hole.

Number of Members in a Joint. The worst case, as shown in Figure 4.31, is that of the two-member joint. In this case, the lack of symmetry in the joint produces considerable twisting. This situation is referred to as *single shear* because the bolt is subjected to shear on a single cross section of the bolt. With more members in the joint, twisting may be eliminated and the bolt is sheared at multiple cross sections.

Ripping Out of the Bolt When Too Close to an Edge. This problem, together with the spacing of bolts, is dealt with by limits given in the NDS.

Nailed Joints

A great variety of nails are used in building construction. For structural fastening, the nail most commonly used is called,

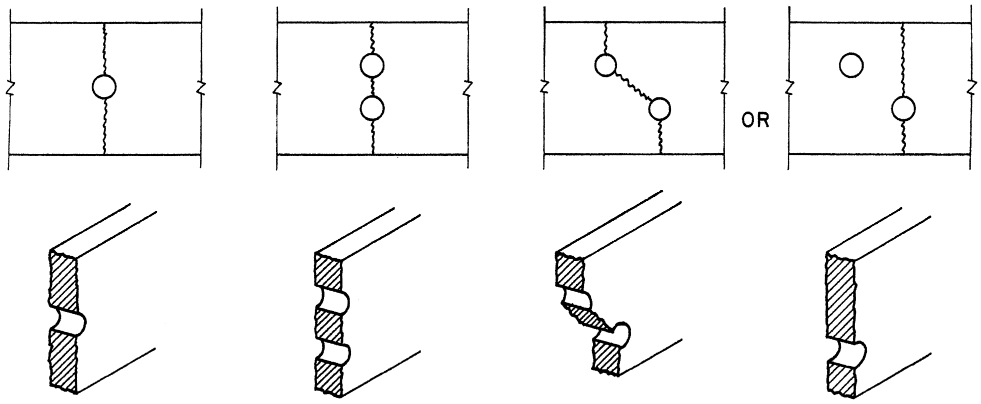


Figure 4.30 Effect of bolt holes on reduction of the cross section in tension.

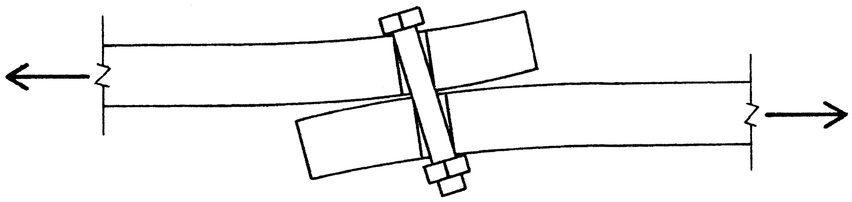


Figure 4.31 Twisting in the two-member bolted joint.

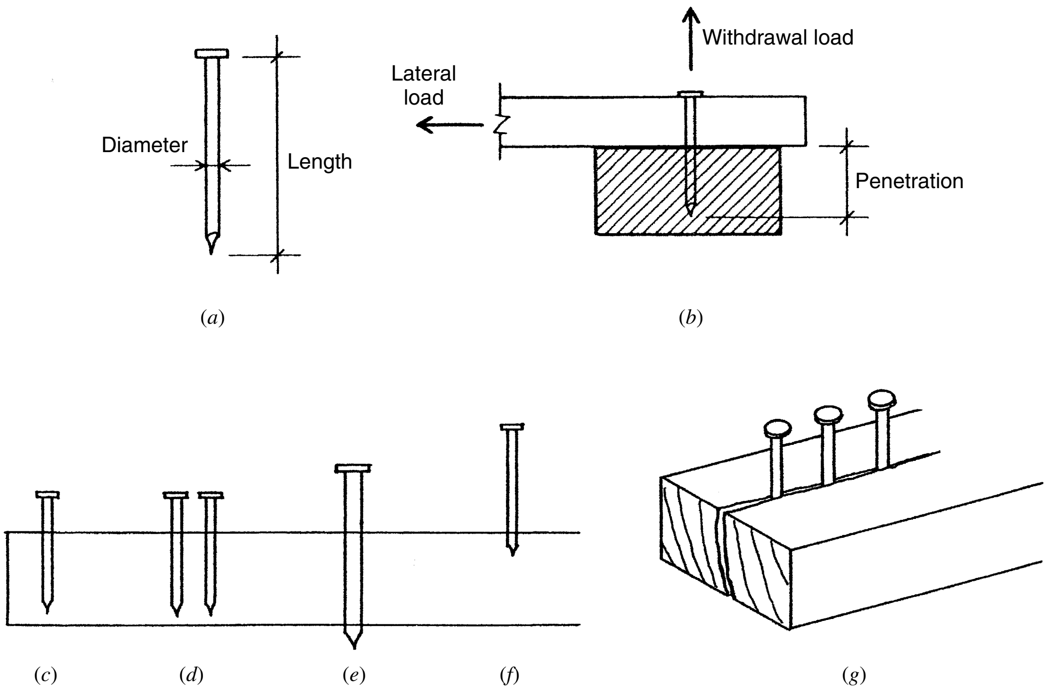


Figure 4.32 Usage considerations for wire nails.

appropriately, the *common wire nail*. As shown in Figure 4.32, the critical concerns for such nails are the following:

Nail Size. Critical dimensions are the diameter and length (see Figure 4.32*a*). Sizes are specified in pennyweight units, designated as 4d, 6d, and so on, and referred to as four penny, six penny, and so on.

Load Direction. Pullout loading in the direction of the nail shaft is called *withdrawal*; shear loading perpendicular to the nail shaft is called *lateral load*.

Penetration. Nailing is typically done through one element and into another, and the load capacity is essentially limited by the amount of the length of embedment of the nail in the second element (see

Figure 4.32*b*). The length of this embedment is called the penetration.

Species and Grade of Wood. The denser and heavier the wood (indicating usually harder, tougher material), the more the load resistance capability.

Design of good nailed joints requires a little engineering and a lot of good carpentry. Some obvious situations to avoid are those shown in Figures 4.32*c* through *g*.

Withdrawal load capacities are given in units of force per inch of penetration. This unit load is multiplied by the actual penetration length to obtain the total force capacity of the nail. For structural connections, withdrawal resistance is relied on only when the nails are perpendicular to the wood grain direction.

Lateral load capacities for common wire nails are given in Table 4.15 for joints with both plywood and lumber side pieces. The NDS contains very extensive tables for many wood types as well as metal side pieces. The following example illustrates the design of a nailed joint using the Table 4.15 data.

Example 14. A joint is formed as shown in Figure 4.33, with members of Douglas fir-larch connected by 16d common wire nails. What is the maximum value for the compression force in the two side members?

Solution. From Table 4.15, nail capacity is 141 lb per nail. (Side member thickness of 1.5 in., 16d nails.) As shown in the illustration, there are 5 nails on each side, or a total of 10 nails in the joint. The total load capacity is thus

$$C = (10)(141) = 1410 \text{ lb}$$

No adjustment is made for direction of load to the grain. However, the basic form of nailing here is so-called side grain nailing, in which the nail is inserted at 90° to the wood grain direction and the load is perpendicular (lateral) to the nails.

Minimum adequate penetration of the nails into the supporting member is also a necessity, but use of the combinations given in Table 4.15 ensures adequate penetration if the nails are fully buried in the members.

Table 4.15 Reference Lateral Load Values for Common Wire Nails (lb/nail)

Side Member Thickness, t_s (in.)	Nail Length, L (in.)	Nail Diameter, D (in.)	Nail Pennyweight	Load per Nail, Z (lb)
Part 1—With Wood Structural Panel Side Members[a] (G = 0.42)				
3/8	2	0.113	6d	48
	2 1/2	0.131	8d	63
	3	0.148	10d	76
15/32	2	0.113	6d	50
	2 1/2	0.131	8d	65
	3	0.148	10d	78
	3 1/2	0.162	16d	92
23/32	2	0.113	6d	58
	2 1/2	0.131	8d	73
	3	0.148	10d	86
	3 1/2	0.162	16d	100
Part 2—With Sawn Lumber Side Members[b] (G = 0.50)				
3/4	2 1/2	0.131	8d	90
	3	0.148	10d	105
	3 1/2	0.162	16d	121
	4	0.192	20d	138
1 1/2	3	0.148	10d	118
	3 1/2	0.162	16d	141
	4	0.192	20d	170
	4 1/2	0.207	30d	186
	5	0.225	40d	205
	5 1/2	0.244	50d	211

Source: Adapted from the *National Design Specification*® (NDS®) *for Wood Construction,* 2005 edition (Ref. 3), with permission of the publisher, American Forest and Paper Association.

[a]Values for single shear joints with wood structural panel side members with G = 0.42 and nails anchored in sawn lumber of Douglas fir–larch with G = 0.50.

[b]Values for single-shear joints with both members of sawn lumber of Douglas fir–larch with G = 0.50.

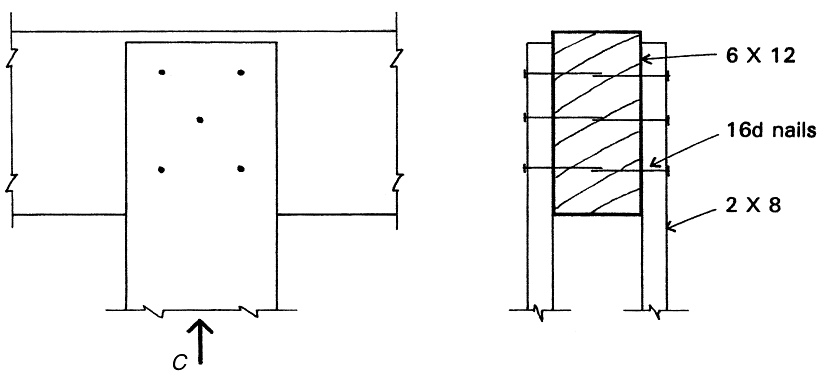


Figure 4.33 Reference for Example 14.

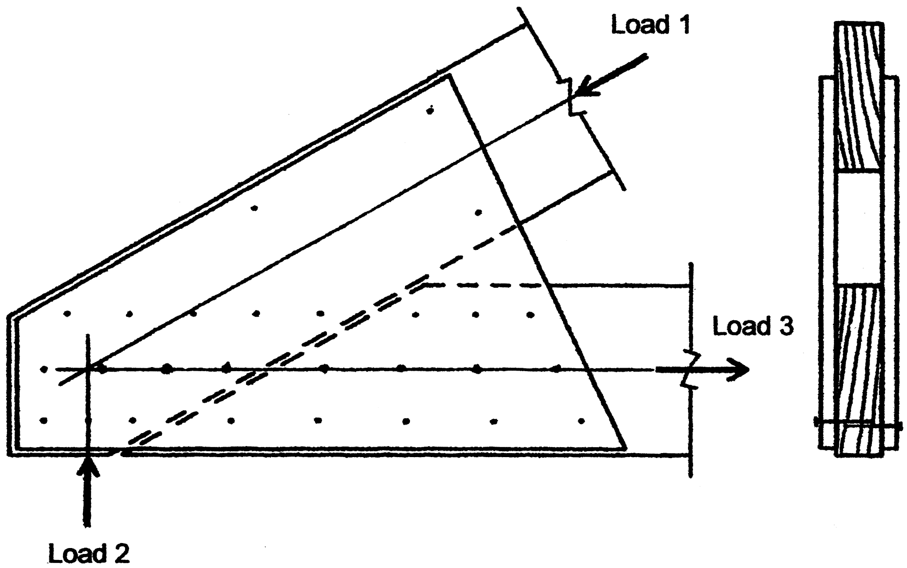


Figure 4.34 Reference for Example 15.

The following example treats a joint for a light wood truss using single lumber members for the truss and connecting panels (called gusset plates) of structural plywood.

Example 15. The truss heel joint shown in Figure 4.34 is made with 2-in.-nominal-thickness lumber truss members and gusset plates of 1/2-in.-thick plywood. Nails are 6d common wire nails with the nail layout shown occurring on both sides of the joint. Find the tension load capacity for the bottom chord member (load 3 in the figure).

Solution. From Table 4.15, the capacity of one nail is 50 lb. With 12 nails on each side of the joint, the total capacity of the joint is thus

$$T = 24 \times 50 = 1200 \text{ lb}$$

Formed Steel Framing Elements

Formed metal framing devices have been used for many centuries for the assembly of structures of heavy timber. In ancient times elements were formed of bronze or cast iron or wrought iron. Later they were formed of forged or bent and welded steel elements. (See Figure 4.35.) Some of the devices commonly used today are essentially the same in function and detail to those used long ago.

For large timber members, connecting elements are now mostly formed of steel plate that is bent and welded to produce the desired shape. (See Figure 4.36.) The ordinary tasks of attaching beams to columns and columns to foundations continue to be required and the simple means of achieving the tasks evolve from practical concerns.

For resistance to gravity loads, connections such as those shown in Figure 4.36 sometimes have no direct structural functions. In theory, it is possible to simply rest a beam on top of a column as is done in some rustic construction. However, for resistance to lateral loads from wind or earthquakes, the tying and anchoring functions of these connecting devices are often quite essential. They also serve a practical function of simply holding the parts together during the construction process.

A development of more recent times is the extension of the use of metal devices for the assembly of light wood frame construction. Devices of thin sheet metal, such as those shown in Figure 4.37, are now commonly used for stud and joist construction employing predominantly wood members of 2 in. nominal dimension thickness. As with the devices used for heavy timber construction, these lighter connectors often serve useful functions of tying and anchoring the structural members. Load transfers between basic elements of a building's lateral bracing system are often achieved with these elements. See discussion in Chapter 9.

Commonly used connection devices of both the light sheet steel type and the heavier steel plate type are readily available from building material suppliers. Many of these devices are approved by building codes for rated structural capacity functions.

Concrete and Masonry Anchors

Wood members supported by concrete or masonry structures must usually be anchored through some intermediate device. The most common attachment is with steel bolts cast into the concrete or masonry. However, there is also a wide variety of devices that may be directly cast into the supports or attached with drilled-in, dynamically anchored, or other elements.

Two common situations are those shown in Figure 4.38. The sill member for a wood stud is typically attached directly with steel anchor bolts that are cast into the supports. These bolts serve to hold the wall securely in position during the

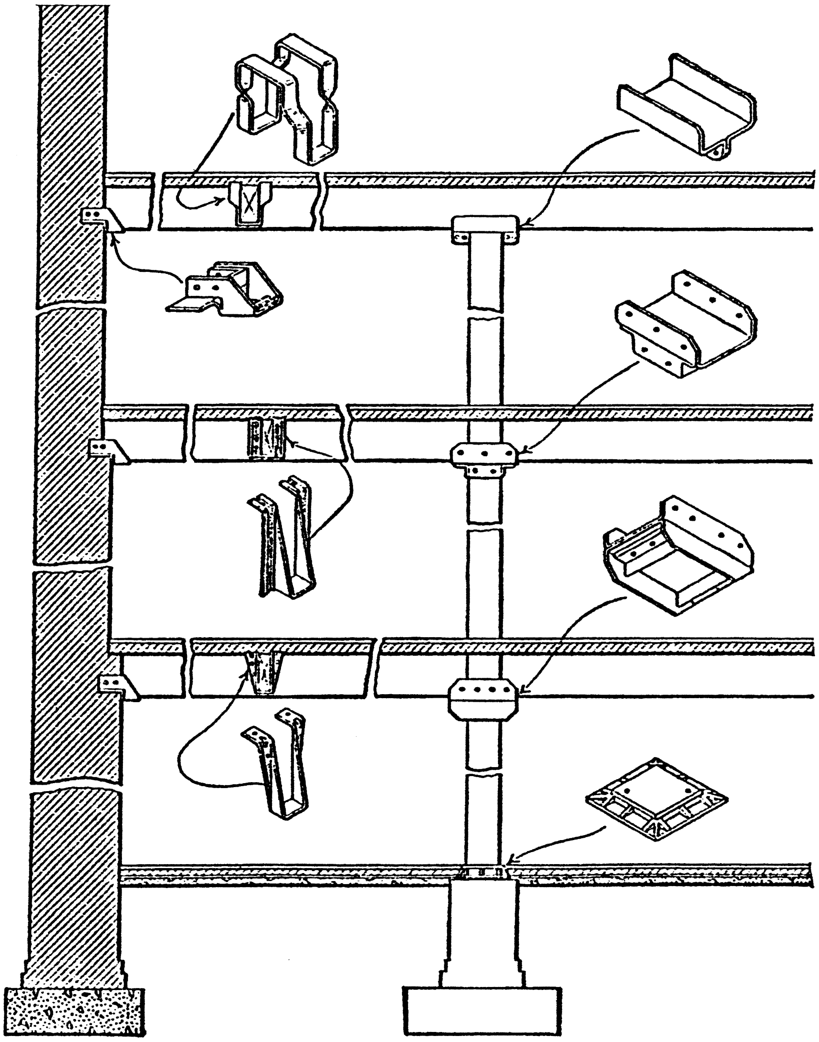


Figure 4.35 Early twentieth-century formed iron and steel connecting devices in timber construction. Reproduced from *Architects and Builders Handbook*, with permission of the publisher, John Wiley & Sons, Hoboken, NJ.

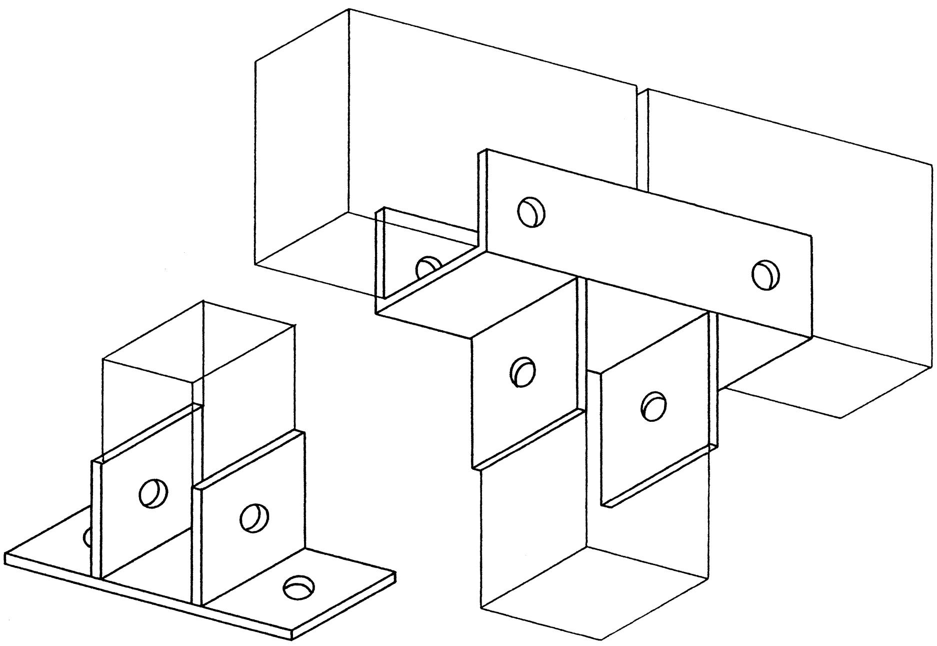


Figure 4.36 Simple connecting devices formed from bent and welded steel plates.

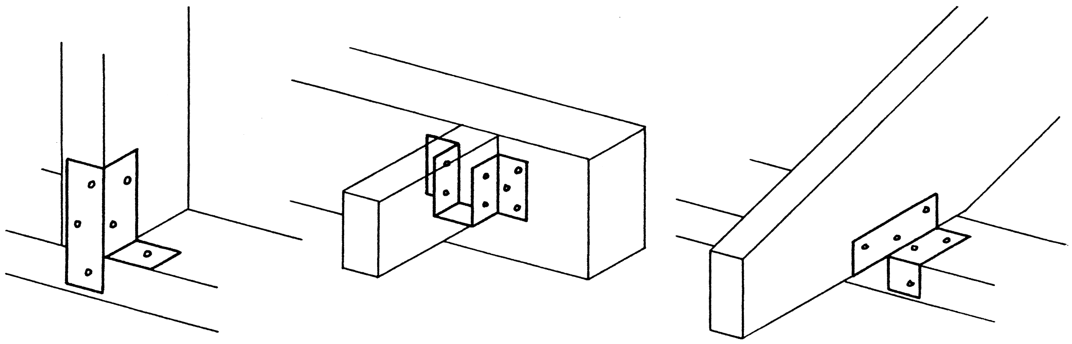


Figure 4.37 Connecting devices used for light wood frame construction formed from bent sheet steel.

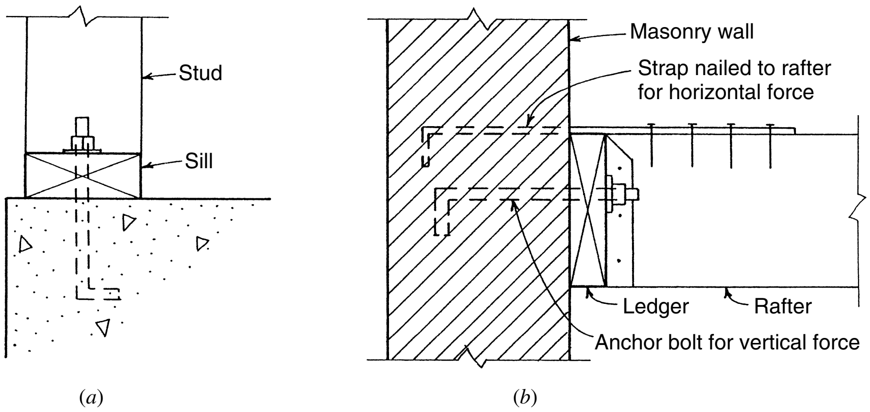


Figure 4.38 Devices for anchoring wood structures to concrete and masonry supports.

construction process. However, they may also serve to anchor the wall against lateral or uplift forces.

Figure 4.38*b* shows a common situation in which a wood-framed roof or floor is attached to a masonry wall through a member bolted to the face of the wall, called a *ledger*. For vertical load transfer the shear effect on the bolt is essentially as described in Section 4.1. For lateral force a problem is the pullout or tension effect on the bolt, although another problem may be the cross-grain bending in the ledger. In zones of high seismic risk, it is usually required to have a separate *horizontal anchor*, such as the strap shown in Figure 4.38*b*.

CHAPTER

5

Steel Structures

Steel is used in some form in every type of construction. Steel nails for wood and steel reinforcement for concrete are indispensable items. This chapter deals essentially with the use of steel as a structural material in its own right. As such, steel is used in the form of industrial products of various types. A principal usage dealt with in this chapter is that of so-called *structural steel*, a term used for products produced by rolling to form a semimolten steel ingot into a linear element with some formed cross section, the product being described as a *rolled section*. Many other types of products are used for building structures and are described here as well as in other chapters.

For building structures, steel has been used for most of the very tall and very long-spanning constructions (see Figure 5.1). It is also useable at more modest scale, stretching over a wide range of building size (see Figure 5.2). A major usage is that for multistory buildings with extensive frameworks of linear beams and columns (see Figure 5.3).

5.1 GENERAL CONCERNS FOR STEEL

Steel is a highly variable material and is used for a wide range of products that serve many purposes for building construction. This section deals with some of the general concerns for use of the basic material and with the use of various common forms of products now produced for structural applications.

Types of Steel Products

Steel itself is formless, coming basically in the form of molten material or a heat-softened lump. The structural products produced derive their basic forms from the general potentialities and limitations of the industrial processes of forming and fabricating. Standard raw stock elements—deriving from the various production processes—are the following:

Rolled Shapes. These are formed by squeezing the heat-softened steel repeatedly through a set of rollers that shape it into a linear element with a constant cross section. Simple forms of round rods and flat bars, strips, plates, and sheets are formed as well as more complex shapes of I, H, T, L, U, C, and Z. Special shapes, such as rails and sheet piling, can also be formed in this manner.

Wire. This is formed by pulling (called *drawing*) the steel through a small opening.

Extrusion. This is similar to drawing, although sections other than simple round ones are formed. Widely used with aluminum and plastics, this is only rarely used with steel.

Casting. This is done by pouring the molten steel into a form (mold). This is limited to objects of three-dimensional shape.

Forging. This consists of pounding the heat-softened steel into a mold until it takes the shape of the mold. This is preferred to casting because of the effects of the working on the properties of the finished material.

Stock elements produced by the basic forming processes may be reworked by various means, such as the following:

Cutting. Shearing, sawing, punching, or flame cutting can be used to trim and shape specific forms.

Machining. This may consist of drilling, planing, grinding, routing, or turning on a lathe.

Bending. Sheets, plates, or linear elements may be bent if made from steel with a ductile character (see the following discussion of steel properties).

Figure 5.1 Tall buildings were developed in Chicago in the late nineteenth century. Pushing ever higher, a major achievement was made in the 1970s with the construction of the Sears Tower (currently known as the Willis Tower). This structure expresses clearly the historically classic post-and-beam steel frame. It also reveals an innovative bracing system for resisting the prairie winds of Illinois. In this view it stands among old masonry bearing wall structures as well as ever-sprouting variations on the high-rise building with steel and reinforced concrete structures.

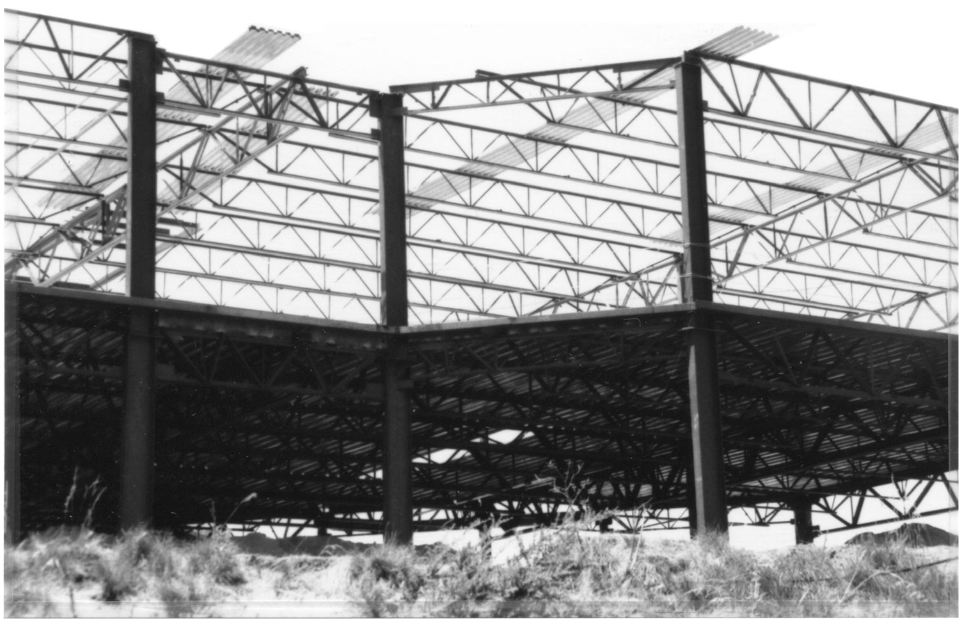


Figure 5.2 Common forms of steel products form this simple structure for a two-story office building. Rolled shapes are used for columns and spandrels and light steel trusses are used for floor joists, rafters, and column-line members. Roof and floor surfaces are developed with formed sheet steel decking.

Figure 5.3 Classic form of the structural frame for a multistory building formed with rolled steel shapes. For fire safety, the steel must be protected from high temperatures, one means being the application of insulating materials shown in the structure on the left.

Stamping. This is similar to forging; in this case sheet steel is punched into a mold that forms the flat material into some three-dimensional shape, such as a hemisphere.

Rerolling. This consists of reworking a linear rolled element into a curved form (arched) or of forming a sheet or flat strip into a formed cross section.

Finally, raw stock or reformed elements can be assembled by various means into objects of multiple parts, such as a manufactured truss or prefabricated wall panel. Basic means of assembling include the following:

Fitting. Threaded parts may be screwed together or various interlocking techniques may be used, such as the tongue-and-groove joint or the bayonet twist lock.

Friction. Clamping, wedging, or squeezing with high-tensile bolts may be used to resist the sliding of parts in surface contact.

Pinning. Overlapping flat elements may have matching holes through which a pin-type device (bolt, rivet, or actual pin) is placed to prevent the slipping of the parts at the contact face.

Nailing or Screwing. Thin elements—mostly with pre-formed holes—may be attached with nails or screws.

Welding. Gas or electric arc welding may be used to produce a bonded connection, achieved partly by melting the contacting elements to fuse them together at contact points.

Adhesive Bonding. This may be used for assembled panels but is not generally used for significant structural connections.

We are dealing here mostly with industrial processes, which at any given time relate to the development of the technology, the availability of facilities, the existence of the necessary craft, and competition with other materials and products.

Properties of Steel

The strength, hardness, and corrosion resistance and some other properties of steel can be varied through a considerable range by changes in the production processes. Hundreds of different steels are produced, although only a few standard products are used for the majority of elements for building structures. Working and forming processes, such as rolling, drawing, machining, and forging, may also alter some properties. Certain properties, such as density (unit weight), stiffness (modulus of elasticity), thermal expansion, and fire resistance, tend to remain constant for all steels.

Basic structural properties, such as strength, stiffness, ductility, and brittleness, can be interpreted from load tests. Figure 5.4 displays typical forms of curves that are obtained by plotting stress and strain values from such tests. An important characteristic of many steels is the plastic deformation (yield) phenomenon. This is demonstrated by curve 1 in Figure 5.4. For steels with this character, there are two different stress values of significance: the yield limit and the ultimate failure limit.

Generally, the higher the yield limit, the less the degree of ductility. Curve 1 in Figure 5.4 is representative of ordinary structural steel [American Society for Testing and Materials (ASTM) specification A36], and curve 2 indicates the typical effect as yield strength is raised a significant amount. Eventually, the significance of the yield phenomenon becomes negligible when the yield strength approaches as much as three times the yield of ordinary steel.

Some of the highest strength steels are produced only in thin-sheet or drawn-wire forms. Bridge strand is made from

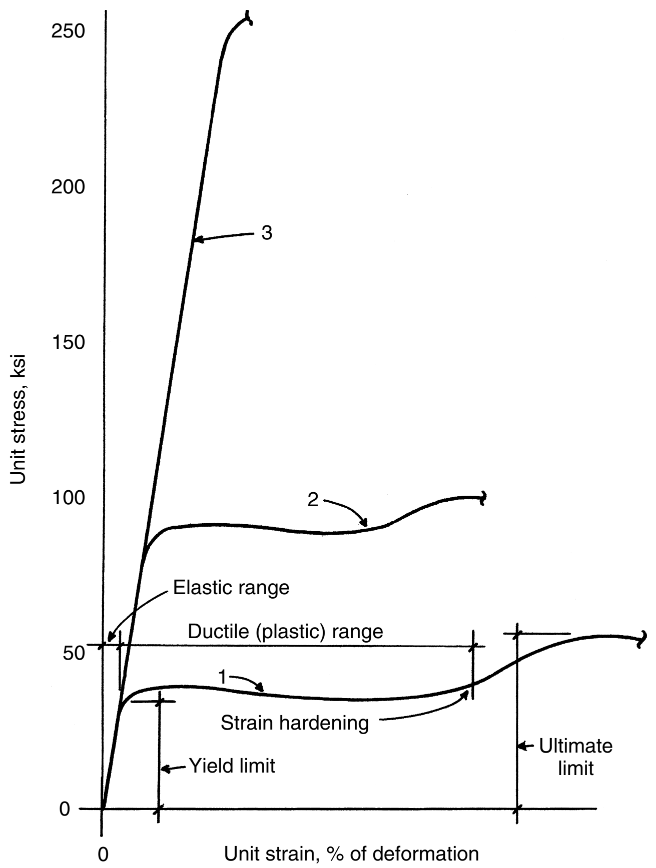


Figure 5.4 Form of the stress–strain response of structural steel: (1) ordinary structural grade, A36; (2) higher grades used for rolled shapes; (3) high-strength strand (wire) used for cables.

wire with tensile strength as high as 300 ksi. At this level yield is almost nonexistent and the wires approach the brittleness of glass rods.

For economical use of the expensive material, steel structures are generally composed of elements with relatively thin parts. This results in many situations in which the limiting strength of elements in bending, shear, and compression is determined by buckling, rather than by the stress limits of the material. Since buckling is a function of the stiffness (modulus of elasticity) of the material, and since this property remains the same for all steels, there is limited opportunity to make effective use of higher strength steels in many situations. The grades of steel most commonly used are to some extent ones that have the optimal effective strength for most structural tasks.

For various applications, other properties will be significant. Hardness affects the ease with which cutting, drilling, planing, and other working can be done. For welded connections, there is a property of weldability of the base material that must be considered. For special applications, additional properties may be important, and special steels may be required.

Resistance to rusting is normally low but can be enhanced by various materials added to the steel; such is the case of stainless steel and so-called rusting (actually, limited rusting) steels that rust at a very slow rate.

Since many structural elements are produced as some manufacturer's product line, choices of materials are often mostly out of the hands of individual building designers. The proper steel for the task—on the basis of many properties—is determined as part of the product design.

Steel that meets the requirements of ASTM A36 is a grade commonly used for rolled elements for building structures. It may be used for bolted, riveted, or welded fabrication and is the grade used primarily for the design examples in this book.

Design Values for Steel

Unit stress values used for structural design are generally based on the limits described in American Institute of Steel Construction (AISC) specifications, as printed in the AISC manual (Ref. 10). Most stresses are specified as some percentage of the yield stress F_y, or the ultimate strength F_u. Explanation of the application of these specifications is presented in the discussion of various design situations through the remainder of this chapter.

Rolled Structural Shapes

The products of the steel rolling mills used as beams, columns, and other structural members are known as *sections* or *shapes*, and their designations are related to the profiles of their cross sections. American Standard I-beams (Figure 5.5*a*) were the first beam sections rolled in the United States and are currently produced in sizes 3 to 24 in. in depth. The wide-flange shapes (Figure 5.5*b*) are a modification of the I cross section and are characterized by parallel flange surfaces as contrasted with the tapered inside flange surfaces of standard I-beams. In addition to the standard I and wide-flange sections, the structural steel shapes most commonly used in building construction are channels, angles, tees, plates, and bars. The tables in Appendix A list the dimensions and weights of some of these shapes with other properties used for design work. Complete tables of structural shapes are given in the AISC manual (Ref. 10).

Wide-Flange Shapes

These shapes, now designated as W, have greater flange widths and relatively thinner webs than standard I-beams and, as noted above, a constant thickness of the flanges. Shapes are designated by the symbol W followed by the nominal depth and the unit weight per foot. Thus the designation W 12 × 26 indicates a shape of nominal 12-in. depth weighing 26 lb per linear foot.

The actual depths of W shapes vary within a nominal depth group. This is a result of the rolling process in which the cross section is increased by spreading the rollers both horizontally and vertically. Additional material is thereby added to the cross section by increasing flange and web thickness as well as flange width. The higher percentage of material in the flanges makes the W shapes more efficient structurally for bending resistance than standard I-beams.

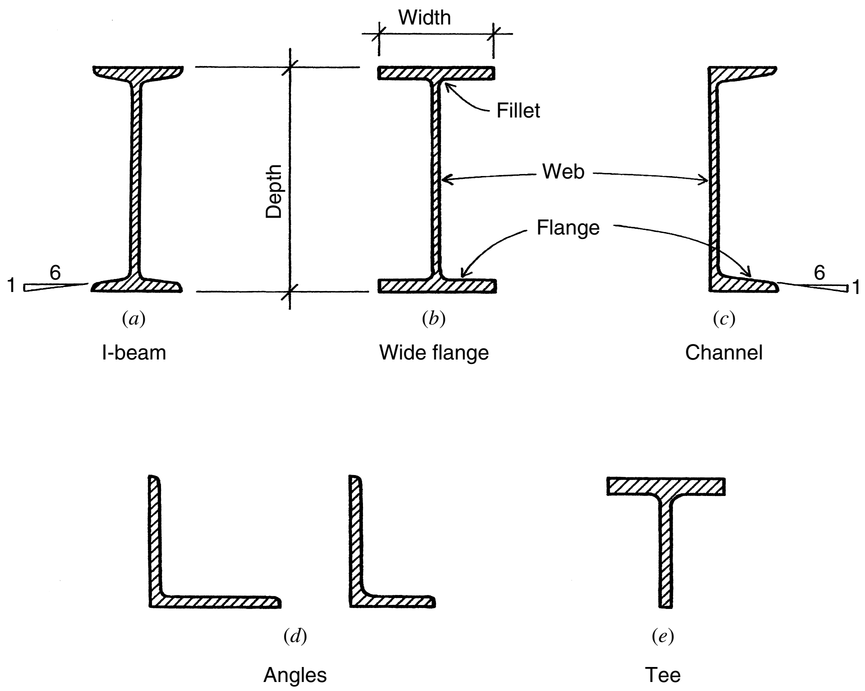


Figure 5.5 Rolled structural shapes.

In addition to shapes with the form shown in Figure 5.5*b*, W sections are also produced with flange widths approximately the same as their depth. The resulting H-form configurations are more suitable for columns than the I-shaped sections.

Standard I-Beams

American Standard I-beams are identified by the alphabetical symbol S, the designation of S 12×35 indicating a standard shape 12-in. deep weighing 35 lb per linear foot. In Appendix A it is shown that this section has an actual depth of 12 in., a flange width of 5.078 in., and a cross-sectional area of 10.3 in.[2] Unlike W sections, I-beams in a given depth group have uniform depths, with increased material created by spreading the rollers in only a horizontal direction.

Standard Channels

The profile of an American Standard channel is shown in Figure 5.5*c*. These shapes are identified by the symbol C, the designation C 12×20 indicating a channel 12 in. deep and weighing 20 lb per linear foot. Like the I-beams, the depth of a size group remains the same with increases in member material. Because of their tendency to twist or buckle, channels are not generally used as single-piece columns or beams. They may, however, be used in built-up members or, if adequately braced, as beams at the edges of openings.

Angles

Structural angles are sections in the shape of the letter L, which is the letter used for their identification. Angle leg widths may be equal or unequal. A designation of L $4 \times 4 \times 1/2$ indicates an equal leg angle with leg width of 4 in. and leg thickness of 1/2 in.

Single angles are often used as lintels and pairs of angles as members of light steel trusses. Angles were formerly used extensively for development of built-up sections such as plate girders and heavy columns. However, the advent of larger rolled shapes has largely eliminated their usefulness for this purpose. Short lengths of angles are used for bracing and other utilitarian tasks.

Structural Tees

A structural tee (Figure 5.5*e*) is made by splitting the web of a W section or a standard I-beam. The cut produces tees with a stem depth approximately equal to one-half the depth of the cut member. Tees cut from W shapes are identified as WT; those cut from I-beams are identified as ST. The designation WT 6×53 indicates a tee with 6-in. depth and a weight of 53 lb/ft. This shape is cut from a W 12×106 shape.

Plates and Bars

Flat plates and bars are made in many different sizes. Flat steel for structural use is generally classified as follows:

Bars. 6 in. or less in width and 0.203 in. or more in thickness; 6 to 8 in. in width and 0.230 in. or more in thickness.

Plates. More than 8 in. in width, 0.230 in. and more in thickness; more than 48 in. in width, 0.180 in. and more in thickness.

Sheets. Thickness less than 0.180 in.

Bars are available in varying widths and in virtually any required thickness and length. The usual practice is to specify bars in increments of 1/4 in. for widths and 1/8 in. for thickness. The standard dimensional sequence when describing bars and plates is thickness × width × length.

Designations for Structural Steel Elements

Table 5.1 lists the standard designations used for rolled shapes. Besides the shapes described previously, there are some special shapes as follows:

M Shapes. These are miscellaneous I-shaped sections that cannot be classified as W or I.

MC Shapes. These are special channel shapes that cannot be classified as the usual C shape.

HP Shapes. These are heavy H-shaped sections, intended primarily for driving of pile foundations.

Also included in Table 5.1 are the designations for steel pipe and for rectangular-shaped steel tubing. These are used primarily for columns and their designations are more fully described in the section on columns (Section 5.3).

Tabulated Data for Steel Products

In general, information for steel structural products must be obtained from steel industry publications. The AISC is a primary source for design information regarding rolled products, which are used principally for beams, columns, large truss members, and so on. Several other industry organizations also publish documents that provide information about particular products, such as manufactured trusses (open-web joists), cold-formed sections, and formed sheet steel decks. Use of data from various sources is illustrated in the following discussions in this chapter.

Sample data from industry and building code sources have been reproduced or abstracted in abbreviated form for use in this book. The reference sources cited should be consulted for more complete and up-to-date information to be used in any design work.

Usage Considerations

The following are some problems that must often be considered in the use of structural components of steel for buildings.

Rust

Exposed to air and moisture, steel will rust at the exposed surface of the material. Rusting will normally continue until the entire steel mass is eventually consumed. Response to this problem may involve one or more of the following actions:

Do nothing if there is essentially no exposure, as when the steel element is fully encased within the general construction.

Coat the surface with rust-inhibiting material.

Use special steels enhanced in production with materials to inhibit critical rusting actions.

Rusting is generally of greater concern when exposure conditions are severe. It is also of greater concern for thinner elements, especially those formed of thin sheet steel, such as formed roof and floor decks.

Fire

As with most materials, the stress and strain response of steel varies with its temperature. The rapid loss of strength at high temperatures coupled with the rapid heat gain due to high conductivity and common use of thin elements makes steel structures highly susceptible to fire. In many instances, the loss of stiffness is most critical, due to the common failures by buckling of thin elements. On the other hand, the material is noncombustible and less critical for some considerations in comparison to constructions with thin elements of wood.

The chief strategy for improving fire safety with steel structures is to prevent the fire (and the rapid heat buildup) from getting to the steel by providing coating or encasement with fire-resistive, insulative materials.

Cost

Steel is relatively expensive on a volume basis. The real cost of concern, however, is the final installed cost, that is, the total cost of the erected structure. Economy concerns begin with attempts to use the least volume of material, but this is applicable only within the design of a single type of item. Rolled structural shapes do not cost the same per pound as other steel products, such as fabricated open-web joists.

Cost concerns in general are discussed in Section 10.1. In this chapter, cost considerations are limited to attempts to find the least-weight member of a specific type in a single design task.

Table 5.1 Standard Designations for Structural Steel Elements

Elements	Designation
American standard I-beams, S shapes	S 12 × 35
Wide flanges, W shapes	W 12 × 27
Miscellaneous shapes, M shapes	M 8 × 18.5
American standard channels, C shapes	C 10 × 20
Miscellaneous channels, MC shapes	MC 12 × 40
Bearing piles, HP shapes	HP 14 × 117
Angles, L shapes	L 5 × 3 × 1/2
Structural tees, WT, ST, MT	WT 9 × 38
Plates	PL 1 1/2 × 10 × 16
Structural tubing	HSS 10 × 6 × 1/2
Pipe, standard weight	Pipe 4 Std
Pipe, extra strong	Pipe 4 X-strong
Pipe, double extra strong	Pipe 4 XX-strong

5.2 STEEL BEAMS, JOISTS, AND DECKS

There are many steel elements that can be used for the basic function of spanning, including rolled sections, cold-formed sections, and fabricated beams and trusses. This section deals with some of the fundamental considerations of use of steel elements for beams, with an emphasis on rolled sections. For simplicity, unless otherwise noted, it is assumed that all the rolled shapes used for the work in this section are of ASTM A36 steel with yield strength of 36 ksi.

Sections and Usage

The shape used most often for beams is the W shape. Its biaxial symmetry and various attributes of its section make it well suited for this task. All of the other shapes in Figure 5.5 can be used for beams for special situations, but the workhorse of the rolled sections is the W shape. It is produced in a wide range of sizes and weights, as an examination of the tables in the AISC manual will reveal.

Rolled steel elements can be combined in various ways to form built-up sections. Some examples of these are shown in Figure 5.6. Very large structural members can be formed in this manner, extending the range of size beyond that available as single-piece elements. However, the built-up section is also used for various special structural tasks.

Design for Flexure

Design for beam use may involve any combination of the following considerations.

Flexural Stress

Flexural stresses generated by bending moments are the primary concern for stress in beams. There are several failure modes for steel beams that define the approach to designing them, but the general equation for the design of bending members is as follows:

$$\phi_b M_n \geq M_u$$

where

$\phi_b = 0.9$ for rolled shapes
$M_n =$ nominal moment capacity of member
$M_u =$ maximum moment due to factored loading

Shear Stress

Whereas shear stress is often critical in wood and concrete beams, it is less often a problem in steel beams, except for situations where buckling of the thin beam web may be critical. For beam shear the general equation for shear design is as follows:

$$\phi_v V_n \geq V_u$$

where

$\phi_v = 0.9$ for rolled shapes
$V_n =$ nominal shear capacity of member
$V_u =$ maximum shear due to factored loading

Buckling

Beams that are not adequately braced may be subject to various forms of buckling. Especially critical are beams with very thin webs or narrow flanges or with cross sections especially weak in the lateral direction (on their minor, or y–y axis). Buckling controls the failure mechanism in inadequately braced members and greatly reduces bending moment capacity. The most effective solution is to provide adequate bracing to eliminate buckling as the initial mode of failure.

Deflection

Although steel is the stiffest of the common structural materials, steel structures tend to be quite flexible; thus the vertical deflection of beams must be carefully considered. A significant value to monitor is the span-to-depth ratio of beams; if this is kept within certain limits, deflection is much less likely to be critical.

Connections and Supports

Framed structures contain many joints between separate pieces, and the details of the connections and supports must be developed for proper construction as well as for the transfer of necessary structural forces through the joints.

Individual beams are often parts of a system in which they play an interactive role. Besides their basic beam functions, there are often design considerations that derive from the overall system actions and interactions. For study purposes, the discussions in this section focus on basic beam functions, but discussions in other parts of this book treat the usage and overall incorporation of beams in structural systems.

Selection of Beams

There are several hundred different W shapes for which properties are listed in the AISC manual (see sample in

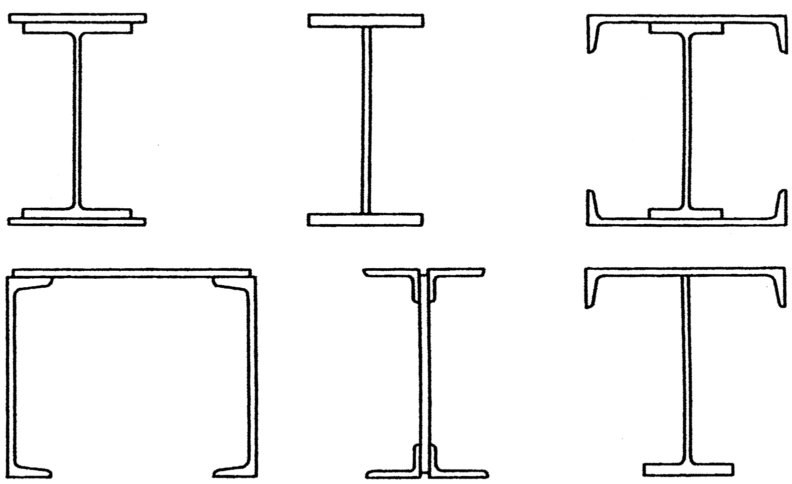


Figure 5.6 Common built-up beam sections, formed with rolled elements.

Table A.3). In addition, there are several other shapes that frequently serve beam functions in special circumstances. Selection of the optimal shape for a given situation involves many considerations; frequently, an overriding consideration is the choice of the most economical shape for the task. In general, the least costly shape is usually the one that weighs the least—other things being equal—because steel is priced by unit weight. In most design cases, therefore, the *least-weight* selection is considered to be the most economical.

Just as a beam may be required to develop other structural actions, such as tension, compression, or torsion, other structural elements may also develop beam actions. Exterior walls may span for bending against wind pressures; columns may receive bending moments as well as compression loads; truss chords may span as beams as well as function for basic truss actions. The beam functions described here may thus be part of the design work for elements other than beams.

Inelastic versus Elastic Behavior

There are two principal competing methods of design for steel: Allowable stress design (ASD) and load and resistance factor design (LRFD). ASD is rooted in elastic behavior and is concentrated on the service load (true maximum load) conditions for the structure. LRFD, on the other hand, considers basically the ultimate failure mode of the structure and relates its response to a theoretical load at something greater than the real (service) loads. This section presents a comparison of the different levels of structural response of steel elements as they are considered for these two design methods.

With the use of ductile steel, a major distinction is made between behaviors that are within the elastic range of the materials and those that involve some inelastic (beyond-the-yield-point) conditions. Although allowable stresses for the ASD method are based on yield and ultimate stress limits, the structural behaviors used for design investigations are mostly based on elastic responses. While some structural failure modes may involve elastic stress conditions—mostly those involving buckling of slender elements—many of the ultimate failure responses involve stress development beyond the elastic range. The following discussion treats the comparison of elastic and inelastic response of beams to flexural actions.

The maximum resisting moment by elastic theory is predicted to occur when the stress at the extreme fiber of a beam reaches the elastic yield value, F_y, and it may be expressed as

$$M_y = F_y \times S$$

in which S is the section modulus for the beam cross section as related to elastic stress–strain responses.

Beyond this limiting moment value, the resisting moment for the cross section can no longer be expressed by elastic theory equations because an inelastic, or *plastic*, stress condition will start to develop on the beam cross section.

Figure 5.7 represents an idealized form of the graph for a load-test response for a specimen of ductile steel. The graph shows that that up to the yield point the deformations (*strains*) are proportional to the applied stress and the form of the graph is that of a straight line of constant slope (predicted by the *modulus of elasticity* of the material). Beyond the yield point deformations will occur at an approximately equal stress for a considerable amount of deformation. The extent of this plastic behavior is called the *plastic range*; for A36 steel, with a value of 36 ksi for the yield stress, this range is approximately 15 times that produced just before yielding occurs. This relative magnitude of the plastic range is the basis for qualification of the material as significantly ductile.

Note that beyond the plastic range, the material once again stiffens, called the *strain-hardening effect*, which indicates a loss of the ductility and the onset of a third range in which additional strain is produced only by an increase in stress. The end of this range establishes the *ultimate stress* limit for the material.

The following example illustrates the application of elastic theory and will be used for comparison with an analysis of plastic behavior.

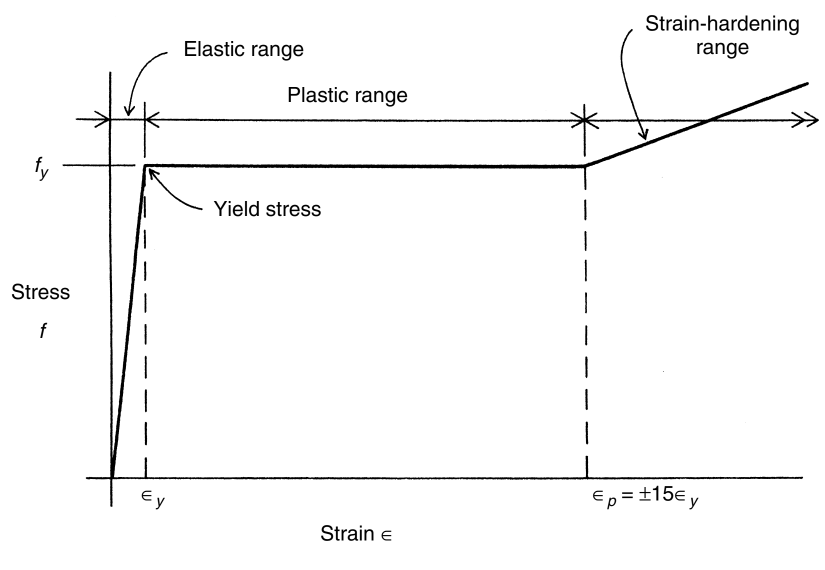


Figure 5.7 Idealized form of the stress–strain graph for the response of ductile steel.

Example 1. A simple beam has a span of 16 ft [4.88 m] and supports a single concentrated load of 18 kips [80 kN] at its center (see Figure 5.8*a*). If the beam is a W 12 × 30, compute the maximum flexural stress.

Solution. For the maximum value of the bending moment,

$$M = \frac{PL}{4} = \frac{18 \times 16}{4} = 72 \text{ kip-ft } [98 \text{ kN-m}]$$

With a value of 38.6 in.^{3} for S (from Table A.3)

$$f = \frac{M}{S} = \frac{72 \times 12}{38.6} = 22.4 \text{ ksi } [154 \text{ MPa}]$$

This stress occurs as shown in Figure 5.8*d*. Note that this stress condition occurs only at midspan. Figure 5.8*e* shows the form of the deformation that accompanies the stress condition. This stress level is well below the elastic stress limit (yield point) of 36 ksi.

The limiting moment for elastic stress occurs when the maximum stress reaches the elastic limit, as shown in Figure 5.9*a*. If the loading (and the bending moment) is increased, a stress condition like that illustrated in Figure 5.9*b* begins to develop as the ductile material deforms plastically. This spread of the yield stress level over the beam cross section indicates the development of a resisting moment in excess of the limiting elastic moment M_y. With an extended range of ductility, a limit for this condition is shown in Figure 5.9*c*, and the limiting moment is described as the *plastic moment*, designated M_p. Although a small percentage of the cross section near the neutral axis remains in an elastic stress condition, its effective contribution to development of the resisting moment is quite negligible. Thus it is commonly assumed that the full plastic limit is developed by the condition shown in Figure 5.9*d*.

Attempts to increase the bending moment beyond the value of M_p will result in a large rotational deformation, with the beam acting as though it were hinged (pinned) at this location. For practical purposes, therefore, the resisting moment capacity of the ductile beam is considered to be exhausted with the attaining of the plastic moment. This location is thus described as a *plastic hinge*, as shown in Figure 5.10.

In a manner similar to that for elastic stress conditions, the value of the plastic moment is expressed as

$$M_p = F_y Z$$

The term Z is called the *plastic section modulus* and its value is determined as follows.

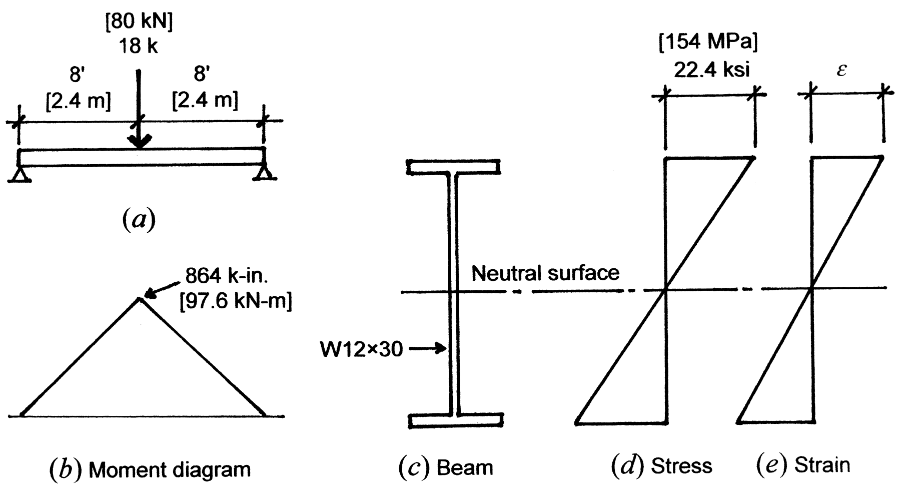


Figure 5.8 Elastic behavior of the beam.

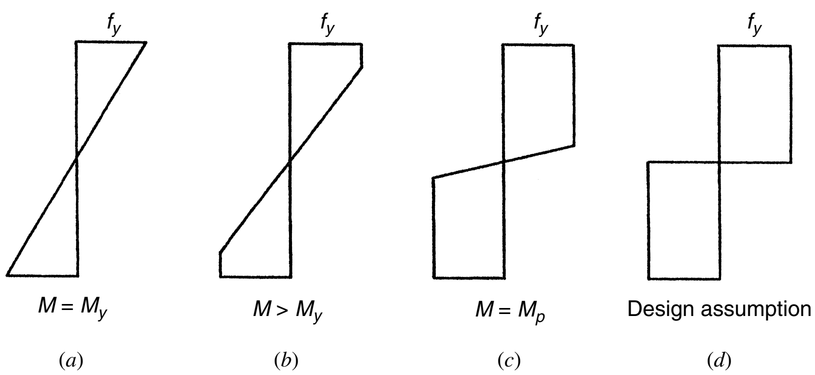


Figure 5.9 Progression of development of flexural stress from the elastic to the plastic range.

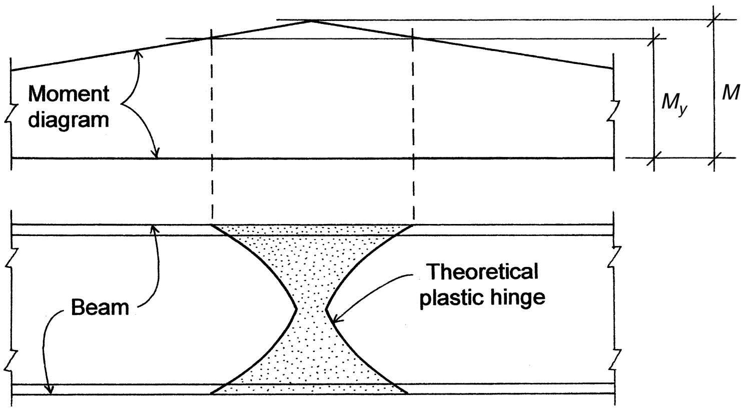


Figure 5.10 Development of the plastic hinge.

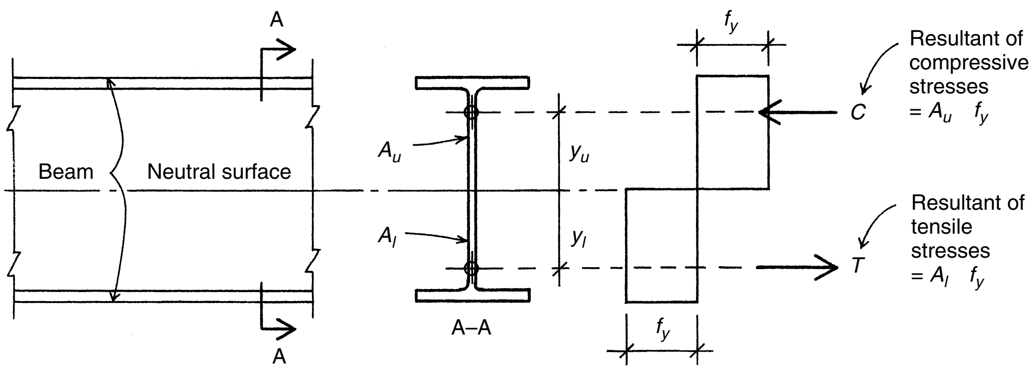


Figure 5.11 Development of the plastic resisting moment.

Referring to Figure 5.11, which shows a W shape subjected to a level of stress corresponding to the fully plastic section (Figure 5.9*d*), *note the following:*

A_u = upper area of cross section above the neutral axis

y_u = distance from centroid of A_u to neutral axis

A_l = lower area of cross section

y_l = distance from centroid of A_l to neutral axis

For equilibrium of the internal forces on the cross section (the resulting forces C and T as shown in Figure 5.11), the condition can be expressed as

$$\sum F_h = 0$$

or

$$[A_u \times (+F_y)] + [A_l \times (-F_y)] = 0$$

and thus

$$A_u = A_l$$

This shows that the plastic stress neutral axis divides the cross section into equal areas, which is apparent for symmetrical sections, but it applies to unsymmetrical sections as well. The resisting moment equals the sum of the moments of the stresses; thus the value for M_p may be expressed as

$$M_p = (C \times y_u) + (T \times y_l)$$

or

$$\begin{aligned} M_p &= (A_u \times F_y \times y_u) + (A_l \times F_y \times y_l) \\ &= F_y[(A_u \times y_u) + (A_l \times y_l)] \\ &= F_y Z \end{aligned}$$

and the quantity $(A_u \times y_u) + (A_l \times y_l)$ is the property of the cross section defined as the plastic section modulus, designated Z.

Using the expression for Z just derived, its value for any cross section can be computed. However, values of Z are tabulated in the AISC manual (Ref. 10) for all rolled shapes used for beams. For W shapes see Table A.3 in Appendix A.

Comparison of the tabulated values for S and Z for the same W shape will show that the values for Z are larger. This presents an opportunity to compare the fully plastic resisting moment to the yield stress limiting moment by elastic stress, that is, the advantage of using plastic analysis.

Example 2. A simple beam consisting of a W 21 × 57 is subjected to bending. Find the limiting moments based on (a) elastic stress conditions with a limiting stress of $F_y = 36$ ksi and (b) full development of the plastic moment.

Solution. From Table A.3, $S_x = 111$ in.^{3} and $Z = 129$ in.^{3} For (a) the limiting moment is

$$M_y = F_y \times S = (36) \times (111) = 3996 \text{ kip-in.}$$

and for (b)

$$M_p = F_y \times Z = (36) \times (129) = 4644 \text{ kip-in.}$$

The increase gained by plastic analysis is thus

$$4644 - 3996 = 648 \text{ kip-in.}$$

or

$$\frac{648}{3996}(100) = 16.2\%$$

Nominal Moment Capacity of Steel Beams

The nominal moment capacity of a steel section (M_n) is based upon the cross-sectional properties, its yield stress, and bracing of the member from out-of-plane buckling. All of these parameters affect how the beam will ultimately fail and thus how much capacity it will have for bending.

The preferred mode of failure is by development of the inelastic plastic hinge, as previously described. If a member is capable of this behavior, it is considered a "compact" cross section. Compact shapes are defined by criteria in the AISC specification. Most rolled shapes used for beams are compact and any that are not are identified in the tables of properties in the AISC manual.

For compact shapes that are adequately laterally supported, the nominal moment capacity is the plastic yield moment; thus

$$M_n = M_p = F_y \times Z$$

Example 3. Determine the moment capacity of an A36 W 24 × 76 beam that is adequately supported against buckling.

Solution. From Table 3.1, $Z = 200 \text{ in.}^3$; thus

$$M_n = F_y \times Z = 36 \times 200 = 7200 \text{ kip-in.}$$
$$= \frac{7200}{12} = 600 \text{ kip-ft}$$

The plastic yield moment applies for laterally unbraced lengths (designated L_b) up to the distance L_p. Beyond L_p the nominal resisting moment capacity must be decreased. The rate of decline of the nominal moment is of a linear form from L_p until the unbraced length reaches a value of L_r, Beyond this point the value for the nominal moment is determined by a formula from the AISC specification; thus

$$M_n = \left(\frac{S_x \times X_1 \times \sqrt{2}}{L_b/r_y}\right) \times \sqrt{1 + \frac{(X_1)^2 \times X_2}{2 \times (L_b/r_y)^2}}$$

Data related to the determination of nominal resisting moments are provided in Table 5.2. This is a sample of more extensive tables in the AISC Manual.

Figure 5.12 shows the form of the relation between M_n and L_b using an example of a W 18 × 50 with $F_y = 50$ ksi. The AISC manual provides a series of such graphs for shapes commonly used as beams. For the graph in Figure 5.12, values for key values of nominal moment and unbraced length are obtained from Table 5.2. Also shown on the graph are moment values relating to unbraced lengths of 18 and 20 ft which were obtained by use of the AISC formula.

The following examples illustrate the process for finding nominal resisting moment capacities using data from Table 5.2.

Example 4. Determine the nominal resisting moment capacity of an A36 W 24 × 76 steel beam with laterally unbraced lengths of (a) 6 ft, (b) 10 ft, and (c) 25 ft.

Solution. From Table 5.2 the following are determined:

$$L_p = 8 \text{ ft}, \quad L_r = 23.4 \text{ ft}, \quad M_p = 600 \text{ kip-ft},$$
$$M_r = 381 \text{ kip-ft}$$

For (a), L_b is less than L_p, so $M_n = M_p = 600$ kip-ft. For (b), L_b is between the limiting values of L_p and L_r, so the value for the resisting moment is determined by proportional analysis using the formula given in Figure 5.12; thus

$$M_n = M_p - (M_p - M_r) \times \left(\frac{L_b - L_p}{L_r - L_p}\right)$$
$$= 600 - (600 - 381)\left(\frac{10 - 8}{23.4 - 8}\right)$$
$$= 572 \text{ kip-ft}$$

For (c), the following formula is used:

$$M_n = \left(\frac{S_x X_1 \sqrt{2}}{L_b/r_y}\right)\sqrt{1 + \frac{(X_1)^2 X_2}{2(L_b/r_y)^2}}$$

For this case, using data from Table 5.2, the moment will be found as 347 kip-ft. Using these relationships in a hands-on solution will quickly demonstrate the advantage of using the graphs in the AISC manual.

Design for Bending

Design for bending usually involves the determination of the ultimate bending moment (M_u) that the beam must resist and the use of resistance factors to determine the required nominal moment capacity (M_n) for the beam. The basic formulation of this process is

$$\phi_b M_n = M_u$$

for which the resistance factor is 0.9.

For a complete design process, all additional data must be used to establish other requirements for the beam; these may include unbraced lengths, values for shear, limits for deflection, and so on. It is common practice, however, to first make a selection based on only bending moment and unbraced length and to then investigate other situations.

Table 5.2 LFRD Selection for Shapes Used as Beams

		$F_y = 36$ ksi				$F_y = 50$ ksi								
Designation	Z_x (in.^{3})	L_p (ft)	L_r (ft)	M_p (kip-ft)	M_r (kip-ft)	L_p (ft)	L_r (ft)	M_p (kip-ft)	M_r (kip-ft)	r_y (in.)	$b_f/2t_f$	h/t_w	X_1 (ksi)	$X_2 \times 10^6$ [(1/ksi)^2]
W 33 × 141	**514**	**10.1**	**30.1**	**1,542**	**971**	**8.59**	**23.1**	**2,142**	**1,493**	**2.43**	**6.01**	**49.6**	**1,800**	**17,800**
W 30 × 148	500	9.50	30.6	1,500	945	8.06	22.8	2,083	1,453	2.28	4.44	41.6	2,310	6,270
W 24 × 162	468	12.7	45.2	1,404	897	10.8	32.4	1,950	1,380	3.05	5.31	30.6	2,870	2,260
W 24 × 146	418	12.5	42.0	1,254	804	10.6	30.6	1,742	1,237	3.01	5.92	33.2	2,590	3,420
W 33 × 118	**415**	**9.67**	**27.8**	**1,245**	**778**	**8.20**	**21.7**	**1,729**	**1,197**	**2.32**	**7.76**	**54.5**	**1,510**	**37,700**
W 30 × 124	408	9.29	28.2	1,224	769	7.88	21.5	1,700	1,183	2.23	5.65	46.2	1,930	13,500
W 21 × 147	373	12.3	46.4	1,119	713	10.4	32.8	1,554	1,097	2.95	5.44	26.1	3,140	1,590
W 24 × 131	370	12.4	39.3	1,110	713	10.5	29.1	1,542	1,097	2.97	6.70	35.6	2,330	5,290
W 18 × 158	356	11.4	56.5	1,068	672	9.69	38.0	1,483	1,033	2.74	3.92	19.8	4,410	403
W 30 × 108	**346**	**8.96**	**26.3**	**1,038**	**648**	**7.60**	**20.3**	**1,442**	**997**	**2.15**	**6.89**	**49.6**	**1,680**	**24,200**
W 27 × 114	343	9.08	28.2	1,029	648	7.71	21.3	1,429	997	2.18	5.41	42.5	2,100	9,220
W 24 × 117	327	12.3	37.1	981	631	10.4	27.9	1,363	970	2.94	7.53	39.2	2,090	8,190
W 21 × 122	307	12.2	41.0	921	592	10.3	29.8	1,279	910	2.92	6.45	31.3	2,630	3,160
W 18 × 130	290	11.3	47.7	870	555	9.55	32.8	1,208	853	2.7	4.65	23.9	3,680	810
W 30 × 90	**283**	**8.71**	**24.8**	**849**	**531**	**7.39**	**19.4**	**1,179**	**817**	**2.09**	**8.52**	**57.5**	**1,410**	**49,600**
W 24 × 103	280	8.29	27.0	840	531	7.04	20.0	1,167	817	1.99	4.59	39.2	2,390	5,310
W 27 × 94	278	8.83	25.9	834	527	7.50	19.9	1,158	810	2.12	6.70	49.5	1,740	19,900
W 14 × 145	260	16.6	81.6	780	503	14.1	54.7	1,083	773	3.98	7.11	16.8	4,400	348
W 24 × 94	254	8.25	25.9	762	481	7.00	19.4	1,058	740	1.98	5.18	41.9	2,180	7,800
W 21 × 101	253	12.0	37.1	759	492	10.2	27.6	1,054	757	2.89	7.68	37.5	2,200	6,400
W 12 × 152	243	13.3	94.8	729	453	11.3	62.1	1,013	697	3.19	4.46	11.2	6,510	79
W 18 × 106	230	11.1	40.4	690	442	9.40	28.7	958	680	2.66	5.96	27.2	2,990	1,880
W 14 × 120	212	15.6	67.9	636	412	13.2	46.2	883	633	3.74	7.80	19.3	3,830	601
W 24 × 76	**200**	**8.00**	**23.4**	**600**	**381**	**6.79**	**18.0**	**833**	**587**	**1.92**	**6.61**	**49.0**	**1,760**	**18,600**
W 16 × 100	200	10.4	42.7	600	384	8.84	29.6	833	590	2.5	5.29	23.2	3,530	947
W 21 × 83	196	7.63	24.9	588	371	6.47	18.5	817	570	1.83	5.00	36.4	2,400	5,250
W 18 × 86	186	11.0	35.5	558	360	9.30	26.1	775	553	2.63	7.20	33.4	2,460	4,060
W 12 × 120	186	13.0	75.5	558	353	11.1	50.0	775	543	3.13	5.57	13.7	5,240	184
W 21 × 68	**160**	**7.50**	**22.8**	**480**	**303**	**6.36**	**17.3**	**667**	**467**	**1.8**	**6.04**	**43.6**	**2,000**	**10,900**
W 24 × 62	**154**	**5.71**	**17.2**	**462**	**286**	**4.84**	**13.3**	**642**	**440**	**1.37**	**5.97**	**49.7**	**1,730**	**23,800**
W 16 × 77	152	10.3	35.4	456	295	8.70	25.5	633	453	2.46	6.77	29.9	2,770	2,460
W 12 × 96	147	12.9	61.4	441	284	10.9	41.4	613	437	3.09	6.76	17.7	4,250	407
W 10 × 112	147	11.2	86.4	441	273	9.48	56.5	613	420	2.68	4.17	10.4	7,080	57
W 18 × 71	146	7.08	24.5	438	275	6.01	17.8	608	423	1.7	4.71	32.4	2,690	3,290
W 14 × 82	139	10.3	42.8	417	267	8.77	29.5	579	410	2.48	5.92	22.4	3,590	849
W 24 × 55	**135**	**5.58**	**16.6**	**405**	**249**	**4.74**	**12.9**	**563**	**383**	**1.34**	**6.94**	**54.1**	**1,570**	**36,500**
W 21 × 57	129	5.63	17.3	387	241	4.77	13.1	538	370	1.35	5.04	46.3	1,960	13,100
W 18 × 60	123	7.00	22.3	369	234	5.94	16.6	513	360	1.68	5.44	38.7	2,290	6,080
W 12 × 79	119	12.7	51.8	357	232	10.8	35.7	496	357	3.05	8.22	20.7	3,530	839
W 14 × 68	115	10.3	37.3	345	223	8.70	26.4	479	343	2.46	6.97	27.5	3,020	1,660
W 10 × 88	113	11.0	68.4	339	213	9.30	45.1	471	328	2.63	5.18	13.0	5,680	132
W 21 × 50	**110**	**5.42**	**16.2**	**330**	**205**	**4.60**	**12.5**	**458**	**315**	**1.3**	**6.10**	**49.4**	**1,730**	**22,600**
W 16 × 57	105	6.67	22.8	315	200	5.66	16.6	438	307	1.6	4.98	33.0	2,650	3,400
W 18 × 50	**101**	**6.88**	**20.5**	**303**	**193**	**5.83**	**15.6**	**421**	**296**	**1.65**	**6.57**	**45.2**	**1,920**	**12,400**
W 21 × 44	**95.4**	**5.25**	**15.4**	**286**	**177**	**4.45**	**12.0**	**398**	**272**	**1.26**	**7.22**	**53.6**	**1,550**	**36,600**
W 18 × 46	90.7	5.38	16.6	272	171	4.56	12.6	378	263	1.29	5.01	44.6	2,060	10,100
W 14 × 53	87.1	8.00	28.0	261	169	6.79	20.1	363	259	1.92	6.11	30.9	2,830	2,250
W 10 × 68	85.3	10.8	53.7	256	164	9.16	36.0	355	252	2.59	6.58	16.7	4,460	334
W 16 × 45	82.3	6.54	20.2	247	158	5.55	15.2	343	242	1.57	6.23	41.1	2,120	8,280
W 18 × 40	**78.4**	**5.29**	**15.7**	**235**	**148**	**4.49**	**12.1**	**327**	**228**	**1.27**	**5.73**	**50.9**	**1,810**	**17,200**
W 12 × 53	77.9	10.3	35.8	234	153	8.77	25.6	325	235	2.48	8.69	28.1	2,820	2,100
W 14 × 43	69.6	7.88	24.7	209	136	6.68	18.3	290	209	1.89	7.54	37.4	2,330	4,880
W 10 × 54	66.6	10.7	43.9	200	130	9.05	30.2	278	200	2.56	8.15	21.2	3,580	778
W 12 × 45	64.2	8.13	28.3	193	125	6.89	20.3	268	192	1.95	7.00	29.6	2,820	2,210
W 16 × 36	**64.0**	**6.33**	**18.2**	**192**	**122**	**5.37**	**14.0**	**267**	**188**	**1.52**	**8.12**	**48.1**	**1,700**	**20,400**
W 10 × 45	54.9	8.38	35.1	165	106	7.11	24.1	229	164	2.01	6.47	22.5	3,650	758

(*Continued*)

Table 5.2 (*Continued*)

		$F_y = 36$ ksi				$F_y = 50$ ksi								
Designation	Z_x (in.^{3})	L_p (ft)	L_r (ft)	M_p (kip-ft)	M_r (kip-ft)	L_p (ft)	L_r (ft)	M_p (kip-ft)	M_r (kip-ft)	r_y (in.)	$b_f/2t_f$	h/t_w	X_1 (ksi)	$X_2 \times 10^6$ [(1/ksi)^2]
W 14 × 34	**54.6**	**6.38**	**19.0**	**164**	**105**	**5.41**	**14.4**	**228**	**162**	**1.53**	**7.41**	**43.1**	**1,970**	**10,600**
W 12 × 35	51.2	6.42	20.7	154	99	5.44	15.2	213	152	1.54	6.31	36.2	2,430	4,330
W 16 × 26	**44.2**	**4.67**	**13.4**	**133**	**83**	**3.96**	**10.4**	**184**	**128**	**1.12**	**7.97**	**56.8**	**1,480**	**40,300**
W 14 × 26	**40.2**	**4.50**	**13.4**	**121**	**76**	**3.82**	**10.2**	**168**	**118**	**1.08**	**5.98**	**48.1**	**1,880**	**14,100**
W 10 × 33	38.8	8.08	27.5	116	76	6.86	19.8	162	117	1.94	9.15	27.1	2,720	2,480
W 12 × 26	**37.2**	**6.29**	**18.1**	**112**	**72**	**5.34**	**13.8**	**155**	**111**	**1.51**	**8.54**	**47.2**	**1,820**	**13,900**
W 10 × 26	**31.3**	**5.67**	**18.6**	**94**	**60**	**4.81**	**13.6**	**130**	**93**	**1.36**	**6.56**	**34.0**	**2,510**	**3,760**
W 12 × 22	**29.3**	**3.53**	**11.2**	**88**	**55**	**3.00**	**8.41**	**122**	**85**	**0.848**	**4.74**	**41.8**	**2,170**	**8,460**
W 10 × 19	**21.6**	**3.64**	**12.0**	**65**	**41**	**3.09**	**8.89**	**90**	**63**	**0.874**	**5.09**	**35.4**	**2,440**	**5,030**

Source: Compiled from data in the *Manual of Steel Construction, Load and Resistance Factor Design*, 3rd ed. (Ref. 10), with permission of the publisher, American Institute of Steel Construction.

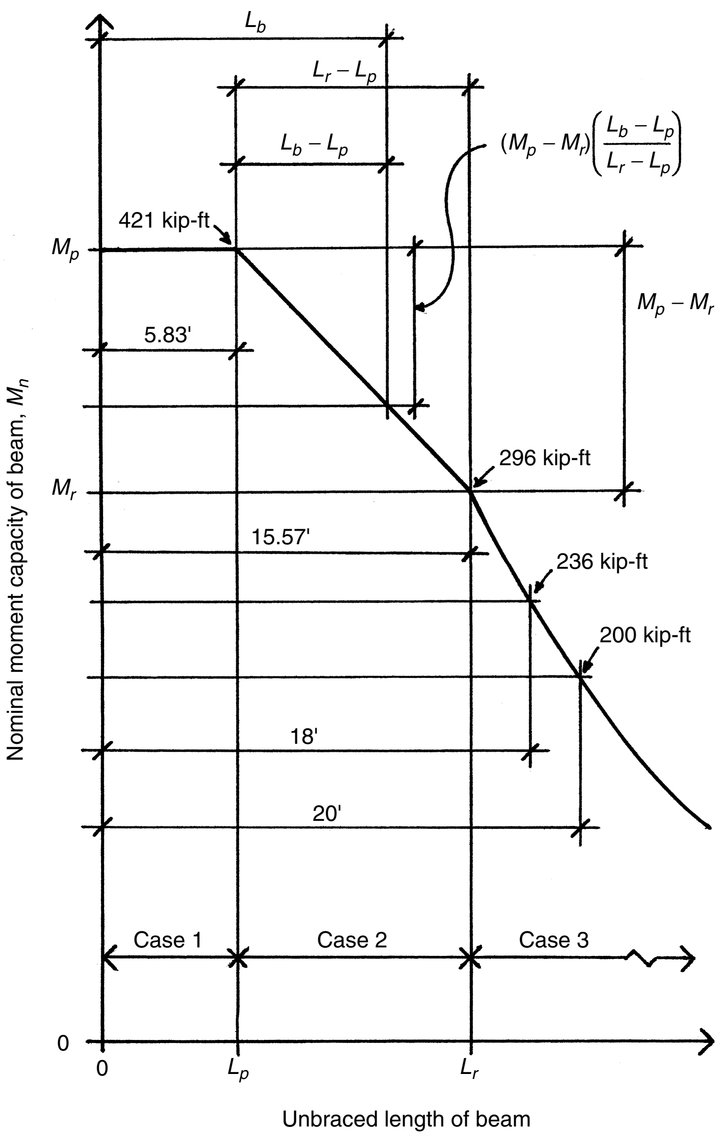


Figure 5.12 Relation between nominal moment capacity and lateral unbraced length for a W 18 × 50 beam with yield strength of 50 ksi.

Design for Plastic Failure Mode

The minimum required plastic section modulus (Z_x) is found for this mode of failure as follows:

$$M_n = \frac{M_u}{\phi_b} = F_y Z_x$$

$$\text{Required } Z_x = \frac{M_n}{F_y}$$

Because weight (and thus cost) is determined by cross-sectional area, not by plastic section modulus, the beam chosen may have more than the required section modulus and still be the most economical choice.

Data in Table 5.2 may be used directly for selection of shapes for this failure mode. At a minimum, input for design includes values for the steel yield stress, the beam span and load, and the unbraced length for lateral bracing. The following example illustrates the process.

Example 5. A simply supported beam of A36 steel spans 24 ft [7.3 m] and carries a live load of 1.5 kips/ft [2.03 kN/m] and a superimposed dead load of 0.50 kips/ft [0.92 kN/m]. Lateral support is continuous throughout the span. Select the lightest acceptable W shape.

Solution. Not including the beam weight, the design ultimate loading is

$$w_u = 1.2(\text{DL}) + 1.6(\text{LL}) = 1.2(0.5) + 1.6(1.5) = 3.0 \text{ kips/ft}$$

[*Note*: See Section 10.1 for a discussion of load factors for the strength design (LRFD) method.]

For the simple beam, the maximum bending moment is

$$M_u = \frac{wL^2}{8} = \frac{3(24)^2}{8} = 216 \text{ kip-ft} \ \ [293 \text{ kN-m}]$$

The required nominal resisting moment is thus

$$M_n = \frac{M_u}{\phi_b} = \frac{216}{0.9} = 240 \text{ kip-ft} \ \ [325 \text{ kN-m}]$$

The required plastic section modulus is

$$Z_x = \frac{M_n}{F_y} = \frac{240 \times 12}{36} = 80.0 \text{ in.}^3 \ \ [1.31 \times 10^6 \text{ mm}^3]$$

The shapes with properties listed in Table 5.2 are listed in order of their Z_x values. Inspection of the table reveals the following possibilities for the beam:

W 10 × 68: $Z_x = 85.3$ in.^{3}

W 14 × 53: $Z_x = 87.1$ in.^{3}

W 16 × 45: $Z_x = 82.3$ in.^{3}

W 18 × 46: $Z_x = 90.7$ in.^{3}

W 21 × 44: $Z_x = 95.4$ in.^{3}

Lacking other design considerations, the least-weight shape is the W 21 × 44. For a final step, the effect of the beam weight must be considered, adding to the unit dead load and slightly increasing the required resisting moment. The 21-in.-deep beam will still be adequate with this requirement.

Actually, a shorter design process is to simply omit the determination of the required Z value and use the required resisting moment directly with Table 5.2. Using the value of 240 kip-ft and scanning the fifth column of Table 5.2 will produce the same results for possible choices.

Note that certain shapes in Table 5.2 have their designations listed in boldface type. These are sections that have an especially efficient bending moment resistance, indicated by the fact that there are other sections of greater weight but the same or smaller section modulus. Thus, for a savings of material cost, these *least-weight* sections offer an advantage. Considerations for other beam design factors, however, may sometimes make this a less important design concern.

The problem data for Example 5 includes the condition of continuous lateral support—a situation that generally applies only for beams that directly support decks. For beams that support other beams, this situation is not true.

Design of Beams for Buckling Failure

As discussed earlier with regard to lateral support, there are two significant unbraced lengths that bracket the determination of resisting moment: L_p and L_r. Values for these lengths are listed in Table 5.2. The solution and choices found in Example 5 are proper only for unbraced lengths up to the plastic yield limit: L_p. Thus, the choice for the W 21 × 44 is adequate only if the unbraced length does not exceed 4.45 ft. However, since the plastic section modulus of this shape considerably exceeds the required value, it is possible that the choice can work for unbraced lengths only slightly exceeding the limit of 4.45 ft.

The following example illustrates a procedure for designing a beam with the inclusion of considerations for lateral buckling.

Example 6. A simply supported beam of A36 steel spans 14 ft [4.7 m] and carries a live load of 3 kips/ft [43.3 kN/m] and a dead load of 2 kips/ft [29.2 kN/m]. There is no lateral support for the beam, so the unbraced length is that of the entire span. Find the least-weight W shape available.

Solution. First determine the ultimate factored design loading, the maximum ultimate moment, and the required resisting moment:

$$w_u = 1.2(2) + 1.6(3) = 7.2 \text{ kips/ft} \ \ [105 \text{ kN/m}]$$

$$M_u = \frac{7.2(14)^2}{8} = 176 \text{ kip-ft} \ \ [239 \text{ kN-m}]$$

$$M_n = \frac{M_u}{\phi_b} = \frac{176}{0.9} = 196 \text{ kip-ft} \ \ [266 \text{ kN-m}]$$

From Table 5.2, the most economical shape not taking into account unbraced length is a W 18 × 40, but this shape has a plastic moment limit for unbraced length of 5.29 ft. A next step is to inspect Table 5.2 for any shapes that have values for L_p of at least 14 ft together with a resisting moment capacity of at least 196 kip-ft. Two shapes that satisfy this criteria are a W 14 × 120 and a W 14 × 145. However, these two shapes have resisting moment capacities considerably greater than required.

Consider the beam whose graph is displayed in Figure 5.12. At the unbraced length of $M_r = 15.57$ ft this beam has a resisting moment capacity of 296 kip-ft and is thus adequate for this case, with a weight of only 50 lb/ft. Other shapes with weights in this range may also be considered. In fact, this process is quite laborious and the advantage of having the curves like that in Figure 5.12 for all shapes would be really helpful, which is the situation for the graphs in the AISC manual. Hand calculations can be used for this problem, but the process is much assisted by the AISC graphs or a computer-aided program.

Shear in Steel Beams

Investigation and design for shear forces are similar to that for bending moment in that maximum factored shear force must be less than the factored shear capacity of the beam. This is expressed as

$$\phi_v V_n \geq V_u$$

where

$\phi_v = 0.90$

V_n = nominal shear capacity of beam

V_u = shear force due to factored beam loading

The internal shear force mechanism is viewed in terms of response to the shear force visualized by the shear diagram, as shown in Figure 5.13*a*. As the shear diagram shows, the shear force for a simply supported beam with uniformly distributed load reaches a maximum value at the beam ends and decreases to zero at midspan.

Figure 5.13*b* shows another loading condition—that of a concentrated load within the beam span. In this case, a major internal shear force is generated over some length of the beam. If the load is closer to one support, the maximum shear force occurs between the load and the closer support.

The distribution of shear stresses over the beam cross section depends on the form and geometric properties of the cross section. For a rectangular cross section, such as that of a wood beam, the stress distribution is as shown in Figure 5.13*c*, taking the form of a parabola with a maximum shear stress value at the neutral axis and a decrease to zero stress at the extreme edges of the section.

For the I-shaped cross section of the typical W-shape steel beam, shear stress distribution takes the form shown in Figure 5.13*d*—referred to as the "derby hat" form. As the figure shows, the major shear force development is taken by the beam web. The traditional shear investigation for the W shape ignores the contribution of the beam flanges entirely and assumes a uniform distribution of shear stress on an effective web area (A_w), as shown in Figure 5.13*e*. This area is computed as the product of the beam depth and the web thickness:

$$A_w = d_b t_w$$

As with flexure, there are various modes of shear failure, depending primarily on the slenderness of the beam web. Slenderness is determined as the ratio of the clear web height (beam depth minus twice the flange thickness, called h) to the web thickness. Three cases of slenderness are defined by the slenderness ratio and control the nominal shear capacity; these are defined as follows:

1. A very stiff (thick) web that may actually reach something close to the full yield stress limit of the

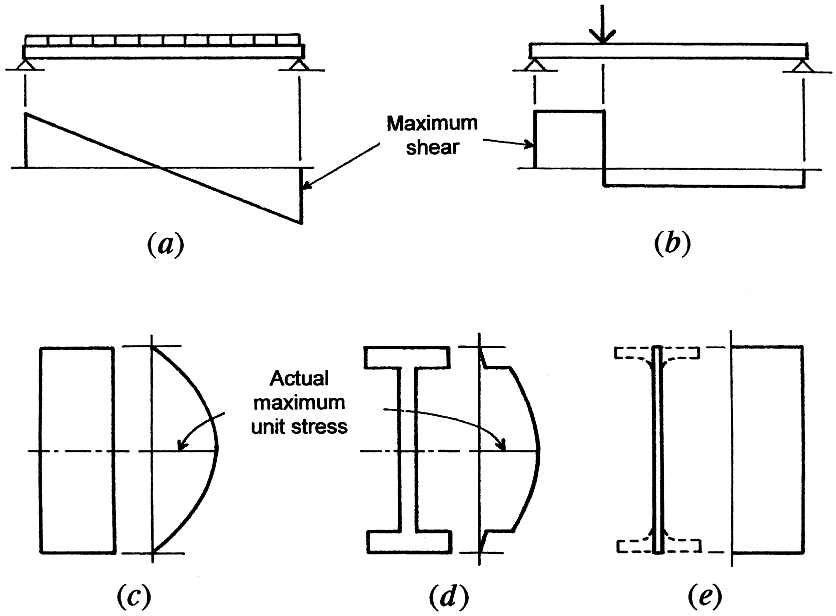


Figure 5.13 Development of shear in beams.

material, where $h/t_w \le 418/\sqrt{F_y}$:

$$V_n = (0.6F_y)A_w$$

2. A somewhat slender web that responds with some combined yield stress and inelastic buckling, where $418/\sqrt{F_y} \le h/t_w \le 523/\sqrt{F_y}$:

$$V_n = (0.6F_y)A_w\left(\frac{418/\sqrt{F_y}}{h/t_w}\right)$$

3. A very slender web that fails essentially in elastic buckling, with $h/t_w > 523\sqrt{F_y}$:

$$V_n = 132{,}000\left(\frac{A_w}{(h/t_w)^2}\right)$$

Example 7. A simple beam of A36 steel is 6 ft long and has a concentrated live load of 36 kips applied 1 ft from one end. It is found that a W 10 × 19 is adequate for the bending moment. Investigate the beam to determine if the shear capacity is adequate for the required maximum shear.

Solution. The factored live load is determined as

$$P_u = 1.6(36) = 57.6 \text{ kips}$$

The reactions are 48 and 9.6 kips and the maximum shear is thus 48 kips.

From Table A.3, for the given shape, $d = 10.24$ in., $t_w = 0.250$ in., and $t_f = 0.395$ in. Then

$$h = d - 2(t_f) = 10.24 - 2(0.395) = 9.45 \text{ in.}$$

$$\frac{h}{t_w} = \frac{9.45}{0.25} = 37.8$$

$$\frac{418}{\sqrt{F_y}} = \frac{418}{\sqrt{36}} = 69.7 > \frac{h}{t_w}$$

Therefore, the shear capacity is determined using the equation associated with the full yield stress limit of the material:

$$A_w = d_b t_w = 10.24 \times 0.250 = 2.56 \text{ in.}^2$$

$$V_n = (0.6F_y)A_w = (0.6 \times 36) \times 2.56 = 55.3 \text{ kips}$$

$$\phi_v V_n = 0.9 \times 55.3 = 49.8 \text{ kips}$$

Because the factored shear capacity of the beam is greater than the factored shear of 48 kips, the shape is adequate.

Shear forces internal to a W-shape beam may be problematic in terms of compression forces on the beam's web. Figure 5.14 shows various situations involving conditions of shear and vertical compression at the ends of beams. Standard framed connections, consisting of a pair of angles as shown in Figure 5.14*a*, are typically welded to the beam web and thus concentrate the shear stress in the web.

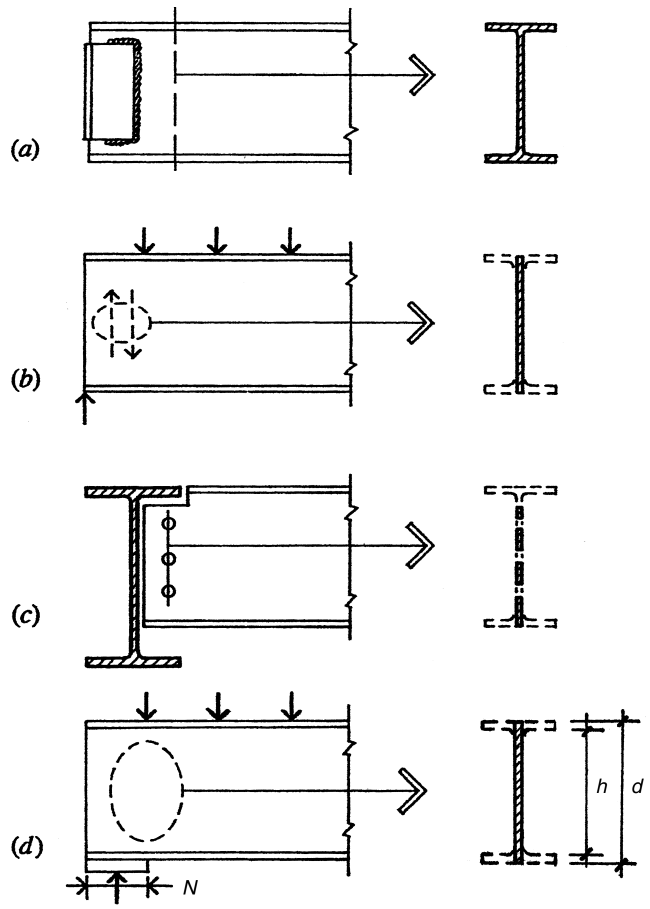


Figure 5.14 Considerations for end support of beams.

For end-bearing supports, as shown in Figure 5.14*b*, the shear force is more fully developed by the whole beam end, thus justifying more fully the concept of the effective web area as used for the specified code analysis for shear.

End connections to other beams are often achieved as shown in Figure 5.14*c*, with the top flange of the supported beam cut back to allow its end to extend closely to the web of the supporting beam. This reduces the total web area for shear and should be considered in investigations. If this reduction is coupled with that produced by use of a bolted connection with accompanying holes in the supported beam's web, further reduction occurs.

End bearing also represents a case of a highly concentrated compression in the beam web, as shown in Figure 5.14*d*; this situation is discussed later with regard to concentrated load effects on beams in general.

The net effect of investigations of all the situations so far described, relating to end shear in beams, may be to influence a choice of beam shape with a web that is sufficient. However, other criteria for selection (for flexure, deflection, framing details, and so on) may indicate an ideal choice that has a vulnerable web. In the latter situation, a possible remedy is the reinforcing of the beam web with added stiffeners—a remedy illustrated in the discussion on concentrated load effects (see Figure 5.26).

Deflection of Beams

Deformation of structures must often be controlled for various reasons. These reasons may relate to the proper functioning of the structure but more often relate to its purposes in facilitating functions of the building occupants, to accommodation of other building services (such as roof drainage), or to relations to the rest of the building construction.

To steel's advantage is the relative stiffness of the material itself. With a modulus of elasticity of 29,000 ksi, it is 8 to 10 times as stiff as average structural concrete and 15 to 20 times as stiff as structural lumber. However, it is often the overall deformation of the whole structural assemblages that must be considered; in this regard, steel structures are often quite flexible. Because of its cost, steel is usually formed into elements of a slender nature with very thin parts. Thus, for general design considerations, deformation of steel structures is often a major concern.

For a beam in a horizontal position, a critical deformation is the maximum sag, called the beam's deflection. For most beams this deflection will be too small to be detected by the eye. However, any load on a beam, beginning with the beam's own weight, will cause some amount of deflection, as shown in Figure 5.15. In the case of a symmetrical, simply supported, single-span beam, the maximum deflection will occur at midspan, and it usually is the only deformation value of concern for design. However, as the beam deflects, its ends rotate unless restrained, and this twisting deformation may also be of concern in some situations.

If deflection is determined to be excessive, the usual remedy is to select a deeper beam. The real aim is to obtain a higher moment of inertia (I), which is the section property most affecting deflection. Formulas for deflection take a typical form that involves variables as follows:

$$\Delta = C\frac{WL^3}{EI}$$

where Δ = deflection, measured vertically, in. [mm]
C = constant related to beam form and span
W = total load on beam
L = span of beam
E = modulus of elasticity of beam material
I = moment of inertia of beam cross section

Note that the magnitude of deflection is directly proportional to the magnitude of the load; double the load and you get twice the deflection. However, the deflection is proportional to the third power of the span; double the span and you get 2^3, or eight times, the deflection. For resistance to deflection, increases in either material stiffness or moment of inertia will cause direct proportional reduction of the deflection.

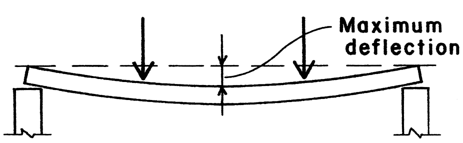


Figure 5.15 Deflection of a simple beam under a symmetrical loading.

Excessive deflection may cause various problems. For roofs, excessive sag may disrupt the intended drainage patterns for generally flat surfaces. For floors, a common problem is the development of some perceivable bounciness.

The form of the beam and its supports may also be a consideration. For the simple-span beam in Figure 5.15, the usual concern is simply the maximum midspan sag. For a beam with a projected (cantilevered) end, however, a problem may be created at the unsupported end; depending on the extent of the cantilever, this may involve a downward deflection (as shown in Figure 5.16*a*) or an upward deflection (as shown in Figure 5.16*b*).

With continuous (multiple-span) beams, a potential problem derives from the fact that a load in any single span causes some deflection in adjacent spans. This is most critical when loads vary in different spans or the spans differ significantly (see Figure 5.16*c*).

Most deflection problems in buildings stem from effects of the structural deformations on adjacent or supported elements. When beams are supported by girders, rotation caused by deflection occurring at the ends of the supported beams can result in cracking or separation of the floor deck that is continuous over the girders, as shown in Figure 5.16*d*. For such a framing system, there is also an accumulative deflection of the deck, beams, and girders, which can cause problems for maintaining a flat floor surface or a desired roof surface profile for drainage.

Figure 5.16*e* shows the case of a beam occurring directly over a solid wall. If the wall is made to fit tightly beneath the beam, any deflection of the beam will cause it to bear on top of the wall—not an acceptable situation if the wall is relatively fragile (a metal and glass curtain wall, for example). A different sort of problem occurs when relatively rigid walls (plastered, for example) are supported by spanning beams, as shown in Figure 5.16*f*. In this case, the supported wall is relatively intolerant of any deformation, so anything significant in the form of sag of the beam is really critical.

For long-span structures (an ambiguous class, usually meaning 50 ft or more span), a special problem is the relatively flat roof surface. In spite of provisions for code-mandated minimum drainage, heavy rain will run off slowly and linger to cause considerable loading. Because this results in deflection of the spanning structure, the sag may form a depressed area that the rain can swiftly turn into a pond (see Figure 5.16*g*). The pond then constitutes an additional load that causes more sag—and a resulting deeper pond. This progression can quickly pyramid into failure of the structure, so that codes (including the AISC specification) now provide design requirements for investigation of potential ponding.

Standard Equations for Deflection

Computation of beam deflection is usually done using standard equations for the most common conditions of

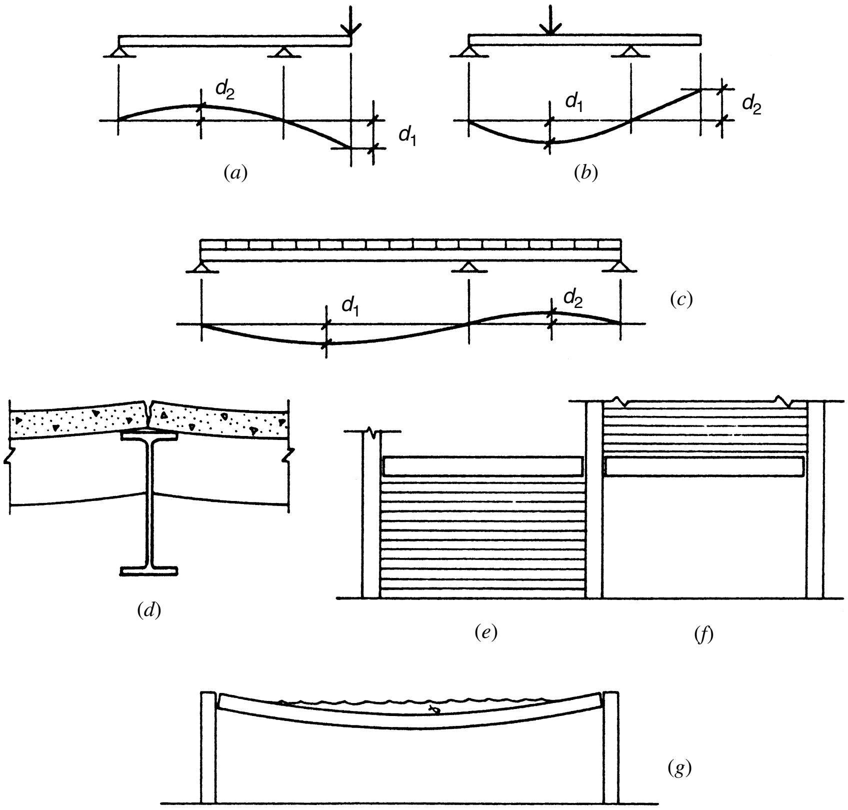


Figure 5.16 Considerations for deflection of beams.

loading and support. These equations are listed in the AISC manual (Ref. 10) with a sample given in Figure 3.8. For a simple-span beam, two commonly used equations are as follows:

$$\text{For uniformly distributed load: } \Delta = \frac{5wL^4}{384EI} \quad \text{or} \quad \frac{5WL^3}{384EI}$$

$$\text{For a midspan point load: } \Delta = \frac{PL^3}{48EI}$$

Example 8. A simple beam has a span of 20 ft with a uniformly distributed load of 1.95 kips/ft. The beam is a W 14 × 34. Find the maximum deflection.

Solution. From Table A.3, the moment of inertia is 340 in.^{4} Using the formula, deflection is

$$\Delta = \frac{5WL^3}{384EI} = \frac{5(1.95 \times 20)(20 \times 12)^3}{384(29{,}000)(340)} = 0.712 \text{ in.}$$

Allowable Deflections

What is acceptable for beam deflection is mostly a matter of judgment by experienced designers. It is difficult to provide any useful guidance for specific limitations to avoid the various problems described in Figure 5.16. Each situation must be investigated individually, and decisions regarding tolerable deformations must be made jointly by the structural designer and the designers of the rest of the building construction.

For spanning beams in ordinary situations, rules of thumb have been derived over many years of experience. These usually consist of limits for maximum degree of beam curvature described in the form of a limiting ratio of the deflection to the beam span, expressed as a fraction of the span. Typical limits currently used are the following:

To avoid visible sag under total load on short to medium spans, $L/150$
For total load deflection of a roof structure, $L/180$
For live-load deflection for a roof, $L/240$
For total-load deflection for a floor structure, $L/240$
For live-load deflection for a floor, $L/360$

Deflection of Uniformly Loaded Simple Beams

The most frequently used beam in flat roof and floor framing systems with wood or steel structures is the single, simple span (no continuity or end restraint) beam with a uniformly distributed loading. Deflection for this beam is easily determined with the standard deflection formula as described.

In Chapter 4 an equation was derived for such a beam, expressing the deflection in a form as follows:

$$\Delta = \frac{5L^2 f_b}{24Ed}$$

In both the ASD and LRFD methods, deflections are computed with service loads, rather than factored loads. The

actual stresses encountered at service load levels are therefore ordinarily close to the allowable stress limits for the ASD method. For an A36 steel beam, this value for bending stress is 24 ksi. If substitutions of 24 ksi for bending stress and 29,000 ksi for modulus of elasticity are made in this equation, the following expression is obtained:

$$\Delta = \frac{5(12L)^2(24)}{24(29{,}000)d} = \frac{0.02483L^2}{d}$$

This formula, involving only span and beam depth, can be used to plot a graph that displays the deflection of a beam of a constant depth for a variety of spans. Figure 5.17 consists of a series of graphs for A36 steel beams from 6 to 36 in. in depth. In a similar manner, Figure 5.18 consists of graphs for deflection of beams with $F_y = 50$ ksi. Values for deflections obtained from these graphs will be very approximate, which is usually not a critical concern, since deflection computations are based on inputs of very approximate data for loads and details of construction.

The real value of the graphs in Figures 5.17 and 5.18 is for use in the design process, where consideration for deflections can be quite easily performed with a minimum of computation. An additional aid for design work is the inclusion of radiating lines on the figures that represent various deflection/span ratios, as discussed with regard to limits for allowable deflection. These lines may be used to establish beam depths for which deflection will not be critical.

Safe-Load Tables

The simple beam with uniformly distributed loading occurs so frequently that it is useful to have a rapid design method for quick selection of shapes based on knowing only the beam load and span. The AISC manual (Ref. 10) provides a series of such tables with data for W, M, S, and C shapes.

Table 5.3 presents load–span data for selected W shapes of A36 steel. Table values are the total factored load capacity for beams with uniformly distributed loading and bracing at points not farther apart than the limiting dimension of L_p for the shape. The table values are determined using the maximum plastic yield moment of the shapes ($M_p = Z_x \times F_y$).

For very short spans, loads are often limited by the beam shear capacity and by support conditions, rather than by

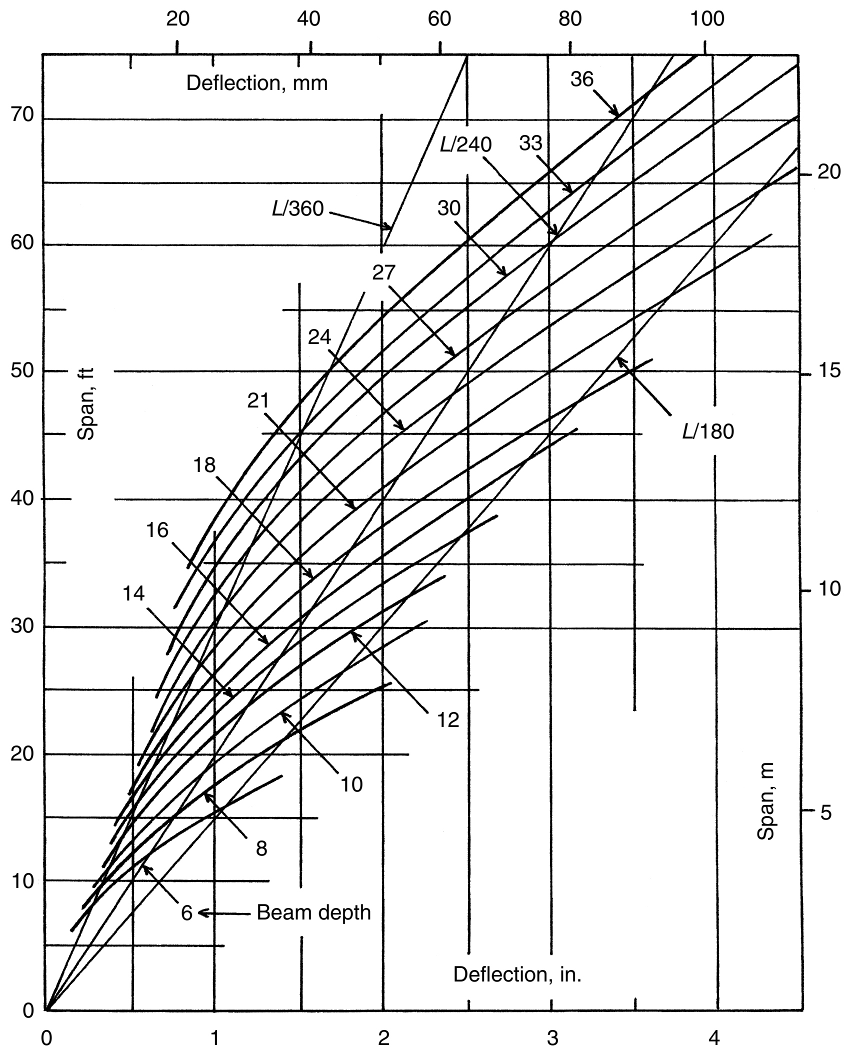


Figure 5.17 Deflection of steel beams with yield stress of 36 ksi under a constant maximum bending stress of 24 ksi.

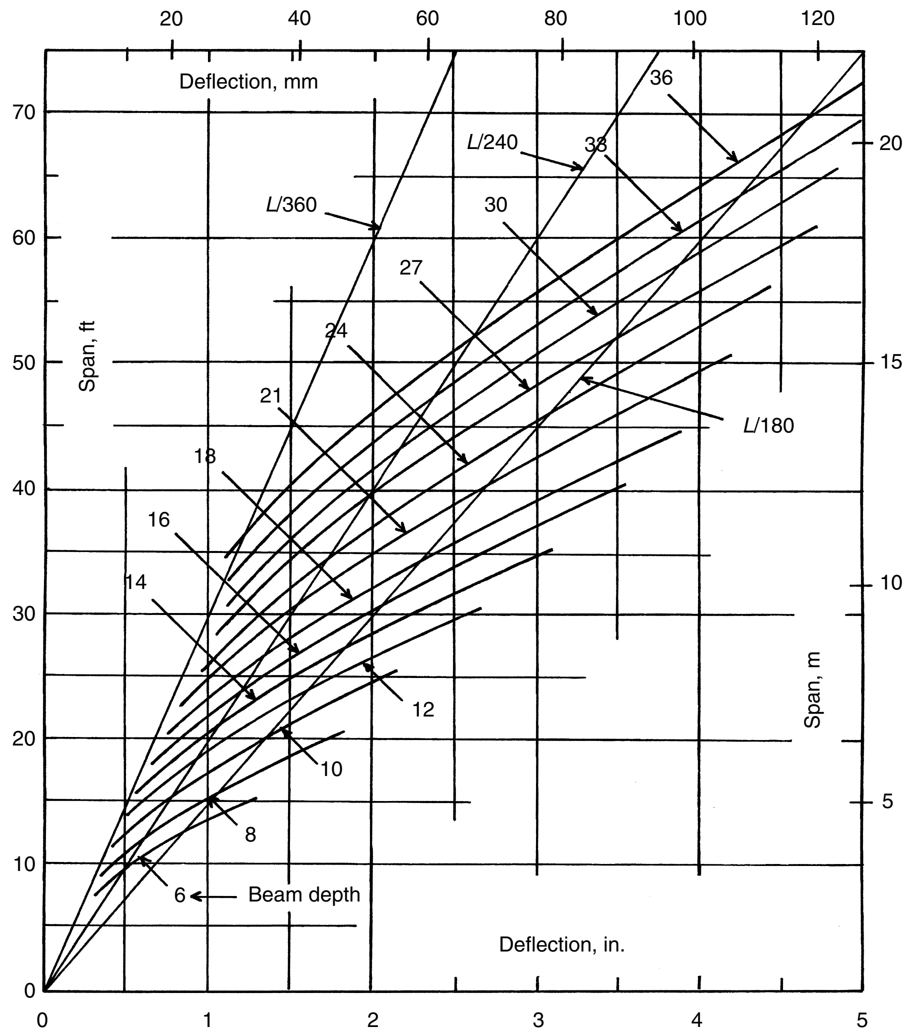


Figure 5.18 Deflection of steel beams with yield stress of 50 ksi under a constant maximum bending stress of 33 ksi.

bending or deflection limits. For this reason, table values are not shown for spans of less than 12 times the beam depth. For long spans, loads are often limited by deflection. Thus table values are not shown for spans exceeding a limit of 24 times the beam depth.

The load values in Table 5.3 are for the total load, including the weight of the beam itself. When selecting a shape for a computed factored, superimposed load, a value slightly larger than the computed load should be used. Once the weight of a selected shape is determined, it can be subtracted from the table value to verify the adequacy of the selection.

The following example illustrates the use of Table 5.3.

Example 9. Design a simply supported A36 steel beam to carry a uniformly distributed live load of 1.33 kips/ft and a superimposed dead load of 0.66 kip/ft on a span of 24 ft. Find (1) the lightest shape permitted and (2) the shallowest (least-depth) shape permitted.

Solution. The factored load to be carried is determined as

$$w_u = 1.2(0.66) + 1.6(1.33) = 2.92 \text{ kips/ft}$$

The total superimposed load is thus

$$W_u = 2.92 \times 24 = 70.1 \text{ kips}$$

From Table 5.3 some optional shapes and their table values are:

W 12 × 79,107 kips

W 14 × 53,78.4 kips

W 16 × 45,74.1 kips

W 18 × 46,81.6 kips

W 21 × 44,85.9 kips

The net load capacity for the W 16 × 45 is

$$W = 74.1 - (1.2 \times 0.045 \times 24) = 74.1 - 1.3 = 72.8 \text{ kips}$$

which indicates that the shape is adequate. The other shapes are obviously also adequate:

For part 1 the W 21 × 44 is the lightest choice.

For part 2 the W 12 × 79 is the shallowest choice.

Table 5.3 Factored Load–Span Values for A36-ksi Beams[a]

	$F_y = 36$ ksi				Span (ft)									
Designation	L_p (ft)	L_r (ft)	M_p (kip-ft)	M_r (kip-ft)	12	14	16	18	20	22	24	26	28	30
W 10 × 17	3.52	11.1	56	35	33.7	28.9	25.2	22.4	20.2					
W 12 × 16	3.22	9.58	60	37	36.2	31.0	27.1	24.1	21.7	19.7	18.1			
W 10 × 19	3.64	12.0	65	41	38.9	33.3	29.2	25.9	23.3					
W 12 × 22	3.53	11.2	88	55	52.7	45.2	39.6	35.2	31.6	28.8	26.4			
W 10 × 26	5.67	18.6	94	60	56.3	48.3	42.3	37.6	33.8					
W 12 × 26	6.29	18.1	112	72	67.0	57.4	50.2	44.6	40.2	36.5	33.5			
W 10 × 33	8.08	27.5	116	76	69.8	59.9	52.4	46.6	41.9					
W 14 × 26	4.50	13.4	121	76		62.0	54.3	48.2	43.4	39.5	36.2	33.4	31.0	
W 16 × 26	4.67	13.4	133	83			59.7	53.0	47.7	43.4	39.8	36.7	34.1	31.8
W 12 × 35	6.42	20.7	154	99	92.2	79.0	69.1	61.4	55.3	50.3	46.1			
W 14 × 34	6.38	19.0	164	105		84.2	73.7	65.5	59.0	53.6	49.1	45.4	42.1	
W 10 × 45	8.38	35.1	165	106	98.8	84.7	74.1	65.9	59.3					
W 16 × 36	6.33	18.2	192	122			86.4	76.8	69.1	62.8	57.6	53.2	49.4	46.1
W 12 × 45	8.13	28.3	193	125	116	99.1	86.7	77.0	69.3	63.0	57.8			
W 10 × 54	10.7	43.9	200	130	120	103	89.9	79.9	71.9					
W 14 × 43	7.88	24.7	209	136		107	94.0	83.5	75.2	68.3	62.6	57.8	53.7	
W 12 × 53	10.3	35.8	234	153	140	120	105	93.5	84.1	76.5	70.1			
W 18 × 40	5.29	15.7	235	148				94.1	84.7	77.0	70.6	65.1	60.5	56.4
W 16 × 45	6.54	20.2	247	158			111	98.8	88.9	80.8	74.1	68.4	63.5	59.3
W 10 × 68	10.8	53.7	256	164	154	132	115	102	92.1					
W 14 × 53	8.00	28.0	261	169		134	118	105	94.1	85.5	78.4	72.4	67.2	
W 18 × 46	5.38	16.6	272	171				109	98.0	89.1	81.6	75.4	70.0	65.3
W 18 × 50	6.88	20.5	303	193				121	109	99.2	90.9	83.9	77.9	72.7
W 16 × 57	6.67	22.8	315	200			142	126	113	103	94.5	87.2	81.0	75.6
W 10 × 88	11.0	68.4	339	213	203	174	153	136	122					
W 14 × 68	10.3	37.3	345	223		177	155	138	124	113	104	95.5	88.7	
W 12 × 79	12.7	51.8	357	232	214	184	161	143	129	117	107			
W 18 × 60	7.00	22.3	369	234				148	133	121	111	102	94.9	88.6
W 14 × 82	10.3	42.8	417	267		214	188	167	150	136	125	115	107	
W 18 × 71	7.08	24.5	438	275				175	158	143	131	121	113	105
W 10 × 112	11.2	86.4	441	273	265	227	198	176	159					
W 12 × 96	12.9	61.4	441	284	265	227	198	176	159	144	132			
W 16 × 77	10.3	35.4	456	295			205	182	164	149	137	126	117	109
W 12 × 120	13.0	75.5	558	353	335	287	251	223	201	183	167			
W 16 × 100	10.4	42.7	600	384			270	240	216	196	180	166	154	144
W 14 × 120	15.6	67.9	636	412		327	286	254	229	208	191	176	164	
W 12 × 152	13.3	94.8	729	453	437	375	328	292	262	239	219			
W 14 × 145	16.6	81.6	780	503		401	351	312	281	255	234	216	201	

	$F_y = 36$ ksi				Span (ft)									
Designation	L_p (ft)	L_r (ft)	M_p (kip-ft)	M_r (kip-ft)	24	26	28	30	32	34	36	38	40	42
W 21 × 44	5.25	15.4	286	177	85.9	79.3	73.6	68.7	64.4	60.6	57.2	54.2	51.5	49.1
W 21 × 50	5.42	16.2	330	205	99.0	91.4	84.9	79.2	74.3	69.9	66.0	62.5	59.4	56.6
W 21 × 57	5.63	17.3	387	241	116	107	99.5	92.9	87.1	82.0	77.4	73.3	69.7	66.3
W 24 × 55	5.58	16.6	405	249	122	112	104	97.2	91.1	85.8	81.0	76.7	72.9	69.4
W 24 × 62	5.71	17.2	462	286	139	128	119	111	104	97.8	92.4	87.5	83.2	79.2
W 21 × 68	7.50	22.8	480	303	144	133	123	115	108	101.6	96.0	90.9	86.4	82.3
W 18 × 86	11.0	35.5	558	360	167	155	143	134	126	118	112			
W 21 × 83	7.63	24.9	588	371	176	163	151	141	132	125	118	111	106	101
W 24 × 76	8.00	23.4	600	381	180	166	154	144	135	127	120	114	108	103
W 18 × 106	11.1	40.4	690	442	207	191	177	166	155	146	138			
W 21 × 101	12.0	37.1	759	492	228	210	195	182	171	161	152	144	137	130
W 24 × 94	8.25	25.9	762	481	229	211	196	183	171	161	152	144	137	131
W 27 × 94	8.83	25.9	834	527			214	200	188	177	167	158	150	143
W 24 × 103	8.29	27.0	840	531	252	233	216	202	189	178	168	159	151	144
W 30 × 90	8.71	24.8	849	531				204	191	180	170	161	153	146
W 24 × 104	12.1	35.2	867	559	260	240	223	208	195	184	173	164	156	149

(*Continued*)

Table 5.3 (*Continued*)

	$F_y = 36$ ksi				Span (ft)									
Designation	L_p (ft)	L_r (ft)	M_p (kip-ft)	M_r (kip-ft)	24	26	28	30	32	34	36	38	40	42
W 18 × 130	11.3	47.7	870	555	261	241	224	209	196	184	174			
W 21 × 122	12.2	41.0	921	592	276	255	237	221	207	195	184	175	166	158
W 24 × 117	12.3	37.1	981	631	294	272	252	235	221	208	196	186	177	168
W 27 × 114	9.08	28.2	1,029	648			265	247	232	218	206	195	185	176
W 30 × 108	8.96	26.3	1,038	648				249	234	220	208	197	187	178
W 18 × 158	11.4	56.5	1,068	672	320	296	275	256	240	226	214			
W 24 × 131	12.4	39.3	1,110	713	333	307	285	266	250	235	222	210	200	190
W 21 × 147	12.3	46.4	1,119	713	336	310	288	269	252	237	224	212	201	192
W 30 × 124	9.29	28.2	1,224	769				294	275	259	245	232	220	210
W 33 × 118	9.67	27.8	1,245	778						264	249	236	224	213
W 24 × 146	12.5	42.0	1,254	804	376	347	322	301	282	266	251	238	226	215
W 24 × 162	12.7	45.2	1,404	897	421	389	361	337	316	297	281	266	253	241
W 30 × 148	9.50	30.6	1,500	945				360	338	318	300	284	270	257
W 33 × 141	10.1	30.1	1,542	971						327	308	292	278	264

Source: Compiled from data in the *Manual of Steel Construction, Load and Resistance Factor Design*, 3rd ed. (Ref. 10), with permission of the publisher, American Institute of Steel Construction.
[a]Table yields total safe uniformly distributed load in kips for indicated spans.

Equivalent Load Techniques

The loads in Table 5.3 are uniformly distributed loads on simple beams. Actually, the table values are determined on the basis of bending moment alone, so that it is possible to use the tables for other loadings for some purposes.

Consider the following situation: a beam with a load consisting of two equal concentrated loads placed at the beam third points—in other words, case 3 in Figure 3.8. For this condition, the figure yields a maximum moment value expressed as $PL/3$. By equating this to the moment value for a uniformly distributed loading, a relationship between the two loads can be derived:

$$\frac{WL}{8} = \frac{PL}{3} \quad \text{or} \quad W = 2.67P$$

This shows that if the value of one of the concentrated loads in case 3 of Figure 3.8 is multiplied by 2.67, the result would be an *equivalent uniform load* (called EUL) that would produce the same magnitude of maximum moment as the true loading condition.

Although the expression "equivalent uniform load" is the general name for this converted loading, when derived to facilitate the use of tabular materials, it is referred to as the e*quivalent tabular load* (called ETL), which is the designation used in Figure 3.8.

This method may also be used for any loading condition, not just the simple cases in Figure 3.8. The process consists of first finding the true maximum moment due to the actual loading; then this is equated to the expression for the maximum moment for a theoretical uniform load, and the EUL is determined; thus

$$M = \frac{WL}{8} \quad \text{or} \quad W = \frac{8M}{L}$$

Of course, once the true maximum moment is determined for any beam, shape selection can also be performed with the data in Table 5.2. So this method simply presents an alternative process.

Bracing for Torsional and Lateral Buckling

Beams are basically intended to resist bending, shear, and deflection. However, as discussed earlier in this section, any beams with a lack of lateral stiffness may be subject to failure by buckling. An important consideration for design, therefore, is what precisely constitutes the mode of failure for the beam.

Lateral buckling is developed primarily by the compression due to bending. This makes the compression side of the beam work somewhat like a column, with column slenderness a critical concern. Since the beam is not likely to buckle on its strong (X–X) axis, the buckling that must be considered is in a direction perpendicular to the beam, referred to as the lateral (sideways) direction.

Torsion (twisting) may be induced in a beam by a number of situations. Torsion due to loading occurs mostly when the vertical axis of the loading does not correspond with the so-called *shear center* of a beam. For the doubly symmetrical I-shaped beams this point is coincident with the center of the member. Examples of off-center loading are shown in Figures 5.19*a* and *b*. Some torsion is also induced by the standard framing shown in Figure 5.19*c*, since the load-transferring element (the connecting angles) are placed on one side of the web of the supporting beam.

A special torsional effect is that of the rotational effect called *torsional buckling*, which can be developed with ordinary bending situations on beams that lack torsional resistance and are not adequately braced against twisting movements.

When torsional effects threaten, they may be designed for, but the preferred method is to brace the beam to prevent

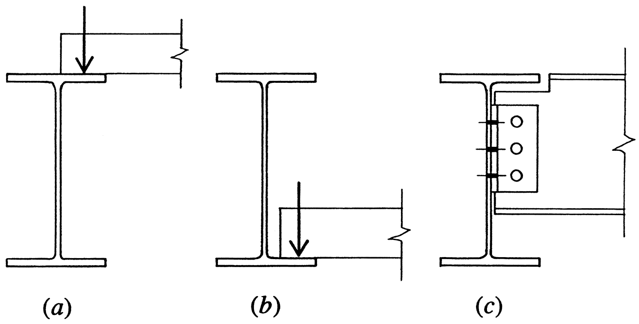


Figure 5.19 Torsion-inducing loads on beams.

the twisting deformation from occurring. In Figure 5.19*c*, for example, the framing of the supported beam will prevent twisting rotation of the supporting beam. Of course, the spacing of such bracing must also be at close enough spacing for full protection.

It is not always a simple matter to determine that a beam is adequately laterally supported. In cases such as that shown in Figure 5.20*a*, lateral support is supplied to the beam by the floor construction attached to it. On the other hand, with the form of construction shown in Figure 5.20*b*, where wood joists simply rest on top of the steel beam, little resistance to lateral buckling is provided.

When the construction is firmly attached, as shown in Figures 5.20*c* through *f*, adequate bracing is usually achieved. These forms of bracing, involving a relatively firm grip to prevent twisting of the top flange of the steel beam, are usually also assumed to provide resistance to torsional failures. However, the form of bracing provided in Figure 5.20*g* is generally the best all-purpose bracing method, preventing both lateral movements and rotational movements of the beam.

Spacing of bracing for some situations is specified; such is the case for wood joists and open-web steel joists (light

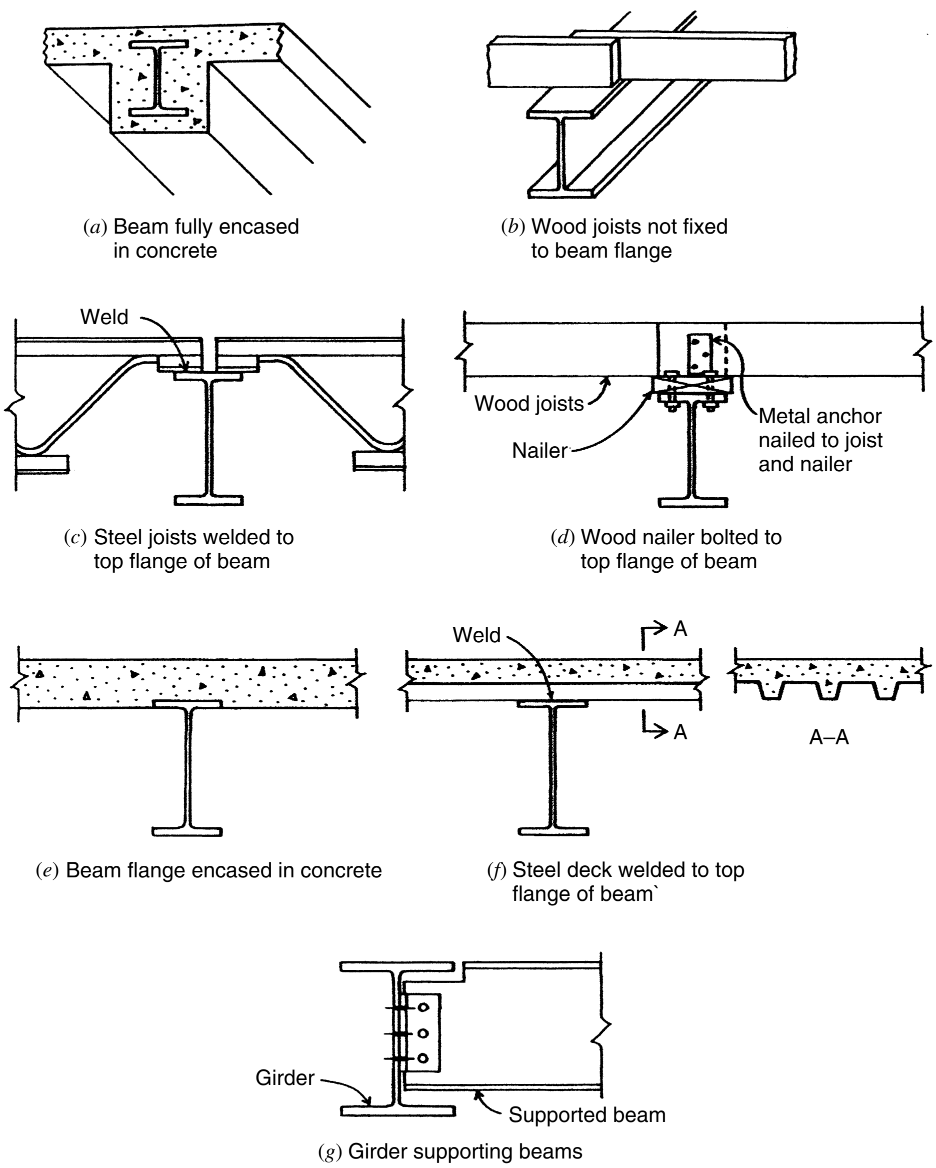


Figure 5.20 Provision of lateral bracing for steel beams.

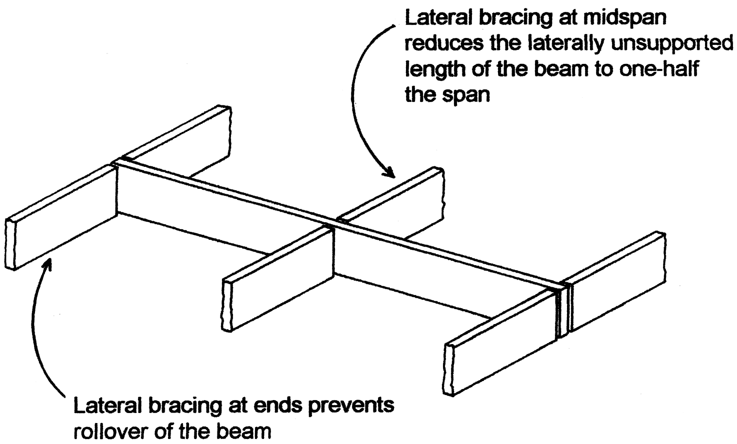


Figure 5.21 Bracing to prevent twisting.

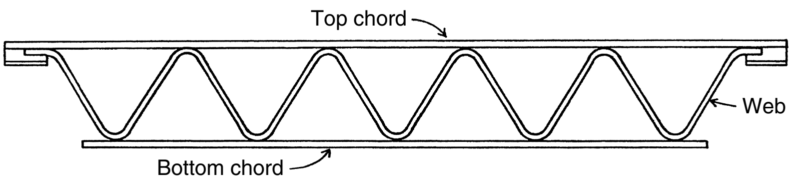


Figure 5.22 Form of a short-span open-web joist.

prefabricated trusses). For the W shapes, spacing relates to the qualified limits for the basic modes of bending failure, as discussed previously in this section.

In Figure 5.21, framing at both midspan and the beam ends will prevent rotation at these points, reducing the lateral unbraced length in this case to one-half of the beam span.

Structural elements providing bracing are most often parts of the framing systems with other functions. Their selection and spacing must be developed in terms of all the tasks they perform.

Manufactured Trusses

Factory-fabricated, parallel-chorded trusses are produced for a wide range of sizes by a number of manufacturers. Most producers comply with the standards of industrywide organizations; for light steel trusses the principal organization is the Steel Joist Institute (SJI). Publications of the SJI are a chief source of general information (see Ref. 11), although the products of the individual manufacturers tend to vary. As with most manufactured structural products, final design must be completed with information obtained directly from the manufacturers or distributors of the products.

Light steel parallel-chorded trusses, called *open-web joists,* have been widely used for years. Early versions used all steel bars for the chords and continuously bent web members (see Figure 5.22), so that they were referred to as *bar joists*. Although other elements are now used for the chords, bent bars are sometimes still used for the webs of smaller joists. The range of size of this basic element has now been stretched considerably, with other forms of joists using a more common form of truss construction.

Table 5.4 is adapted from a table in one of the SJI's publications (Ref. 11). It lists a number of sizes in the K series, which includes the lightest group of joists. Joists are identified by a three-unit designation. The first number indicates the overall nominal depth of the joist, the letter indicates the series, and the second number indicates the class of size of the members—the higher the number, the heavier and stronger the joist.

Table 5.4 can be used to select the proper joist for a determined load-and-span condition. Span values in the table are based on a definition of the span as given by the SJI and as shown in Figure 5.23. There are two entries in the table for each span–joist combination; the first number represents the total load capacity of the joist in pounds per foot of the span; the number in parentheses is the load that will cause a deflection of 1/360 of the span.

For a given load-and-span combination, there will generally be a single joist that represents the least-weight member. However, there will usually also be other choices—of different depths—that are also acceptable and within a close variation of weight. Where deflection is a critical concern—notably for floors—it is advisable to use the *deepest*

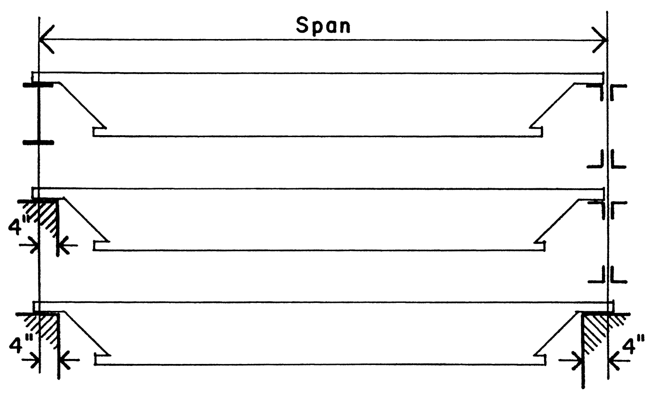


Figure 5.23 Definition of span for open-web joists.

Table 5.4 Safe Factored Loads for K-Series Open-Web Joists[a]

Joist Designation:	12K1	12K3	12K5	14K1	14K3	14K6	16K2	16K4	16K6	18K3	18K5	18K7	20K3	20K5	20K7
Weight (lb/ft):	5.0	5.7	7.1	5.2	6.0	7.7	5.5	7.0	8.1	6.6	7.7	9.0	6.7	8.2	9.3
Span (ft)															
20	357 (142)	448 (177)	607 (230)	421 (197)	528 (246)	729 (347)	545 (297)	732 (386)	816 (426)	687 (423)	816 (490)	816 (490)	767 (517)	816 (550)	816 (550)
22	295 (106)	369 (132)	500 (172)	347 (147)	435 (184)	641 (259)	449 (222)	602 (289)	739 (351)	567 (316)	769 (414)	816 (438)	632 (393)	816 (490)	816 (490)
24	246 (81)	308 (101)	418 (132)	291 (113)	363 (141)	537 (199)	377 (170)	504 (221)	620 (269)	475 (242)	644 (318)	781 (382)	530 (302)	720 (396)	816 (448)
26				246 (88)	310 (110)	457 (156)	320 (133)	429 (173)	527 (211)	403 (190)	547 (249)	665 (299)	451 (236)	611 (310)	742 (373)
28				212 (70)	267 (88)	393 (124)	276 (106)	369 (138)	454 (168)	347 (151)	472 (199)	571 (239)	387 (189)	527 (248)	638 (298)
30							239 (86)	320 (112)	395 (137)	301 (123)	409 (161)	497 (194)	337 (153)	457 (201)	555 (242)
32							210 (71)	282 (92)	346 (112)	264 (101)	359 (132)	436 (159)	295 (126)	402 (165)	487 (199)
36										209 (70)	283 (92)	344 (111)	233 (88)	316 (115)	384 (139)
40													188 (64)	255 (84)	310 (101)

Joist Designation:	22K4	22K6	22K9	24K4	24K6	24K9	26K5	26K7	26K9	28K6	28K8	28K10	30K7	30K9	30K12
Weight (lb/ft):	8.0	8.8	11.3	8.4	9.7	12.0	9.8	10.9	12.2	11.4	12.7	14.3	12.3	13.4	17.6
Span (ft)															
28	516 (270)	634 (328)	816 (413)	565 (323)	693 (393)	816 (456)	692 (427)	816 (501)	816 (541)	813 (543)	816 (543)	816			
30	448 (219)	550 (266)	738 (349)	491 (262)	602 (319)	807 (419)	601 (346)	730 (417)	816 (459)	708 (439)	816 (500)	816 (500)	816 (543)	816 (543)	816 (543)
32	393 (180)	484 (219)	647 (287)	430 (215)	530 (262)	709 (344)	528 (285)	641 (343)	770 (407)	620 (361)	764 (438)	815 (463)	743 (461)	815 (500)	815 (500)
36	310 (126)	381 (153)	510 (201)	340 (150)	417 (183)	559 (241)	415 (199)	504 (240)	607 (284)	490 (252)	602 (306)	723 (366)	586 (323)	705 (383)	723 (392)
40	250 (91)	307 (111)	412 (146)	274 (109)	337 (133)	451 (175)	337 (145)	408 (174)	491 (207)	395 (183)	487 (222)	629 (284)	473 (234)	570 (278)	650 (315)
44	206 (68)	253 (83)	340 (109)	227 (82)	277 (100)	372 (131)	277 (108)	337 (131)	405 (155)	326 (137)	402 (167)	519 (212)	390 (176)	470 (208)	591 (258)
48				190 (63)	233 (77)	313 (101)	233 (83)	282 (100)	340 (119)	273 (105)	337 (128)	436 (163)	328 (135)	395 (160)	542 (216)
52							197 (65)	240 (79)	289 (102)	233 (83)	286 (100)	371 (128)	279 (106)	335 (126)	498 (184)
56										200 (66)	246 (80)	319 (102)	240 (84)	289 (100)	446 (153)
60													209 (69)	250 (81)	389 (124)

Source: Data adapted from more extensive tables in the *Guide for Specifying Steel Joists with LRFD*, (Ref. 12), with permission of the publisher, Steel Joist Institute. The Steel Joist Institute publishes both specifications and load tables; each of these contains standards that are to be used in conjunction with one another.

[a]Loads in pounds per foot of joist span. First entry represents the total factored joist capacity; entry in parentheses is the load that produces a deflection of 1/360 of the span. See Figure 5.23 for definition of span.

joist if possible. The following examples illustrate the use of data from Table 5.4 for design of a roof joist and a floor joist.

Example 10. Open-web steel joists are to be used to support a roof with a unit live load of 20 psf and a unit dead load of 15 psf, not including the joist weight, on a span of 40 ft. Joists are placed at 6 ft center to center. Select the lightest joist if deflection under live load is limited to 1/360 of the span.

Solution. The first step is to determine the unit load per foot on the joists; thus,

$$\text{Live load:} \quad 6(20) = 120 \text{ lb/ft}$$

$$\text{Dead load:} \quad 6(15) = 90 \text{ lb/ft}$$

$$\text{Total factored load:} \quad 1.2(90) + 1.6(120) = 108 + 192 = 300 \text{ lb/ft}$$

Table 5.5 Possible Choices for the Roof Joist

Load Condition	Required Capacity (lb/ft)	Capacity of the Indicated Joists (lb/ft)		
		22K9	24K6	26K5
Factored total capacity		412	337	337
Joist weight from Table 5.4		11.3	9.7	9.8
Factored joist weight		14	12	12
Net usable capacity	300	398	325	325
Load for deflection of 1/360	120	146	133	145

This yields the two numbers that can be used to scan Table 5.4. Note that joist weight is included in the table entry and must be deducted once the joist is chosen in order to find its actual carrying capacity of superimposed load. We note from Table 5.4 the possible choices listed in Table 5.5. Although the joist weights are quite close, the lightest choice is the 24K6.

Example 11. Open-web steel joists are to be used for a floor with a unit live load of 75 psf and a unit dead load of 40 psf (not including the joist weight) on a span of 30 ft. Joists are 2 ft on center and deflection is limited to 1/240 of the span under total load and 1/360 of the span under live load only. Determine the lightest possible joist and the lightest joist of least depth possible.

Solution. As in Example 11, the unit loads for the joist are determined first; thus,

Live load: $2(75) = 150$ lb/ft
Dead load: $2(40) = 80$ lb/ft
Total service load $= 150 + 80 = 230$ lb/ft
Total factored load $= 1.2(80) + 1.6(150) = 336$ lb/ft

To satisfy the deflection criteria for total load, the limiting load value for deflection in the table should not be greater than $(240/360)(230) = 153$ lb/ft. Because this is slightly larger than the live load, it becomes the value to look for in the table. Possible choices from Table 5.4 are listed in Table 5.6, from which we obtain the following:

The lightest joist is the 18K5.
The shallowest depth joist is also the 18K5.

Table 5.6 Possible Choices for the Floor Joist

Load Condition	Required Capacity (lb/ft)	Capacity of the Indicated Joists (lb/ft)		
		18K5	20K5	22K4
Factored total capacity		409	457	448
Joist weight from Table 5.4		7.7	8.2	8.0
Factored joist weight		10	10	10
Net usable capacity	336	399	447	438
Load for deflection	153	161	201	219

In some situations, it may be desirable to select a deeper joist, even though its load capacity may be somewhat redundant. Total sag, rather than an abstract curvature limit, may be of more significance for a flat roof structure. For example, for the 40-ft span in Example 10, a sag of 1/360 of the span $= (1/360)(40)(12) = 1.33$ in. The actual effect of this dimension on roof drainage or in relation to interior partition walls must be considered. For floors, a major concern is for bounciness, and the very lightweight open-web joist construction is highly vulnerable in this regard. Therefore, many designers prefer to choose the deepest feasible joist for floor structures in order to get all the possible stiffness to reduce deflection and bounciness.

Lateral stability is a concern for these elements. Bracing by other construction may help, but the bottom chords must be braced by X-bracing or other means.

One means of assisting stability deals with the typical end-support detail, in which the joists are typically hung by their top chords, which prevents the rollover form of buckling at the supports, as shown in Figure 3.10*c*. For construction detailing, however, this adds a dimension to the overall depth of the construction, in comparison with a system of all rolled shapes. This added dimension (the depth of the extended end of the joist is typically 2.5 in. for small joists and 4 in. for larger joists.

For development of a complete truss system, a special truss is used, described as a *joist girder.* A common form of joist girder is shown in Figure 5.24, together with the customary form for designation of the girder, which includes indications of the girder span, the number of spaces between loads, and the panel point load.

Predesigned joist girders may be selected from catalogs in a manner similar to that for joists. The procedure is usually as follows:

The designer determines the joist spacing, the total joist load, and the girder span.
The designer uses this information to specify the girder using the standard form of designation.
Complete design of the joist girder is accomplished by the manufacturer—generally the same one who supplies the associated open-web joists.

Examples of the use of open-web joists and joist girders are given in the building design cases in Chapter 10.

As mentioned previously, joists are available in other series for heavier loads and longer spans. The SJI, as well as individual manufacturers and suppliers, has considerably more information regarding suggested specifications, installation details and procedures, finished bracing, need for temporary bracing during construction, and modifications for cantilevered ends or accommodation of holes. As with other large structural elements, these products are usually supplied by local manufacturers, which should be used as sources for design information.

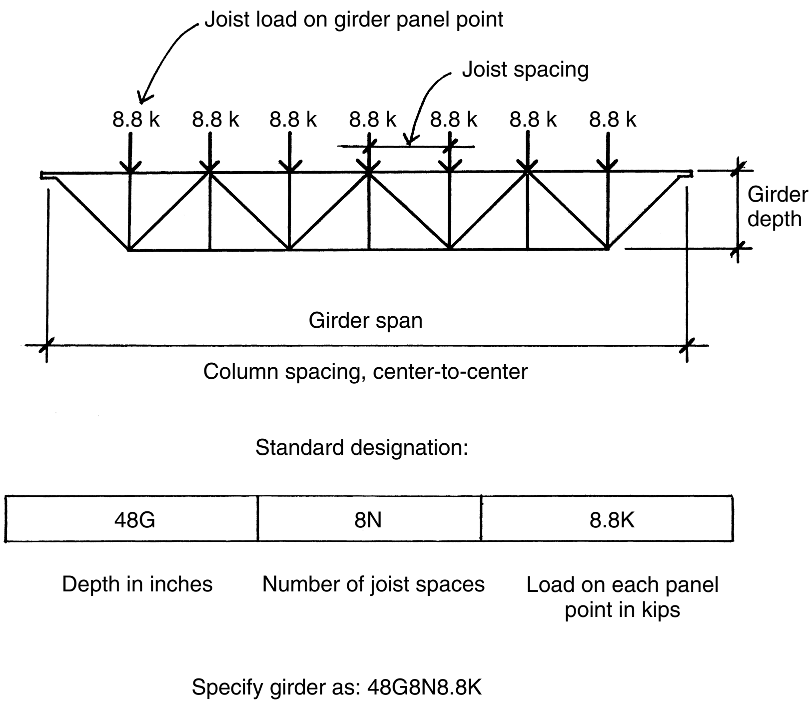


Figure 5.24 Considerations for the layout of joist girders.

Concentrated Load Effects in Beams

An excessive end reaction for a bearing-supported beam or an excessive concentrated load on top of a beam may cause web yielding or web crippling (buckling). The two actions must be investigated separately as follows (see Figure 5.25):

For web yielding: When the force to be resisted is a concentrated load producing tension or compression, applied at a distance from the member end that is greater than the depth of the member,

$$P_n = (5k + N)f_{yw}t_w$$

When the force to be resisted is a concentrated load applied at or near the end of the member,

$$R_n = (2.5k + N)f_{yw}t_w$$

where

R_n = reaction, kips
P_n = concentrated load, kips
t_w = thickness of web, in.
N = length of bearing, in.
k = distance from flange face to web fillet, in.
F_{yw} = yield stress of web, ksi

For web crippling: When the concentrated load is applied at a distance not less than $d/2$ from the end of the beam,

$$P_n = 0.6(t_w)^2\left[1 + 3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}}$$

When the concentrated load is applied at a distance of less than $d/2$ from the end of the member,

$$P_n = 0.3(t_w)^2\left[1 + 3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}}$$

where

F_{yw} = yield stress of beam web, ksi
d = depth of beam, in.
t_f = flange thickness, in.
E = modulus of elasticity of steel, 29,000 ksi

If the beam web is overstressed, one solution is to provide stiffeners, as shown in Figure 5.26.

Any design or investigation work related to these issues is, as usual, aided significantly by materials provided in the AISC manual (Ref. 10). Shortcuts, especially involving beam end reactions, are provided in the load/span tables for rolled sections in the manual.

Problems of this kind arise primarily because the W shapes are basically developed for optimal resistance to bending, specifically to development of resistance to plastic yield moment. For other functions there are many behaviors affected by the four basic dimensions: beam depth, flange width, web thickness, and flange thickness. Each dimension is of concern, but various combinations and relationships are also of concern. For example, the effective area for shear is the combination of beam depth and web thickness.

Roof and Floor Systems

Steel elements may be used to produce a variety of horizontal-spanning floor structures and horizontal or sloping roof

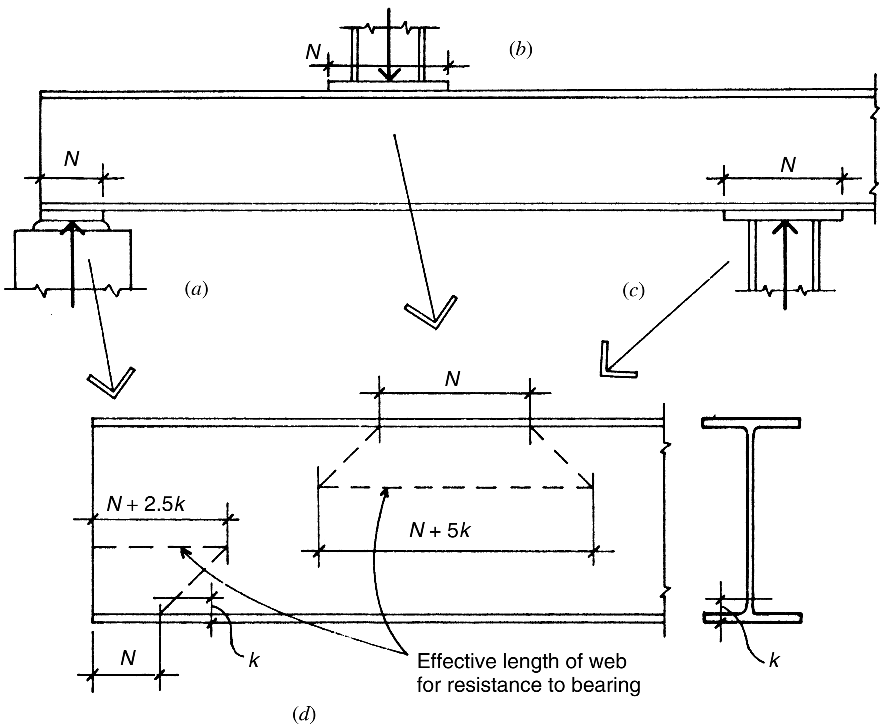


Figure 5.25 Bearing considerations for web failure.

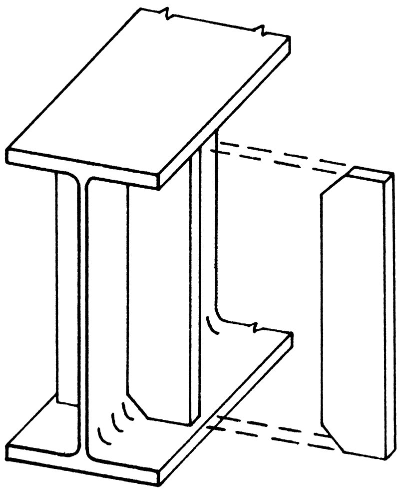


Figure 5.26 Stiffeners to prevent web buckling.

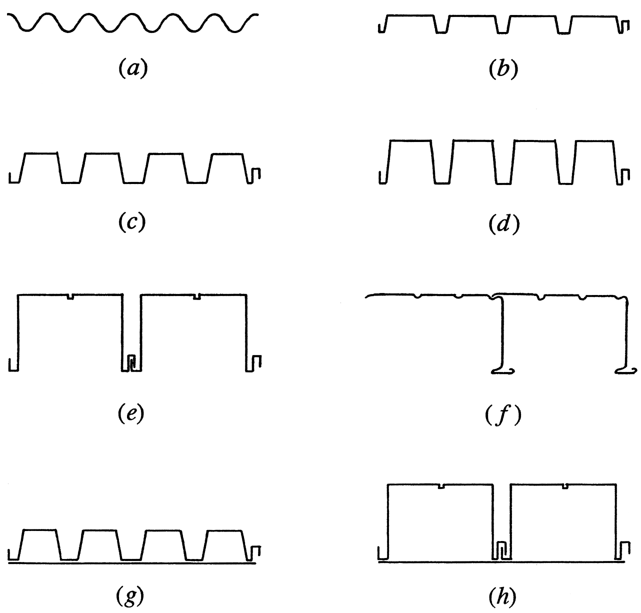


Figure 5.27 Formed sheet steel deck units.

structures. Choice of steel may be related to a need for exceptional spans but is often due to a requirement for noncombustible construction in order to satisfy fire resistance requirements. Design examples of roof and floor systems are presented in Chapter 10.

Decks

Steel decks consist of formed sheet steel produced in a variety of configurations, as shown in Figure 5.27. The simplest is the corrugated sheet, shown in Figure 5.27*a*. This may be used as the single, total surface for walls or roofs for utilitarian buildings. For more serious buildings, it is often used in pairing with light steel joist systems where the joist spacing is quite close and a cast or preformed filler is placed on top of the steel deck.

A widely used product is that shown in three variations in Figures 5.27*b*–*d*. When used for roof deck, where loads are light and a flat top surface is desired, the unit shown in Figure 5.27*b* is popular, with the overall deck thickness being only 1.5 in. The deeper decks shown in Figures 5.27*c* and *d* can be used for longer spans for either roof or floor applications.

There are also steel deck units produced as proprietary items by individual manufacturers. Figures 5.27*e* and *f* show two units of considerable depth capable of achieving quite long spans. These are most likely to be used with larger steel frame elements that are widely spaced.

Although less used now, with the advent of other wiring products and systems, a possible use for the steel deck is as a conduit for power or communication wiring. This is accomplished by closing the deck cells with a flat sheet of steel, as shown in Figures 5.27*g* and *h*. This provides for wiring in one direction in a wiring grid; the perpendicular wiring being achieved in conduits buried in a concrete fill on top of the deck.

Closing of the larger units shown in Figure 5.27*h* creates cells that may be used for an HVAC air circulation system. Without insulation, these would most likely be used for a return air system.

Decks vary in form, including ones not shown in Figure 5.27, and in gauge (thickness) of the sheet steel. Choice relates primarily to the load and span conditions for the deck and to the general development of the construction. Other concerns include fire requirements, exposure to moisture (for rusting), and use of decks as horizontal diaphragms for lateral forces on the building.

Roof decks are most often made with either a very low density poured concrete fill or preformed rigid foam insulating panels on top of the steel deck. The steel deck itself is the functioning structure in these cases. For floor decks, however, the concrete fill is usually of sufficient strength to perform structural tasks, for which the steel units may work in one of three ways:

As the Basic Structure. In this case the concrete is an inert fill, forming the floor surface.

As a Form for the Cast Concrete. In this case the concrete is designed as a spanning structural slab and the steel units serve only as forms for the casting.

As Part of a Composite Structure. In this case the steel and concrete interact, with the steel performing the task usually fulfilled by the bottom reinforcement in the spanning concrete slab.

Table 5.7 presents data relating to the use of the type of deck unit shown in Figure 5.27*b* for roof structures. This

Table 5.7 Safe Service Load Capacity of Formed Steel Roof Deck

Deck	Span	Weight[b]	Total Safe Service Load (Dead + Live)[c] for Spans Indicated in Feet												
Type[a]	Condition	(lb/ft)^2	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
NR22	Simple	1.6	73	58	47										
NR20		2.0	91	72	58	48	40								
NR18		2.7	125	99	80	66	55	47							
NR22	Two	1.6	80	63	51	42									
NR20		2.0	97	76	62	51	43								
NR18		2.7	128	101	82	68	57	48	42						
NR22	Three +	1.6	100	79	64	53	44								
NR20		2.0	121	96	77	64	54	46							
NR18		2.7	160	126	102	85	71	61	52	45					
IR22	Simple	1.6	84	66	54	44									
IR20		2.0	104	82	67	55	46								
IR18		2.7	142	112	91	75	63	54	46	40					
IR22	Two	1.6	90	71	58	48	40								
IR20		2.0	110	87	70	58	49	41							
IR18		2.7	145	114	93	77	64	55	47	40					
IR22	Three +	1.6	113	89	72	60	50	43							
IR20		2.0	137	108	88	72	61	52	45						
IR18		2.7	181	143	116	96	81	69	59	52	45	40			
WR22	Simple	1.6			90	70	56	46							
WR20		2.0			113	88	70	57	48	40					
WR18		2.7			159	122	96	77	64	54	46	40			
WR22	Two	1.6			96	79	67	57	49	43					
WR20		2.0			123	102	86	73	63	55	48	43			
WR18		2.7			164	136	114	98	84	73	64	57	51	46	41
WR22	Three +	1.6			119	99	83	71	61	53	47	41	36		
WR20		2.0			153	127	107	91	79	68	58	50	43		
WR18		2.7			204	169	142	121	105	91	79	67	58	51	43

Source: Adapted from the *Steel Deck Institute Design Manual for Composite Decks, Form Decks, and Roof Decks* (Ref. 13), with permission of the publisher, the Steel Deck Institute.

[a]Letters refer to rib type (see Fig. 5.28). Numbers indicate gauge (thickness) of deck sheet steel.

[b]Approximate weight with paint finish; other finishes available.

[c]Total safe allowable service load in lb/ft^2. Loads in parentheses are governed by live-load deflection not in excess of 1/240 of the span, assuming a dead load of 10 lb/ft^2.

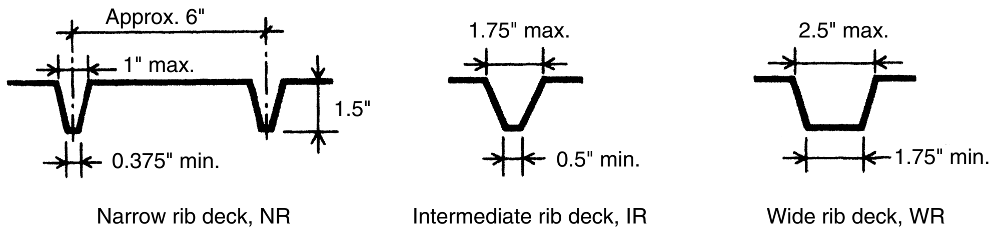


Figure 5.28 Reference for Table 5.7.

information is adapted from a publication distributed by an industrywide organization referred to in the table notes. In general, data for these products is best obtained from the manufacturers of the products. While most products will conform to industry standards, there is room for modifications that may alter some structural properties. This is especially so for usage in a composite system, for which special units may be developed that create an enhanced bond between the concrete and the steel deck units.

In general, economy is obtained with the use of the thinnest sheet steel (highest gauge number) that can be utilized for a particular application. This holds true for actions related to gravity loads, but use of the deck for diaphragm actions may require a heavier, stiffer, unit. Where rusting may be a problem, the use of the thinnest material may not be advisable, although protection of the deck surface is probably more important. With highly corrosive conditions, special considerations are required.

Rusting is a critical problem for all thin steel elements. Use of rust-resisting steel may be considered but is usually not an option for sheet steel decks. The common solution is usually some rust-inhibiting finish applied to the deck units, such as paint or a galvanized treatment. For the deck weights in Table 5.7, a painted surface is assumed, which is usually the most economical solution. For decks with concrete fill, the top is considered protected.

Figure 5.29 shows four possibilities for a floor deck used in conjunction with a framing system of rolled steel shapes. When a wood deck is used, it is usually nailed to series of wood joists or trusses that are supported by the steel beams.

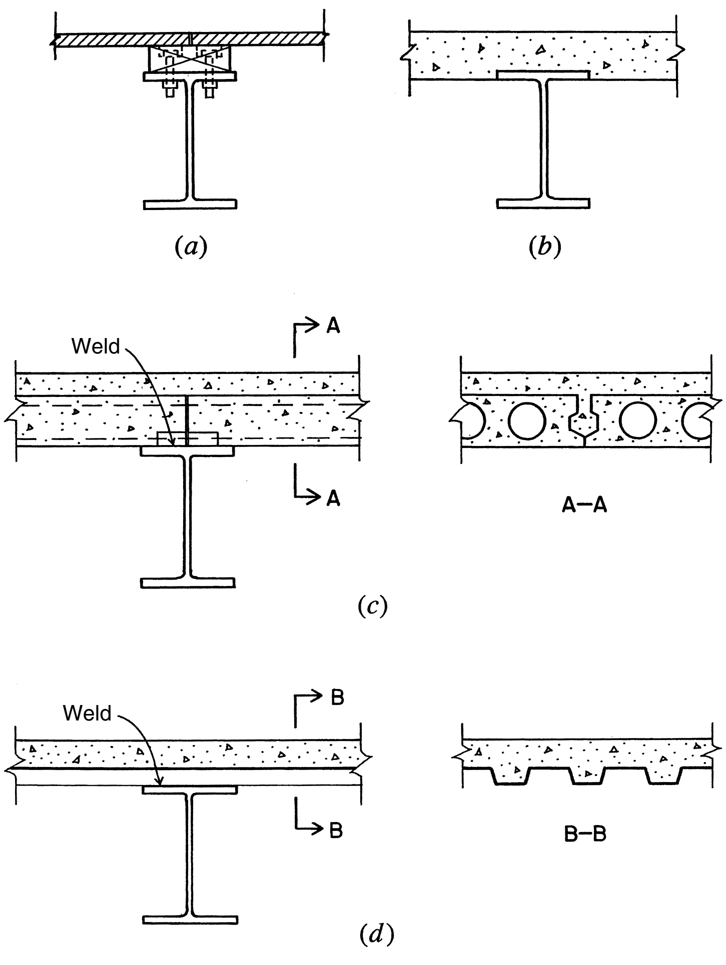


Figure 5.29 Typical floor decks with steel framing.

Where the supports are steel, the deck is usually nailed to wood pieces that are bolted to the steel members, as shown in Figure 5.29*a*.

A sitecast concrete slab may be used, with the slab forms placed on the underside of the top flanges of the steel beams, producing the detail shown in Figure 5.29*b*. It is common in this case to use steel devices welded to the top of the beams to develop composite action of the steel beams and the concrete slab.

Concrete may also be used in the form of precast units that are welded to the steel beams using steel devices cast into the concrete units. These precast units are usually prestressed and their sag is only partly controllable, producing a top surface that can be quite irregular. A concrete fill is therefore used on top of the precast units to develop a smooth top surface. The fill is usually bonded to the top of the precast units to develop an enhanced spanning system.

Figure 5.29*d* shows the most typical deck system, created simply with the formed sheet steel units and a concrete fill. The deck units are welded directly to the tops of the steel supports.

Figure 5.30 shows three possibilities for a roof deck used in conjunction with a steel framing system. A fourth possibility is that of the plywood deck shown in Figure 5.29*a*. Many of the issues discussed for the floor decks also apply here. However, roof loads are usually lighter, and there is less concern for bounciness due to people walking; thus some lighter systems are feasible here and not for floors.

Formed steel units, such as those presented in Table 5.7, are normally used with a rigid insulation, as shown in Figure 5.30*a*, or a poured concrete fill, as shown in Figure 5.30*b*.

To facilitate roof drainage, concrete fill may be varied in thickness, allowing the deck units to remain flat. Rigid insulation units can also be obtained in modular packages with tapered pieces. This is only possible for a few inches of elevation change; the structure must be tilted for substantial slopes.

When the concrete fill works only as an inert fill on top of a structural steel deck, it may consist of very lightweight concrete, creating a more insulative material and substantially reducing dead load. A special deck is shown in Figure 5.30*c*, consisting of a low-density concrete fill poured on top of a forming system of inverted steel tees and lay-in rigid panel units. The panel units can be used to form the finished ceiling surface where it is possible to leave the steel framing uncovered.

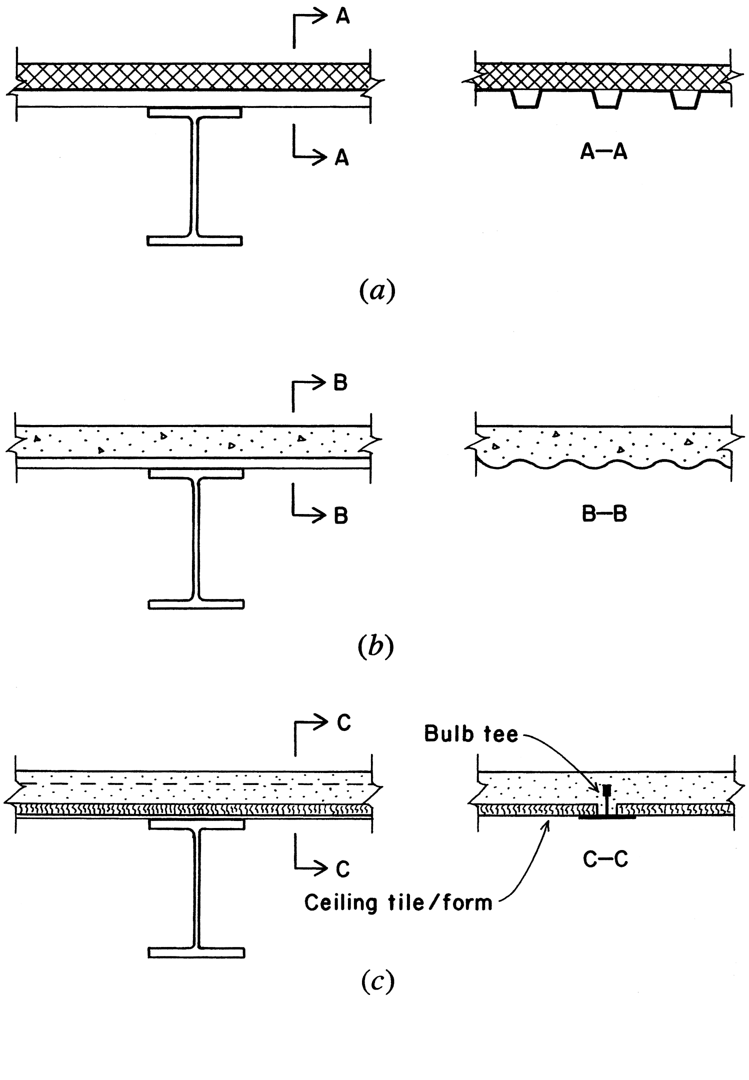


Figure 5.30 Typical roof decks with steel framing.

System Planning

The planning of the layout of the structural framing system for a roof or floor must respond to many concerns. Figure 5.31 shows a framing plan layout for a system that has four basic components. Development of the layout for this system involves considerations such as the following:

Deck Span. The type of deck, as well as its potential variations (thickness of plywood, gauge of sheet steel, etc.), will relate to the deck span.

Joist Spacing. This determines the deck span and the magnitude of load on the joists. The type of joist selected may limit the spacing, based on the joist capacity. The type and spacing of joists must therefore be coordinated with the selection of the deck.

Beam Span. For systems with some plan regularity, the joist spacing should be some full number division of the beam span.

Column Spacing. The spacing of the columns determines the spans for the beams and joists and is thus related to the planning of modules for all the other components.

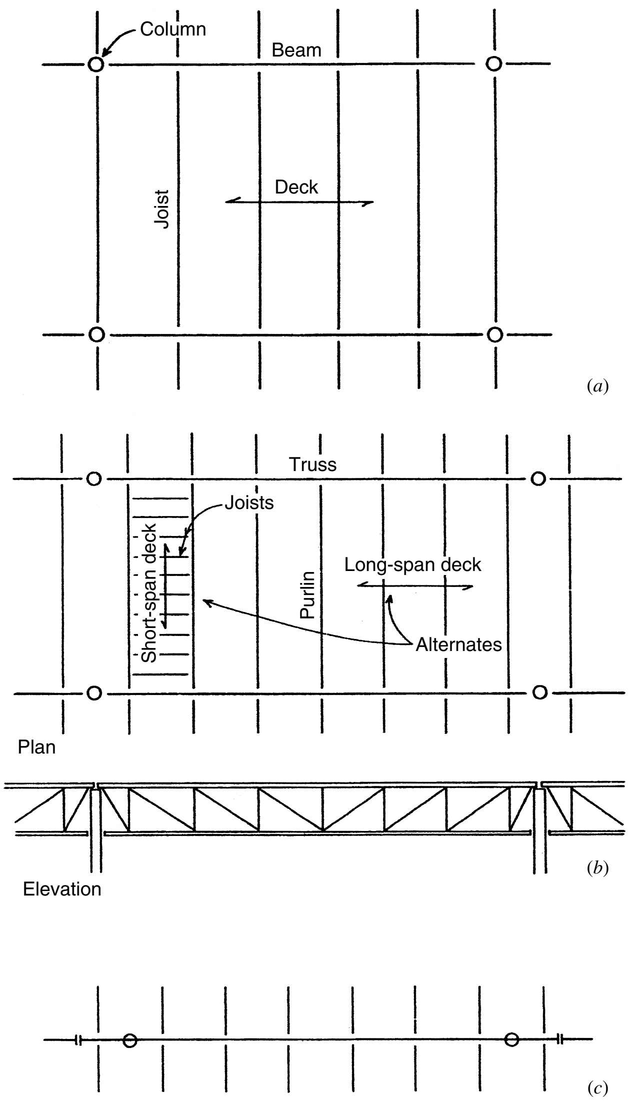


Figure 5.31 Considerations for framing layouts.

For a system such as that shown in Figure 5.31*a,* the basic planning starts with the location of the system supports, usually columns or bearing walls. The character of the spanning system is closely related to the magnitude of the spans it must achieve. Decks are mostly quite short in span, requiring relatively close spacing of the elements that provide their direct support. Joists and beams may be small or large, depending mostly on their spans. The larger they are, the less likely they will be closely spaced. Thus very long span systems may have several levels of components before ending with the elements that directly support the deck. If the spans of the beams and joists in Figure 5.31*a* are quite long, there will most likely be another set of framing elements between joists in order to keep the deck span low.

Figure 5.31*b* shows a plan and elevation of a system that uses trusses for the main spanning elements. If the trusses are very large and the purlin spans are quite long, the purlins may have to be quite widely spaced. A constraint on the purlin spacing is the desire to have them correspond to the panel points of the truss to avoid direct loading on the truss top chords. In the latter case, it may be advisable to use an additional set of framing elements, such as the joists shown in the plan. This decision would be coordinated with that of the choice for the deck. In any event, the truss span and panel module, the purlin span and spacing, the joist span and spacing, and the deck span are interrelated and the selection of the components of the system is a highly interactive exercise.

Development of the layouts of these systems also involves decisions regarding the positioning of the elements in the system. Figure 5.31*a* shows the beams spanning the long dimension of the column bay, which in this case will produce major sizes for the beams.

For all framing systems some consideration must be given to the various intersections and connections of the components. For the framing plan shown in Figure 5.31*a*, there is a five-member intersection at the column, involving the column, the two beams, and the two joists—plus possibly an upper column if the building is multistory. Depending on the materials and forms of the members, the forms of connections, and the types of force transfers at the joint, this may be a routine matter of construction or a real mess. Some relief of the traffic congestion may be achieved by the plan layout shown in Figure 5.31*b*, in which the module of the joist spacing is offset at the columns, leaving only the column and beam connections. A further reduction possible is that shown in Figure 5.31*c*, where the beam is made continuous through the column, with a beam splice occurring off the column. In the latter case, the connections are all limited to two-member relationships: column to beam, beam to beam, and beam to joist.

Bridging, blocking, and cross-bracing for trusses must also be planned with care. These elements may interfere

with ducting or other building elements as well as create connection problems similar to those just discussed.

In the end, structural planning must be carefully coordinated with the general planning of the building and its various service subsystems. Optimization of the structure may need to yield to other building design concerns.

Cantilevered Edges

A situation that occurs frequently is that of providing for the extension of the horizontal structure beyond the plane of the building's exterior walls. This most often occurs with an overhanging roof but can also be required to provide for balconies or exterior walkways for a floor. A simple solution for the cantilever is shown in Figure 5.32*a*, consisting of extension of the joists that are perpendicular to the wall—the joists bearing on top of an edge beam or a bearing wall between columns.

Another means for achieving a cantilever is shown in Figure 5.32*b*, in which beams are extended over columns to support a beam at the roof edge. The edge beam then provides support for the extended joists, relieving the wall below from any bearing requirement.

A special problem with the cantilevered edge is that occurring at a building corner when both sides of the building have the cantilevered edge, as shown in Figure 5.33*a*. With the framing system shown in Figure 5.20*b*, one supporting edge beam is cantilevered from the corner column to support the edge on the other side. The joists (3) on one side are then as shown in Figure 5.32*a*, while the joists on the other side (1) are supported by the extended beam. The corner itself is supported by extension of the joist (1) and the edge member (2).

For the system shown in Figure 5.33*c*, the cantilevered edge on one side is formed as shown in Figure 5.32*b*, with beams extended over the columns on one side and supporting an edge member which in turn supports the cantilevered joist. The joist at the corner on the other side is then supported by the extended edge member (1) and the extended beam (2).

Another possibility for the corner is the use of a diagonal member, as shown in Figure 5.33*d*. This layout is frequently used in wood structures and is a common solution used for sloping roofs, where the diagonal member defines a ridge when the roof slopes both ways at the corner.

Miscellaneous Framing Concerns

General planning concerns for structures are discussed in Chapter 10. The following are some special issues that relate to the design of framing systems.

Ceilings

Where ceilings exist, they are generally provided for in one of three ways: by direct attachment to the overhead structure, by some independent structure, or by suspension from the overhead structure. Suspended ceilings are quite common, as the space created between the ceiling and the overhead structure is used for ducting, registers, and other equipment of the HVAC system, for wiring and fixtures of the lighting system, for piping for fire sprinklers, and so on. If framing members are closely spaced (4 ft or less), the structure for the ceiling may be suspended from the framing members. Another means of suspension is from the deck, which frees the ceiling layout from the framing layout.

Roof Drainage

Providing for drainage of roof surfaces is always an issue for roof framing. The most direct means is by simply tilting of the framing, although complex drainage patterns are difficult to achieve in this manner. Another possibility is to keep framing flat but vary the thickness of the deck, possible only with poured decks and only capable of achieving a few inches of slope. If rigid insulation units are used, they may be obtainable in tapered form to achieve specific drainage slopes.

For some types of structural members—such as manufactured trusses—it is possible to slope the top of the member while keeping the bottom flat. This makes it possible to have a sloped roof surface and a directly attached ceiling.

Dynamic Behavior

Roof structures may be optimized for light weight to provide reduction of dead load for the structure and its supports. Lightweight floors, on the other hand, tend to be bouncy.

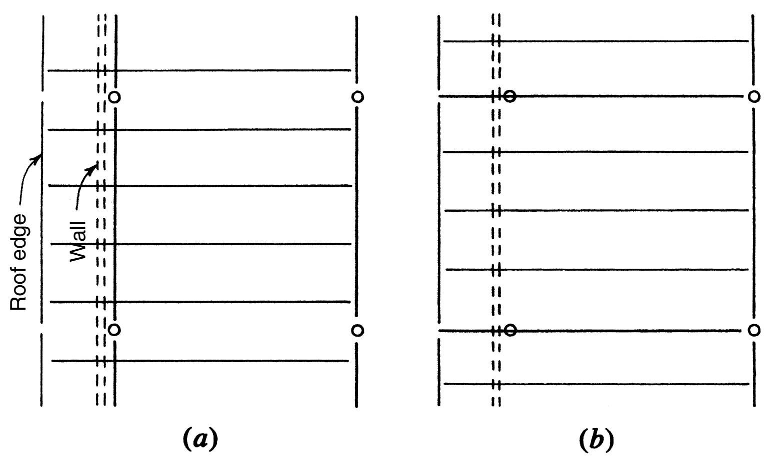


Figure 5.32 Framing of cantilevered edges.

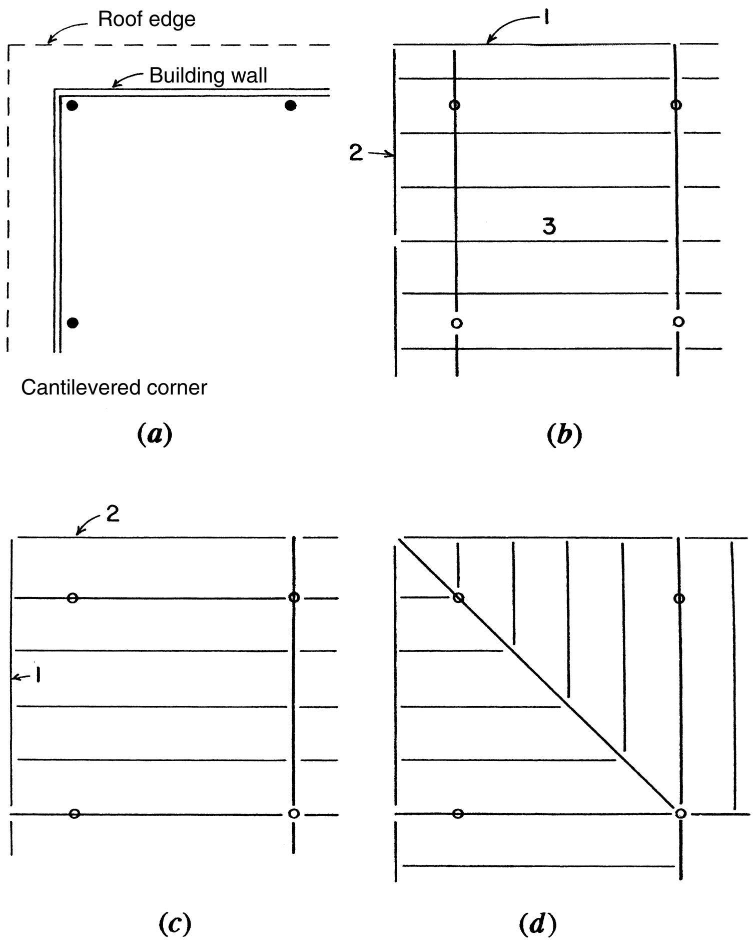


Figure 5.33 Framing of cantilevered corners.

Bounciness can also happen when span–depth ratios are high. Experience is the primary guide, but some general rules are the following:

Restrict live-load deflections to $L/360$ or less.
Limit span–depth ratios to 20 for rolled shapes, 15 for trusses.
Use a very stiff deck for its load-distributing effect.
Do not use decks for the longest spans listed.

A major factor in reducing bounciness is the concrete fill on top of steel decks. This fill is now often used on wood decks as well for reduction of bounciness and for acoustic separation.

Holes

Both roof and floor decks are commonly pierced by a number of passages for various items: stairs, elevators, ducts, and so on. The structure must be planned to accommodate these openings, which entails some of the following considerations:

Location of Openings. Openings may often be required at locations not convenient for framing layouts. For structures that utilize column line bents for lateral bracing, the integrity of the bents generally requires that openings be kept off of the column lines. For regularly spaced framing systems in general, the layout of the framing and the location of openings should be coordinated to maintain a maximum regularity of the system.

Size of Openings. Large openings usually have some framing around their perimeters, which are often the location of heavy walls. Very small openings (for single pipes, for example) may simply pierce the deck with no special structural provision. For sizes of openings between these extremes, accommodation depends on the form and size of framing elements of the structure. When one of a series of closely spaced joists is interrupted, it is usually necessary to double the joists on each side of the opening.

Openings near Columns. For efficiency in architectural planning, it is sometimes convenient to locate duct shafts or chases for piping or wiring next to a structural column. If this can be done without interrupting a major spanning member, it may not present a problem. If the opening must be on the column line, spanning members may be offset from the column line (see Figure 5.31*b*) or doubled members may be used to straddle the column.

Loss of Diaphragm Effectiveness. Presence of large openings must be considered with regard to effects on the compromising of the roof or floor deck as a horizontal diaphragm for lateral bracing. This issue is discussed in Chapter 9 and in some of the design case examples in Chapter 10.

Remodeling of buildings for usage changes often requires some new holes. Different forms of construction facilitate such changes differently; steel framing is quite tolerant.

5.3 STEEL COLUMNS

Steel columns in buildings vary from small, single-piece members (pipe, tube, W shape, etc.) to large assemblages of many elements for large towers. The basic column function is the development of axial compression force, but many columns must also sustain some amount of bending, shear, or torsion. This section presents a general discussion of column design issues and procedures, with an emphasis on the simple, single-piece column. For a general discussion of column behavior, see Section 3.3.

Column Shapes and Usage

The most common building columns are the round pipe, the rectangular tube, and the squarelike W shapes. Pipes and tubes are useful for one-story buildings in which attachment to supported framing is usually achieved by setting the spanning members on top of the columns. Framing that requires spanning members to be attached to the side of a column is more easily achieved with W-shape members. The most commonly used columns for multistory buildings are the approximately square W shapes of nominal 10-, 12-, and 14-in. depths. These are available in a wide range of flange and web thicknesses, up to some of the heaviest rolled shapes.

Pipes and tubes are ideally suited by the geometry of their cross sections for resistance to axial compression. The W shape has a strong axis (the X–X axis) and a weak axis (the Y–Y axis), although the close-to-square shapes have the least difference in stiffness and radius of gyration on the two principal axes. The W shape is well suited to the development of bending in combination with the axial force if it is bent about its strong axis.

It is sometimes necessary to make a column section by assembling two or more individual steel elements. Figure 5.34 shows some commonly used assemblages. These are used where a particular shape or size is not available from the inventory of rolled shapes. These built-up shapes are somewhat less used now, as the range of size and shape of stock sections steadily increases. The customized fabrication of built-up sections is usually costly, so a single piece is typically preferred if one is available.

A widely used section is the double angle, shown in Figure 5.34*f*. This is often used for truss members and for bracing elements but seldom as a column.

Stability Concerns

Columns are primarily designed for a uniformly distributed compression stress. The usable value of this stress is determined from formulas in the AISC specification with variables that include yield strength, modulus of elasticity, relative stiffness, and conditions of restraint of the column.

As a measure of resistance to buckling, a basic property of a column is its slenderness, computed as L/r, in which L is the unbraced length and r the radius of gyration of the

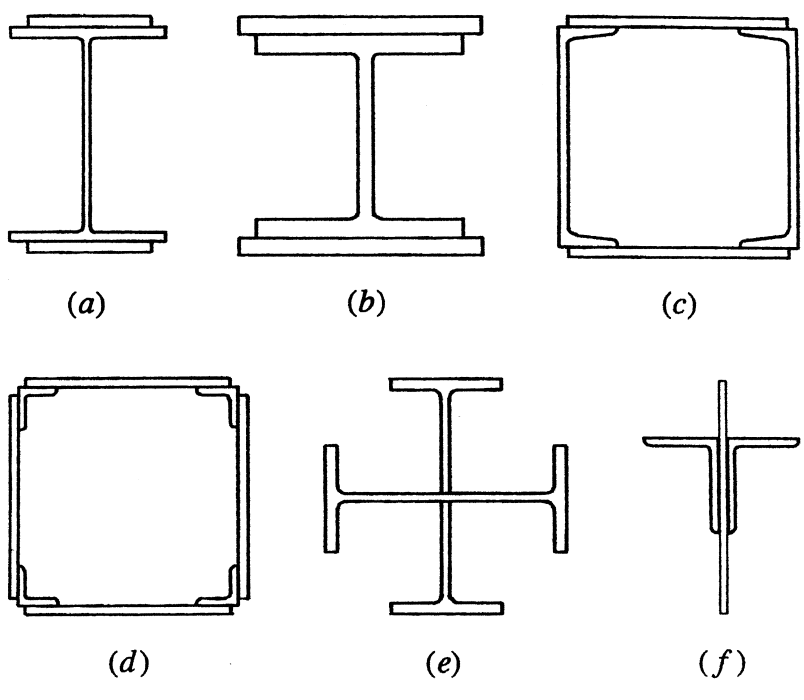


Figure 5.34 Built-up column sections.

column cross section. Effect of various conditions, as shown in Figure 5.35, is considered by use of a modifying factor (K) multiplied by the column length to produce a design value called the *effective column length*.

Figure 5.36 is a graph that yields the allowable axial compression stress for columns with a yield stress of 36 ksi [250 MPa] or 50 ksi [350 MPa], as determined from the AISC formulas. Values for full number increments of KL/r are also given in Table 5.8.

It is generally recommended that building columns not have slenderness greater than 120.

Investigation for Axial Compression

The design strength in axial compression for a column is computed by multiplying the design stress ($\phi_c F_c$) by the cross-sectional area of the column; thus

$$P_u = \phi_c P_n = \phi_c F_c A$$

where

P_u = maximum factored load
ϕ_c = 0.85 for columns
P_n = nominal capacity of column
F_c = critical compressive stress, based on KL/r
A = area of column cross section

The following examples demonstrate the process.

For single-piece columns, a more direct process consists of using column load tables. For built-up sections, however, it is necessary to compute the properties of the section to determine the radius of gyration (r) for KL/r.

Example 12. A W 12 × 53 of A36 steel is used as a column with an unbraced length of 16 ft [4.88 m]. Compute the maximum factored load (P_u).

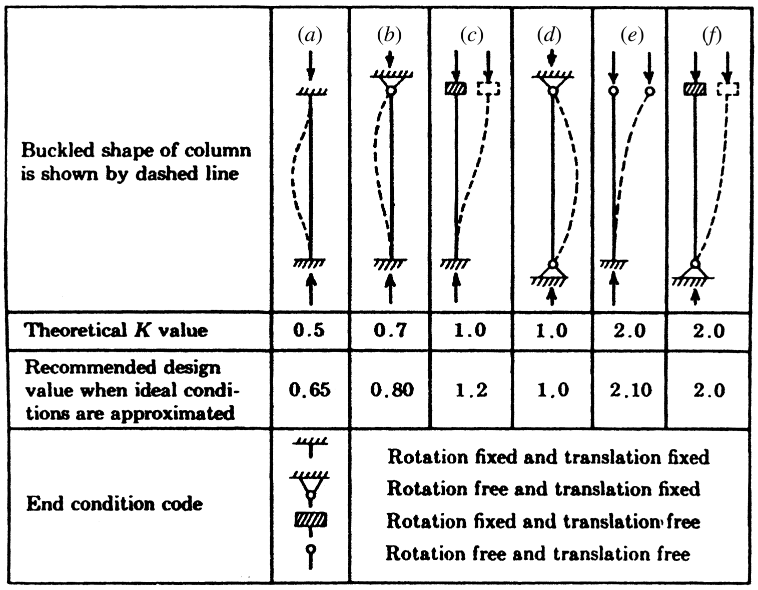


Figure 5.35 Determination of effective column length KL. Reproduced from the AISC manual (Ref. 10), with permission of the publisher, American Institute of Steel Construction.

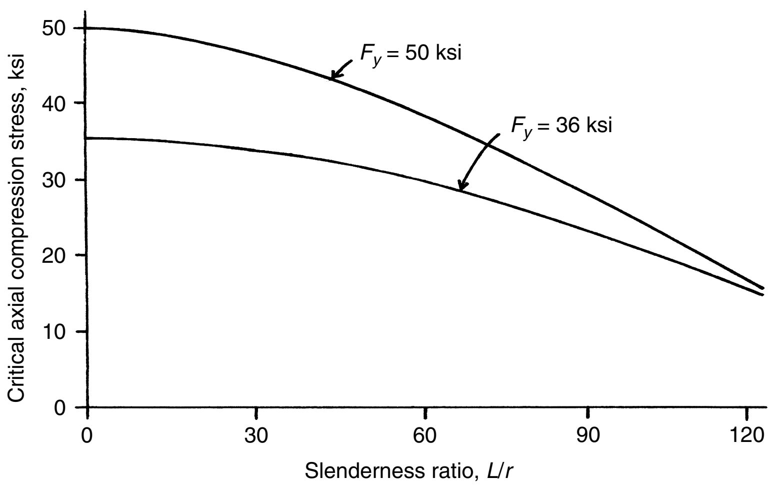


Figure 5.36 Critical unfactored compressive stress for columns of A36 steel with yield of 36 ksi and high-strength steel with yield of 50 ksi.

Table 5.8 Critical Unfactored Compressive Stress for Columns, F_c[a]

	Critical Stress, F_c			Critical Stress, F_c			Critical Stress, F_c	
KL/r	$F_y = 36$ ksi	$F_y = 50$ ksi	*KL/r*	$F_y = 36$ ksi	$F_y = 50$ ksi	*KL/r*	$F_y = 36$ ksi	$F_y = 50$ ksi
1	36.0	50.0	41	33.0	44.2	81	25.5	30.9
2	36.0	50.0	42	32.8	43.9	82	25.3	30.6
3	36.0	50.0	43	32.7	43.7	83	25.0	30.2
4	36.0	49.9	44	32.5	43.4	84	24.8	29.8
5	36.0	49.9	45	32.4	43.1	85	24.6	29.5
6	35.9	49.9	46	32.2	42.8	86	24.4	29.1
7	35.9	49.8	47	32.0	42.5	87	24.2	28.7
8	35.9	49.8	48	31.9	42.2	88	23.9	28.4
9	35.8	49.7	49	31.7	41.9	89	23.7	28.0
10	35.8	49.6	50	31.6	41.6	90	23.5	27.7
11	35.8	49.6	51	31.4	41.3	91	23.3	27.3
12	35.7	49.5	52	31.2	41.0	92	23.1	26.9
13	35.7	49.4	53	31.1	40.7	93	22.8	26.6
14	35.6	49.3	54	30.9	40.4	94	22.6	26.2
15	35.6	49.2	55	30.7	40.1	95	22.4	25.8
16	35.5	49.1	56	30.5	39.8	96	22.2	25.5
17	35.5	49.0	57	30.3	39.4	97	21.9	25.1
18	35.4	48.8	58	30.2	39.1	98	21.7	24.8
19	35.3	48.7	59	30.0	38.8	99	21.5	24.4
20	35.2	48.6	60	29.8	38.4	100	21.3	24.1
21	35.2	48.4	61	29.6	38.1	101	21.0	23.7
22	35.1	48.3	62	29.4	37.7	102	20.8	23.4
23	35.0	48.1	63	29.2	37.4	103	20.6	23.0
24	34.9	47.9	64	29.0	37.1	104	20.4	22.7
25	34.8	47.8	65	28.8	36.7	105	20.1	22.3
26	34.7	47.6	66	28.6	36.4	106	19.9	22.0
27	34.6	47.4	67	28.4	36.0	107	19.7	21.6
28	34.5	47.2	68	28.2	35.7	108	19.5	21.3
29	34.4	47.0	69	28.0	35.3	109	19.3	21.0
30	34.3	46.8	70	27.8	34.9	110	19.0	20.6
31	34.2	46.6	71	27.6	34.6	111	18.8	20.3
32	34.1	46.4	72	27.4	34.2	112	18.6	20.0
33	34.0	46.2	73	27.2	33.9	113	18.4	19.7
34	33.9	45.9	74	27.0	33.5	114	18.2	19.3
35	33.8	45.7	75	26.8	33.1	115	17.9	19.0
36	33.6	45.5	76	26.6	32.8	116	17.7	18.7
37	33.5	45.2	77	26.3	32.4	117	17.5	18.3
38	33.4	45.0	78	26.1	32.0	118	17.3	18.0
39	33.2	44.7	79	25.9	31.7	119	17.1	17.7
40	33.1	44.5	80	25.7	31.3	120	16.9	17.4

Source: Developed from data in the *Manual of Steel Construction, Load and Resistance Factor Design*, 3rd ed. (Ref. 10), with permission of the publisher, American Institute of Steel Construction.
[a]Usable design stress limit in ksi for obtaining the nominal strength (unfactored) of steel columns.

Solution. Referring to Table A.3, $A = 15.6$ in.^{2}, $r_x = 5.23$ in., and $r_y = 2.48$ in. If the column is unbraced on both axes, it is limited by the lower r value for buckling. With no stated end conditions, case (*d*) in Figure 5.35 is assumed, for which $K = 1.0$; that is, no modification is made. Thus, the relative slenderness is computed as

$$\frac{KL}{r} = \frac{1.0 \times 16 \times 12}{2.48} = 77.4$$

In design work, it is usually considered acceptable to round the slenderness ratio off to the nearest whole number. Thus, with a *KL/r* value of 77, Table 5.8 yields a value for F_c of 26.3 ksi. The maximum factored load capacity of the column is then

$$P_u = \phi_c F_c A = 0.85 \times 26.3 \times 15.6$$
$$= 349 \text{ kips } [1.55 \times 10^3 \text{ kN}]$$

Example 13. Compute the maximum factored load for the column in Example 12 if the top is pinned but prevented from lateral movement but the bottom is totally fixed.

Solution. Referring to Figure 5.35, note that the case for this is (*b*) and the modifying factor K is 0.8. Then

$$\frac{KL}{r} = \frac{0.8 \times 16 \times 12}{2.48} = 62$$

From Table 5.8, F_c is 29.4 ksi and

$$P_u = \phi_c F_c A = 0.85 \times 29.4 \times 15.6$$
$$= 390 \text{ kips} \ \ [1.73 \times 10^3 \text{ kN}]$$

The following example illustrates the situation where a W shape is braced differently on its two axes.

Example 14. Figure 5.37 shows an elevation of the steel framing at the location of an exterior wall. The column is laterally restrained but rotation free at both its top and bottom. With respect to the X axis of the section, the column is laterally unbraced for its full height. However, horizontal framing in the wall plane provides lateral bracing at a point between the top and bottom with respect to the Y axis of the section. Referring to the figure, $L_1 = 30$ ft and $L_2 = 18$ ft. If the column is a W 12 × 53 of A36 steel, what is the allowable compression load?

Solution. The procedure here is to find the KL/r values for both axes and to use the highest value obtained. This is the same column as in Example 12, for which properties were previously obtained. Considering the x axis first,

$$\frac{KL_1}{r_x} = \frac{1 \times 30 \times 12}{5.23} = 68.8, \quad \text{say } 69$$

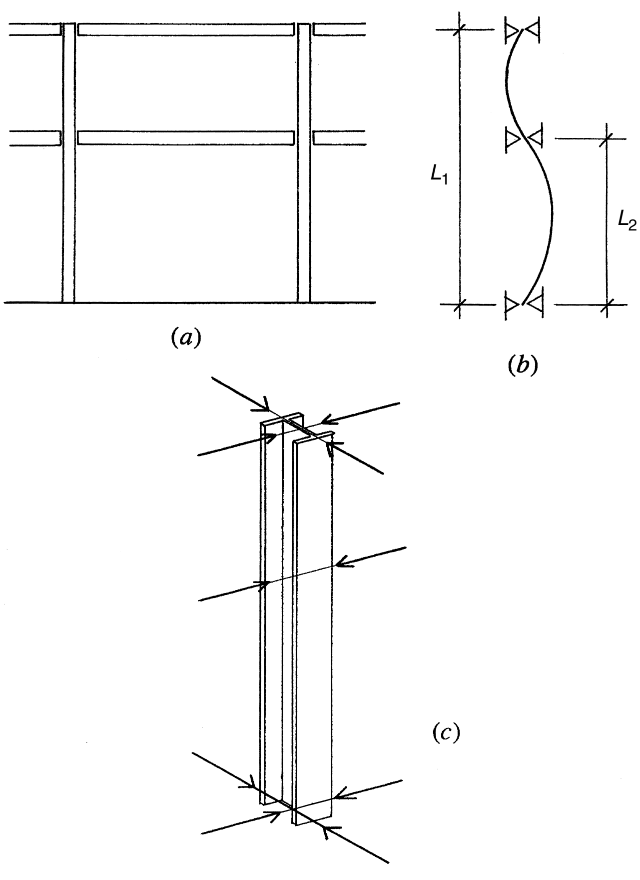


Figure 5.37 Biaxial bracing for a steel column.

For the y axis, the situation is also assumed to be case (d) from Figure 5.35, except that the deformation occurs in two parts (see Figure 5.37b). Thus,

$$\frac{KL_2}{r_y} = \frac{1 \times 18 \times 12}{2.48} = 87.1, \quad \text{say } 87$$

Despite the bracing in the wall plane, the column is still critical on its weak axis. From Table 5.8 the value for F_c is 24.2 ksi and the load limit is thus

$$P_u = \phi_c F_y A = 0.85 \times 24.2 \times 15.6$$
$$= 321 \text{ kips} \ \ [1.43 \times 10^6 \text{ kN}]$$

As with all column design work, it is very difficult to adapt the investigative formulas to ones that easily yield requirements for design selection. There are simply too many variables and too many interrelationships between the variables. The following discussion treats a simplified approach to design of steel columns.

Design of Steel Columns

Unless a computer-supported procedure is used, design of steel columns is mostly accomplished through the use of tabulated data. The following discussions consider a process using materials from the AISC manual (Ref. 10).

The principal value of the safe-load tables for single rolled shapes is the ability to use them directly once only the factored load, unbraced height, and K factor are known. In many cases, the simple, axial compression capacity of the column is all that is involved in its design selection. However, columns are also sometimes subjected to bending and shear and in some cases to torsion. In combined actions, however, the axial load capacity is usually included, so its singular determination is still a design factor.

W-Shape Columns

The single rolled shape most commonly used as a column is the squarish W shape with a nominal depth of 8 in. or more and relatively wide flanges.

The AISC manual (Ref. 10) provides extensive tables with factored loads for W and M shapes. The tables also yield considerable other data for the listed shapes. Table 5.9 summarizes data for shapes ranging from the W 8 × 24 to the W 14 × 211 for steel with yield strength of 36 ksi. Table values are based on the r value for the y axis, with $K = 1.0$. Also included in this table are values for the bending factor (m), which is used for an approximate design for combined compression and bending, as discussed later in this section.

To illustrate one use of Table 5.9, refer to Example 12, in which a safe load was determined for the W 12 × 53 with an unbraced height of 16 ft. For this situation, Table 5.9 yields a value of 348 kips, which agrees quite closely with the computed value found in the example.

Table 5.9 Safe Factored Loads for Selected A36 W Shapes[a]

	Ratio	Effective Length (KL) in feet																		
$F_y = 36$ ksi	r_x/r_y	0	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
W 14 × 211	1.61	1897	1866	1842	1812	1776	1734	1687	1636	1580	1520	1458	1392	1325	1257	1187	1118	1048	980	912
W 14 × 176	1.60	1585	1559	1538	1512	1482	1446	1406	1362	1314	1263	1210	1154	1097	1039	980	922	863	805	748
W 14 × 145	1.59	1307	1284	1267	1246	1220	1190	1156	1119	1079	1036	992	945	898	849	800	751	703	655	608
W 14 × 120	1.67	1080	1059	1043	1023	999	971	940	906	870	831	791	749	706	663	620	577	535	494	454
W 14 × 82	2.44	734	703	679	649	615	577	536	493	449	404	361	319	279	243	214	189	169	151	137
W 14 × 68	2.44	612	585	565	540	511	479	444	408	371	334	297	262	229	199	175	155	138	124	112
W 14 × 53	3.07	477	443	418	389	355	319	282	245	210	176	148	126	109	95	83				
W 12 × 336	1.85	3023	2956	2904	2839	2761	2672	2573	2465	2350	2229	2104	1975	1845	1716	1587	1460	1337	1218	1102
W 12 × 279	1.82	2506	2447	2402	2345	2278	2201	2115	2021	1922	1818	1710	1600	1490	1379	1270	1164	1061	960	866
W 12 × 230	1.75	2072	2021	1982	1933	1875	1809	1735	1656	1571	1482	1391	1298	1204	1111	1020	931	845	761	687
W 12 × 190	1.79	1707	1664	1631	1589	1540	1483	1421	1353	1281	1206	1129	1051	973	895	819	745	674	605	546
W 12 × 152	1.77	1368	1332	1304	1270	1229	1182	1130	1074	1015	954	891	827	763	700	638	578	520	467	421
W 12 × 120	1.76	1080	1051	1028	1000	966	928	886	841	793	743	692	640	589	538	489	442	395	355	320
W 12 × 96	1.76	863	839	820	797	770	739	704	667	628	588	546	505	463	422	383	345	308	276	249
W 12 × 79	1.75	710	689	674	654	631	605	576	545	512	479	444	409	375	341	308	277	247	221	200
W 12 × 53	2.11	477	457	441	422	400	375	348	320	292	263	235	207	181	158	139	123	110	98	89
W 12 × 45	2.64	401	373	353	328	301	271	241	210	181	152	128	109	94	82	72				
W 10 × 112	1.74	1007	969	941	906	865	819	768	715	660	604	548	493	440	389	342	303	270	242	219
W 10 × 88	1.73	793	762	739	710	677	639	599	556	511	466	422	378	336	295	259	230	205	184	166
W 10 × 68	1.71	612	588	569	547	520	490	458	424	389	354	319	285	252	221	194	172	153	138	124
W 10 × 54	1.71	483	464	449	431	409	385	360	332	304	276	248	221	195	170	150	133	118	106	96
W 10 × 45	2.15	407	380	361	337	311	282	252	222	192	164	138	118	102	88	78				
W 10 × 33	2.16	297	276	261	243	222	200	177	155	133	112	94	80	69	60	53				
W 8 × 58	1.74	523	492	469	441	409	374	337	300	263	228	194	165	143	124	109	97			
W 8 × 40	1.73	358	335	319	298	275	251	225	198	173	148	125	107	92	80	70	62			
W 8 × 31	1.72	279	261	248	232	214	194	173	153	133	114	96	82	70	61	54				
W 8 × 24	2.12	217	195	180	162	142	122	102	84	68	56	47	40							
Bending Factor (m)			2.2	2.1	2.0	1.9	1.8	1.7	1.6	1.5	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3

Source: Developed from data in the *Manual of Steel Construction, Load and Resistance Factor Design*, 3rd ed. (Ref. 10), with permission of the publisher, American Institute of Steel Construction.
[a]Factored nominal strength of columns in kips.

The real value of Table 5.9, however, is for a quick design selection. The basic equation for design of a steel column in LRFD is

$$\phi_c P_n \geq P_u$$

where

$\phi_c = 0.85$
$P_n =$ nominal column strength
$P_u =$ maximum factored load

Example 15. Using Table 5.9, select a W-shape A36 steel column shape for an axial load of 100 kips [445 kN] dead load and 150 kips [667 kN] live load if the unbraced height is 24 ft [7.32 m] and the end conditions are pinned at top and bottom.

Solution. First, the maximum factored load is determined:

$$P_u = 1.2(100) + 1.6(150) = 360 \text{ kips } [1600 \text{ kN}]$$

From Table 5.9, some possible choices are as follows:

Section	*Design Load* ($\phi_c P_n$)
W 10 × 88	422 kips [1880 kN]
W 12 × 79	444 kips [1970 kN]
W 14 × 82	361 kips [1610 kN]

Steel Pipe Columns

Round steel pipe columns most frequently occur as single-story columns, supporting wood or steel beams that sit in top of the columns. Pipe is available in three weight categories: *standard* (Std), *extra strong* (XS), and *double extra strong* (XXS). Pipe is designated by its nominal diameter, which is slightly less than the outside diameter. Table 5.10 gives safe loads for pipe columns of steel with a yield stress of 35 ksi.

Example 16. Using Table 5.10, select a standard weight steel pipe column to carry a dead load of 15 kips [67 kN] and a live load of 26 kips [116 kN] if the unbraced height is 12 ft [3.66 m].

Solution. The maximum factored load for the column is determined as

$$P_u = 1.2(15) + 1.6(26) = 59.6 \text{ kips } [265 \text{ kN}]$$

For the height of 12 ft, the table yields a value of 95 kips as the design load for a 5-in. pipe. A 4-in. pipe is just short of the required load.

Most selected pipe columns are of the standard weight classification as these are most available and most efficient. The heavier weight columns are mostly used only when a column carries a heavy load but placement of the column in the building construction is a dimensional problem.

Table 5.10 Safe Factored Loads for Steel Pipe Columns[a]

				Effective Length (KL) in feet																		
$F_y = 35$ ksi			Area (in.^{2})	0	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
Pipe	12	XS	19.2	571	563	557	549	540	529	517	503	488	472	455	438	420	401	382	363	343	324	305
Pipe	12	Std.	14.6	434	428	424	418	411	403	394	384	372	361	348	335	321	307	293	279	264	249	235
Pipe	10	XS	16.1	479	469	462	453	442	429	415	400	383	365	347	328	309	290	270	251	232	214	196
Pipe	10	Std.	11.9	354	347	342	335	327	318	308	297	284	272	258	245	231	216	202	188	174	161	148
Pipe	8	XXS	21.3	634	612	596	575	551	524	495	463	430	397	363	329	297	265	235	208	186	166	150
Pipe	8	XS	12.8	381	369	360	348	335	320	303	286	267	248	228	209	190	171	153	136	121	109	98
Pipe	8	Std.	8.4	250	242	237	229	221	211	201	190	178	165	153	140	128	116	104	93	83	75	67
Pipe	6	XXS	15.6	464	436	415	390	361	330	298	264	232	200	170	145	125	109	96	85			
Pipe	6	XS	8.4	250	236	226	214	200	185	169	152	135	119	103	88	76	66	58	52	46		
Pipe	6	Std.	5.6	166	158	151	144	135	125	114	104	93	82	72	62	53	47	41	36	32		
Pipe	5	XXS	11.3	336	307	287	262	235	206	178	150	124	102	86	73	63						
Pipe	5	XS	6.1	182	168	158	146	133	119	104	90	76	63	53	45	39	34					
Pipe	5	Std.	4.3	128	119	112	104	95	85	75	65	56	47	39	33	29	25					
Pipe	4	XXS	8.1	241	209	187	163	137	112	88	70	56	47									
Pipe	4	XS	4.4	131	116	106	94	81	68	55	44	36	30	25								
Pipe	4	Std.	3.2	94	84	77	68	59	50	41	33	27	22	19								
Pipe	3.5	XS	3.7	109	94	83	71	59	47	37	29	23										
Pipe	3.5	Std.	2.7	80	69	61	53	44	36	28	22	18	15									
Pipe	3	XXS	5.5	163	128	106	83	62	46	35												
Pipe	3	XS	3.0	90	73	62	51	40	30	23	18											
Pipe	3	Std.	2.2	66	54	47	38	30	23	17	14											

Source: Developed from data in the *Manual of Steel Construction, Load and Resistance Factor Design*, 3rd ed. (Ref. 10), with permission of the publisher, American Institute of Steel Construction.
[a]Factored nominal strength of columns in kips.

Structural Tubing Columns

Rectangular-shaped steel tubing is used for building columns and for members of steel trusses and trussed rigid frame bents. The rectangular shapes are somewhat more popular for fitting into the building general construction.

Tubes are available in a considerable range of sizes with a variation of wall thickness for each size. Both square and oblong shapes are available, although the square tubes are mostly used for freestanding columns. For building columns, sizes used range upward from the 3-in. square tube to as large as 48 in. square, although most columns are in the range from 4 to 12 in. Tubing may be specified in various grades of steel; that used mostly is steel with a yield strength of 46 ksi. Tubes are not rolled but rather are formed from bent steel plates with a welded seam on one side. Because of the limits of the bending process, outside corners are slightly rounded.

Table 5.11 yields design strengths for square tubes from 3 to 12 in. in outside dimension. Unlike pipe, tubes are the exact dimension of their designated size. Use of the table is similar to that for the steel pipes.

Double-Angle Compression Members

Matched pairs of angles are frequently used for truss members or for braces in frames. The common form consists of the two angles placed back to back but separated a short distance to achieve joints by use of gusset plates or by sandwiching the angles around the web of a structural tee. Compression members that are not columns are frequently called *struts*.

The AISC manual (Ref. 10) contains safe-load tables for double angles with an assumed average separation distance of $\frac{3}{8}$ in. [9.5 mm]. For angles with unequal legs, two back-to-back arrangements are possible, described either as *long legs back to back* or as *short legs back to back*. Table 5.12 has been adapted from data in the AISC manual for selected pairs of double angles with long legs back to back. Note that separate data are provided for the variable situation of either axis being used for the determination of the effective unbraced length and slenderness (*KL/r*). If conditions relating to the unbraced length are the same for both axes, then the lower value for the safe load from Table 5.12 should be used. Properties for the cross sections for selected pairs of angles are given in Table A.6.

Like other members that lack biaxial symmetry, such as the structural tee, there may be some reduction applicable due to the slenderness of the thin elements of the cross section. This reduction is incorporated in the values provided in the safe-load tables in the AISC manual.

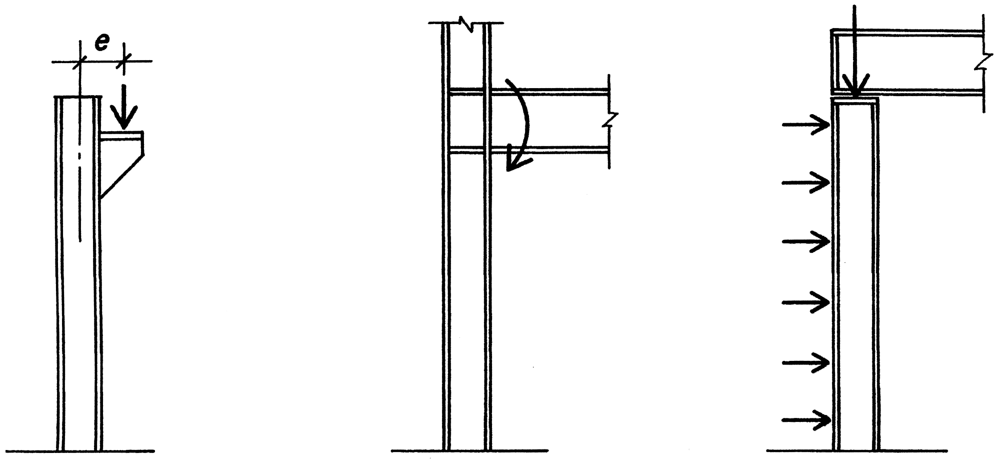
Columns with Bending

Columns must frequently sustain bending in addition to the usual axial compression. Figures 5.38*a* through *c* show three of the most common situations that result in this combined effect. When loads are supported on a bracket at the column face, the eccentricity of the applied load adds a bending effect (Figure 5.38*a*). When moment-resistive connections are used to produce a rigid frame, any loads on the beams will induce a twisting effect (bending) on the column (Figure 5.38*b*). Columns built into exterior walls may become involved in the spanning effect of the wall in resisting wind forces (Figure 5.38*c*).

Table 5.11 Safe Factored Loads for Square Steel Tube Columns[a]

		Effective Length (KL) in feet																		
$F_y = 46$ ksi	Area (in.^{2})	0	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
HSS 12 × 12 × 5/8	25.7	1005	989	976	960	941	919	895	867	838	807	774	739	704	668	631	595	558	522	486
HSS 12 × 12 × 3/8	16	626	616	609	599	588	575	560	544	526	507	488	467	446	424	402	379	357	335	313
HSS 12 × 12 × 1/4	10.8	422	416	411	405	397	389	379	368	357	344	331	317	303	289	274	259	244	230	215
HSS 10 × 10 × 1/2	17.2	673	657	645	630	612	592	569	545	519	491	462	433	404	375	346	317	290	263	237
HSS 10 × 10 × 5/16	11.1	434	424	417	408	397	384	370	355	338	321	303	285	266	248	229	211	193	176	160
HSS 10 × 10 × 3/16	6.76	264	259	254	249	242	235	226	217	207	197	186	176	164	153	142	131	121	110	100
HSS 8 × 8 × 1/2	13.5	528	508	494	475	454	430	404	376	347	318	289	260	232	205	181	160	143	128	116
HSS 8 × 8 × 5/16	8.76	343	331	322	310	297	282	266	249	231	212	194	176	158	141	124	110	98	88	79
HSS 8 × 8 × 3/16	5.37	210	203	197	191	183	174	164	154	143	132	121	110	99	89	79	70	62	56	50
HSS 7 × 7 × 5/8	14	547	519	499	473	444	412	377	342	306	271	237	204	176	153	135	119	107	96	86
HSS 7 × 7 × 3/8	8.97	351	334	322	307	289	270	249	227	205	183	162	142	123	107	94	83	74	67	60
HSS 7 × 7 × 1/4	6.17	241	230	222	212	201	188	174	159	145	130	115	101	88	77	68	60	53	48	43
HSS 6 × 6 × 1/2	9.74	381	355	336	313	288	260	231	203	175	148	125	106	92	80	70	62	55		
HSS 6 × 6 × 5/16	6.43	251	236	224	210	194	176	158	140	122	104	88	75	65	56	50	44	39	35	
HSS 6 × 6 × 3/16	3.98	156	146	139	131	121	111	100	89	78	68	58	49	42	37	32	29	26	23	
HSS 5 × 5 × 3/8	6.18	242	219	202	183	162	140	119	98	80	66	56	47	41	36					
HSS 5 × 5 × 1/4	4.3	168	153	142	130	116	101	86	72	59	49	41	35	30	26	23				
HSS 5 × 5 × 1/8	2.23	87	80	75	68	61	54	47	39	33	27	23	19	17	15	13				
HSS 4 × 4 × 3/8	4.78	187	159	140	119	98	78	60	47	38	32	27								
HSS 4 × 4 × 1/4	3.37	132	113	101	87	72	58	45	36	29	24	20								
HSS 4 × 4 × 1/8	1.77	69	60	54	47	40	32	26	20	16	14	11	10							
HSS 3 × 3 × 5/16	3.52	138	116	101	85	69	54	41	32	26	22									
HSS 3 × 3 × 3/16	2.24	88	75	66	57	47	37	29	23	18	15	13								

Source: Developed from data in the *Manual of Steel Construction, Load and Resistance Factor Design*, 3rd ed. (Ref. 10), with permission of the publisher, American Institute of Steel Construction.
[a]Factored nominal strength of columns in kips.



(a) (b) (c)

Figure 5.38 Situations involving bending in steel columns.

Adding bending to a direct compression effect results in a combined stress, or net stress, condition with something other than an even distribution of stress across the column cross section. The two stresses may be investigated separately and the stresses added to determine this net effect. However, for columns, the two *actions*—compression and bending—are essentially different, so that a combination of the separate actions, not just the stresses, is more significant. This combination is accomplished with the so-called *interaction* analysis that takes the form

$$\frac{P_u}{\phi_c P_n} + \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \leq 1$$

This formula in its pure form describes a straight line, but various other considerations—for lack of straightness, for buckling, for lack of perfect alignment of loads, and so on—make the relationships, in reality, somewhat more complex.

Table 5.12 Safe Factored Loads for Double-Angle Compression Members of A36 Steel[a]

Size (in.)			4 × 3			3.5 × 2.5			3 × 2		2.5 × 2	
Thickness (in.)			3/58	5/16		5/16	1/4		5/16	1/4	5/16	1/4
Weight (lb/ft)			16.9	14.2		12.2	9.88		10.1	8.18	8.97	7.30
Area (in.^{2})			4.98	4.19		3.58	2.90		2.96	2.40	2.64	2.14
r_x			1.26	1.27		1.11	1.12		0.945	0.953	0.774	0.782
r_y			1.30	1.29		1.09	1.08		0.897	0.883	0.943	0.930
Effective Buckling Length in Feet (L/r) with Respect to Indicated Axis	$X–X$ Axis	0	152	128	0	110	86	0	90.6	73.4	80.8	65.5
		4	141	119	2	107	84	2	87.6	71.0	76.8	62.3
		6	128	108	4	99	78	3	83.9	68.1	72.1	58.6
		8	112	95	6	88	69	4	79.1	64.3	66.0	53.7
		10	94	80	8	74	59	5	73.3	59.6	58.9	48.0
		12	77	65	10	59	48	6	66.7	54.4	51.2	41.9
		14	60	51	12	45	37	8	52.6	43.0	35.9	29.6
		16	46	39	14	33	27	10	38.8	31.9	23.4	19.4
		18	36	31	16	25	21	12	27.2	22.4	16.3	13.5
		20	29	25	18	20	17	14	20.0	16.5	—	—
	$Y–Y$ Axis	0	152	128	0	110	86	0	90.6	73.4	80.8	65.5
		4	130	104	2	96	71	2	80.7	62.4	74.4	58.6
		6	118	95	4	89	66	3	76.8	59.5	71.0	55.9
		8	104	84	6	78	59	4	71.6	55.6	66.5	52.4
		10	88	72	8	65	50	5	65.4	50.9	61.1	48.2
		12	71	58	10	51	40	6	58.5	45.7	55.1	43.5
		14	55	46	12	38	30	8	43.9	34.4	42.2	33.4
		16	43	35	14	29	22	10	31.9	24.9	31.5	24.8
		18	34	28	16	23	17	12	22.3	17.5	22.0	17.4
		20	28	23	18	18	14	14	16.5	12.9	16.2	12.8

Size (in.)			8 × 6			6 × 4			5 × 3.5		5 × 3	
Thickness (in.)			3/4	1/2		1/2	3/8		1/2	3/8	3/8	5/16
Weight (lb/ft)			68.0	46.3		32.3	24.6		27.2	20.8	19.5	16.4
Area (in.^{2})			20.0	13.6		9.50	7.22		8.01	6.10	5.73	4.81
r_x			2.52	2.55		1.91	1.93		1.58	1.59	1.60	1.61
r_y			2.47	2.43		1.64	1.61		1.48	1.46	1.22	1.21
Effective Buckling Length in Feet (L/r) with Respect to Indicated Axis	$X–X$ Axis	0	612	379	0	291	201	0	245	183	172	134
		10	543	341	10	236	167	6	220	165	155	122
		12	515	325	12	216	154	8	202	152	143	113
		14	484	308	14	193	140	10	181	137	129	103
		16	451	289	16	171	125	12	158	120	113	91
		20	380	248	18	148	110	14	135	103	97	79
		24	308	206	20	127	96	16	113	86	82	68
		28	240	165	22	106	82	18	91	70	67	57
		32	184	128	26	76	59	22	61	47	45	38
		36	145	101	30	57	44	26	44	34	32	27
	$Y–Y$ Axis	0	612	379	0	291	201	0	245	183	172	134
		10	493	276	6	231	148	4	213	149	135	100
		12	467	264	10	194	128	6	199	140	123	92
		14	438	251	12	172	115	8	180	127	108	82
		16	406	236	14	148	101	10	158	113	90	70
		20	338	203	16	125	87	12	135	97	72	57
		24	269	167	18	102	73	14	111	81	55	45
		28	205	131	20	83	60	16	89	65	43	35
		32	158	102	22	69	50	18	71	52	34	28
		36	126	82	26	50	36	22	48	35		

Source: Developed from data in the *Manual of Steel Construction, Load and Resistance Factor Design*, 3rd ed. (Ref. 10), with permission of the publisher, American Institute of Steel Construction.

[a]Factored nominal axial compression strength for members in kips.

For steel columns, issues include slenderness of column flanges and webs (for W shapes), ductility of the steel, and overall column slenderness that affects potential buckling in both axial compression and bending. Understandably, therefore, the AISC formulas are considerably more complex than the simple interaction formula. The AISC provides for interaction of compression and bending as follows: If $P_u/\phi_c P_n \geq 0.2$,

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left[\frac{M_{ux}}{\phi_b M_{ux}} + \frac{M_{uy}}{\phi_b M_{uy}}\right] \leq 1$$

If $P_u/\phi_c P_n < 0.2$,

$$\frac{P_u}{2\phi_c P_n} + \left[\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{uy}}\right] \leq 1$$

For preliminary design, an approximate equivalent axial load can be used using a bending factor (m), which is listed here for the W shapes in Table 5.9. (See Ref. 15.) Using this factor, a modified load (P'_u) for use in the safe-load table is determined as follows:

$$P'_u = P_u + mM_{ux} + 2mM_{uy}$$

If $P_u/P'_u < 0.2$, recalculate P'_u using the equation

$$P'_u = \frac{P_u}{2} + \frac{9}{8}(mM_{ux} + 2mM_{uy})$$

In these interaction formulas the variables are defined as follows:

P_u = factored compression load, kips

P_n = nominal axial compression capacity, kips

P'_u = equivalent factored compression load, kips

M_{ux} = moment due to factored load, x axis, kip-ft

M_{uy} = moment due to factored load, y axis, kip-ft

M_{nx} = nominal moment capacity, x axis, kip-ft

M_{ny} = nominal moment capacity, y axis, kip-ft

m = bending factor for column shape

$\phi_c = 0.85$

$\phi_b = 0.90$

The following example illustrates the use of the approximation method.

Example 17. A 10-in. W shape is to be used for a column in a situation such as that shown in Figure 5.39. The factored axial load from above is 175 kips [778 kN], and the factored beam load at the column face is 35 kips [156 kN]. The column has an unbraced height of 16 ft [4.88 m] and a K factor of 1.0. Select a trial shape for the column.

Solution. From Table 5.9, the bending factor m is 1.7 for the 16-ft unbraced height. The equivalent factored axial compression load is thus determined as

$$\begin{aligned} P'_u &= P_u + mM_{ux} \\ &= (175 + 35) + (1.7 \times 35 \times 5/12) \\ &= 210 + 25 = 235 \text{ kips } [1050 \text{ kN}] \end{aligned}$$

From Table 5.9, a trial selection is for a W 10 × 45. This selection may then be verified by use of the proper equation for interaction.

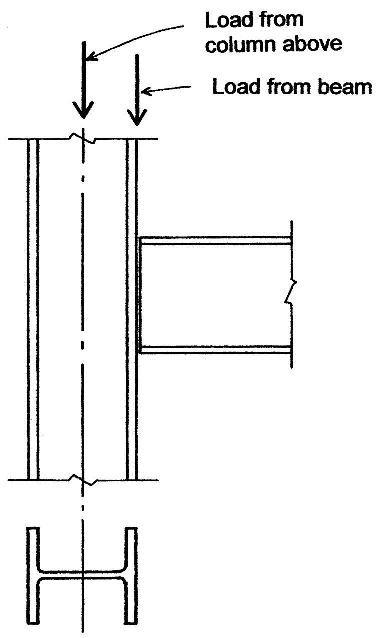


Figure 5.39 Development of an eccentric loading.

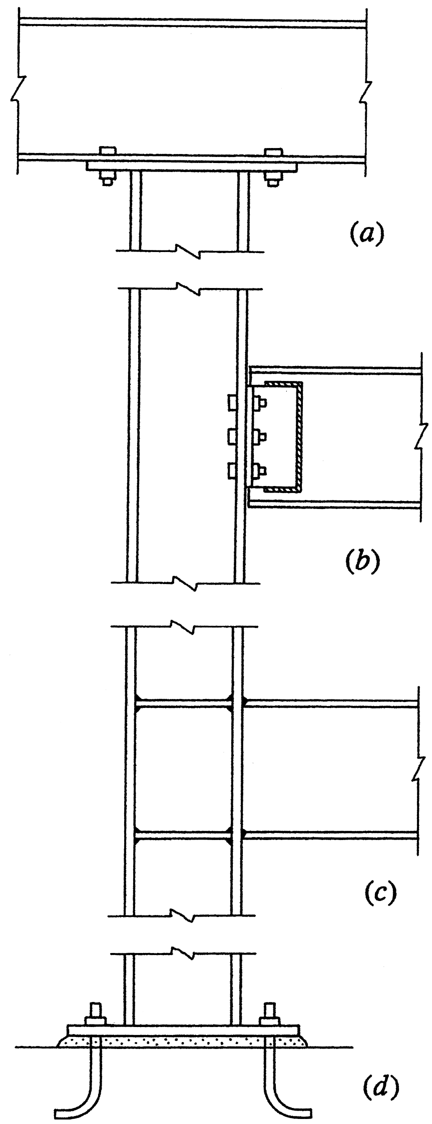


Figure 5.40 Typical connection details for light steel frames.

When bending occurs on both axes, as it does in full three-dimensional rigid frames, all three parts of the interaction formulas must be used.

Column Framing and Connections

Connection details for columns must be developed with considerations of the column shape and size; the shape, size, and orientation of other framing; and the particular structural functions of the joints. Some common forms of simple connections for light frames are shown in Figure 5.40. The usual means of attachment are by welding, by bolting with high-strength bolts, or with anchor bolts embedded in concrete or masonry supports. Welding is the preferred connection for work achieved in the fabrication shop and bolting for connections achieved at the erection site.

When beams sit directly on top of a column (Figure 5.40*a*), the usual solution is to weld a bearing plate on top of the column and bolt the bottom flange of the beam to the plate. For this, and for all connections, it is necessary to consider what parts of the connection are achieved in the fabrication shop and what is achieved as part of the erection process at the building site (called the *field*). In this joint the plate serves no special structural function, because the beam could theoretically bear directly on top of the column. However, field assembly of the frame works better with the plate, and the plate also probably helps to spread the bearing stress more fully over the column cross section.

In many situations, beams frame directly into the side of a column. If simple transfer of vertical load is all that is required, a common solution is the connection shown in Figure 5.40*b*, in which a pair of steel angles is used to connect the beam web to the column face. With minor variation, this form of connection can also be used to connect a beam to the column web, which requires a relatively deep column shape—a situation that makes W shapes of at least 10-in. depth most popular for frames.

If bending moment must be transferred between a beam's end and its supporting column, a common solution is to weld the ends of the beam flanges directly to the column face, as shown in Figure 5.40*c*. Because the bending must be developed in both flanges of the column and the beam directly grabs only one flange, the filler plates shown are often used for a more effective transfer of the bending from the beam. This leaves the beam web as yet unconnected, so some attachment must also be made there, because the beam web actually carries most of the beam shear force. Although common for many years and still used for gravity and wind loads, this form of connection has recently received a lot of scrutiny because of poor performance in earthquakes, and some refinements are in order if it is used for this load condition.

At the column bottom, where bearing is commonly on top of a concrete pier or footing, the major concern is for reduction of the bearing pressure on the soft concrete. With upward of 20 ksi or more compression in the column steel and possibly little over a usable value of 1000 psi of resistance in the concrete, the contact bearing area must be considerably spread out. For this reason, as well as the simple practical one of holding the column bottom in place, the common solution is a steel bearing plate attached to the column. For modest-size columns (not for super–high rises) the bearing plate is attached by welding to the column in the fabricating shop and bears on a filler material on top of the foundation concrete. The filler serves to make a uniform bearing surface to level out the bearing pressure. Attachment of the bearing plate to the foundation is achieved with anchor bolts cast into the concrete.

5.4 BOLTED CONNECTIONS FOR STEEL STRUCTURES

Elements of structural steel are often connected by mating flat parts with common holes and inserting a pin-type device to hold them together. In times past, the pin device was usually a rivet; today it is a bolt. A great number of types and sizes of bolts are available, as are many connections in which they are used.

Structural Actions of Bolted Connections

Figures 5.41*a* and *b* show plan and section views of a connection between two steel bars that functions to transfer tension between the bars. Although this is a tension transfer connection, it is also described as a shear connection because of the manner in which the connecting device (the bolt) works in the connection (see Figure 5.41*c*).

Connections such as that shown in Figures 5.41*a* and *b* are now mostly achieved with so-called *high-strength bolts*, which are special bolts that are tightened in a controlled manner that induces development of yield stress in the bolt shaft. For a connection of this type, many types of failure must be considered, including the following.

Bolt Shear

The failure of the bolt involves a slicing (shear) effect that is developed as a shear stress on the bolt cross section. With the size of the bolt and the grade of steel known, it is a simple matter to establish this load limit. In some types of connections, it may be necessary to slice the same bolt more than once to separate the connected parts. This is the case in the connection shown in Figure 5.41*f*, which shows that the bolt must be sliced twice to make the joint fail. When the bolt develops shear on only one section (Figure 5.41*c*), it is said to be in *single shear*; when it develops shear on two sections (Figure 5.41*f*), it is said to be in *double shear*. It is possible for additional sections to be involved, but these two cases cover most structural joints for steel construction.

Bearing

If the bolt tension (due to tightening of the nut) is relatively low, the bolt serves primarily as a pin in the matched holes, bearing against the sides of the holes, as shown in Figure 5.41*d*. The connected parts must be of sufficient thickness to develop the contact bearing of the bolts.

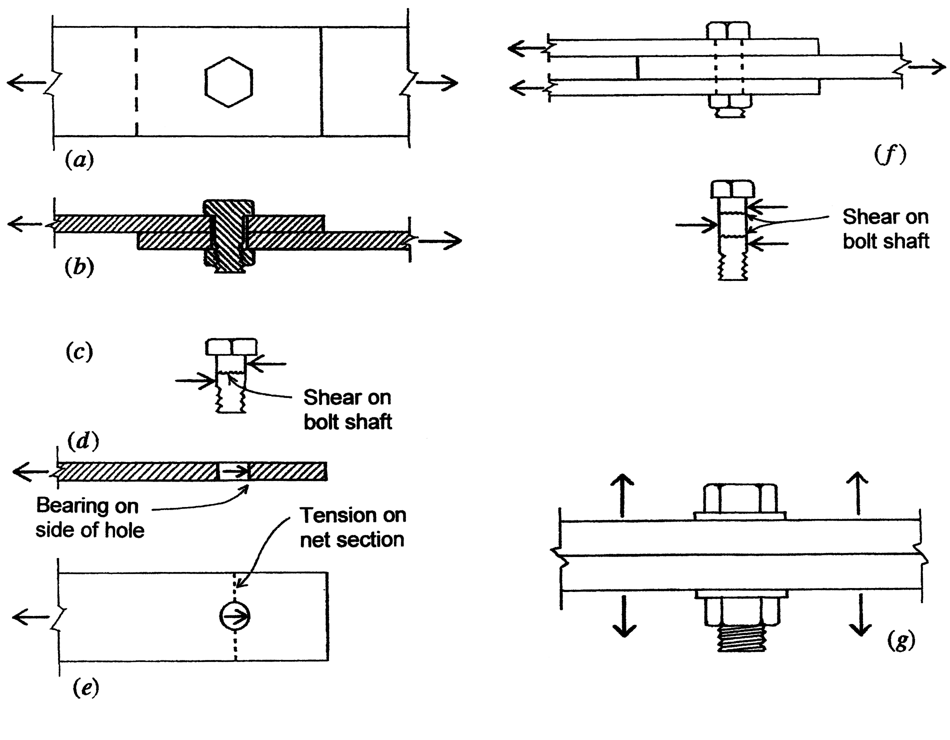


Figure 5.41 Actions of bolted connections.

Tension on Net Section of Connected Parts

For the connected bars in Figure 5.41*b*, the tension stress in the bars will be a maximum at a section across the bar at the location of the hole. This reduced section is called the *net section* for tensile resistance.

Bolt Tension

Some connections employ bolts for their resistance in tension, as shown in Figure 5.41*g*. For the threaded bolt, the maximum tension stress is developed at the net section through the cut threads. However, the bolt can also have extensive elongation if yield stress develops in the bolt shaft. Bolt tension resistance is established on the basis of data from destructive tests.

Bending in the Connection

Whenever possible, bolted connections are designed to have a bolt layout that is symmetrical with regard to the applied forces. This is not always possible because, in addition to the direct force actions, the connection may be subjected to twisting due to a bending moment or torsion induced by the loads. Figure 5.42 shows some examples of this situation.

In Figure 5.42*a*, two bars are connected by bolts, but the bars are not aligned in a way to transmit tension directly between the bars. A twisting action is thus developed with a moment equal to the product of the tension force and the eccentricity of the bar alignments. Shear on the bolts will be increased by this action and bending will be added to the tension in the bars.

Figure 5.42*b* shows the single shear connection, in which a twisting moment will be induced on both the bars and the bolts. This twisting action increases with thicker bars, but steel bars will usually be thin; nevertheless, it is not a favored form of connection for major force transfers.

Figure 5.42*c* shows a side view of a beam end with a typical form of connection that employs a pair of angles. As shown, the angles grasp the beam web between their legs and turn the other legs out to fit against the flat side of a column or the web of a supporting beam. Vertical load from the beam, vested in the shear in the web of the beam, is transferred to the angles by the bolts connecting the angles to the beam web. This load is then transferred from the angles at their outwardly turned face, resulting in a separated set of forces caused by the eccentricity shown. This twisting action must be considered in the design of the connection.

Slipping of Connected Parts

Highly tensioned, high-strength bolts develop a very strong clamping action on the mated flat parts being connected, analogous to the situation shown in Figure 5.43*a*. As a result, there is a strong development of friction at the slip face, which is the initial form of resistance in the shear-type joint. Development of bolt shear, bearing, and even tension on the net section will not occur until this slipping is allowed. For service-level loads, therefore, this is the *usual* form of resistance, and the bolted connection with high-strength bolts is considered to be a very rigid form of joint.

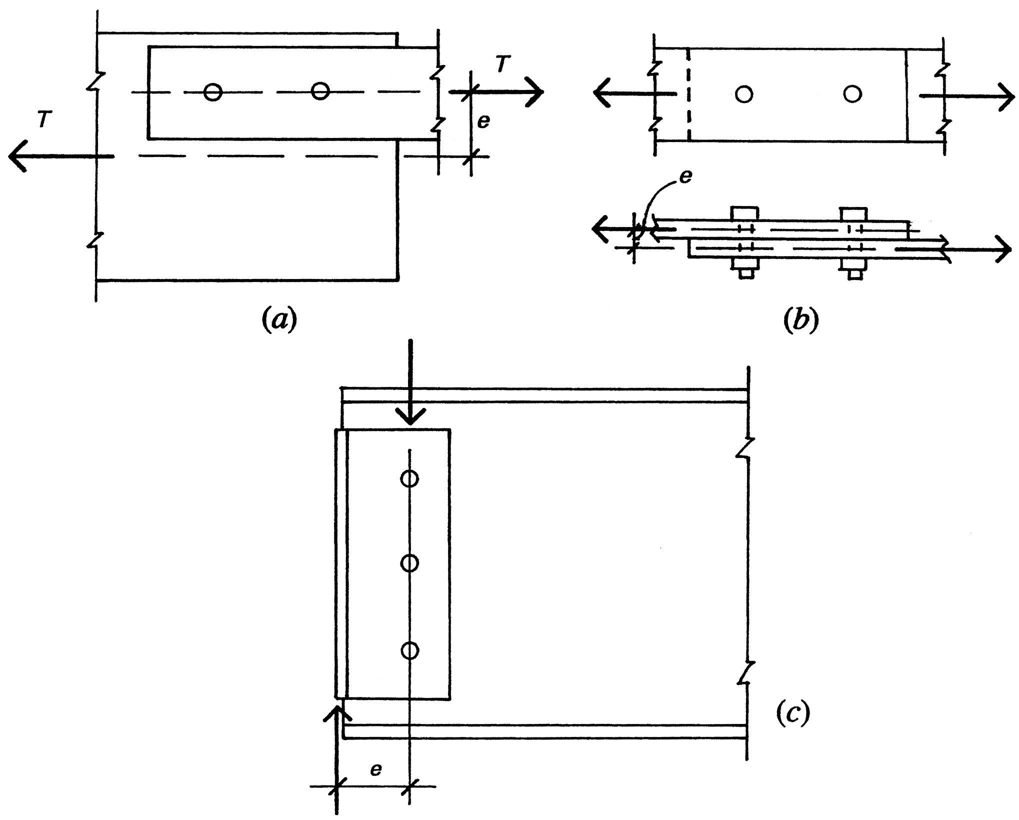


Figure 5.42 Development of bending in bolted connections.

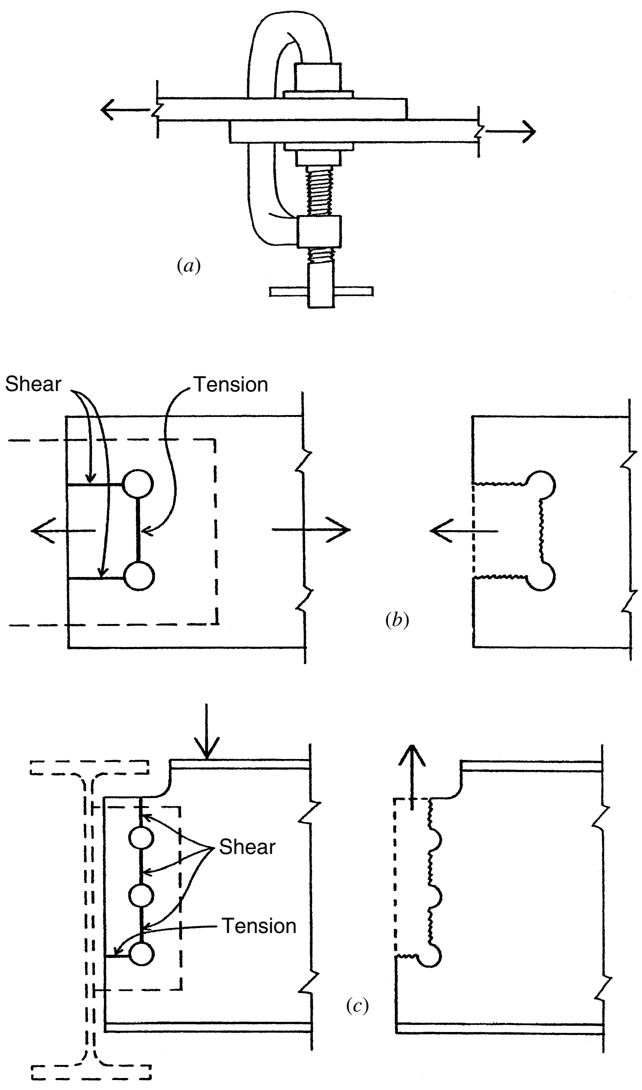


Figure 5.43 Special actions of bolted connections.

Block Shear

One possible form of failure in a bolted connection is that of tearing out the edge of the attached members. This is called a *block shear* failure. The diagrams in Figure 5.43*b* show this potentiality of failure in a connection of two plates. The failure in this case involves a combination of shear and tension to produce the torn-out form shown. The total tearing force is computed as the sum of both forms of failure. The design strength for the tension tear is computed as for the tension on a net section. The design strength for the shear tears is determined as $0.75F_vA_c$, where A_c is the total cross-sectional area experiencing shear stress.

With the edge distance, hole spacing, and diameter of the holes known, the net widths for tension and shear are determined and multiplied by the thickness of the part in which tearing occurs. These areas are then multiplied by the appropriate stresses to find the tearing force that can be resisted.

Another case of potential tearing is the common situation for the end framing of a beam in which support is provided by another beam, whose top is aligned with that of the supported beam, as shown in Figure 5.43*c*. The end portion of the top flange of the supported beam must be cut back to allow the beam web to extend to the side of the supporting beam. With the use of a bolted connection, a potential tearing condition is developed.

Types of Steel Bolts

Bolts used for the connection of structural steel members come in two basic types. Bolts designated A307 and called *unfinished bolts* have the lowest load capacity of the structural bolts. The nuts for these bolts are tightened just enough to secure a snug fit of the attached parts; because of the resulting low resistance to slipping plus the routine oversizing of the holes to achieve practical assemblage, there is some movement in the connection in the development of full resistance. These bolts are generally not used for major structural connections, especially when joint movement or loosening under vibration or repeated loading may be a problem. They are, however, used for temporary connections during the erection of frames.

Bolts designated A325 and A490 are called *high-strength bolts*. The nuts of these bolts are tightened to produce a considerable tension in the bolt shaft, which results in a high degree of friction resistance between the attached parts.

When loaded in shear-type connections, bolt capacities are based on the development of shearing action in the connection. The capacity of a single bolt is designated as S for single shear or D for double shear, as discussed previously.

Bolts are ordinarily installed with a washer under both head and nut. Special high-strength bolts have heads or nuts single formed with washers, eliminating the need for the separate washer. When a washer is used, it is sometimes a limiting dimensional factor in detailing for bolt placement in tight locations, such as close to the fillet (inside radius) of angles or other rolled shapes.

For a bolt with a given diameter, a minimum thickness is required for the bolted parts in order to develop the full shear capacity of the bolt. This required thickness is based on the bearing pressure between the bolt and the side of the hole, as discussed previously. The allowable pressure for this investigation may be based on the steel of the bolt or of the connected parts. However, bolts are usually of a very high strength, so the strength of the parts is most likely to define the limit.

Although there is a wide range of available bolt sizes, the work of assembling frames is made much easier if a limited number of sizes are used. This requires some coordination of the design for the many different connections ordinarily involved in steel frameworks.

There are also many other types of fasteners available for special situations, including bolts of different grades of steel. Steel bolts are also used in assemblages of structural elements of wood and precast concrete.

Considerations for Bolted Connections

Layout of Bolted Connections

The design of bolted connections generally involves a number of considerations in the dimensional layout of the bolt-hole patterns for the attached structural members.

Figure 5.44*a* shows the layout of a bolt pattern with bolts placed in two parallel rows. The two basic dimensions for this layout are limited by the size (nominal diameter) of the bolt. The first is the center-to-center spacing of the bolts, usually called the *pitch*.

The second critical layout dimension is the *edge distance*, which is the distance from the centerline of the bolt to the nearest edge of the member containing the bolt hole. There are also specified limits for this distance as a function of the bolt size and the nature of the edge, the latter referring to whether the edge is formed by rolling or by cutting. Edge distances may also be limited by edge tearing in block shear, as discussed previously.

In some cases bolts are staggered in parallel rows (Figure 5.44*b*). In this case, the diagonal distance labeled *m* in the figure must also be considered. For staggered bolts, the spacing of the bolts in the direction of the row is referred to as the pitch, but the spacing of the rows is called the *gauge*. The usual reason for staggering the bolts is that sometimes the rows must be placed closer than the minimum spacing for the bolts. However, staggering the bolt holes also helps to create a slightly less critical net section for tension stress in the connected members.

Location of bolt holes is often related to the size and type of structural members being attached. This is especially true for bolts placed in the legs of angles or in the flanges of other rolled shapes. Figure 5.44*c* shows the placement of holes in the legs of angles. When a single row is placed in a leg, its recommended location is at the distance labeled g from the back of the angle. For two rows, the first row is placed at the distance g_1, and the second row is spaced the distance g_2 from the first. When placed at the recommended locations in rolled shapes, bolts will end up a certain distance from the edge of the member. Based on the recommended edge distance for rolled edges, it is thus possible to determine the maximum size of bolt that can be accommodated. For angles, the maximum size of fastener may be limited by the edge distance, especially when two rows are used; however, other factors may in some cases be more critical. The distance from the center of the bolts to the inside fillet of the angle defines the range for the flat surface of the angle leg. This limits the space available for a washer. Another consideration may be the stress on the net section of the angle, especially if the load is assumed to be taken by the attached leg, as discussed in the following section.

Tension Connections

When tension members have reduced cross sections, two stress investigations must be considered. This is the case for members with holes for bolts. For the member with a hole, the design strength of the section at the hole is defined as $\phi_t P_n = \phi_t F_u A_e$. The resistance at the net section must be compared with the resistance at the unreduced section of the member for which the reduction factor is 0.9 and the usable stress is the yield strength of the member.

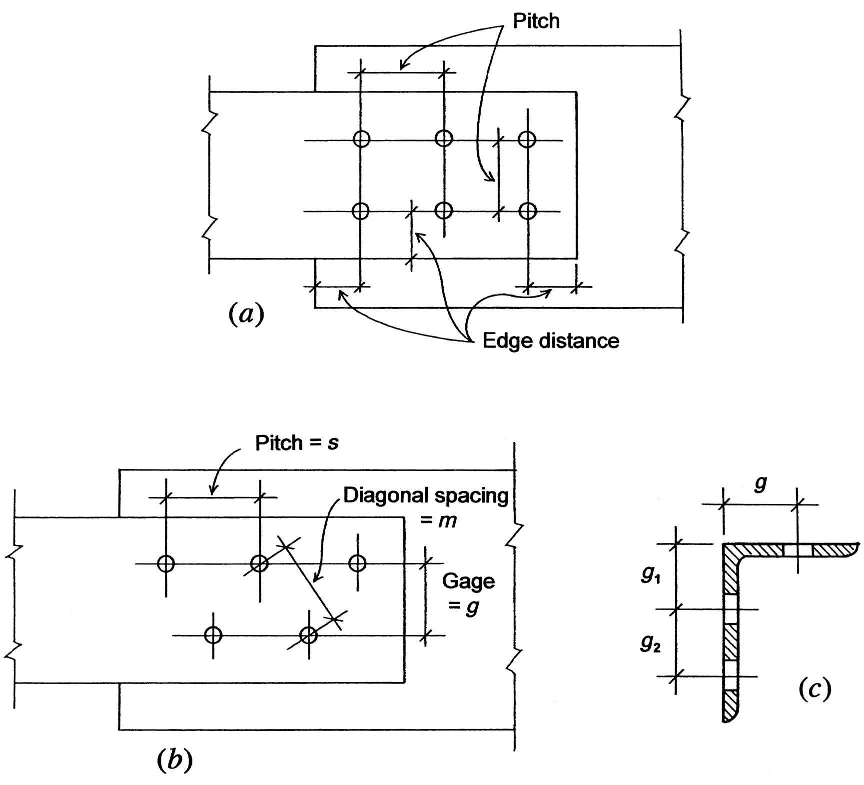


Figure 5.44 Layout considerations for bolted connections.

Angles used in tension are often connected by only one leg. In a conservative design, the effective net area is only that of the connected leg less the area of the holes.

Framing Connections

Framing connections often involve both welding and bolting in a single connection. In general, welding is favored for fabrication in the shop, and bolting is preferred for erection in the field. If this practice is recognized, the connections must be developed with a view to the overall fabrication and erection process and decisions made with regard to what is done where: in the shop or the field.

Developing connection details is particularly critical for structures in which a great number of connections appear. The truss is one such structure.

Framed Beam Connections

The connection shown in Figure 5.45 is the type used most commonly in the development of framed structures that consist of I-shaped beams and H-shaped columns. This device is referred to as a *framed beam connection*, for which there are several design considerations.

Type of Fastening. The angles can be attached to the supported beam and to the supporting member with welds or any of several types of bolt. The most common practice is to weld the angles to the supported beam's web in the fabricating shop and to bolt the angles to the support (the column face or the supporting beam's web) in the field.

Number of Fasteners (Bolts). If bolts are used, this consideration refers to the number of bolts used on the supported beam's web, with twice this number in the angles' outstanding legs. The capacities are matched, however, because the web bolts are in double shear and the others in single shear.

Size of the Angles. For smaller beams or for light loads, angle leg sizes are typically narrow, being just wide enough to accommodate a single row of bolts, as shown in Figure 5.45*b*. However, for large beams and greater loads, a wider leg may be used to accommodate two rows of bolts. Leg width and thickness depend on the size of bolts and the magnitude of loads. Width of the outstanding legs may also depend on the space available, especially if attachment is to a W shape with a narrow flange or shallow depth.

Length of the Angles. The length must be that required to accommodate the number of bolts. The standard layout is shown in Figure 5.45*b* with bolts at 3-in. spacing and edge distances of 1.25 in. This will accommodate up to 1-in.-diameter bolts. However, the angle length is also limited to the distance available on the flat portion of the beam web. (see Figure 5.45*a*).

The AISC manual (Ref. 10) provides data to support the design of these connections. Information is provided relating to magnitudes of allowable loads and to the form and size of connections for beams of various size. Use of this data is highly recommended to avoid the laborious process of designing individual connections.

Although there is no specified limit for the minimum size of a framed connection to be used for a given beam, a general rule is to use one with the angle length at least one-half of the beam depth. This rule is intended, for the most part, to ensure some stability against rotational (torsional) effects in the supporting beam.

For very shallow beams (8 in. and less in depth), the special connector shown in Figure 5.45*c* may be used. Regardless of the load, the angle leg at the beam web must accommodate two bolts, simply for stability of the angles.

There are many structural effects to consider for these connections. One concern is for the bending in the connection that occurs as shown in Figure 5.42*c*. The moment arm for this twisting action is the gauge distance of the angle leg, dimension *g* as shown in Figure 5.45*b*. It is a reason for choosing a relatively narrow angle leg.

If the top flange of the supported beam is cut back, as it commonly is to clear the flange of a supporting beam, shear at the net section or block shear may be critical. Both conditions will be aggravated when the supported beam has a very thin web, which is a frequently occurring situation because the

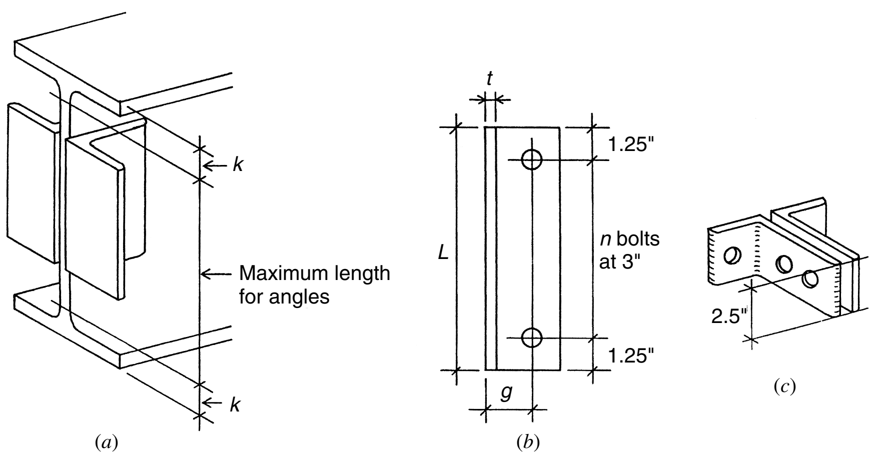


Figure 5.45 Framed beam connections for rolled shapes.

most efficient shapes are usually the lightest shapes in their nominal depth categories.

Another concern for the thin beam web is critical bearing stress at the bolt holes when a choice for a large bolt is combined with one for a thin beam web.

Framed beam connections as shown here may usually be selected from the tables in the AISC manual, where data are provided for most of the issues of concern raised here. The framed beam connection is essentially considered to have negligible moment resistance, as no attachment is made to the beam flanges. In joints required to transmit moment, there is typically a two-part connection development: one for the moment transfer and another (possibly basically a framed beam connection) for the transfer of shear from the beam web.

Bolted Truss Connections

A major factor in the design of trusses is the development of truss joints. Because a single truss typically has several joints, the joints must be relatively easy to produce and economical, especially if there is a large number of trusses of a single type in the building structural system. With respect to the design of connections for the joints, the truss configuration, member shapes and sizes, and fastening method—usually welding or high-strength bolts—must be considered.

Trusses are usually fabricated in the shop in the largest units possible, which means the whole truss for modest spans or the maximum-sized unit that can be transported for large trusses. Bolting is mostly used for connections made at the building site. For the small truss, bolting is usually done only for the connections to supports and to supported elements or bracing. For the large truss, bolting may also be done at splice points between shop-fabricated units. All of this is subject to many considerations relating to the nature of the rest of the building structure, the particular location of the site, and the practices of local fabricators and erectors.

Two common forms for light steel trusses are shown in Figure 5.46. In Figure 5.46*a* the truss members are pairs of angles and steel gusset plates are attached to the members to form the joints. For top and bottom chords, the angles are often made continuous through the joint, reducing the number of connectors required and the number of separate cut pieces of the angles. For flat-profiled, parallel-chord trusses of moderate size, the chords are sometimes made from tees, with interior truss members fastened directly to the tee stem, eliminating the need for gusset plates (Figure 5.46*b*).

Figure 5.47 shows a layout for several joints of a light roof truss, employing the system shown in Figure 5.46*a*. In the past, this form was commonly used for roofs with high slopes, with many short-span trusses fabricated in a single piece in the shop, usually with riveted connections. Trusses of this form now either are welded or use high-strength bolts.

Development of the joint designs for the truss shown in Figure 5.47 involves many considerations, including the following:

Truss Member Size and Member Force Magnitudes. These conditions determine the choice for size and type of fastener (bolt) required, based on individual fastener capacities.

Angle Leg Size. This relates to the maximum diameter of bolt that can be used, based on angle gauges and minimum edge distances.

Thickness and Shape of Gusset Plates. The preference is to reduce cost for the steel and to have the lightest weight added to the structure.

Layout of Members at Joints. The aim is to have the action lines of the forces (vested in rows of bolts) all meet at a single point, thus avoiding twisting in the joints.

Many of the points mentioned are determined by data. Minimum edge distances for bolts can be matched to usual gauge dimensions for angles. Forces in members can be related to bolt capacities, the general attempt being to reduce the number of bolts and the required size of gusset plates.

The truss shown in Figure 5.47 has some features that are quite common for small trusses. All member ends are connected by only two bolts (the minimum), indicating that size and type of bolt chosen have the capacity required for all internal forces in the truss with only two bolts in the connection. At the top-chord joint between the support and the truss peak, the top-chord member is shown as being continuous through the joint, a cost savings in member fabrication as well as connections.

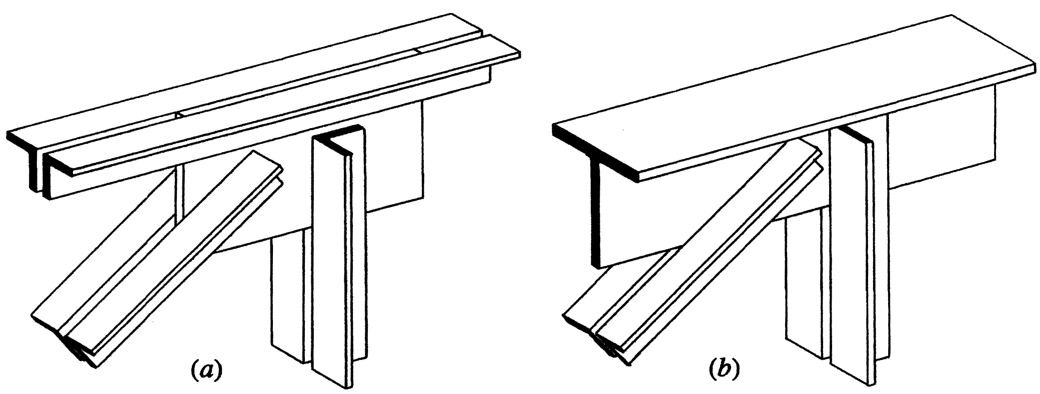


Figure 5.46 Common connection details for light steel trusses.

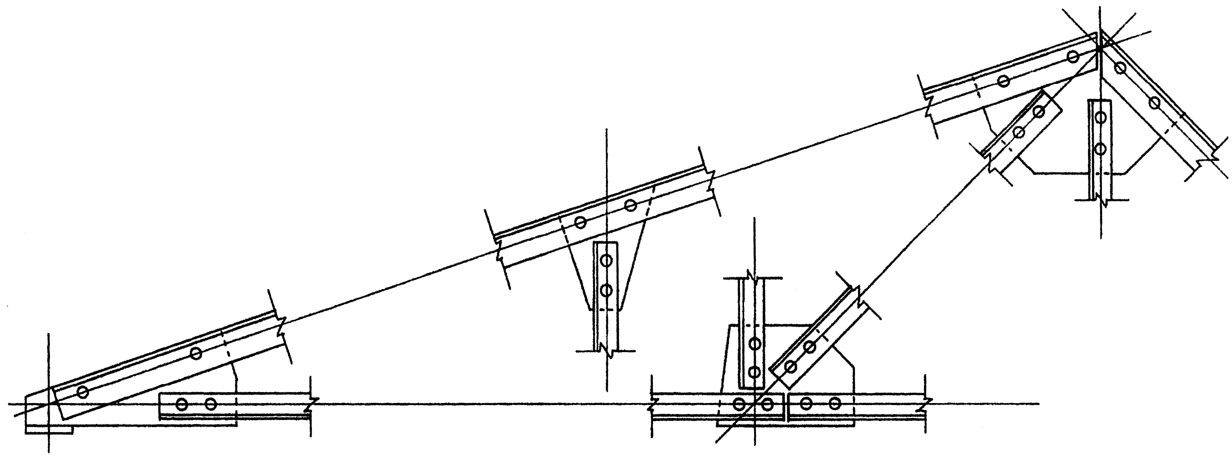


Figure 5.47 Typical form of a light steel truss with double-angle members and bolted connections with gusset plates.

There are, of course, other options for this truss, including one with all welded joints and tees for the top and bottom chords, as shown in Figure 5.46*b*.

5.5 STEEL TRUSSES

When iron and steel emerged as major industrial materials in the eighteenth and nineteenth centuries, an early application to spanning structures was for trusses. One reason for this was the early limit on the size of members that could be produced. In order to create a reasonably large structure, therefore, it was necessary to link a large number of small parts.

A major technical problem for such assemblages was the achievement of the many joints, which need to be both economical and practical to form. Various connecting devices or methods have been employed; a major one used for building structures in earlier times was the use of hot-driven rivets. This process consists of matching up holes in members to be connected, placing a heat-softened pin in the hole, and then beating the heck out of the protruding ends of the pin to form a rivet.

Basic forms developed for early riveted connections are still widely used. Today, however, the joints are mostly achieved by welding or by using highly tightened bolts in place of rivets.

Truss forms relate to the particular structural use (bridge, gable-form roof, arch, flat-span floor, etc.), the magnitude of the span, and the materials and methods of construction. Some typical forms for trusses used in steel construction are shown in Figure 5.48. The forms in widest use are the parallel-chorded types, shown in Figures 5.48*a* and *b*; these are often produced as manufactured products. Variations of the gable-form truss (Figure 5.48*c*) can be produced for a wide range of spans.

As planar elements, trusses are quite unstable in a lateral direction (perpendicular to the plane of the truss). Structural systems employing trusses require attention to this issue, which mostly concerns development of bracing. The *delta truss* (Figure 5.48*d*) is a special, self-stabilizing form that is frequently used for towers or for spanning trusses that do not require added bracing to achieve lateral stability.

Various aspects of trusses are discussed in Section 3.4.

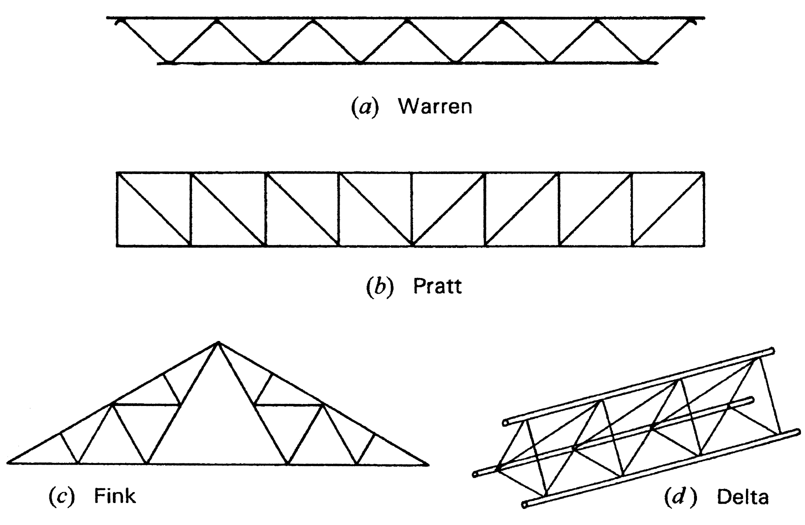


Figure 5.48 Common forms of light steel trusses.

CHAPTER

6

Concrete Structures

The process of generating a solid substance by using a binder to adhere a mass of loose material is applied in various ways to produce filled plastics, asphalt pavements, particleboard, and plaster as well as what we call concrete. Although the term has broader generic meaning, we usually apply the word *concrete* to the material that is produced in rocklike form with a binder of water and portland cement and a loose filler consisting of sand and gravel.

Forms of concrete made with natural binders were used by ancient builders, but modern concrete, as we use it today, dates primarily from the development of calcined (burned) portland cement in the early nineteenth century. The potential for this highly improved material was first not fully recognized, and concrete continued to be used mostly in the old ways—for crude, filler functions in massive construction. Eventually, designers and builders began to experiment with the new material and to find ways to make better use of its refined quality. Basic systems and construction methods developed in a few decades in the late nineteenth and early twentieth centuries continue largely unchanged in form as major uses for building structures.

6.1 GENERAL CONCERNS FOR CONCRETE

Concrete is a somewhat complex material, and its use involves many concerns, such as those for mixing, forming, reinforcing, finishing, and curing of the cast material. This section deals with some of these critical concerns for the material and its production in forms for building structures. Considerations of its structural functions and the process of design must be built on some understanding of these general concerns. As compared to wood, steel, or masonry, concrete structures offer greater degree of freedom variability and require greater responsibility in terms of control of the finished product (see Figure 6.1).

Usage Considerations

Most of the concrete produced in the United States does not go into buildings but rather goes into pavements, bridges, dams, retaining walls, waterways, tunnels, and other types of structures. Indeed, most of the concrete used for buildings goes into foundations and grade-level pavements; almost every building has these elements, while only a relatively few have a structure above ground made of concrete (see Figure 6.2). This is said only so that it may be appreciated that the concrete industry is not oriented principally to the production of building structures.

On the other hand, concrete does lend itself to the possible production of all the basic structural components—foundations, roof and floor framing, walls, and columns—as well as a great range of various systems, including arches, domes, shells, and space frames. It is also generally the most inert and durable construction material, resisting aging, weather effects, rot, insects, fire, and most chemical change and decomposition. Given the right circumstances, it is a very usable material (see Figure 6.3).

Most concrete is produced by pouring the semifluid mixed material into a hole or a forming mold at the building site. For above-ground construction, forming costs often exceed that of the basic material itself. This has led to use of factory or on-site precasting of units that are then erected much like ordinary steel and wood elements.

Of course, a major early use of concrete was for precast masonry units: bricks and hollow-cored blocks. Today, most structural masonry is produced with concrete blocks—now called CMUs, for concrete masonry units.

A major structural limitation for concrete is its low resistance to tension. Compensation consists of using steel

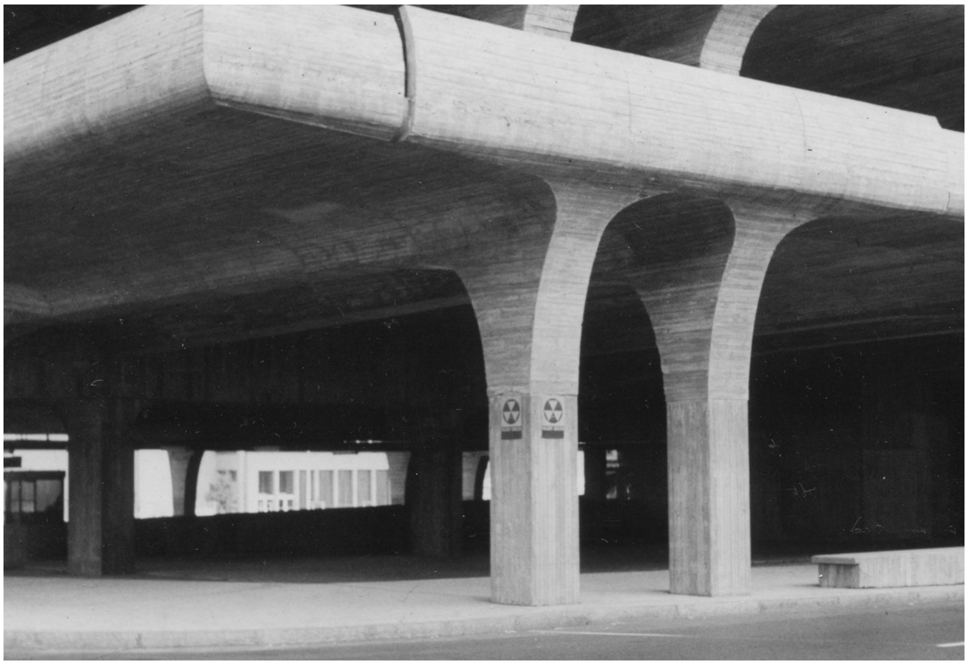


Figure 6.1 A carefully crafted version of the ordinary post, beam, and deck system; shaped here in a flowing manner with member connections continuously molded and with exterior exposed surfaces textured by forming with thin wood strips. New Haven Municipal Parking Garage, Hew Haven, Connecticut; Paul Rudolph, architect.

Figure 6.2 Concrete crudely formed by carved soil for a building foundation system with individual column footings and a continuous, strip-form grade beam between footings.

Figure 6.3 Highly exposed concrete structure, with columns and spandrel beams dominating the viewed exterior of a multistory building. Resistance to weather and fire permits this exposure of the raw concrete structure.

reinforcement, compression with prestressing, or addition of fibrous materials to the concrete mix. Use of inert steel rods cast into the concrete produces the product called *reinforced concrete*. For a building structure, just about every component of the system will be of reinforced concrete. A major part of the work of designing concrete structures involves the determination of the type, size, amount, and placement of the steel reinforcing bars.

Practical Considerations

Producing the finished concrete structure as a sitecast object involves attention to several concerns. If the structure is exposed to view, its appearance will be vitally dependent on careful consideration of these matters. Even for the covered structure, however, the integrity of the finished construction may be critical to structural behavior. Principal factors that must be considered are the following.

Forming

The shapeless, fluid-mixed concrete must be poured into a mold (called the *form*) of the desired shape and held until it is adequately hardened. For footings and paving slabs the mold is mostly the dug and leveled ground surface. For walls, columns, beams, and spanning slabs, the forms must be built and removed after the concrete is hardened.

Built forms must be reasonably watertight and strong enough to support the wet concrete without bulging or sagging. It is not unusual for forming costs to exceed the cost of the concrete material itself, especially when the forms must be completely hand built, used once, and discarded. Use of reusable forms, repetitive shapes, and precast forming units are means of overcoming forming costs (see Figures 6.4 and 6.5).

The finished cast surface of the concrete will reflect the form, making accuracy of the form construction and finish quality of the form surfaces a design concern.

Installation of Reinforcing

For reinforced concrete, the steel bars must be placed in the forms and held firmly in place—literally in midair—during pouring of the concrete—Not an easy task.

Pouring

Getting the concrete into the forms and working it into place are a challenge when the forms are intricate and are filled with reinforcement and the accessories needed to hold the reinforcement and brace the forms. Distances between bars (called *spacing*) and distances between the bars and the forms (called *cover*) must be controlled to allow pouring as well as to satisfy structural and fire protection considerations.

Finishing (Wet)

The top surface of the concrete in a single pour must be finished. This may consist of merely leveling it to a desired flat surface. For floor slabs the surface may be made quite smooth if it constitutes a finished floor or deliberately roughened if bonding to an additional surfacing is required.

Finishing (Dry)

Surfaces may be reworked after the concrete is hardened. If this is a reduction type of working (chipping, sand-blasting,

Figure 6.4 Concrete waffle: a structure uniquely formed with the cast concrete material, using forming units that are removable and reusable. Continuity of the material eliminates the need for the extensive connections that would be required between members with wood or steel construction.

Figure 6.5 Conventional concrete post-and-beam structure for a multistory building, partially achieved by use of precast, hollow concrete forms for the exterior columns and spandrel beams. The hollow units are filled with concrete during the casting of the conventionally formed interior structure. Factory casting permits the development of exceptionally high quality surfaces on the precast units.

grinding, etc.), the surface dimension lost must be considered in establishing required cover for the reinforcement.

Curing

Ordinary concrete hardens within a few hours after being mixed but does not attain significant strength for several days or weeks. Various means may be used to slow down or speed up these actions, depending on the circumstances. What is critical is that the concrete be kept moist and within some range of temperature during the curing period. If allowed to dry out, to freeze, or to heat excessively from its own chemical reactions or the sun, it may become damaged or may not attain its potential quality.

Much of what must be done to achieve good concrete falls on the suppliers, the builders, and the craft capabilities of the workers. However, the designer must exercise some decision making and specify actions that set up a situation that favors good construction. Informed choice of shapes, finishes, and construction detailing requires a lot of sensitivity to the realities of construction work.

Materials for Concrete Structures

Consideration must be given to the various ingredients of structural concrete and to the factors that influence the physical properties of the finished concrete. The following discussion treats many of these issues.

Common Forms of Structural Concrete

For structural usage, concrete must attain significant strength, stiffness, surface hardness, and other necessary properties. While the mixture used to obtain concrete can be almost endlessly varied, the mixes used for structural applications are developed within a quite limited set of variables. The most commonly used mix contains ordinary portland cement, clean water, medium-to-coarse sand, and a considerable volume of some fairly large pieces of rock (gravel). This common form of concrete will be used as a basis for comparison of mixes for special purposes.

Figure 6.6 shows the composition of ordinary structural concrete. The binder consists of water and cement, whose chemical reaction results in the hardening of the mass. The binder is mixed with aggregate (loose, inert particles) so that the binder coats the surfaces and fills the voids between the particles of the aggregate.

For materials such as grout, plaster, and stucco, the aggregate consists of reasonably fine-grained sand. For concrete the grain size is extended into the category of gravel, with the maximum particle size limited primarily by the size of the concrete elements. The end product—the hardened concrete—is highly variable, due to the choices for individual ingredients; to modifications in the mixing, handling, and curing processes; and to possible addition of special ingredients.

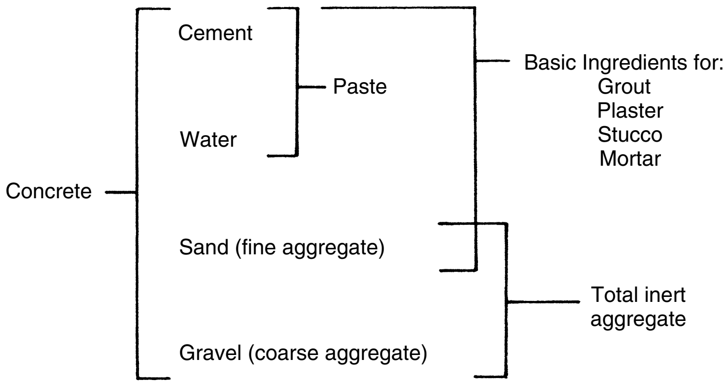


Figure 6.6 Composition of the concrete material.

Cement

The cement used most extensively is portland cement. Of the five standard types of portland cement available, two types account for most of the cement used for building construction. These are a general-purpose cement for use in concrete designed to reach its required strength in about 28 days and a high-early-strength cement for use in concrete that attains its design strength in a period of a week or less.

Cements set and harden by reacting with water, a hydration process accompanied by heat. A low-heat cement is used where this is a problem.

Mixing Water

Mixing water must be reasonably clean and free of oil, organic matter, and any substance that may affect the hardening, curing, or general finish quality of the concrete. Drinking-quality (potable) water is usually adequate.

A critical concern for good concrete is the amount of water used. In this regard, there are three principal concerns:

Having enough water to react chemically with the cement so that the hardening and strength gain of the concrete proceeds over time until the desired quality of material is obtained.

Having enough water to facilitate good mixing of the ingredients and allow for handling of the concrete for casting and finishing.

Having the amount of water low enough so that the combination of water and cement (the paste) is not too low in cement to perform the necessary bonding action. This is a major factor in producing high-grade concrete for structural applications.

Stone Aggregate

The most common aggregates are sand, crushed stone, and pebbles. Particles smaller than $\frac{3}{16}$ in. in diameter constitute the *fine aggregate*. There should be only a small amount of very fine materials to allow for the free flow of the water–cement mixture between aggregate particles. Material larger than $\frac{3}{16}$ in. is called *coarse aggregate*. The maximum size of the coarse aggregate particle is limited on the basis of the thickness of cast elements, on spacing and cover for reinforcement, and on some considerations for surface finishing.

In general, the aggregate should be well graded, with some portion of large to small over a range to permit the smaller particles to fill the spaces between larger ones. The volume of the concrete is thus mostly composed of the aggregate, with the water and cement going into the very small spaces remaining. The weight of the concrete is determined largely by the weight of the coarse aggregate. Strength of the finished concrete is also dependent, to a considerable degree, on the structural integrity of the large aggregate particles.

Special Aggregates

Aggregates other than stone may be used to impart particular modified properties to the concrete. Some of these desired properties and the types of aggregates used to achieve them are as follows:

Weight Reduction. Concrete structural elements tend to be thick and reduction of the ensuing dead load is usually an advantage, especially for spanning structures. Since the coarse aggregate usually constitutes as much as three-fourths of the concrete volume, this is a major factor for weight. Various natural and synthetic materials may be used as substitutes for ordinary stone, but if reasonable strength and stiffness are critical, the maximum attainable reduction is usually around 30%.

Better Fire Resistance. Individual types of stone have different actions when exposed to the extreme heat of fires. Specific materials may be selected for this property and may be natural stone or a synthetic product.

Fiber Aggregate. Fibrous materials may be added to concrete, usually for the increased tension resistance they provide for the concrete itself. For concrete structures, these usually constitute a small volume and generally do not reduce the required steel reinforcement any significant amount. Reduction of surface cracking on exposed structures is one major advantage gained.

Use of Local Materials. Transporting coarse aggregate materials for great distances is usually prohibitive. There is thus a great advantage in using local materials.

Clam shells and crushed glass have been used for some projects. In some cases, it may be necessary to settle for the best available stone and to limit the quality of concrete to some minimal quality for design.

Additions to the Basic Concrete Mix

Substances added to concrete to improve its workability, accelerate its set, harden its surface, and increase its waterproof qualities are known as *admixtures.* The term embraces all materials other than the cement, water, and aggregates that are added during mixing.

Air-entrained concrete is produced by using special cement or an admixture to produce microscopic bubbles throughout the concrete mass. These minute voids enhance resistance to freezing, improve workability of the wet mix, and permit lower water content for improved strength and durability of the hardened concrete.

Significant Structural Properties of Concrete

Strength

The primary index of strength of concrete is the *specified compressive strength,* designated f'_c. This is the key value used in the design of structural elements. It is usually given in units of psi, with the quality of the concrete ordinarily referred to with the numerical unit alone: 3000 lb concrete, for example. Allowable stresses are mostly based on this value, and strength design uses it singularly in expressing resistance of structural elements.

Hardness

The hardness of concrete refers essentially to its surface density. This is dependent primarily on its basic strength. However, surfaces may often be softer than the central mass of concrete, owing to early drying at the surfaces. Some techniques are used to deliberately harden surfaces, especially those of the tops of pavements and spanning slabs. Fine troweling will tend to draw a very cement-rich material to the surface, resulting in enhanced local density. Chemical hardeners can also be used as well as sealing compounds that trap surface water and slow the drying of the concrete mass in general. However, the most commonly used method is to simply keep the exposed surfaces wet for as long as possible during the curing period.

Stiffness

Stiffness of structural materials is a measure of the resistance to deformation under stress. For tension and compression stress resistance, stiffness is measured by the *modulus of elasticity*, designated E. The value of E is established by tests and is the ratio of unit stress to unit strain. Since unit strain has no unit designation (expressed as a percentage, e.g., inch per inch), the unit for E thus becomes that used for stress, usually psi or ksi.

The magnitude of the modulus of elasticity for concrete, E_c, depends on the density (weight) of the concrete and its strength. For values of unit weight between 90 and 155 pcf, the value for E_c is

$$E_c = w^{1.5} 33\sqrt{f'_c}$$

The unit weight for ordinary stone−aggregate concrete is usually assumed to be an average of 145 pcf. Using this value in the equation produces a simpler value for the concrete modulus of

$$E_c = 57{,}000\sqrt{f'_c}$$

For metric units, with stress measured in megapaseals, the expression becomes

$$E_c = 4730\sqrt{f'_c}$$

Variation of the distribution of stresses and strains in reinforced concrete is dependent on the concrete modulus, the steel modulus being a constant of 29,000 ksi. This issue is discussed in Section 6.2. A factor sometimes used is the ratio of the steel modulus to the concrete modulus, designated n; thus $n = E_s/E_c$.

Creep

When subjected to long-duration stress at a high level, concrete has a tendency to *creep*, a phenomenon in which strain increases over time under constant stress. This has effects on deflection and on the distribution of stresses between the concrete and the reinforcement. Some of the implications of this are discussed in the sections that deal with beams and columns.

Other Significant Properties of Concrete

In addition to the basic structural properties, there are various properties of concrete that bear on its use as a construction material and in some cases on its structural integrity.

Workability

This term refers to the ability of the wet concrete to be handled, placed in forms, and finished while still in a semifluid state. A certain degree of workability is essential to the proper casting and finishing of the material. However, the fluid nature of the mix is largely determined by the amount of water present, and the easiest way to make it more workable is to add water. Up to a point, this may be acceptable, but the extra water usually means less strength, greater porosity, and more shrinkage—all generally not desirable properties. Use is made of vibration, admixtures, and other techniques to facilitate handling without increasing the water content.

Watertightness

Raw concrete is porous; it soaks up water and allows it to pass through the mass of the concrete—the rate of water seepage is called the porosity. However, porosity occurs in various degrees, it being generally desirable not to have an excessively porous finished concrete. In general, well-mixed, high-quality concrete is less porous. Nevertheless, moisture or waterproof barriers must be used where water penetration must be prevented.

Fire Resistance

Concrete is noncombustible and its insulative, fire protection character is used to protect the steel reinforcement. However, under long exposure to fire, popping and cracking of the material will occur, resulting in actual structural collapse or a much diminished capacity that requires repair or replacement after a major fire. Design for fire resistance involves the following basic concerns:

Thickness of Parts. Thin walls or slabs may crack quickly, permitting penetration and spread of fire and gases.

Cover of Reinforcement. More insulative protection by the concrete is required for higher fire rating of the construction.

Character of the Aggregate. Some stone materials are more vulnerable to high temperatures.

Shrinkage

Water-mixed materials such as plaster, mortar, and concrete tend to shrink during the hardening process. For ordinary concrete, the shrinkage averages about 2% of the volume. Actual dimensional change of structural members is usually less due to the presence of steel bars; however, some consideration must be given to the shrinkage effects. Stresses caused by shrinkage are in some ways similar to those caused by thermal change, the combination resulting in specifications for minimum two-way reinforcement in walls and slabs. For the structure in general, shrinkage is usually dealt with by limiting the extent of the individual pours of concrete because the major shrinkage occurs quite rapidly as the concrete hardens. For special situations, it is possible to modify the concrete with admixtures or special cements that cause a slight expansion to compensate for the usual shrinkage.

Steel Reinforcement

The steel used in reinforced concrete consists of round bars, mostly of the deformed type, with lugs or projections on their surfaces. The surface deformations help to develop a better bond between the bars and the enclosing concrete mass. The essential purpose of steel reinforcement is to reduce cracking of the concrete due to tension stress. Structural actions are investigated for the development of tension in the structural members, and steel reinforcement in the proper amount is placed within the concrete mass to resist the tension. In some situations reinforcement may also be used to increase compressive resistance since the ratio of magnitudes of strength of the two materials is quite high; thus, the steel displaces a much weaker material and the member gains significant strength.

Tension stress can also be induced by the shrinkage of the concrete during its drying out from the initial wet mix. Temperature variations may also induce tension in many situations. A minimum amount of reinforcement is therefore used in surface-type members, such as walls and slabs, even if no specific structural action is computed.

Yield strength of the ductile steel used for reinforcement is the primary structural property of concern. Reinforcing bars are specified by yield strength, common grades used being 40, 50, and 60 ksi. For columns, higher grades are sometimes used, usually in combination with very high concrete strengths.

The yield strength of the steel is of primary interest for two reasons. Plastic yielding of the steel generally represents the limit of its practical utilization for reinforcing of the concrete because the extensive deformation of steel in its plastic range results in major cracking of the concrete. Thus, for service load conditions, it is desirable to keep the stress in the steel within its elastic range of behavior where deformations are minimal.

Standard reinforcing bars are produced in a range of sizes identified by a number, the range being from No. 3 to No. 18. See Table 6.1. For bars numbered from 3 through 8, the cross-sectional area is equivalent to a round bar having a diameter of as many eighths of an inch as the bar number. Thus, a No. 4 bar is equivalent to a round bar of $\frac{4}{8}$, or 0.5, in. in diameter. Bars numbered from 9 up lose this identity and are essentially identified by the tabulated properties in a referenced document, such as the American Concrete Institute (ACI) code (Ref. 16).

The bars in Table 6.1 are developed in U.S. units but can, of course, be used with their properties converted to metric units. However, a new set of bars has been developed, deriving their sizes more logically from metric units. The work in this book uses the inch-based bars.

General Requirements for Steel Reinforcement

The following are some general requirements for steel reinforcement in reinforced concrete structures. Specific requirements for individual types of structural elements are also presented in other parts of this chapter.

Minimum Reinforcement

In the design of most reinforced concrete members, the amount of steel reinforcement required is determined from

Table 6.1 Properties of Deformed Reinforcing Bars

Bar Size Designation	Nominal Weight		Nominal Dimensions: Diameter		Cross-Sectional Area	
	lb/ft	kg/m	in.	mm	in.^{2}	mm^2
No.3	0.376	0.560	0.375	9.5	0.11	71
No.4	0.668	0.994	0.500	12.7	0.20	129
No.5	1.043	1.552	0.625	15.9	0.31	200
No.6	1.502	2.235	0.750	19.1	0.44	284
No.7	2.044	3.042	0.875	22.2	0.60	387
No.8	2.670	3.974	1.000	25.4	0.79	510
No.9	3.400	5.060	1.128	28.7	1.00	645
No.10	4.303	6.404	1.270	32.3	1.27	819
No.11	5.313	7.907	1.410	35.8	1.56	1006
No.14	7.650	11.390	1.693	43.0	2.25	1452
No.18	13.600	20.240	2.257	57.3	4.00	2581

computations and represents the amount determined to be necessary to resist the force in the member. In various situations, however, there is a minimum reinforcement required which may exceed that determined by computations. This applies to beams, spanning slabs, columns, and walls. The required minimum reinforcement may be specified as a minimum percentage of the member cross-sectional area, as a minimum number of bars, or as a minimum bar size.

Shrinkage and Temperature Reinforcement

The basic purpose of steel reinforcement is to prevent the cracking of the concrete due to tension stresses. Investigation is made for structural actions that will produce tension stress, primarily the actions of bending, shear, and torsion. However, tension is also induced by shrinkage and temperature variations in the monolithic concrete structural mass. Reinforcing of members of the structure will help for this, but it is a three-dimensional concern, which is not always addressed in basic structural actions. Thus, in walls, spanning slabs, and slabs used for pavements, reinforcement in two directions is required, with quantities specified by design codes.

Cover

The steel reinforcement needs to be fully engaged by the concrete mass with which it interacts. It also needs protection from fire and from the air and water combination that causes rusting. The dimension from the edge of a reinforcing bar to the outside face of the concrete, called the *cover*, provides for these concerns. The amount of cover required for various situations is specified by the design codes.

Spacing of Steel Bars

Where multiple bars are used in members (which is the common situation), there are both upper and lower limits for the spacing of the bars. Lower limits are intended to permit adequate development of the concrete-to-steel stress transfers and to facilitate the flow of the wet concrete during pouring. For columns, the minimum clear distance between bars is 1.5 times the bar diameter or a minimum of 1.5 in. For other situations the minimum is one bar diameter, or a minimum of 1 in.

For walls and slabs, maximum center-to-center spacing is specified as three times the wall or slab thickness or a maximum of 18 in. This applies to reinforcement required for computed stresses. For shrinkage and temperature reinforcement, the maximum spacing is five times the thickness or a maximum of 18 in.

For adequate flow of the concrete during pouring, the largest size of the coarse aggregate should not be greater than three fourths of the clear distance between bars.

Bending of Reinforcement

In various situations, it is necessary to bend reinforcing bars. Bending should be done in the highly controlled conditions in the fabricating shop instead of at the job site. The bend diameter (Figure 6.7) should be adequate to avoid cracking of the bars. As the yield stress of the bars increases, bending—which involves yield stress development—becomes increasingly difficult.

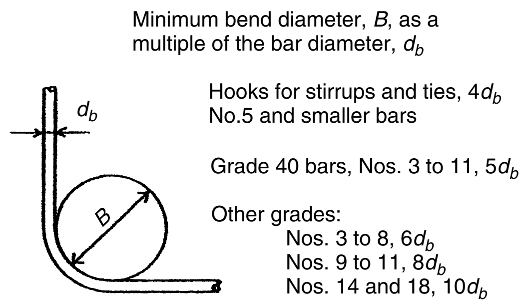


Figure 6.7 Bending requirements for steel reinforcing bars.

Bending should be avoided when the yield stress exceeds 60 ksi and, when necessary, should be done with diameters slightly greater that those given in Figure 6.7. The largest sizes of bars are mostly produced with high-yield steel, and the combination of bar size and yield make for very difficult bending. Fabricators should be consulted regarding the feasibility of the bending of large bars.

Bending of bars is sometimes done in order to secure anchorage for the bars. The code defines such a bend as a "standard hook," and the requirements for the details of this type of bend are discussed in Section 6.2 under the topic of bar development.

Design Process for Concrete Structures

Use of strength design for concrete structures has been well in place for many years. General development of the load and resistance factor design (LRFD) method was largely first developed for concrete, although it is now fully developed for almost all structural design work.

The LRFD method in general is described in Section 10.1. It involves primarily the use of load factors and specified load combinations for determining design loads and the use of resistance factors to qualify resistance of structural members to various force actions.

Resistance Factors

For the resistance factor method, a structural member's ultimate resistance is first determined in a form of unit that relates to the ultimate load-generated action required. This resistance value is then modified (mostly reduced) by the resistance factor applicable to the type of structural action, the type of structural member, and the material of the structure. There are thus different factors for wood, steel, and concrete and for the various conditions of behavior or type of structural element involved. For concrete, the resistance factor ϕ is as follows:

0.90 for flexure, axial tension, and combinations of flexure and tension

0.75 for columns with spirals
0.70 for columns with ties
0.85 for shear and torsion
0.70 for compression bearing
0.65 for other reinforced members
0.55 for plain (not reinforced) concrete

Application of these factors is illustrated in the following sections of this chapter.

6.2 REINFORCED CONCRETE FLEXURAL MEMBERS

The primary concerns for flexural members (beams and spanning slabs) relate to their necessary resistance to bending and shear and some limitations on their deflection. For wood or steel beams, the usual concerns are only for the singular maximum values of bending and shear in a given beam. For concrete members, on the other hand, it is necessary to provide for the values of bending and shear as they vary along the entire length of the member.

This situation is true even through multiple spans in the case of continuous beams, which are a common occurrence in concrete structures. For simplification in design work, it is necessary to consider the actions of a beam at a specific location along its length, but one must bear in mind that this singular action must be integrated with all the other effects on the beam throughout its length.

When a member is subjected to bending, such as the beam shown in Figure 6.8*a*, internal resistive actions of two basic kinds are required. Internal actions are visualized with a cut section, such as that taken at X–X in Figure 6.8*a*. When the portion of the beam to the left of the cut section is removed, its free-body actions are as shown in Figure 6.8*b*. At the cut section, consideration of static equilibrium requires the development of the internal shear force V and the internal bending moment M, with the moment generated by the opposition of the net compression force C and the net tension force T, as shown in the figure.

If the beam in Figure 6.8*a* is a typical reinforced concrete member with a section as shown in Figure 6.8*c*, the net compression force C is developed by compression stresses distributed on the shaded upper portion of the beam. The net tension force T is largely vested in the steel reinforcement. As in all flexural members, there is a neutral axis in the cross section where the internal stresses reverse from compression to tension.

Concrete Spanning Structures

Spanning structures function basically as shown in Figure 6.8, developing internal shear and bending moment in opposition to the external effects of the loads and support reactions. For study purposes, we will use the ordinary beam for considerations of the basic effects of internal shear and bending. However, these actions also occur in spanning slabs and in columns subjected to combined compression and bending; these cases will be considered in later discussions.

Behavior of Reinforced Concrete Beams

If a beam consists of a simple rectangular section with tension reinforcing only, as shown in Figure 6.8*c*, the development of the internal bending resistance is developed by opposition of the force couple: C and T. The compression force is vested in some distribution of compression stress on the portion of the section above the neutral axis of the cross section. Although the concrete does develop a low level of tension stress near the neutral axis, this is generally ignored and the tension force is considered to be concentrated at the location of the steel bars. Quantification of the tension force is simply expressed as the product of the steel stress times its cross-sectional area. Quantification of the compression is a bit more complex.

At moderate levels of stress, the internal resisting moment is visualized as shown in Figure 6.9*a*, with a linear variation of compressive stress from zero at the neutral axis to a maximum

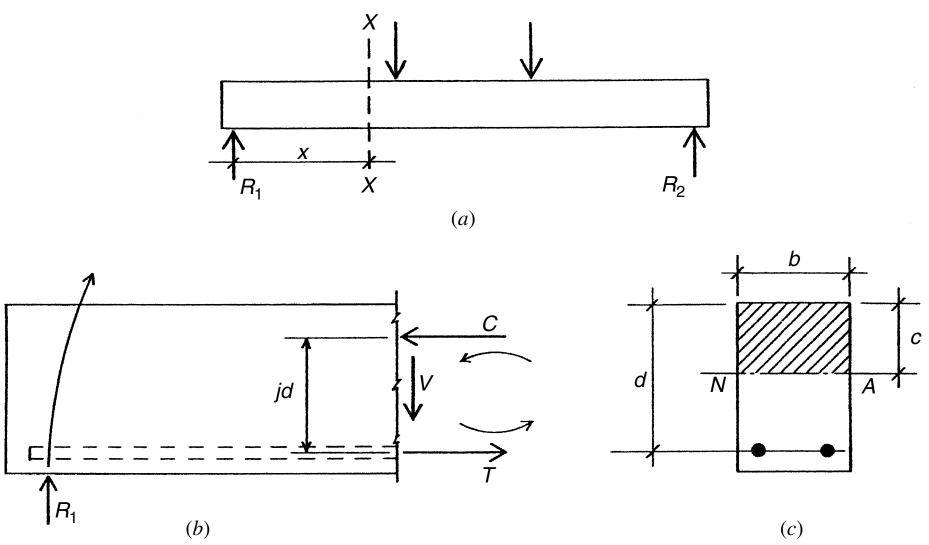


Figure 6.8 Bending action in a reinforced concrete beam.

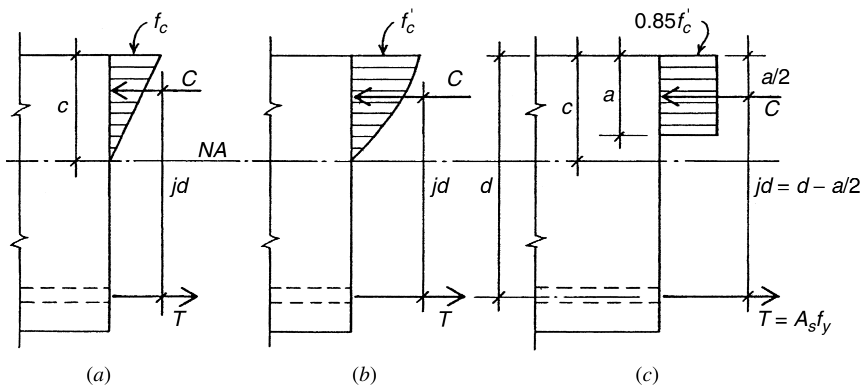


Figure 6.9 Development of bending stress actions in a reinforced concrete beam.

value of f_c at the top edge of the section. As stress levels increase, however, the nonlinear stress–strain character of concrete becomes more significant, and it becomes necessary to acknowledge a more realistic form for the compression stress variation, such as that shown in Figure 6.9*b*.

As stress levels approach the limit of the concrete, the compression becomes vested in almost constant magnitude of unit stress, concentrated near the top of the section. With strength design, for the ultimate moment capacity, it is common to assume the form of stress distribution shown in Figure 6.9*c*, with the limit for the concrete stress set at $0.85f'_c$.

Response of the steel reinforcement is more simply visualized and expressed. Since the steel in tension is concentrated at a small location with respect to the size of the beam, the stress in the bars is considered to be a constant. Thus, at any level of stress, the total value of the internal tension force may be expressed as

$$T = A_s f_s$$

and for the practical ultimate limit of T,

$$T = A_s f_y$$

Investigation and Design for Flexure

The following is a presentation of the formulas and procedures used in the strength method. The discussion is limited to a rectangular beam section with tension reinforcement only. Referring to Figure 6.10, the following are defined:

b = width of concrete compression zone

d = effective depth of section for stress analysis; from the center of the steel to the edge of the compression zone

h = overall depth (height) of section

A_s = cross-sectional area of steel bars

p = percentage of reinforcement, defined as

$$p = \frac{A_s}{bd}$$

Figure 6.9*c* shows the rectangular "stress block" that is used for analysis of the rectangular section with tension reinforcement only by the strength method. This is the basis for investigation and design as provided for in the ACI code (Ref. 16).

The rectangular stress block is based on the assumption that a concrete stress of $0.85f'_c$ is uniformly distributed over the compression zone, which has dimensions equal to the beam width b and the distance a which locates a line parallel to and above the neutral axis. The value of a is determined from the expression $a = \beta_1 \times c$, where β_1 (beta one) is a factor that varies with the compression strength of the concrete and c is the distance from the extreme fiber to the neutral axis (see Figure 6.9*c*). For concrete having f'_c equal to or less than 4000 psi, the ACI code gives a maximum value for a of $0.85c$.

With the rectangular stress block, the magnitude of the compression force in the concrete is expressed as

$$C = (0.85f'_c)(b)(a)$$

and it acts at a distance of $a/2$ from the top of the beam.

The moment arm of the resisting force couple then becomes $d - (a/2)$, and the developed resisting moment as governed by the concrete is

$$M_c = C\left(d - \frac{a}{2}\right) = 0.85f'_c ba\left(d - \frac{a}{2}\right)$$

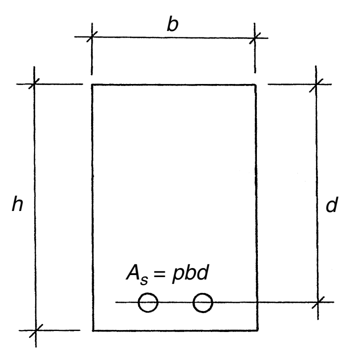


Figure 6.10 Reference for beam notation.

With T expressed as $A_s f_y$, the developed moment as governed by the reinforcement is

$$M_t = T\left(d - \frac{a}{2}\right) = A_s f_y \left(d - \frac{a}{2}\right)$$

A formula for the dimension a of the stress block can be derived by equating the compression and tension forces; thus,

$$0.85 f_c' ba = A_s f_y, \quad a = \frac{A_s f_y}{0.85 f_c' b}$$

By expressing the area of steel in terms of a percentage p, the formula for a may be modified as follows:

$$p = \frac{A_s}{bd}, \quad A_s = pbd$$

$$a = \frac{(pbd) f_y}{0.85 f_c' b} = \frac{pdf_y}{0.85 f_c'} \quad \text{or} \quad \frac{a}{d} = \frac{pf_y}{0.85 f_c'}$$

A useful reference is the so-called *balanced section*, which occurs when the exact amount of reinforcement results in the simultaneous development of the limiting stresses in the concrete and steel. Actually, the balanced section for strength design is visualized in terms of strain rather than stress. The limit for a balanced section is expressed in the form of the percentage of steel required to produce balanced conditions. The formula for this percentage is

$$p_b = \frac{0.85 f_c'}{f_y} \times \frac{87}{87 + f_y}$$

in which f_c' and f_y are in units of ksi.

Returning to the formula for the developed resisting moment, as expressed in terms of the steel, we can derive a useful formula as follows:

$$\begin{aligned} M_t &= A_s f_y \left(d - \frac{a}{2}\right) \\ &= (pbd)(f_y)\left(d - \frac{a}{2}\right) \\ &= (pbd)(f_y)(d)\left(1 - \frac{a}{2d}\right) \\ &= (bd^2)\left[pf_y\left(1 - \frac{a}{2d}\right)\right] \end{aligned}$$

Thus,

$$M_t = Rbd^2$$

where

$$R = pf_y\left(1 - \frac{a}{2d}\right)$$

With the resistance factor ϕ applied, the design moment for a section is limited to nine-tenths of the theoretical resisting moment.

Values for the balanced section factors (p, R, and a/d) are given in Table 6.2 for various combinations of f_c' and f_y. The balanced section is not necessarily a practical one for design. In most cases, economy will be achieved by using less than the balanced reinforcement for a given concrete section.

Table 6.2 Balanced Section Properties for Rectangular Sections with Tension Reinforcement Only

f_y		f_c'				R	
ksi	MPa	ksi	MPa	p	a/d	ksi	kPa
40	276	2	13.8	0.0291	0.685	0.766	5280
		3	20.7	0.0437	0.685	1.149	7920
		4	27.6	0.0582	0.685	1.531	10600
		5	34.5	0.0728	0.685	1.914	13200
60	414	2	13.8	0.0168	0.592	0.708	4890
		3	20.7	0.0252	0.592	1.063	7330
		4	27.6	0.0335	0.592	1.417	9770
		5	34.5	0.0419	0.592	1.771	12200

In special circumstances it may also be possible, or even desirable, to use compressive reinforcement in addition to the tension reinforcement. Nevertheless, the balanced section is often a useful point of reference for design work.

Beams with reinforcement less than that required for balanced conditions are called *underbalanced sections* or *underreinforced sections*. If a beam must carry moment in excess of the balanced moment for the section, it is necessary to provide some compressive reinforcement. This is a very rare circumstance, as the balanced section is already quite heavily reinforced.

In the design of concrete beams, two situations commonly occur. The first occurs when the beam is entirely undetermined; that is, both the concrete section dimensions and the reinforcement must be established.

The second situation occurs when the concrete sections are determined, but the reinforcement of the section must be determined for a specific bending moment. This is a quite frequent situation, since most concrete beams are continuous through more than a single span, and the requirement of reinforcement must be determined at several points along the beam length.

The following examples illustrate the procedures for both of these situations.

Example 1. The service load bending moments on a beam are 58 kip-ft [78.6 kN-m] for dead load and 38 kip-ft [51.5 kN-m] for live load. The beam is 10 in. wide [254 mm], f_c' is 3000 psi [20.7 MPa], and f_y is 60 ksi [414 MPa]. Determine the depth of the beam and the tension reinforcing required.

Solution. The first step is to determine the required moment using the load factor. Thus,

$$\begin{aligned} U &= 1.2(\mathrm{D}) + 1.6(\mathrm{L}) \\ M_u &= 1.2(M_{\mathrm{DL}}) + 1.6(M_{\mathrm{LL}}) \\ &= 1.2(58) + 1.6(38) \\ &= 130.4 \text{ kip-ft } [177 \text{ kN-m}] \end{aligned}$$

With capacity reduction applied (resistance factor $\phi = 0.90$), the desired moment capacity of the section is determined as

$$M_t = \frac{M_u}{0.90} = \frac{130.4}{0.90} = 145 \text{ kip-ft } [197 \text{ kN-m}]$$

or

$$145 \times 12 = 1740 \text{ kip-in.}$$

The reinforcement ratio for a balanced section, as given in Table 6.2, is $p = 0.0252$. If the balanced section is used, the area for steel may thus be determined as $A_s pbd$. Using the value for R from Table 6.2, the required depth for a balanced section can be determined as follows:

$$M_t = Rbd^2 = 1.063(bd^2)$$

This formula can be transposed to yield a formula for the required depth of the section; thus,

$$d = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{1740}{1.063 \times 10}} = 12.8 \text{ in. } [325 \text{ mm}]$$

If this value is used for d, the required steel area may be found using the value for p from Table 6.2; thus,

$$A_s = pbd = (0.0252)(10 \times 12.8) = 3.23 \text{ in.}^2 \ [2084 \text{ mm}^2]$$

There may well be other considerations for selection of the section dimensions in a real design situation. However, for this exercise we will consider only the data given. If the beam is of the ordinary form shown in Figure 6.11, the specified construction dimension is usually that given as h. Assuming the use of a No. 3 U-stirrup, a cover of 1.5 in., and an average bar size of 1 in. diameter, the design dimension d will be less than h by 2.375 in. Lacking other considerations, the overall height h will be chosen as 16 in.

Next, consider the selection of a set of bars to obtain the required area. For the purpose of the example, select bars all of a single size (see Table 6.1); the number required will be:

No. 6 bars: $3.23/0.44 = 7.3$, or 8
No. 7 bars: $3.23/0.60 = 5.4$, or 6
No. 8 bars: $3.23/0.79 = 4.1$, or 5
No. 9 bars: $3.23/1.00 = 3.3$, or 4
No. 10 bars: $3.23/1.27 = 2.5$, or 3
No. 11 bars: $3.23/156 = 2.1$, or 3

In real design situations, there are always additional considerations that influence the choice of the bars. One general desire is that of having the bars in a single layer, as this keeps the centroid of the steel as close as possible to the edge (bottom in this case) of the member, giving the greatest value for d with a given height h. With the section as shown in Figure 6.11, a beam width of 10 in. will yield a net width of 6.25 in. inside the No. 3 stirrups. Applying the code criteria for minimum spacing, the required width for the various bar combinations can be determined. Two examples for this are shown in Figure 6.12. It will be found that none of the possible bar choices will fit; thus the beam width must be increased.

If there are reasons, as there often are, for not selecting the least deep section with the greatest amount of reinforcement, a slightly different procedure is used, as illustrated in the following example.

Example 2. Using the same data as in Example 1, find the reinforcement required if the beam section has $b = 10$ in. and $d = 18$ in.

Solution. The first two steps in this situation are the same as in Example 1—to determine M_u and M_t. The next step is to determine whether the given section is larger than, smaller than, or equal to a balanced section. Since this investigation has already been done in Example 1, we observe that the 10-by-18 in. section is larger than a balanced section. Thus, the actual value for a/d will be less than the balanced section of 0.592. We next select an estimated value for a/d; let us try 0.3. Then

$$a = 0.3d = 0.3(18) = 5.4 \text{ in. } [137 \text{ mm}]$$

With this value for a, we find a required value for A_s as follows: Referring to Figure 6.8,

$$M_t = T(jd) = (A_s f_y)\left(d - \frac{a}{2}\right)$$

$$A_s = \frac{M_t}{f_y(d - a/2)} = \frac{1740}{(60)(15.3)} = 1.89 \text{ in.}^2 \ [1220 \text{ mm}^2]$$

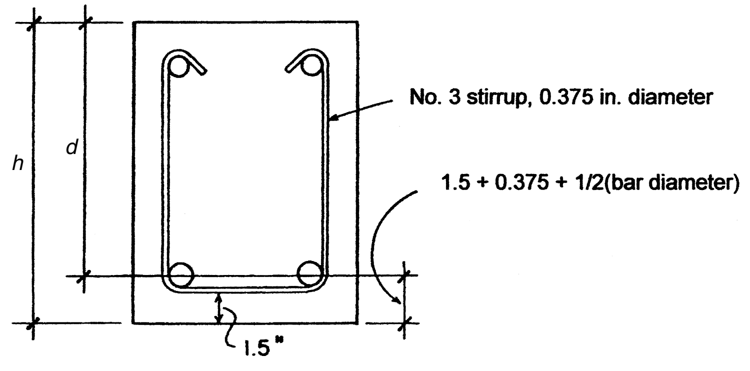


Figure 6.11 Common form of a reinforced concrete beam.

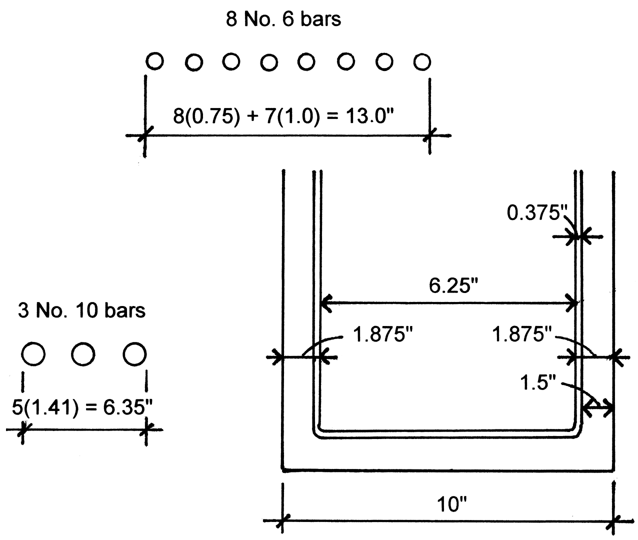


Figure 6.12 Consideration of beam width for proper spacing of a single layer of reinforcing bars.

Next, test to see if the estimate for *a/d* was close by finding *a/d* from the previously derived formula; thus,

$$p = \frac{A_s}{bd} = \frac{1.89}{10 \times 18} = 0.0105$$

$$a = \frac{pdf_y}{0.85f'_c} = \frac{(0.0105)(18)(60)}{(0.85)(3)} = 4.45 \text{ in.}$$

$$jd = d - \frac{a}{2} = 18 - \frac{4.45}{2} = 15.775 \text{ in. [399 mm]}$$

If this vale for $d - a/2$ is used to replace that used earlier, the required value for A_s will be slightly reduced. In this example, the correction will be only a few percent. If the change is significant, a second estimate should be made and a corrected area determined.

For beams that are classified as underreinforced (section dimensions larger than the limit for a balanced section), a check should be made for the minimum reinforcement. For a rectangular section, the ACI code specifies that a minimum area be

$$A_s = \frac{3\sqrt{f'_c}}{f_y}(bd)$$

but not less than

$$A_s = \frac{200}{f_y}(bd)$$

On the basis of these requirements, values for minimum reinforcement for rectangular sections with tension reinforcement only are given in Table 6.3 for two grades of steel and three concrete strengths.

For this example, with a concrete strength of 3000 psi and f_y of 60 ksi, the minimum area of steel is

$$A_s = 0.00333(bd) = 0.00333(10 \times 18) = 0.60 \text{ in.}^2 \text{ [387 mm}^2\text{]}$$

which is clearly not critical in this case.

Table 6.3 Minimum Required Tension Reinforcement for Rectangular Sections[a]

f'_c (psi)	$f_y = 40$ ksi	$f_y = 60$ ksi
3000	0.0050	0.00333
4000	0.0050	0.00333
5000	0.0053	0.00354

[a]Required A_s equals table value times *bd* of the beam section.

Beams in Sitecast Systems

In sitecast construction, it is common to cast as much of the total structure as possible in a single, continuous pour. The length of the workday, the size of the available work crew, and other factors may affect this decision. Other considerations involve the nature, size, and form of the structure. For example, a convenient single-cast unit may consist of the whole floor structure at one level for a multistory building if it can be cast in a single workday.

Planning of the concrete construction itself is a major design task. Sitecast work typically involves continuous beams and slabs, versus the common condition of simple-span elements in wood and steel construction. The design of continuous-span elements involves more complex investigation for behavior to determine the internal force actions.

In the upper part of Figure 6.13 the condition of a simple-span beam subjected to a uniformly distributed loading is

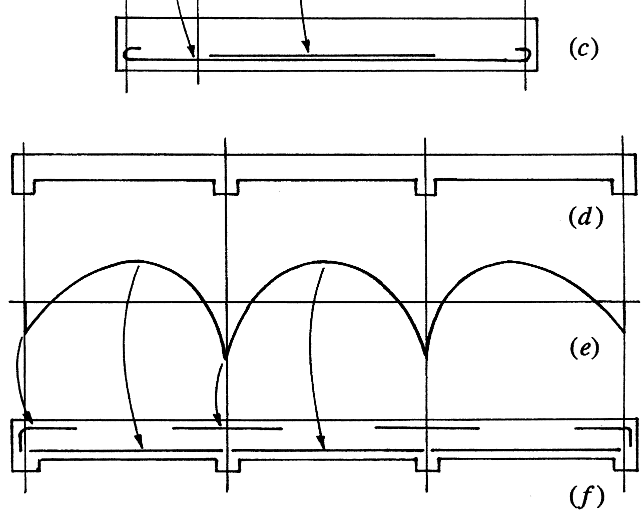


Figure 6.13 Utilization of reinforcement in concrete beams as related to the distribution of bending moments.

shown. The typical moment diagram showing the variation of bending moment along the beam length takes the form of a parabola, as shown in Figure 6.13*b*. As with a beam of any material, the maximum effort in bending resistance must respond to the maximum bending moment—here occurring at midspan. For a concrete beam, the concrete section and its reinforcement must be designed for this moment value. However, for a long span and a large beam with a lot of reinforcement, it may be possible to reduce the amount of steel at points nearer to the beam ends. That is, some bars may be full length in the beam, while some others are only partial length and occur only in the midspan portion of the beam (see Figure 6.13*c*).

Figure 6.13*d* shows the typical situation for a continuous beam in a sitecast slab and beam framing system. For a single uniformly distributed loading, the moment diagram takes the form as shown in Figure 6.13*e*, with positive moments near the beam's midspan and negative moments at the supports. Locations for reinforcement that respond to the sign of these moments are shown on the beam elevation in Figure 6.13*f*. Note that bottom bars are usually full length in the span while the top bars are only partial length.

For the continuous beam, it is obvious that separate requirements for the beam's moment resistance must be considered at each of the locations of peak values on the moment diagram. However, there are many additional concerns as well. Principal considerations include the following:

T-Beam Action. At points of positive moment (midspan) the slab and beam monolithic construction must be considered to function together, giving a T-shaped form for the portion of the beam section that resists compression.

Use of Compression Reinforcement. If the beam section is designed to resist the maximum bending moment with tension reinforcement only for the maximum bending moment, which occurs at only one point along the beam length, the section will be overstrong for all other locations. For this or other reasons, it may be advisable to use compression reinforcement to reduce the beam size at the singular points of maximum bending moment. Since these points ordinarily occur at supports, a simple way of providing the compression reinforcement is to extend the bottom bars through the supports.

Spanning Slabs. Design of sitecast beams must usually be done in conjunction with the design of the slabs they support. The slabs must span on their own but are also parts of the T-beam section for the beams.

Beam Shear. While consideration for bending is a major issue, beams must also be designed for shear effects. The usual design process includes design of the concrete for a portion of the shear resistance and, when that is not sufficient, to provide steel reinforcement. The most common form of reinforcement is the so-called *U-stirrup*, as shown in Figure 6.11.

Development of Reinforcement. This refers to the proper anchorage of the steel bars in the concrete so that their resistance to tension can be developed. At issue is the exact location of the end cutoffs of the bars and some details such as those for the hooked bar ends shown in Figures 6.13*c* and *f*.

T-Beams

When a slab and its supporting beams are cast at the same time, the result is a monolithic construction in which a portion of the slab on each side of the beam serves as the flange of a T-beam. The part of the section that projects below the slab is called the *web* or *stem* of the T-beam. This type of beam is shown in Figure 6.14*a*.

For positive moment, the top of the T-beam is in compression, with a range of possibilities for the location of the neutral axis, as shown in Figures 6.14*b* and *c*. In the case of negative moment, however, compression is in the bottom and there is essentially no T action; instead, the beam behaves as a rectangular section, as shown in Figure 6.14*d*. For a continuous beam, the dimensions of the T web are usually established on the basis of the negative-moment response.

The effective flange (slab) width b_f to be used in the design of the T-beam is limited to one-fourth the span length of the beam. In addition, the overhanging width on either side

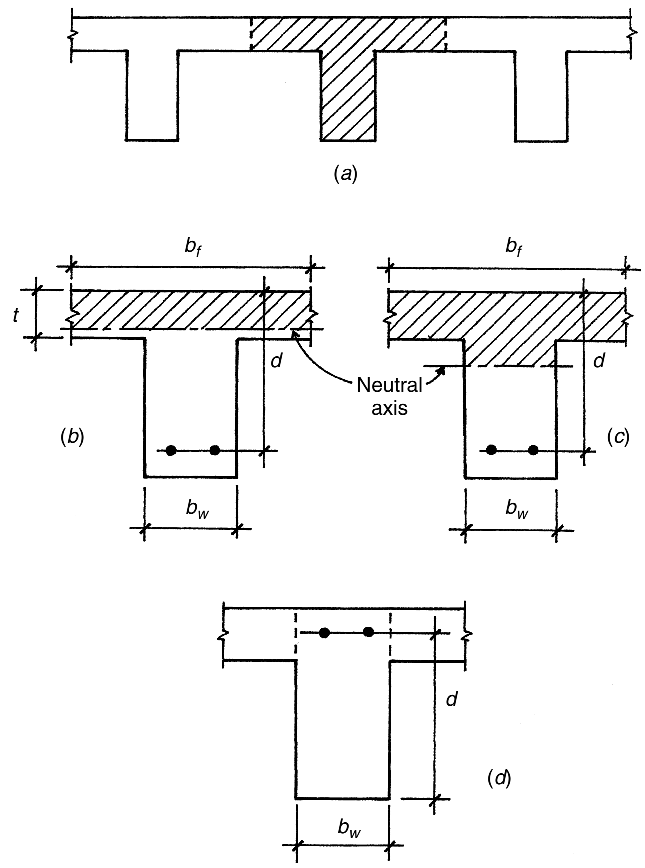


Figure 6.14 Considerations for T-beams.

of the web is limited to eight times the slab thickness, or one-half the clear distance between beam stems.

In monolithic construction with beams and one-way-spanning slabs, the T flange is usually quite thick and is quite capable of resisting the compressive stresses caused by positive bending moments. With a large flange area, the neutral axis usually occurs quite high in the beam web. The compressive force is thus vested primarily in the flange. An approximate analysis of the T section by the strength method that avoids the need to find the location of the neutral axis and the centroid of the trapezoidal stress zone consists of the following:

Determine the effective flange width.
Ignore compression in the web and assume a constant value of stress in the flange (see Figure 6.15).

Thus,

$$jd = d - \frac{t}{2}$$

Then, find the required steel area as

$$M_t = \frac{M_u}{0.9} = T(jd) = A_s f_y\left(d - \frac{t}{2}\right)$$

$$A_s = \frac{M_t}{f_y\,(d - t/2)}$$

To check the compression stress in the flange (assumed as a constant value), compare the computed value to the limit, as follows:

$$f_c = \frac{C}{b_f t} \leq 0.85 f_c'$$

where

$$C = \frac{M_r}{jd} = \frac{M_r}{d - t/2}$$

T-beams ordinarily function for positive moments in continuous beams. Because these moments are usually less than the negative moments at supports, the beam web is usually designed for the higher moment assuming a rectangular beam action. This usually results in T-beam actions that use very minimal reinforcement, thus making it necessary to consider the issue of required minimum reinforcement. The ACI code

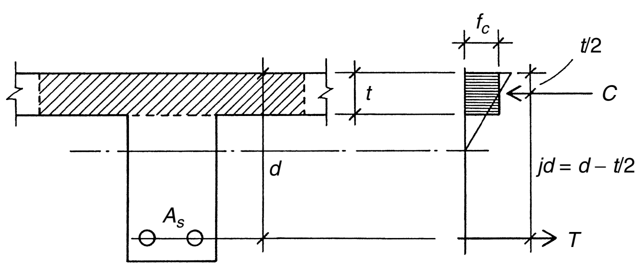


Figure 6.15 Simplified analysis of a T-beam.

provides special requirements for this for T-beams, for which the minimum area required is defined as the greater value of

$$A_s = \frac{6\sqrt{f_c'}}{f_y}\quad(b_w d)\quad\text{or}\quad A_s = \frac{3\sqrt{f_c'}}{f_y}\quad(b_f d)$$

where

b_w = width of beam web
b_f = effective width of flange

The following example illustrates the use of this procedure. It assumes a typical design situation in which the basic dimensions of the T are set by other considerations, and the design of the T itself is reduced to the determination of the required reinforcement.

Example 3. A T section is to be used for a beam to resist positive moment. The following data are given: beam span is 18 ft [5.49 m], beams are 9 ft [2.74 m] on center, slab thickness is 4 in. [102 mm], beam stem width is 15 in. [381 mm], effective depth d is 22 in. [559 mm], f_c' = 4 ksi [27.6 MPa], and f_y = 60 ksi [414 MPa]. Find the required area of steel and select the reinforcing bars for a dead-load moment of 125 kip-ft [170 kN-m} plus a live-load moment of 100 kip-ft [136 kN-m].

Solution. Determine the effective flange width b_f:

$$b_f = \frac{\text{span}}{4} = \frac{18 \times 12}{4} = 54\,\text{in.}$$

or

$$b_f = \text{ beam spacing } = 9 \times 12 = 108\,\text{in.}$$

or

$$b_f = b_w + 16t = 15 + (16 \times 4) = 79\,\text{in.}$$

The limiting value is 54 in. [1.37 m]. Next, find the required steel area:

$$M_u = 1.2(125) + 1.6(100) = 310\text{ kip-ft [420 kN-m]}$$

$$M_r = \frac{M_u}{0.9} = 344\text{ kip-ft [466 kN-m]}$$

$$A_s = \frac{M_r}{f_y\,(d - t/2)} = \frac{344 \times 12}{60\,(22 - 4/2)} = 3.44\,\text{in.}^2\ [2219\,\text{mm}^2]$$

Compare this to the minimum reinforcement required:

$$A_s = \frac{6\sqrt{f_c'}}{f_y}(b_w d) = \frac{6\sqrt{4000}}{60{,}000}(15 \times 22) = 2.09\,\text{in.}^2$$

or

$$A_s = \frac{3\sqrt{f_c'}}{f_y}(b_f d) = \frac{3\sqrt{4000}}{60{,}000}(54 \times 22) = 3.76\,\text{in.}^2$$

Thus, the minimum requirement prevails and the required area is 3.76 in.^{2} Optional choices for the bars of a single size

Table 6.4 Options for the T-Beam Reinforcement

Bar Size	No. of Bars	Actual Area Provided (in.^{2})	Width Required[a] (in.)
7	6	3.60	14
8	5	3.95	13
9	4	4.00	12
10	3	3.81	11
11	3	4.68	11

[a]From Table 6.11.

are given in Table 6.4. Note that the No. 7 bar choice requires a slightly wider section.

Check the concrete stress:

$$C = \frac{M_r}{jd} = \frac{344 \times 12}{20} = 206.4 \text{ kips}$$

$$f_c = \frac{C}{b_f t} = \frac{206.4}{54 \times 4} = 0.956 \text{ ksi [6.59 MPa]}$$

Compare this to the limiting stress of

$$0.85 f'_c = 0.85(4) = 3.4 \text{ ksi [23.4 MPa]}$$

Thus, compression stress is not critical.

The approximate investigation is reasonably adequate for beams that occur in ordinary sitecast slab and beam systems, where the T-beam flange (slab) thickness is usually not less than one-eighth of the beam depth. For very thin flanges, a more exact analysis is required.

Beams with Compression Reinforcement

There are many situations in which steel reinforcement is used on both sides of the neutral axis in a bending member. When this occurs, the steel on one side of the axis will be in tension and that on the other side in compression. Such a member is referred to as a *double reinforced section.* The most common occasions for such reinforcement include when:

The desired resisting moment exceeds that for which the concrete alone is capable of the necessary compression force.

Other functions of the section require the use of reinforcement on both sides of the member; such is the case for columns and members subjected to torsion.

It is desired to reduce deflections by increasing the stiffness of the compressive side of the member, an example being deflections caused by creep.

The combination of loading conditions on the structure result in reversal moments on the member section; that is, the section sometimes must resist one sign of moment and at other times moment of the opposite sign (for example, both positive and negative moments on a beam).

Anchorage requirements (for development of reinforcement) require that the bottom bars in a beam be extended a significant distance into the supports.

The precise investigation and accurate design of double-reinforced sections are quite complex and are beyond the scope of the work in this book. The following discussion presents an approximation method that is adequate for preliminary design of a double-reinforced section. For real design situations, this method may be used to establish a first trial design, which may then be more precisely investigated using more rigorous methods.

For the beam with double reinforcement, as shown in Figure 6.16*a*, consider the total resisting moment for the section to be the sum of the following two component moments:

M_1 (Figure 6.16*b*) is developed by a section with tension reinforcement only (A_{s1}). This section is subject to the usual procedures for design.

M_2 (Figure 6.16*c*) is developed by two opposed steel areas (A'_s and A_{s2}) that function in simple force couple action, similar to the flanges of a steel beam or the top and bottom chords of a truss.

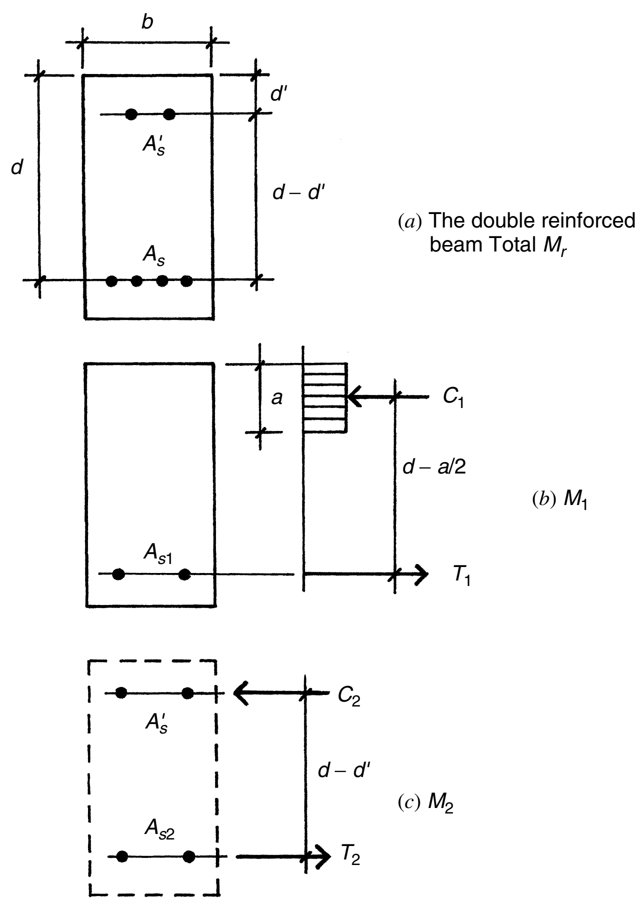


Figure 6.16 Basis for the simplified analysis of a double-reinforced beam.

The limit for M_1 is the so-called balanced section moment. Given the dimensions for the concrete section, this limit can be determined using the value of R from Table 6.2. If the capacity of the section as thus determined is less than the value of the required moment, compression reinforcement is required. Design then consists of a two-stage process to determine separately the values of A_{s1}, A'_s, and A_{s2}. If the balanced resistance is larger than the required moment, compression reinforcement is not required, although it may be used for any of the previously described reasons. The approximate procedure that follows starts with the assumption that the concrete dimensions describe a section whose balanced moment exceeds the required moment.

For this condition, it is really not necessary to find separate values for the tension reinforcement. This is because of an additional assumption that the value for a is equal to $2 \times d'$. Thus, the moment arms for both A_{s1} and A_{s2} are the same and the value for the total tension reinforcement can be easily determined as

$$A_s = \frac{\text{Required } M_r}{f_y \times (d - d')}$$

With the tension reinforcement established, the next step is to determine the compression reinforcement. Compression reinforcement in beams usually ranges from 0.2 to 0.4 times the total tension reinforcement. For this approximation method, we will determine the area to be $0.3A_s$.

The following example illustrates the procedure.

Example 4. A concrete section with $b = 18$ in. [457 mm] and $d = 21.5$ in. [546 mm] is required to resist service load moments as follows: dead load moment = 175 kip-ft [237 kN-m], live load moment = 160 kip-ft [217 kN-m]. Using strength methods, find the required reinforcement for a double-reinforced beam using $f'_c = 3$ ksi [20.7 MPa] and $f_y = 60$ ksi [414 MPa].

Solution. The required factored moment for the section is

$$M_u = 1.2(175) + 1.6(160) = 466 \text{ kip-ft } [632 \text{ kN-m}]$$

and the required factored resisting moment is

$$M_r = \frac{M_u}{0.9} = \frac{466}{0.9} = 518 \text{ kip-ft } [702 \text{ kN-m}]$$

Using $R = 1.063$ from Table 6.2, the balanced moment for the section is

$$M_b = Rbd^2 = \frac{1.063}{12}(18)(21.5)^2 = 737 \text{ kip-ft } [999 \text{ kN-m}]$$

As this is considerably larger than the required moment, it is reasonable to use the simplified procedure; thus, for the tension reinforcement

$$A_s = \frac{\text{required } M_r}{f_y(d - d')} = \frac{518 \times 12}{60(21.5 - 2.5)} = 5.45 \text{ in.}^2 \; [3520 \text{ mm}^2]$$

A reasonable assumption for compression reinforcement is

$$A'_s = 0.3A_s = 0.3(5.45) = 1.63 \text{ in.}^2 \; [1050 \text{ mm}^2]$$

Bar combinations may now be chosen for these areas.

Spanning Slabs

Concrete slabs are frequently used as spanning roof or floor decks, often occurring in sitecast slab and beam framing systems. There are two basic types of slabs: one-way spanning and two-way spanning. The spanning condition is not determined so much by the slab as by its support conditions. The following discussion relates to the design of one-way spanning slabs using procedures developed for the design of rectangular beams.

Slabs are designed by considering the slab to consist of a series of 12-in.-wide planks. Thus, the procedure consists of designing a beam section with a predetermined width of 12 in. Once the depth of the slab is established, the required area of steel is determined and specified on a per-foot-of-width basis.

Reinforcing bars are selected from a limited range of sizes, appropriate to the slab thickness. For thin slabs (4 to 6 in. thick), bars may be of a size from No. 3 to No. 6 or so. The bar size selection is related to the bar spacing, the combination resulting in the amount of area provided per foot of slab width. Spacing of the computed steel area is limited to a maximum of three times the slab thickness. There is no minimum spacing, other than that required for proper placing of the concrete during pouring. A very close spacing also results in a larger number of bars and a more laborious chore for the bar installers.

Every slab must be provided with two-way reinforcement, because of considerations for temperature change and concrete shrinkage. If computed reinforcing is required in only one direction, the reinforcing in the other direction is specified as a minimum percentage p, based on the gross cross section of the slab, as follows:

For grade 40 or grade 50 bars, $p = 0.002$.

For grade 60 bars, $p = 0.0018$

Center-to-center spacing of this minimum reinforcement is limited to five times the slab thickness or 18 in.

Minimum cover for slab reinforcement is normally 0.75 in., although exposure conditions or need for a higher fire rating may require additional cover. For a thin slab reinforced with large bars, there will be a considerable difference between the slab thickness t and the effective depth d, as shown in Figure 6.17. Thus, the practical efficiency of the slab in flexural resistance decreases rapidly as the slab thickness is decreased. For this and for other reasons, very thin slabs (less than 4 in. thick) are often reinforced with welded wire fabric rather than sets of loose bars.

Shear reinforcement is seldom used in slabs, which is generally not an issue since unit shear stress is usually quite low.

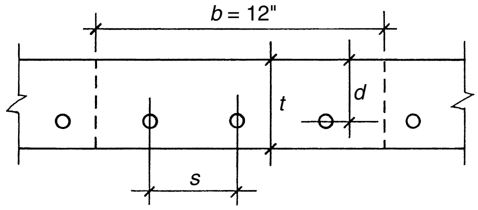


Figure 6.17 Reference for slab design.

Table 6.5 gives data that are useful for slab design, as illustrated in the following example. Table values indicate the average amount of steel area per foot of slab width that is provided by various combinations of bar size and spacing. Table entries are determined as follows:

$$A_s/\text{ft} = (\text{single bar area})\ \frac{12}{\text{bar spacing}}$$

Thus, for No. 5 bars at 8-in. centers,

$$A_s/\text{ft} = (0.31)\left(\frac{12}{8}\right) = 0.465\ \text{in.}^2/\text{ft}$$

The table entry for this combination is rounded off to 0.46 in.^{2}/ft.

Example 5. A one-way concrete slab is to be used for a simple span of 14 ft [4.27 m]. In addition to its own weight, the slab carries a superimposed dead load of 30 psf [1.44 kPa] plus a live load of 100 psf [4.79 kPa]. Using $f_c' = 3$ ksi [20.7 MPa] and $f_y = 40$ ksi [276 MPa], design the slab for minimum overall thickness.

Solution. Using the procedure for design of a rectangular beam, we first determine the required slab thickness. Thus, for deflection, from Table 6.7,

$$\text{Minimum } t = \frac{L}{25} = \frac{14 \times 12}{25} = 6.72\ \text{in.}\ [171\ \text{mm}]$$

For flexure, first determine the value of the maximum bending moment. The loading must include the weight of the slab, for which the thickness required for deflection may be used as an estimate. Assuming a 7-in.-thick slab, weight is (7/12)(150 pcf) = 87.5 psf, say 88 psf, and the total dead load is 30 + 88 = 118 psf [5.65 kPa]. The factored load is thus

$$U = 1.2(118) + 1.6(100) = 302\ \text{psf}\ [14.45\ \text{kPa}]$$

The maximum bending moment on a 12-in.-wide strip of slab thus becomes

$$M_u = \frac{wL^2}{8} = \frac{(302)(14)^2}{8} = 7399\ \text{ft-lb}\ [10.0\ \text{kN-m}]$$

and the required factored resisting moment is

$$M_r = \frac{7399}{0.9} = 8221\ \text{ft-lb}\ [11.2\ \text{kN-m}]$$

Table 6.5 Areas Provided By Spaced Reinforcement

Bar Spacing	Area Provided (in.^{2}/ft width)									
(in.)	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9	No. 10	No. 11
3	0.20	0.44	0.80	1.24	1.76	2.40	3.16	4.00		
3.5	0.17	0.38	0.69	1.06	1.51	2.06	2.71	3.43	4.35	
4	0.15	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4.5	0.13	0.29	0.53	0.83	1.17	1.60	2.11	2.67	3.39	4.16
5	0.12	0.26	0.48	0.74	1.06	1.44	1.89	2.40	3.05	3.74
5.5	0.11	0.24	0.44	0.68	0.96	1.31	1.72	2.18	2.77	3.40
6	0.10	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
7	0.08	0.19	0.34	0.53	0.75	1.03	1.35	1.71	2.18	2.67
8	0.07	0.16	0.30	0.46	0.66	0.90	1.18	1.50	1.90	2.34
9	0.07	0.15	0.27	0.41	0.59	0.80	1.05	1.33	1.69	2.08
10	0.06	0.13	0.24	0.37	0.53	0.72	0.95	1.20	1.52	1.87
11	0.05	0.12	0.22	0.34	0.48	0.65	0.86	1.09	1.38	1.70
12	0.05	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
13	0.05	0.10	0.18	0.29	0.40	0.55	0.73	0.92	1.17	1.44
14	0.04	0.09	0.17	0.27	0.38	0.51	0.68	0.86	1.09	1.34
15	0.04	0.09	0.16	0.25	0.35	0.48	0.63	0.80	1.01	1.25
16	0.04	0.08	0.15	0.23	0.33	0.45	0.59	0.75	0.95	1.17
18	0.03	0.07	0.13	0.21	0.29	0.40	0.53	0.67	0.85	1.04
24	0.02	0.05	0.10	0.15	0.22	0.30	0.39	0.50	0.63	0.78

For a balanced section, Table 6.2 yields the following properties: $a/d = 0.685$ and $R = 1.149$ (in kip and inch units). Then the minimum value for bd^2 is

$$bd^2 = \frac{M_r}{R} = \frac{8.221 \times 12}{1.149} = 85.9\,\text{in.}^3\ [1{,}400{,}000\,\text{mm}^3]$$

and, since $b = 12$ in.,

$$d = \sqrt{\frac{85.9}{12}} = \sqrt{7.16} = 2.68\ \text{in.}\ [68\,\text{mm}]$$

Assuming a No. 6 bar and cover of $\frac{3}{4}$ in., the minimum required slab thickness based on flexure becomes

$$t = d + \frac{\text{bar diameter}}{2} + \text{cover}$$
$$= 2.68 + 0.375 + 0.75 = 3.8\,\text{in.}$$

This means that the deflection limit controls and the minimum value for t is 6.72 in. Staying with the 7-in. overall thickness, the actual effective depth with a No. 6 bar is

$$d = 7.0 - 1.125 = 5.875\,\text{in.}\ [149\,\text{mm}]$$

Since this d is larger than that required for a balanced section, the value for a/d will be smaller than 0.685, as found in Table 6.2. Assume a value of 0.4 for a/d and determine the required area of reinforcement as follows:

$$a = 0.4d = 0.4(5.875) = 2.35\,\text{in.}$$
$$A_s = \frac{M_r}{f_y\,(d - a/2)} = \frac{8.221 \times 12}{40(5.875 - 1.175)} = 0.525\,\text{in.}^2$$

Using data from Table 6.5, the optional bar combinations are shown in Table 6.6. Spacing values shown are within the requirements.

For the temperature and shrinkage reinforcement the required area is

$$A_s = 0.0020(7 \times 12) = 0.168\,\text{in.}^2/\text{ft}$$

From Table 6.5, this requirement can be satisfied with No. 3 bars at 7-in. centers or No. 4 bars at 14-in. centers.

Although simply supported slabs are sometimes encountered, the majority of slabs occur in slab-and-beam systems, with the slabs continuous through several spans. Design of such a slab is given in the example case in Section 10.8.

Table 6.6 Alternatives for the Slab Reinforcement

Bar Size	Spacing of Bars Center to Center (in.)	Average A_s in a 12-in. Width
5	7	0.53
6	10	0.53
7	13	0.55
8	18	0.53

Deflection Control

Deflection of spanning slabs and beams in sitecast systems is controlled primarily through use of recommended thicknesses expressed as a percentage of the span. Table 6.7 is adapted from a table in The ACI code (Ref. 16) and yields minimum thickness as a fraction of the span. Table values apply only for concrete of normal weight (made with sand and gravel) and for reinforcement with yield strengths of 40 and 60 ksi.

Deflection of concrete structures presents a number of problems. Flexural action normally results in some tension cracking of the concrete at points of maximum bending moment. Thus, the presence of cracks in the bottom of beams at midspan points and in the top over supports is to be expected. If visible cracking is objectionable, more conservative depth-to-span ratios should be used.

Creep of concrete results in additional deflections over time. This is caused by the sustained loads—essentially the dead load of the construction. Creep, as well as other deflection, can be partly reduced by use of compressive reinforcement.

Shear in Beams

From general considerations, as developed in the science of mechanics of materials, the following observations can be made:

- Shear is an ever-present phenomenon produced directly by slicing actions, by lateral loading in beams, and on oblique sections in tension and compression members.
- Shear forces produce shear stress in the plane of the force and equal unit shear stresses in planes that are perpendicular to the shear force.
- Diagonal stresses of tension and compression, having magnitudes equal to that of the shear stress, are produced in directions of 45° from the plane of the shear force.
- Direct slicing shear force produces a constant magnitude of shear stress on affected sections, but beam shear action produces shear stress that varies on the affected sections, having magnitude of zero at the edges of the section and a maximum value at the centroidal axis of the section.

In the discussions that follow it is assumed that the reader has a general familiarity with these relationships.

Consider the case of a simple beam with uniformly distributed load and end supports that provide vertical support but no rotational resistance. The distribution of internal shear force and bending moment are shown in Figure 6.18*a*.

For flexural resistance, it is necessary to provide longitudinal reinforcing bars near the bottom of the beam. These bars are oriented for primary effectiveness in resistance to tension stresses that develop on a vertical plane, which is the case at the center of the span, where the bending moment is a maximum and the shear approaches zero.

Under the combined effects of shear and bending, the beam tends to develop tension cracks as shown in

Table 6.7 Minimum Thickness of Slabs or Beams Unless Deflections Are Computed[a]

Type of Member	End Conditions of Span	Minimum Thickness of Slab or Height of Beam: $f_y = 40$ ksi [276 MPa]	$f_y = 60$ ksi [414 MPa]
Solid one-way slabs	Simple support	L/25	L/20
	One end continuous	L/30	L/24
	Both ends continuous	L/35	L/28
	Cantilever	L/12.5	L/10
Beams or joists	Simple support	L/20	L/16
	One end continuous	L/23	L/18.5
	Both ends continuous	L/26	L/21
	Cantilever	L/10	L/8

Source: Adapted from material in *Building Code Requirements for Structural Concrete* (ACI 318–08) (Ref. 16), with permission of the publisher, American Concrete Institute.

[a]Refers to overall vertical dimension of concrete section. For normal weight concrete (145 pcf) only; code provides adjustment for other weights. Valid only for members not supporting or attached rigidly to partitions or other construction likely to be damaged by large deflections.

Figure 6.18*b*. Near the center of the span, where the bending is predominant and the shear approaches zero, these cracks approach 90° from the horizontal. Near the support, however, where the shear predominates and bending approaches zero, the critical tension plane approaches 45°, and the horizontal bars are only partly effective in resisting cracking.

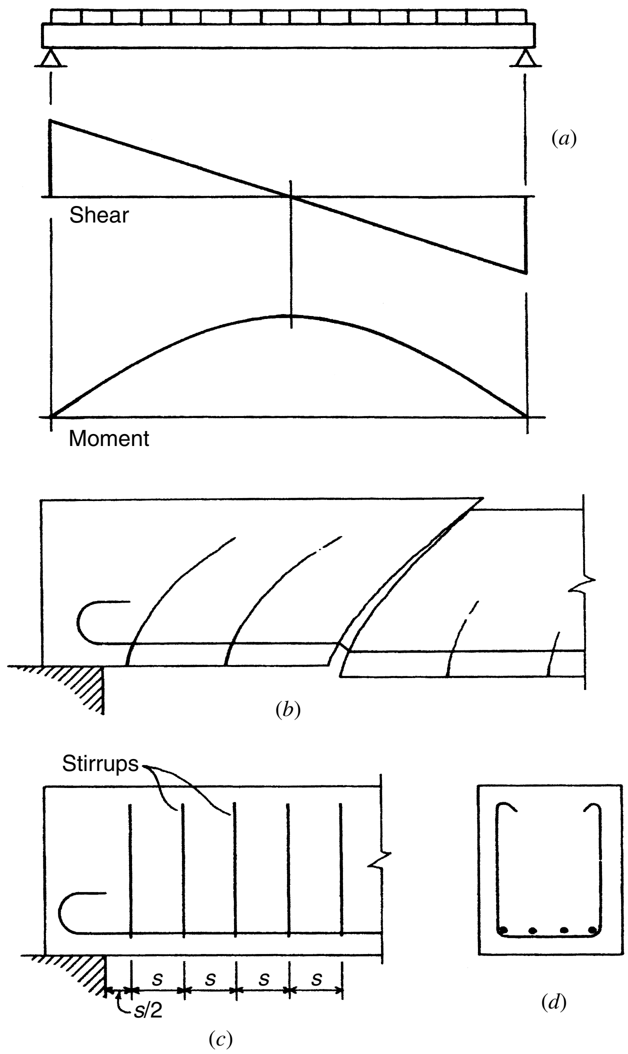


Figure 6.18 Considerations for shear in concrete beams.

Shear Reinforcement for Beams

For beams, the most common form of shear reinforcement is a U-shaped bent bar, as shown in Figure 6.18*d*, placed vertically and spaced in sets along the beam span, as shown in Figure 6.18*c*. These bars, called *stirrups,* provide a vertical component of resistance, working in conjunction with the horizontal bars.

The simple-span beam and the rectangular section occur only infrequently in building structures. The most common case is that of the beam section shown in Figure 6.19*a*, which occurs in sitecast construction. For negative moments near supports, compressive stress is in the bottom of the beam, occurring in the T-beam as shown in Figure 6.19*a*. For negative bending moments and for beam end shear, the T-section is considered to be as shown in Figure 6.19*b*; thus, there is little difference in consideration for shear between the simple- and continuous-span beams. It is important, however, to understand the relationships between shear and moment in the continuous beam.

Figure 6.20 illustrates the typical condition for shear and bending moment in a continuous beam with uniformly distributed loading. Referring to the portions of the beam span numbered 1, 2, and 3:

In zone 1, the high negative bending moment requires flexural reinforcement consisting of horizontal bars near the top of the beam.

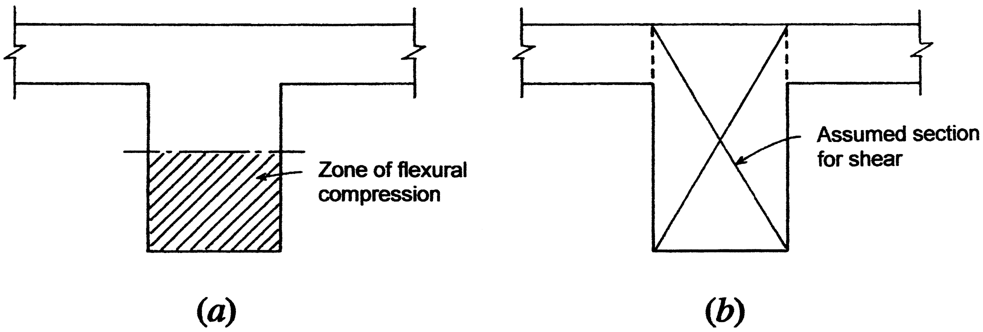


Figure 6.19 Development of negative bending and shear in concrete T-beams.

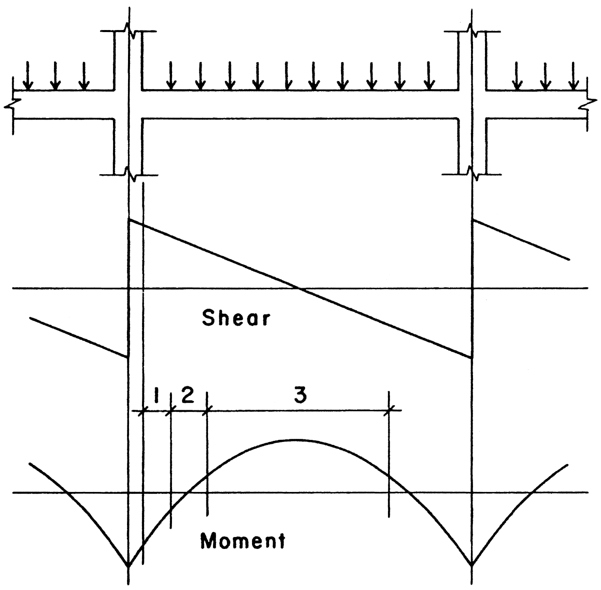


Figure 6.20 Shear and bending in continuous beams.

In zone 2, the moment reverses sign; moment magnitudes are low; and, if shear stress is high, the design for shear is a predominant concern.

In zone 3, shear consideration is reduced, and the predominant concern is for a positive moment requiring flexural reinforcement in the bottom of the beam.

Vertical U-shaped stirrups, similar to those shown in Figure 6.21*a*, may be used in the T-beam. An alternate detail for the stirrup is shown in Figure 6.21*b*, in which the top hooks are turned outward; this makes it possible to spread the top moment-resisting bars for better spacing.

Figures 6.21*c* and *d* show possibilities for stirrups in L-shaped beams that occur at the edges of the structure. This form of stirrup helps for development of negative moment resistance in the slab. So-called *closed stirrups*, similar to ties in columns, are sometimes used, as shown in Figures 6.21*c* through *f*, resulting in improved resistance to torsion.

Stirrup forms are often modified by designers or by the reinforcement fabricator to simplify the fabrication and/or the field installation. The stirrups shown in Figures 6.21*d* and *f* are modifications intended to achieve these purposes.

The following are considerations and code requirements that apply to design for beam shear:

Concrete Capacity. Whereas the tensile strength of the concrete is ignored in design for flexure, the concrete is assumed to take some portion of the shear in beams. If the shear capacity of the concrete is not exceeded—as it is sometimes for lightly loaded beams—there may be no need for steel reinforcement. The more typical case, however, is shown in Figure 6.22, where the maximum shear V exceeds the capacity of the concrete alone (V_c) and the steel reinforcement is required to absorb the excess shear force, indicated as the shaded portion in the shear diagram.

Minimum Shear Reinforcement. Even when the maximum computed shear falls below the capacity of the concrete, the code requires the use of some minimum amount of shear reinforcement. Exceptions are made in some cases, such as for slabs and for very shallow beams. The objective is essentially to toughen the structure with a small investment in additional reinforcement.

Type of Stirrup. The most common form for stirrups is the single U shape, although for special situations other forms may be used, such as with very narrow beams, very wide beams, and beams with very high shear forces.

Size of Stirrups. Beams of moderate size, the most common being the No. 3 bar, are easily bent to create the tight corners of the stirrup form. For slightly larger beams, No. 4 bars may be used, generating almost twice the capacity of the No. 3 stirrup.

Spacing of Stirrups. Stirrup spacing is computed on the basis of the magnitude of the shear force at the location of the stirrup. A maximum spacing permitted is one-half of the effective beam depth d, which assures that at least one stirrup occurs at the location of a diagonal crack (see Figure 6.18). When the shear force is exceptionally high, the maximum spacing is limited to one fourth the beam depth.

Critical Maximum Design Shear. Although the actual maximum shear force occurs at the beam end, the code permits the use of the force at a distance of d from the beam end as the critical maximum force

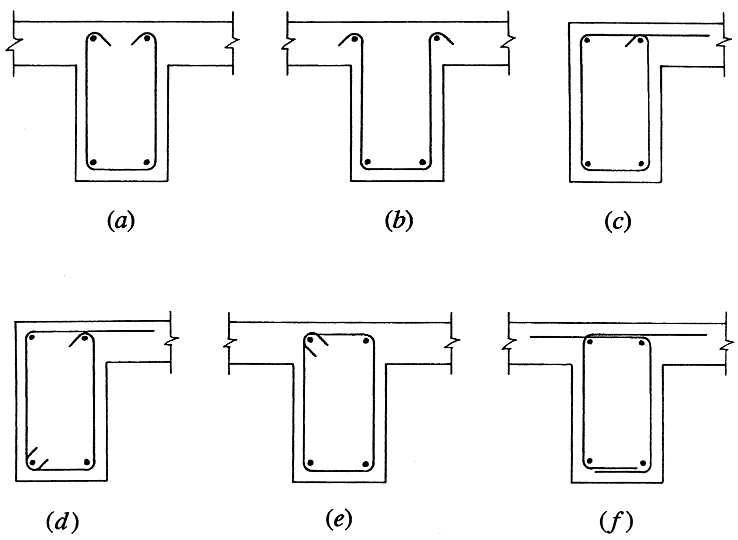


Figure 6.21 Forms for vertical stirrups.

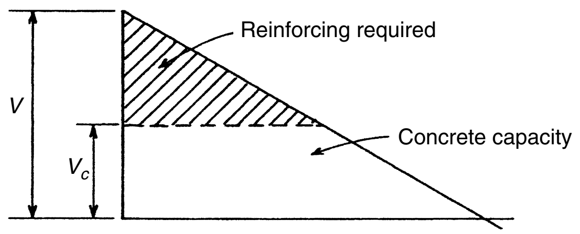


Figure 6.22 Sharing of shear resistance in reinforced concrete beams.

for stirrup design. Thus, as shown in Figure 6.23, the shear requiring reinforcement is slightly different than that shown in Figure 6.22.

Total Length for Shear Reinforcement. On the basis of computed shear forces, reinforcement must be provided along the beam length for the distance defined by the shaded portion of the shear diagram shown in Figure 6.23. For the center portion of the span, the concrete is theoretically capable of the necessary shear resistance without the assistance of reinforcement. However, the code requires that some shear reinforcement be provided for a distance beyond the computed cutoff point. Earlier codes required that stirrups be provided for a distance equal to $d/2$ beyond the cutoff. Currently, the required distance is extended to the point where the computed shear is reduced to one-half of the concrete shear resistance. However it is established, the total range over which reinforcement must be provided is indicated as R in Figure 6.23.

Design for Beam Shear

The ultimate shear force V_u at any cross section along a beam due to factored loading must be equal to or less than the reduced shear capacity at the section. Mathematically, this is represented as

$$V_u \leq \phi_v(V_c + V_s)$$

where

V_u = ultimate design shear force at section
$\phi_v = 0.75$
V_c = shear capacity of concrete
V_s = shear capacity of reinforcement

For beams of normal-weight concrete, subjected only to flexure and shear, shear force in the concrete is limited to

$$V_c = 2\sqrt{f_c'}bd$$

where

f_c' = specified strength of concrete, psi
b = width of cross section,in.
d = effective depth of beam, in.

Required spacing of shear reinforcement is determined as follows. Referring to Figure 6.24, note that the capacity in tensile resistance of a single, two-legged stirrup is equal to the product of the total steel cross-sectional area A_v times the steel yield stress. Thus

$$T = A_v f_y$$

This resisting force opposes part of the shear force required at the location of the stirrup, referred to as V_s'. Equating the stirrup tension capacity to this force, an equilibrium equation for one stirrup is obtained; thus,

$$A_v f_y = V_s'$$

The total shear force capacity of the beam in excess of the concrete is determined by the number of stirrups encountered by the shear force acting at a 45° angle through the beam. The number of stirrups will be d/s. Thus, the equilibrium equation for the beam is

$$\left(\frac{d}{s}\right) A_v f_y = V_s'$$

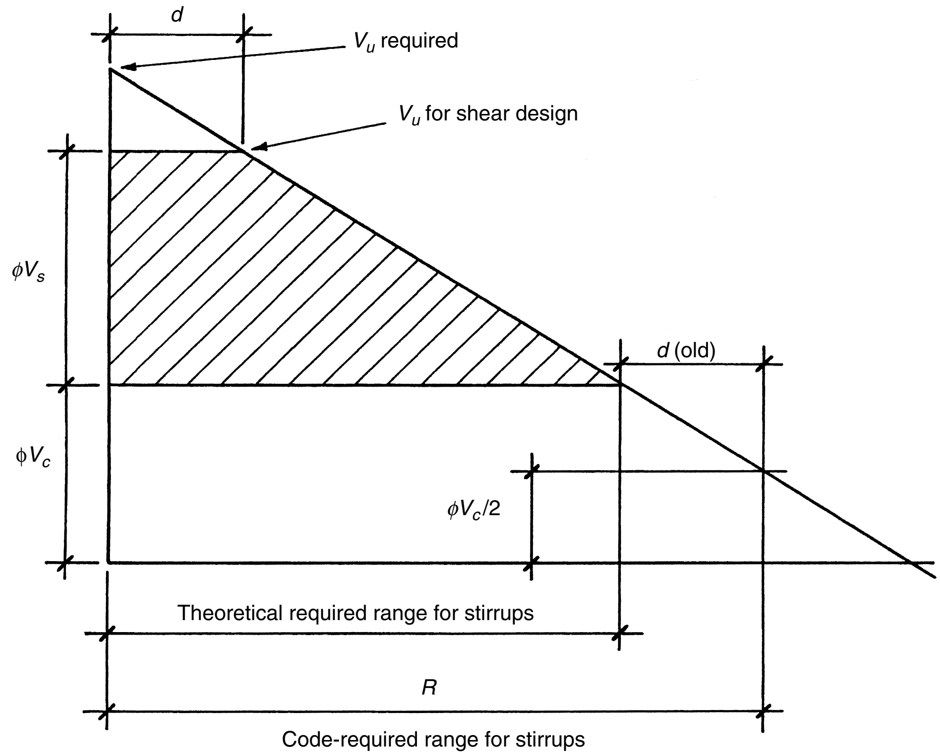


Figure 6.23 Layout for shear analysis, ACI code requirements (Ref. 16).

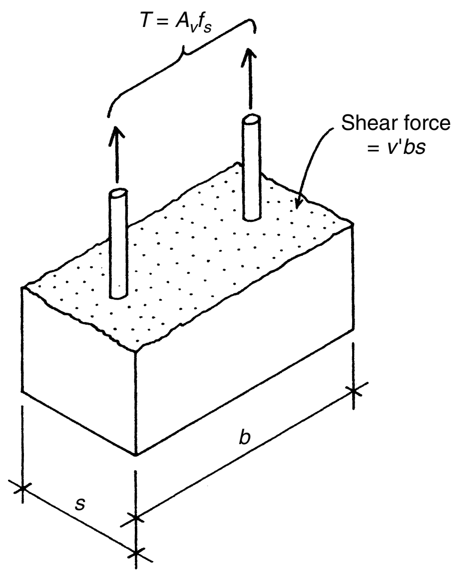


Figure 6.24 Consideration for spacing of a single stirrup.

From this equation, an expression for the required spacing can be derived; thus,

$$s \leq \frac{A_v f_y d}{V_s'}$$

The following example illustrates the procedure for the design of stirrups for a simple beam.

Example 6. Design the required shear reinforcement for the beam shown in Figure 6.25. Use $f_c' = 3$ ksi [20.7 MPa], $f_y = 40$ ksi [276 MPa], and single No. 3 U-stirrups.

Solution. First, the loading must be factored in order to determine the ultimate shear force:

$$w_u = 1.2(2) + 1.6(3) = 7.2 \text{ klf } [105 \text{ kN/m}]$$

The maximum shear force is equal to the end reaction force; thus,

$$\text{Maximum } V_u = \frac{7.2 \times 16}{2} = 57.6 \text{ kips } [256 \text{ kN}]$$

Referring to the shear diagram in Figure 6.25*c*, the critical shear force for design is at 24 in. from the support. Using proportionate triangles, this value is

$$V_u = \left(\frac{72}{96}\right)(57.6) = 43.2 \text{ kips } [192 \text{ kN}]$$

The shear capacity of the concrete is

$$\begin{aligned} \phi_v V_c &= 0.75(2\sqrt{f_c'}bd) \\ &= 0.75\{(2\sqrt{3000})(12 \times 24)\} \\ &= 23{,}700 \text{ lb} = 23.7 \text{ kips } [105 \text{ kN}] \end{aligned}$$

At the point of critical force, therefore, there is an excess shear force of $43.2 - 23.7 = 19.5$ kips that must be carried by the reinforcement. Observe that the excess shear condition extends to 56.7 in. from the support.

Shear reinforcement must be used wherever the shear force exceeds one-half of ϕV_c, which extends the range R to

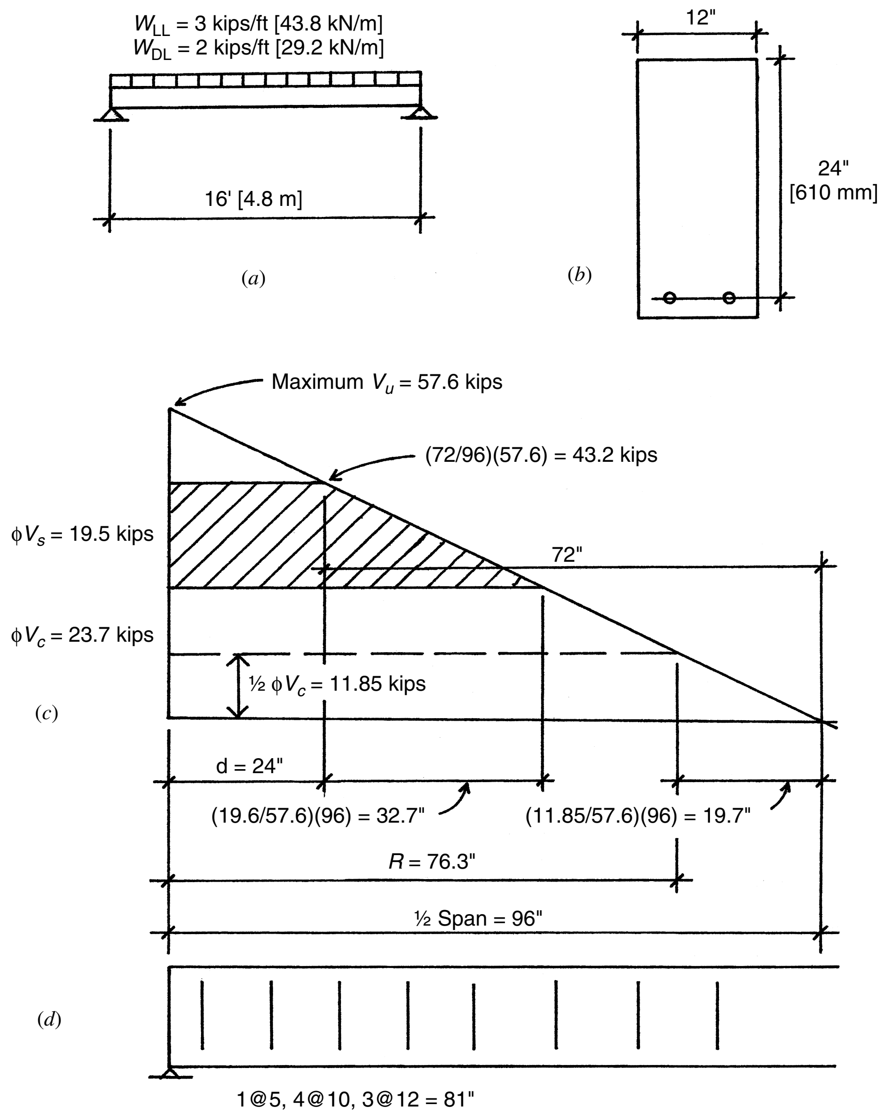


Figure 6.25 Reference for Example 6.

76.3 in. from the support. The code further stipulates that the minimum cross-sectional area of this reinforcement be

$$A_v = 50\left(\frac{b \times s_{\max}}{f_y}\right) = 50\left(\frac{12 \times 12}{40{,}000}\right) = 0.18\ \text{in.}^2\ [116\ \text{mm}^2]$$

which is less than the area provided by the No. 3 U-stirrup.

For the maximum V_s value of 19.5 kips, the maximum spacing permitted at the critical point 24 in. from the support is

$$s = \frac{A_v f_y d}{V_s'} = \frac{(0.22)(40)(24)}{19.5} = 10.8\ \text{in.}\ [274\ \text{mm}]$$

Since this is less than the maximum allowable spacing of one-half the depth or 12 in., it is best to calculate at least one more spacing at a short distance farther from the support. For example, at 36 in. from the support the shear force is

$$V_u = \left(\frac{60}{96}\right)(57.6) = 36.0\ \text{kips}$$

and the value of V_s' at this point is computed as

$$36.0 - 23.7 = 12.3\ \text{kips}$$

The spacing required at this point is thus

$$s = \frac{(0.22)(40)(24)}{12.4} = 17.2\ \text{in.}\ [437\ \text{mm}]$$

This indicates that the required spacing drops to less than the maximum allowed spacing at less than 12 in. from the critical point at 24 in. from the support.

A possible choice for the stirrup spacings is shown in Figure 6.25*d*, with a total of eight stirrups that extend over a range of 81 in. from the support. There are thus a total of 16 stirrups in the beam, 8 at each end. Note that the first stirrup is placed at 5 in. from the support, which is one-half the critical required spacing; this is a common practice.

It frequently happens that the critical required stirrup spacing near the beam end is greater than the minimum of

one-half the beam depth. The following example illustrates such a case.

Example 7. Determine the required number and spacings for No. 3 U-stirrups for the beam shown in Figure 6.26. Use $f_c' = 3$ ksi [20.7 MPa], $f_y = 40$ ksi [276 MPa].

Solution. In this case the maximum critical design shear force is found to be 28.5 kips; thus, the required force for reinforcement is found to be 12.1 kips, and the maximum spacing is determined as

$$s = \frac{(0.22)(40)(20)}{12.1} = 14.5 \text{ in. } [368\,\text{mm}]$$

Since this value exceeds the maximum limit of $d/2 = 10$ in., the stirrups may all be placed at the limiting spacing of 10 in. A possible arrangement is shown in Figure 6.26*d* using 8 stirrups for a range of 75 in., slightly greater than the computed value of $R = 74.1$ in.

Occasionally, with very lightly loaded beams, the required shear force may fall below the value of shear resistance for the beam. In this case, where no reinforcement requirement can be computed based on forces, the situation is reduced to one of satisfying the code minimum requirement for reinforcement, as previously discussed. The following example illustrates such a condition.

Example 8. Determine the required number and spacings for No. 3 U-stirrups for the beam shown in Figure 6.27. Use $f_c' = 3$ ksi [20.7 MPa], $f_y = 40$ ksi [276 MPa].

Solution. As shown on the beam shear diagram in Figure 6.27*c*, the critical maximum design shear force of 14.2 kips falls below the computed value for the shear capacity of the concrete, as determined in Example 7. Thus, in theory, no reinforcement is required. To comply with the code requirement for minimum reinforcement, however, provide stirrups at the maximum permitted spacing of 10 in. out to the point where the shear force drops to 8.2 kips. As shown in Figure 6.27*c*, the required range R for the stirrups is 52 in. The chosen spacings shown in Figure 6.27*d* slightly exceed this distance using a total of six stirrups.

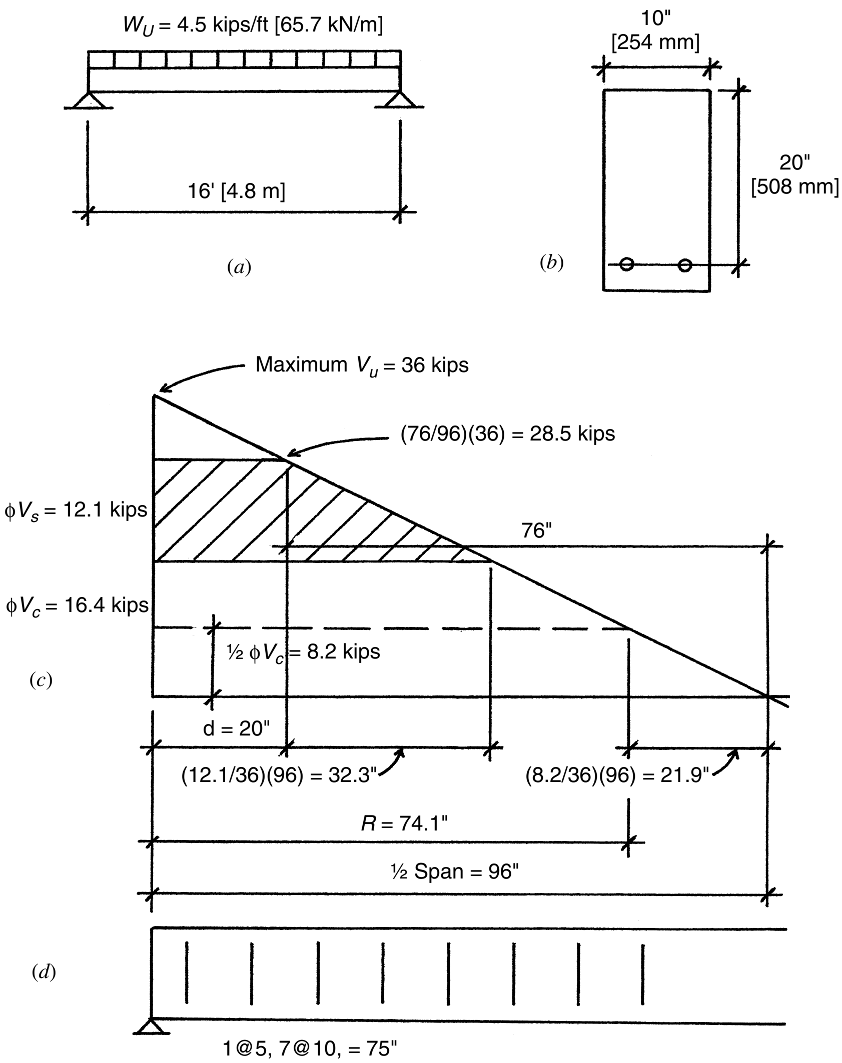


Figure 6.26 Reference for Example 7.

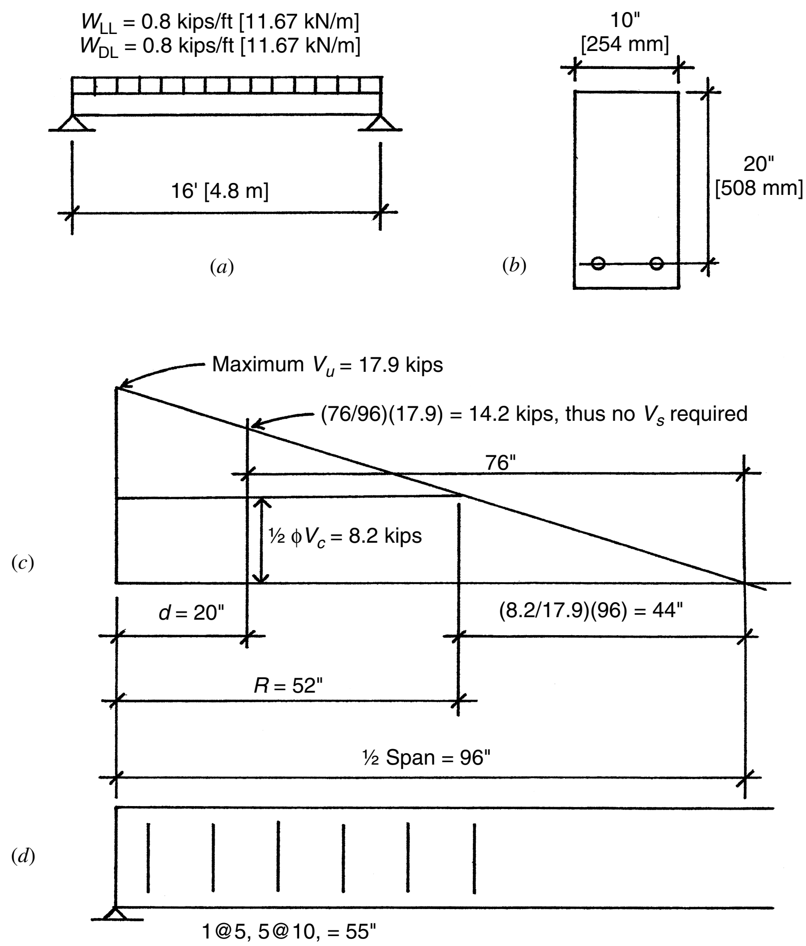


Figure 6.27 Reference for Example 8.

Examples 6 through 8 have illustrated what is generally the simplest case for beam shear design—that of a beam with uniformly distributed load and with sections subjected only to flexure and shear. When concentrated loads or unsymmetrical loadings produce other forms for the shear diagram, these must be used for design of the shear reinforcement.

Anchorage and Development of Reinforcement

The nature of reinforced concrete depends very primarily on the interactive relationship between the steel reinforcing bars and the concrete mass within which they are encased. Loads are mostly applied directly to the concrete structure, that is, to the concrete mass. Stress developed in the steel must be accomplished through engagement between the steel and concrete, which occurs at their interface—the surface of the steel bars. This sliding friction stress is called *bond* or *bonding* stress.

At present the code deals with this issue as one of *development length*, defined as the length of embedment required to develop the design strength of the reinforcement at a critical section. For beams, critical sections occur at points of maximum flexural stress and at points where some of the reinforcement terminates. The section at which the major tension occurs in the reinforcing bars is viewed as the face of a concrete mass, into which the bars must extend a sufficient distance for critical embedment.

In the simple beam, the bottom reinforcement required for the maximum moment at midspan is not entirely required as the moment decreases toward the end of the span (see Figure 6.13). It is thus sometimes the practice to make only part of the reinforcement continuous for the whole beam length. When beams are continuous through the supports, top reinforcement is required for the negative moments at the supports. These top bars must be investigated for the development lengths in terms of the distance they extend from the supports.

For tension reinforcement consisting of No. 11 bars and smaller, the code specifies a minimum length for development (L_d) as follows:

For No. 6 bars and smaller:

$$L_d = \frac{f_y\, d_b}{25\sqrt{f'_c}}$$

but not less than 12 in.

For No. 7 bars and larger:

$$L_d = \frac{f_y\, d_b}{20\sqrt{f'_c}}$$

In these formulas, d_b is the bar diameter.

Modification factors for L_d are specified for various situations as follows:

For top bars in horizontal members with at least 12 in. of concrete below the bars: increase by 30%.
For flexural reinforcement that is provided in excess of that required by computations: decrease by a ratio of (required A_s)/(provided A_s).

Additional modification factors are given for lightweight concrete, for bars with epoxy coating, for bars encased in spirals, and for bars with yield strength in excess of 60 ksi. The maximum value to be used for $\sqrt{f_c'}$ in the formulas is 100.

Table 6.8 gives values for minimum development lengths for tensile reinforcement based on the requirements of the ACI code. The values listed under "other bars" are the unmodified length requirements; those listed under "top bars" are increased by 30%. Values are given for combinations of two steel strengths and two concrete strengths.

There is no resistance factor for ultimate design related to development length. As presented, the formulas for development length relate only to bar size, concrete design strength, and steel yield strength. The computed development lengths are thus equally applicable to stress or strength design.

Example 9. The bending moment in the short cantilever shown in Figure 6.28 is resisted by the bars in the top of the beam. Determine whether the development of the reinforcement is adequate for the No. 6 bars without hooked ends if $L_1 = 48$ in. and $L_2 = 36$ in. Use $f_c' = 3$ ksi and $f_y = 60$ ksi.

Solution. At the face of the support, anchorage must be achieved on both sides: within the support and in the top of the beam. In the beam the condition is one of "top bars"; thus from Table 6.8, a length of 43 in. is required for L_d, which is adequately provided.

Within the support, the condition is one of "other bars." For this, the required length is 33 in., which is also adequately provided.

Hooked ends are not required in this example, although many designers would probably hook the bars in the support, just for the added security of the anchorage. Hooks may also be required in some cases, and their design is treated in the next discussion.

Table 6.8 Minimum Development Length for Tensile Reinforcement (in.)[a]

	$f_y = 40$ ksi [276 MPa]				$f_y = 60$ ksi [414 MPa]			
	$f_c' = 3$ ksi [20.7 MPa]		$f_c' = 4$ ksi [27.6 MPa]		$f_c' = 3$ ksi [20.7 MPa]		$f_c' = 4$ ksi [27.6 MPa]	
Bar Size	Top Bars[b]	Other Bars	Top Bars[b]	Other Bars	Top Bars[b]	Other Bars	Top Bars[b]	Other Bars
3	15	12	13	12	22	17	19	15
4	19	15	17	13	29	22	25	19
5	24	19	21	16	36	28	31	24
6	29	22	25	19	43	33	37	29
7	42	32	36	28	63	48	54	42
8	48	37	42	32	72	55	62	48
9	54	42	47	36	81	62	70	54
10	61	47	53	41	91	70	79	61
11	67	52	58	45	101	78	87	67

[a]Lengths are based on requirements of *Building Code Requirements for Structural Concrete* (ACI 318–08) (Ref. 16).
[b]Horizontal bars with more than 12 in. of concrete cast below them in the member.

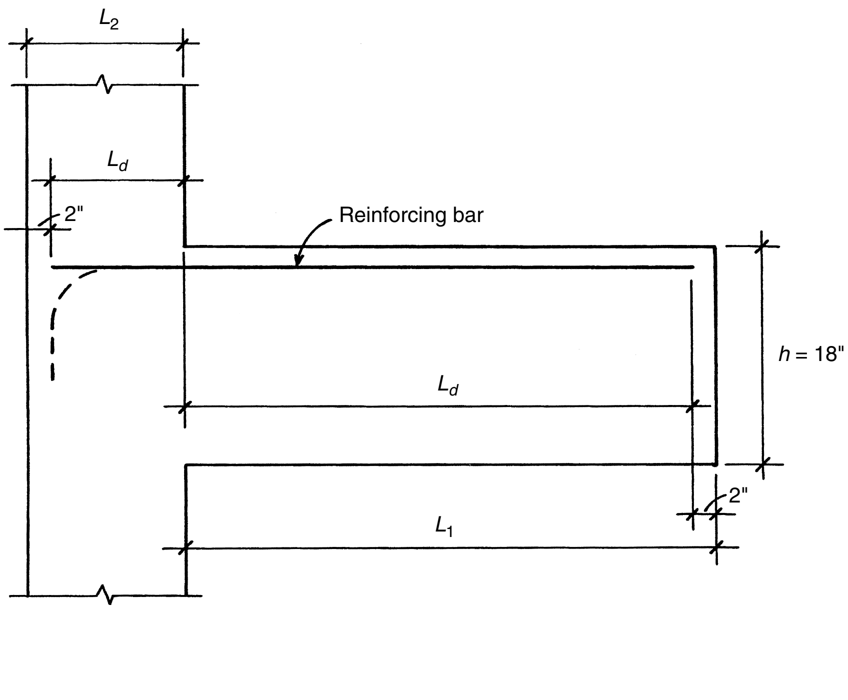


Figure 6.28 Reference for Example 9.

Hooks

When details of the construction restrict the ability to extend bars sufficiently to produce required development lengths, anchorage can sometimes be assisted by use of hooked ends on bars. So-called *standard hooks* may be evaluated for tension force capacity on the basis of a required development length, L_{dh}, as shown in Figure 6.29. Bar ends may be bent at 90°, 135°, or 180° to produce a hook.

Table 6.9 gives values for development lengths with standard hooks using the same variables that are used in Table 6.8. Note that the table values are for 180° hooks and that required lengths may be reduced by 30% for 90° hooks. The following example illustrates the use of the data from Table 6.9.

Table 6.9 Required Development Length L_{dh} for Hooked Bars (in.)[a]

	$f_y = 40$ ksi [276 MPa]		$f_y = 60$ ksi [414 MPa]	
Bar Size	$f'_c = 3$ ksi [20.7 MPa]	$f'_c = 4$ ksi [27.6 MPa]	$f'_c = 3$ ksi [20.7 MPa]	$f'_c = 4$ ksi [27.6 MPa]
3	6	6	9	8
4	8	7	11	10
5	10	8	14	12
6	11	10	17	15
7	13	12	20	17
8	15	13	22	19
9	17	15	25	22
10	19	16	28	24
11	21	18	31	27

[a]See Fig. 6.29. Table values are for a 180° hook; values may be reduced by 30% for a 90° hook.

Example 10. For the bars in Figure 6.27, determine the required length for the bars if a 90° hook is used for the bars extending into the support. Use the same concrete and steel strengths as in Example 9.

Solution. From Table 6.9, the required length for the data given is 17 in. This may be reduced for the use of the 90° to a length of

$$L = 0.70(17) = 11.9\text{ in.}$$

Various details are used to further enhance the anchorage capacity of hooks. One detail frequently used is to hook the bar around a reinforcing bar that is vertically continuous (as in a column) or horizontally continuous (as in a beam) in the support structure. This enhancement is required for structures designed for seismic forces.

Bar Development in Continuous Beams

Critical locations for bar development occur at points of maximum stress and at points within a beam span at which some reinforcement terminates. When beams are continuous through their supports, the negative moments at the supports will require that bars be placed in the top of the beams.

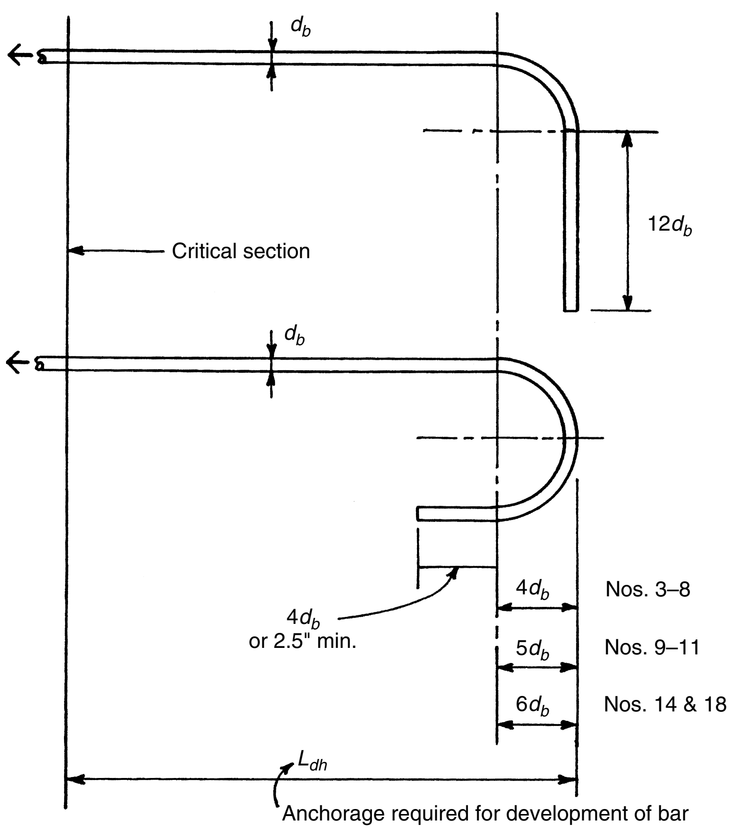


Figure 6.29 Detail requirements for standard hooks for use of values in Table 6.9.

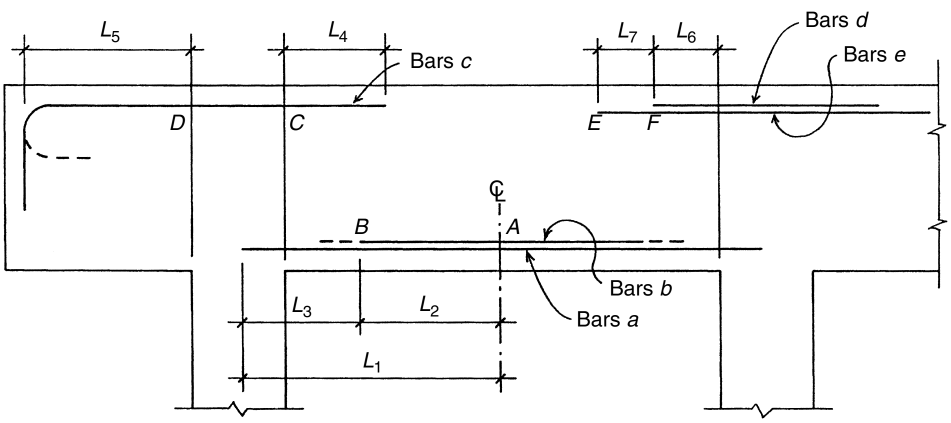


Figure 6.30 Development length in continuous beams.

Within the span, bars will be required in the bottom of the beams for positive moments. Although the positive moment will go to zero at some distance from the supports, the code requires that some of the positive-moment reinforcement be extended for the full length of the span and for a short distance into the support.

Figure 6.30 shows a possible layout for reinforcement in a beam with continuous spans and a cantilevered end at the first support. Referring to the notation in the illustration:

Bars a and b are provided for the maximum positive moment. If all of these bars are made full length, the length L_1 must be sufficient for development. If bars b are partial length, then L_2 must be sufficient to develop bars b and L_3 must be sufficient to develop bars a. The partial-length bars must actually extend beyond the theoretical cutoff point (B in the illustration), and the true length must include the dashed portions indicated.

For the bars at the cantilevered end, the distances L_4 and L_5 must be sufficient for development of bars c. L_4 is required to extend beyond the theoretical cutoff point of negative moment. If the available length for development is not adequate in the cantilever, it may be necessary to use a hook as shown.

At the interior support, L_6 must be adequate for development of bars d, and L_7 must be adequate for development of bars e.

Splices in Reinforcement

In various situations in reinforced concrete structures, it becomes necessary to transfer stress between steel bars in the same direction. Continuity of force in the bars is achieved by splicing, which may be effected by welding, by mechanical means, or by the lapped splice as shown in Figure 6.31. Because a lapped splice is usually made with bars in contact, the lapped lengths must be somewhat greater than those ordinarily required for development.

For a simple tension-lapped splice, the full development of the bars usually requires a lapped length of 1.3 times that required for simple development of the bars. Use of lapped splices is generally limited to bars of No. 11 size or smaller.

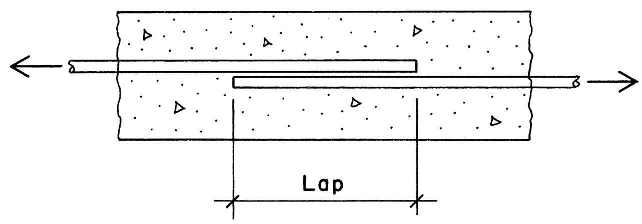


Figure 6.31 Lapped splice for steel reinforcing bars.

Development of Compressive Reinforcement

Development length in compression is a factor in column design and in design of beams with compressive reinforcement. The usual absence of tension cracks permits shorter development length in compression than in tension. The ACI code prescribes that development length in compression be

$$L_d = \frac{0.02 f_y\, d_b}{\sqrt{f_c'}}$$

but not less than $0.0003 f_y\, d_b$ or 8 in., whichever is smaller.

Table 6.10 lists compression bar development lengths for a few combinations of data.

In concrete columns, the concrete and the steel bars share the compression force. For ordinary construction, various situations of bar development must be considered. Figure 6.32 shows a multistory column supported on a concrete footing. For this situation, we note the following:

The construction is ordinarily produced in multiple pours, with construction joints between separate pours occurring as shown in the figure.

Load is transferred from the concrete of the column to the footing in direct bearing. Load from the steel bars must be developed by extension of the bars into the footing: distance L_1 in the figure. Dowels are usually placed in the footing and spliced to the column bars, with the two development lengths considered if concrete strength is different in the column and the footing.

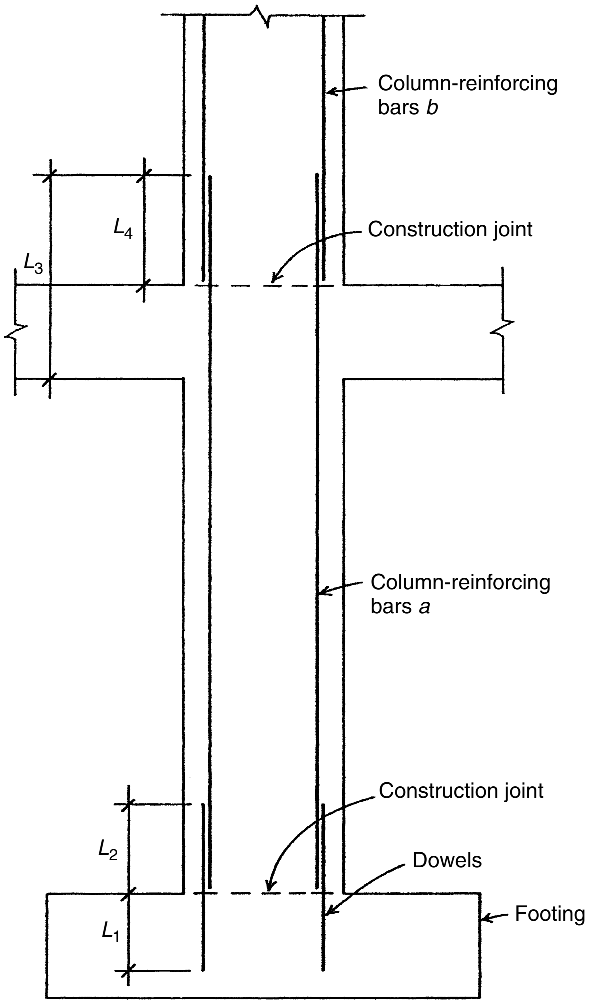


Figure 6.32 Development lengths in multistory columns.

In a similar manner, the bars in the lower column will be extended into the upper column to develop a lapped splice in the upper column.

Choices for bar sizes and their placement in the columns must be made as part of the column design. The process for this is illustrated for the building design case in Section 10.8.

Flat-Spanning Concrete Structures

There are many different systems that can be used to achieve flat spans. These are used most often for floor structures, which typically require a dead flat form. However, in buildings with an all-concrete structure, they may also be used for roofs. Sitecast concrete systems generally consist of one of the following basic types:

One-way solid slab and beam
Two-way solid slab and beam
One-way joist construction
Two-way flat slab or flat plate without beams
Two way joists, called *waffle construction*

Table 6.10 Minimum Development Length for Compressive Reinforcement (in.)

	$f_y = 40$ ksi [276 MPa]		$f_y = 60$ ksi [414 MPa]		
Bar Size	$f'_c = 3$ ksi [20.7 MPa]	$f'_c = 4$ ksi [27.6 MPa]	$f'_c = 3$ ksi [20.7 MPa]	$f'_c = 4$ ksi [27.6 MPa]	$f'_c = 5$ ksi [34.5 MPa]
3	8	8	8	8	7
4	8	8	11	10	9
5	10	8	14	12	11
6	11	10	17	15	13
7	13	12	20	17	15
8	15	13	22	19	17
9	17	15	25	22	20
10	19	17	28	25	22
11	21	18	31	27	24
14			38	33	29
18			50	43	39

Each system has its own distinct advantages and limits and some range of logical use, depending on spans, general layout of supports, magnitude of loads, required fire ratings, and cost limits for design and construction.

Slab-and-Beam Systems

The most widely used and most adaptable sitecast concrete floor system utilizes one-way solid slabs supported by one-way spanning beams. This system may be used for single spans, but it occurs more frequently with multiple-span slabs and beams in a system such as that shown in Figure 6.33.

In the example shown, the continuous slabs are supported by a series of beams that are spaced at 10 ft center to center. The beams, in turn, are supported by a girder-and-column system with columns at 30-ft centers, every third beam being

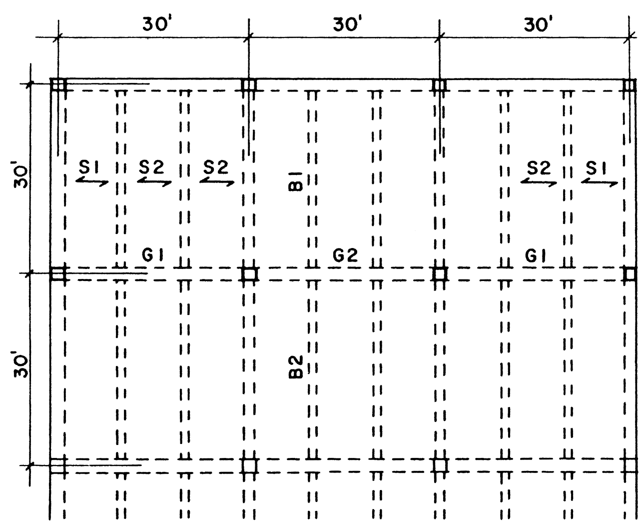


Figure 6.33 Framing plan for a slab-and-beam system.

supported directly by the columns, and the remaining beams being supported by the girders.

Because of the regularity and symmetry of the system shown in Figure 6.33, there are relatively few different elements in the basic system. Special members must be designed for conditions at the building edge and for required openings, but the general portion of the system requires only six basic elements: S1, S2, B1, B2, G1, and G2, as shown in the framing plan.

In computations for single-span concrete beams, span lengths are taken as the distance from center to center of supports. For continuous beams, span length is taken as the clear distance between faces of supports.

In continuous beams, negative bending moments are developed at the supports and positive moments at or near midspan. This may be observed from the exaggerated deformation profile of Figure 6.34*a*. The exact values of the bending moments depend on several factors, but in the case of approximately equal spans with uniformly distributed loads, when live load does not exceed three times dead load, the bending moment values given in Figure 6.34 may be used for design. The values given in Figure 6.34 are adapted from data in the ACI code (Ref. 16). These values have been adjusted to account for partial live loading of multiple-span beams.

Design moments for continuous-span slabs are given in Figure 6.34*e*. With large beams and short slab spans, the torsional stiffness of supporting beams tends to minimize the continuity effect in adjacent slab spans. Thus, most slab spans in the slab-and-beam systems tend to function much like individual spans with fixed ends, which is the basis for the moment factors for interior spans as given in Figure 6.34.

Design of a One-Way Continuous Slab

The design of a simple-span slab was illustrated in a previous example (Example 5) in this chapter. The following example illustrates a procedure for the design of a one-way continuous-span solid slab using the approximate coefficients from Figure 6.34.

Example 11. A one-way solid slab is to be used for a system similar to that in Figure 6.33. Column spacing is 30 ft, with evenly spaced beams occurring at 10 ft center to center. Superimposed loads on the slab (live load plus other construction dead load) are a dead load of 38 psf [1.82 kPa] and a live load of 100 psf [4.79 kPa]. Use concrete strength of 3 ksi [20.7 MPa] and steel yield strength of 40 ksi [276 MPa]. Determine the thickness for the slab and select its reinforcement.

Solution. To find the slab thickness, consider three factors: the minimum thickness for deflection, the minimum depth for flexure, and the minimum depth for shear. The span of the slab is taken as the 10 ft center-to-center beam spacing minus the width of the beams. Assuming 12 in. beam width, this produces a slab span for design of 9 ft.

For deflection, consider the end span to have a free end at the edge beam; then, from Table 6.7, the appropriate factor is $L/30$, and the required t is

$$\text{Minimum } t = \frac{L}{30} = \frac{9 \times 12}{30} = 3.6 \text{ in. } [91.4\,\text{mm}]$$

Assume here that fire requirements make it desirable to have a relatively thick slab of 5 in., for which the dead load of the slab is

$$w = \frac{5}{12} \times 150 = 62\,\text{psf}\,[2.97\,\text{kPa}]$$

The total dead load is thus $62 + 38 = 100$ psf, and the factored total load is

$$w_u = 1.2(100) + 1.6(100) = 280\,\text{psf}\,[13.4\,\text{kPa}]$$

For flexure, the maximum moment factor from Figure 6.34 is $\frac{1}{10}$, the maximum moment is

$$M_u = \frac{w_u L^2}{10} = \frac{280(9)^2}{10} = 2268\,\text{ft-lb}\,[3.08\,\text{kN-m}]$$

and the required resisting moment for the slab is

$$M_r = \frac{M_u}{\phi_b} = \frac{2268}{0.9} = 2520\ \text{ft-lb}\,[3.42\,\text{kN-m}]$$

This moment value is compared to the balanced moment capacity of the slab. For this computation the effective depth d must be determined, which is the overall thickness minus the cover and one-half the bar diameter. Assuming a No. 4 bar,

$$d = t - 1.0 = 5 - 1.0 = 4.0\,\text{in.}$$

Then, with R from Table 6.2, the balanced moment for a 12-in.-wide piece of slab is

$$M_r = Rbd^2 = (1.149)(12)(4)^2 = 221 \text{ kip-in. } [25\,\text{kN-m}]$$
$$= 221 \times \frac{1000}{12} = 18{,}400\,\text{ft-lb}\,[25\,\text{kN-m}]$$

Because this is considerably in excess of the required moment, the slab is more than adequate for flexural stress in the concrete.

It is not practical to use shear reinforcement in one-way slabs, and consequently, the maximum shear force must be kept below the capacity of the concrete. For the interior spans, the maximum shear will be $wL/2$, but for the end span it is usual practice to consider some unbalanced condition for the distribution of shear. We will thus use the recommendation given for a beam in Figure 6.33*d*: maximum shear $= 1.15(wL/2)$. Thus,

$$\text{Maximum } V_u = 1.15 \times \frac{wL}{2} = 1.15 \times \frac{280 \times 9}{2}$$
$$= 1449\,\text{lb}\,[6.45\,\text{kN}]$$

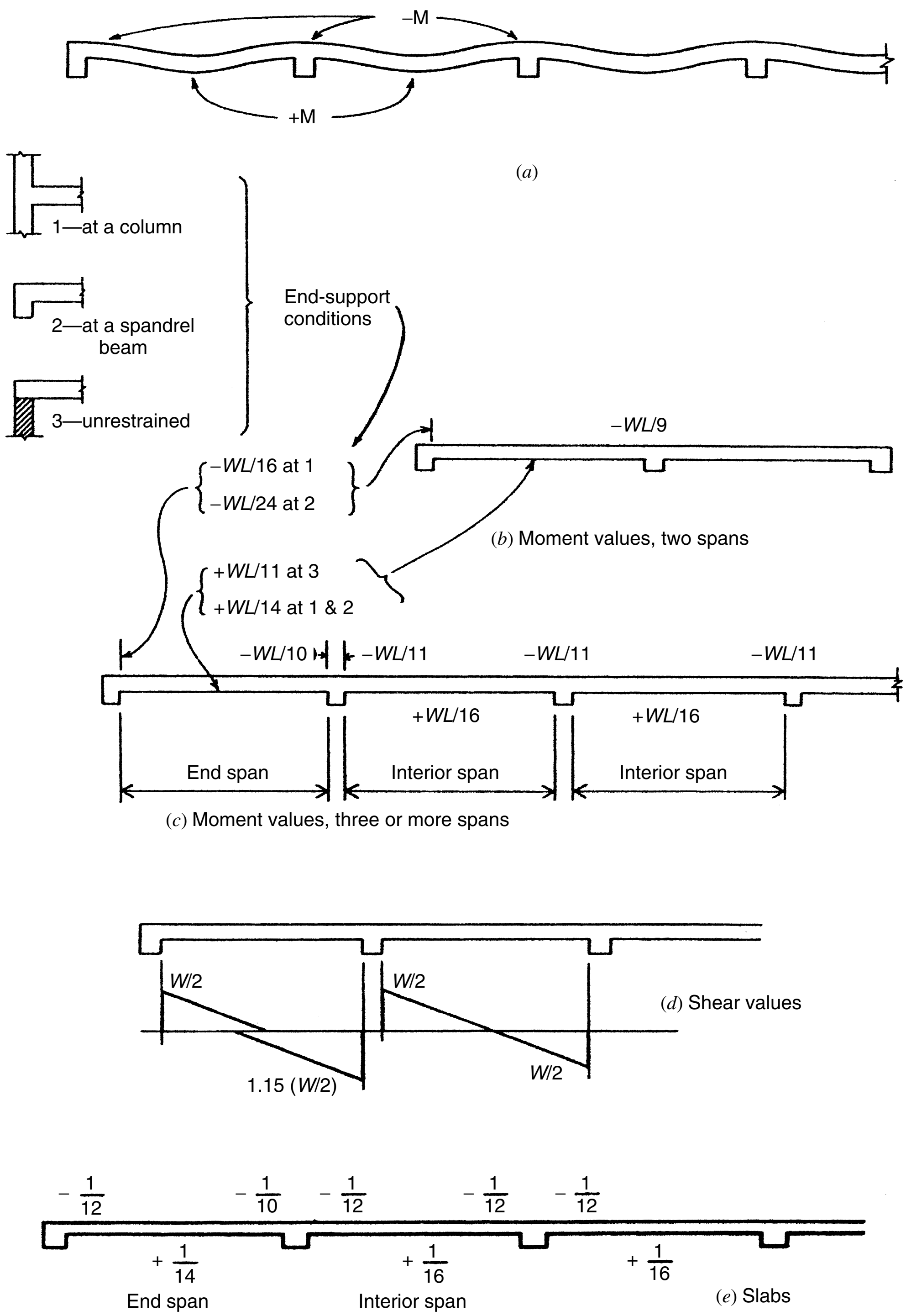


Figure 6.34 Approximate design factors for concrete beams and slabs.

Then,

$$\text{Required } V_r = \frac{V_u}{\phi_v} = \frac{1449}{0.75} = 1932\,\text{lb [8.59 kN]}$$

For the slab section with $b = 12$ in. and $d = 4$ in.,

$$V_c = 2\sqrt{f_c'}bd = 2\sqrt{3000}(12 \times 4) = 5258\,\text{lb [23.4 kN]}$$

As this is considerably greater than the required shear resistance, the selected thickness is not critical for shear.

For flexure, reference to Table 6.2 yields a value of 0.685 for *a/d*. However, for all moment values for this slab design, the section will be *underbalanced* or *underreinforced*; thus, the true value for *a/d* will be much smaller. We will assume a conservative value of 0.4, making the value of $a = 0.4\,d = 0.4(4) = 1.6$ in.; thus for the computation of required reinforcement

$$d - \frac{a}{2} = 4 - \frac{1.6}{2} = 3.2\text{ in. [81.3 mm]}$$

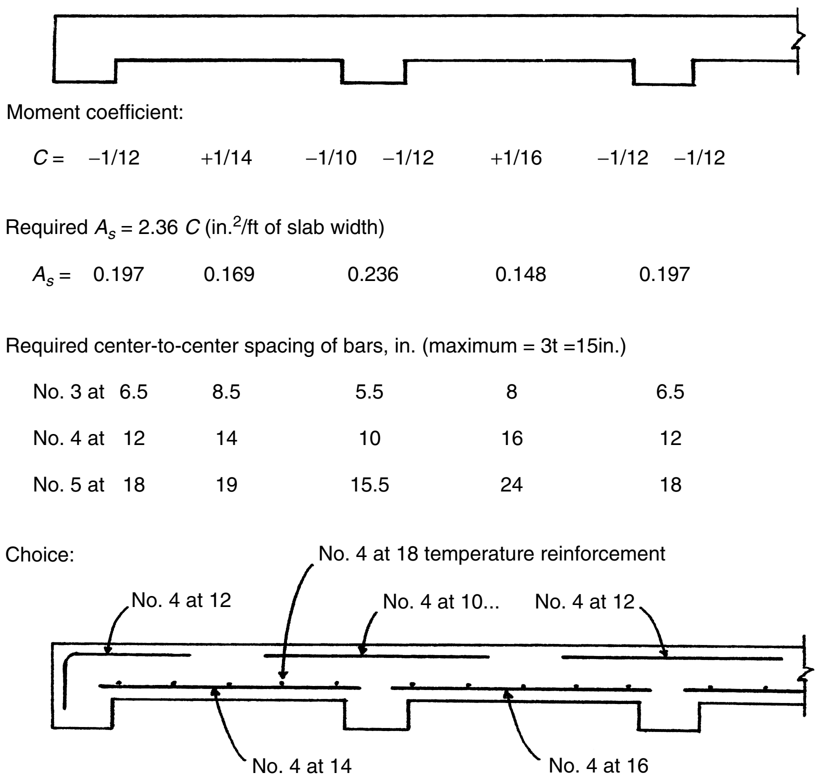


Figure 6.35 Summary of design for the continuous slab in Example 11.

Referring to Figure 6.34, note that there are five critical locations for which moment must be determined and the required steel area computed. The design for these conditions is summarized in Figure 6.35. For the data in the figure, note the following:

Maximum spacing of reinforcing bars:

$$s = 3t = 3(5) = 15\text{ in.}$$

Maximum required bending moment:

$$M_r = \text{(moment factor } C)\left(\frac{wL^2}{\phi_b}\right)$$

$$= C\left(\frac{280(9)^2}{0.9}\right) \times 12 = 302{,}400C$$

Note that the use of the factor 12 gives this value for the moment in inch-pound units.

The required area of reinforcement is given as

$$A_s = \frac{M}{f_y(d - a/2)} = \frac{302{,}400 \times C}{40{,}000 \times 3.2} = 2.36C$$

Using data from Table 6.5, Figure 6.35 shows required spacings for Nos. 3, 4, and 5 bars. A possible choice for the slab reinforcement, using all No. 4 bars, is shown at the bottom of Figure 6.35. For the required temperature reinforcement,

$$A_s = 0.002\,bt = 0.002(12 \times 5) = 0.012\text{ in.}^2\text{ft of slab width}$$

Using data from Table 6.5, possible choices are for No. 3 at 11 in. or No. 4 at 18 in.

General Considerations for Beams

The design of a single beam involves a large number of pieces of data, most of which are established for the system as a whole, rather than individually for each beam. Systemwide decisions include those for the type of concrete and its aggregate, concrete design strength (f'_c), the grade of steel for reinforcement (f_y), the cover required, and various details for forming and finishing of the concrete and installation of the reinforcement.

Most beams occur in conjunction with solid slabs that are cast monolithically with the beams. Slab thickness is established by the structural requirements of the spanning action and by concerns for fire rating, acoustic separation, type of reinforcement, and so on. Design of a single beam is usually limited to determination of the following:

- Choice of shape and dimensions of the beam cross section
- Selection of the type, size, and spacing of shear reinforcement
- Selection of the flexural reinforcement to satisfy requirements based on the variation of bending moment along the beam length

The following are some factors that must be considered in effecting these decisions.

Beam Shape

Figure 6.36 shows the most common shapes used for beams in sitecast construction. The single, rectangular shape is uncommon but does sometimes occur.

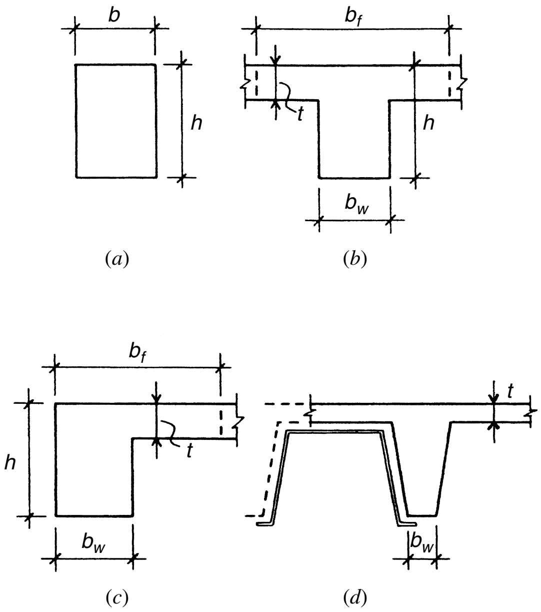


Figure 6.36 Common shapes for concrete beams.

As mentioned previously, beams occur most often in conjunction with monolithic slabs, resulting in the typical T shape shown in Figure 6.36*b* or the L shape shown in Figure 6.36*c*. The full T shape occurs at interior portions of the system, while the L shape occurs at the building edge and at the edges of large openings. As shown in the illustration, there are four basic dimensions for the T and L that must be established in order to fully define the beam section.

The slab thickness t, which is ordinarily established on its own, rather than as a part of the single-beam design

The overall beam stem depth h, corresponding to the same dimension for the rectangular section

The beam stem width b_w, which is critical for consideration of shear and the problem of fitting reinforcement into the section

The so-called *effective width* (b_f), which is the portion of the slab assumed to work with the beam

A special beam shape is that shown in Figure 6.36*d*. This occurs in concrete joist or waffle construction when forms of steel or reinforced plastic are used to form the concrete, the taper of the beam section being required for easy removal of the forms. The smallest width of the tapered stem is ordinarily used for the beam design in this situation.

Beam Width

Consideration of the formulas for flexure will show that the beam width dimension affects the bending resistance in a linear relationship (double the width, and you double the moment resistance). On the other hand, the resisting moment is affected by the *square* of the effective beam depth. Thus, efficiency, in terms of beam weight or concrete volume, will be obtained by striving for deep, narrow beams, instead of shallow, wide ones—just as a 2-by-8 wood joist is more efficient than a 4-by-4 one.

Table 6.11 Minimum Beam Widths[a]

Number	Bar Size								
of Bars	3	4	5	6	7	8	9	10	11
2	10	10	10	10	10	10	10	10	10
3	10	10	10	10	10	10	10	10	11
4	10	10	10	10	11	11	12	13	14
5	10	11	11	12	12	13	14	15	17
6	11	12	13	14	14	15	17	18	19
7	13	14	15	15	16	17	19	20	22
8	14	15	16	17	18	19	21	23	25
9	16	17	18	19	20	21	23	25	28
10	17	18	19	21	22	23	26	28	30

[a]Minimum width in inches for beams with 1.5-in. cover, No. 3 U-stirrups, clear spacing between bars of one bar diameter or minimum of 1 in. General minimum practical width for any beam with No. 3 U-stirrups is 10 in.

Beam width also relates to various other factors, however, and these are often critical in establishing the minimum width for a given beam. The formula for shear capacity indicates that the beam width is equally as effective as the depth in shear resistance. Placement of reinforcing bars is sometimes a problem with narrow beams. Table 6.11 gives minimum beam widths required for various bar combinations based on considerations of bar spacing, minimum cover of 1.5 in., placement of the bars in a single layer, and use of a No. 3 stirrup. Situations requiring additional concrete cover, use of larger stirrups, or the intersection of beams with columns may necessitate widths greater than those given in Table 6.11.

Beam Depth

Although selection of beam depth is partly a matter of satisfying structural requirements, it is typically constrained by other considerations in the building design. Figure 6.37 shows a section through a typical building floor/ceiling with a concrete slab and beam structure. In this situation, the

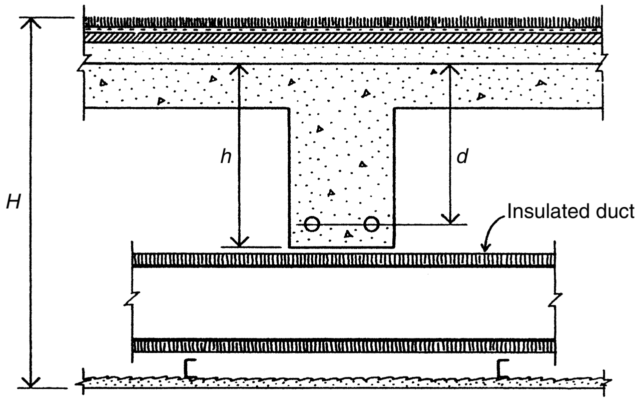


Figure 6.37 Concrete beam in typical multistory construction.

critical depth from a general building design point of view is the overall thickness of the construction, shown as H in the illustration. In addition to the concrete structure, this includes allowances for the floor finish, the ceiling construction, and the passage of an insulated air duct. The net usable portion of H for the structure is shown as the dimension h, with the effective structural depth d being something less than h. Because the space defined by H is not highly usable for the building occupancy, there is a tendency to constrain it, which works to limit any extravagant use of d.

Most concrete beams tend to fall within a limited range in terms of the ratio of width to depth. The typical range is for a width-to-depth ratio between 1 : 1.5 and 1 : 2.5, with an average of 1 : 2. This is not a code requirement or a magic rule; it is merely the result of satisfying typical pragmatic requirements for flexure, shear, deflection, bar spacing, and economy of use of steel.

Steel reinforcement constitutes a significant portion of the total cost of a concrete structure. For the reinforcement, cost factors include the weight of the steel, the cutting and shaping of bars, and the field installation. For the weight, beam depth is a major consideration.

Intersecting Structural Members

Although most of the considerations for beams thus far treated have dealt with the two-dimensional aspect of the beam cross section, beams do indeed exist in a three-dimensional structural framework in most cases. The form of a beam and its dimensions need to be coordinated with those of other structural members, most significantly any supporting beams or columns. A beam usually should not be wider than a supporting column or deeper than its supporting beam.

Another consideration in this regard is that of the intersection of reinforcing bars. Bars normally extend through the intersections of concrete structural members, meeting the extended bars from other members at the intersection. This can get to be a traffic jam when individual members have closely spaced bars, which is a quite common situation. The design of each structural member must be considered in terms of the size, number, and spacing of the bars as well as their precise positioning in the members. Some of these issues are discussed for the example building in Section 10.8.

Other Flat-Spanning Systems

While the slab-and-beam system is the most used and most adaptable sitecast concrete framing system, there are other common systems that may be used when circumstances permit. The following discussions treat two of these systems.

Two-Way Spanning Solid-Slab Construction

If reinforced appropriately in both directions, the solid concrete slab may span in two directions as well as one. The widest use of such a slab is in flat-slab or flat-plate construction. In flat-slab construction, beams are used only at points of discontinuity, with the typical system consisting

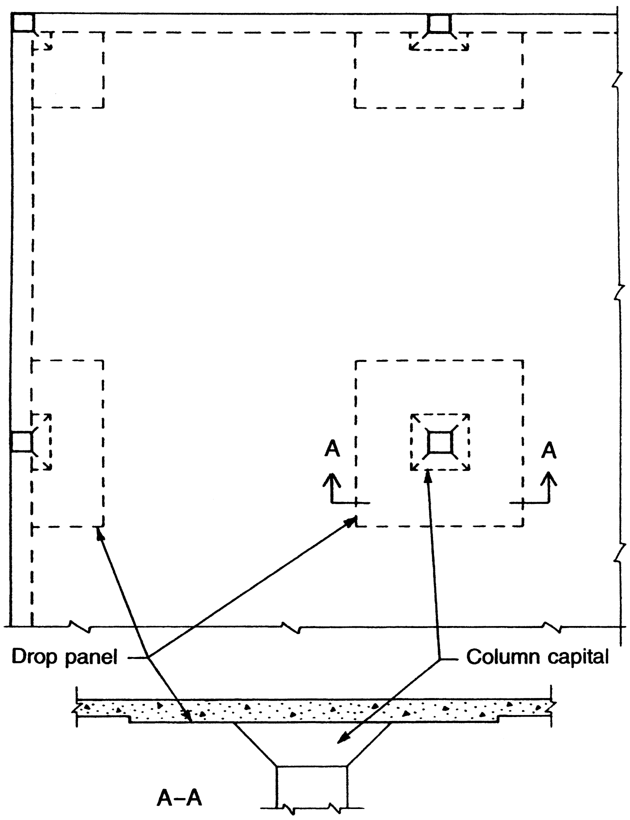


Figure 6.38 Framing plan and details for a flat-slab construction.

only of the slab and the strengthening elements at column supports. Typical details for a flat-slab system are shown in Figure 6.38.

Drop panels consisting of thickened portions of the slab, square in plan, are used to give additional resistance to the high shear and bending moment that develop at the column supports. Enlarged portions are also sometimes provided at the tops of columns (called *column capitals*) to reduce the stresses in the slab further, especially the punching shear effect.

Two-way slab construction consists of bays of two-way solid spanning slabs with edge supports consisting of bearing walls or of column-line beams. Typical details for such a system are shown in Figure 6.39.

Two-way spanning slab construction is generally favored over waffle construction where higher fire rating is required for the unprotected structure or where spans are short and loads are high. As with all types of two-way spanning structures, solid-slab systems function most efficiently where the spans in both direction are approximately the same.

For investigation and design, the flat slab is considered to consist of a series of one-way spanning solid-slab strips. Each of these strips spans through multiple bays in the manner of a continuous beam and is supported either by columns or by the strips that span in the direction perpendicular to it. The analogy for this is shown in Figure 6.40*a*.

As shown in Figure 6.40*b*, the slab strips are divided into types: those passing over the columns (called *column strips*) and those passing between columns (called *middle strips*). The

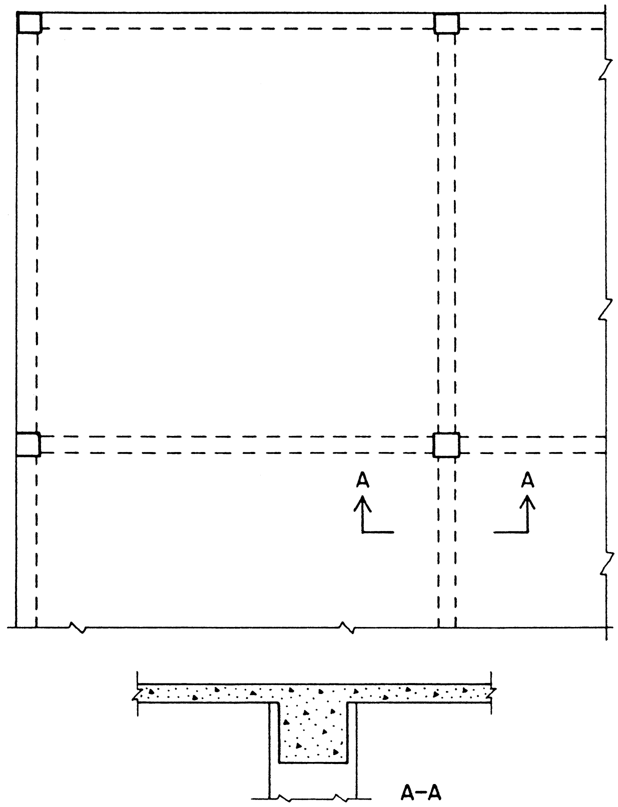


Figure 6.39 Framing plan and details for a two-way slab with edge supports.

complete structure consists of the intersecting series of these strips, as shown in Figure 6.40*c*.

For the flexural action of the system, there is two-way reinforcing in the slab in each of the boxes defined by the intersections of the strips. In box 1 in Figure 6.40*c*, both sets of bars are in the bottom portion of the slab, due to the positive moment in both intersecting strips. In box 2, the middle-strip bars are in the top (for negative moment), while the column strip bars are in the bottom (for positive moment). In box 3, the bars are in the top in both directions.

As with most continuous spanning structures, the highest magnitude of bending moments occur over the supports. Thus the most reinforcement is usually required in box 3 in Figure 6.40*c*; however, with the thickened slab at this location, as shown in Figure 6.38, the bars here may not be so extensive.

Composite Construction: Concrete with Structural Steel

Figure 6.41 shows a section detail of a type of construction generally referred to as *composite construction*. This consists of a sitecast concrete spanning slab supported by structural steel beams, the two being made to interact by the use of shear developers welded to the top of the beams and embedded in the cast concrete. The concrete may be formed by plywood sheets placed against the underside of the beam flange, resulting in the detail shown.

It is common to refer to this construction as composite construction; however, the term is itself more general, referring to any structural member in which parts of different materials are made to interact. By the general definition, therefore, ordinary reinforced concrete is "composite," with the concrete interacting with the steel reinforcement. The AISC manual (Ref. 10) contains data and design examples for the type of construction shown in Figure 6.41.

6.3 CONCRETE COLUMNS

In view of the ability of concrete to resist compression and its weakness in tension, it would seem to be apparent that its most logical use is for structural members whose primary task is the resistance of compression. This observation ignores the use of reinforcement to a degree but is nevertheless noteworthy. Indeed, major use is made of concrete for columns, piers, pedestals, posts, and bearing walls—all basically compression members. This section presents discussion of concrete columns, which also often exist in combination with concrete beam systems, forming rigid frames with vertical planar bents.

Effects of Compression Force

When concrete is subjected to a direct compressive force, the most obvious response in the material is one of compressive stress, as shown in Figure 6.42*a*. This response may be the essential concern, as it would be in a wall composed of flat, precast concrete bricks stacked on top of each other. Direct compressive stress in the individual bricks and in the mortar joints between bricks would be a primary situation for investigation.

However, if the concrete member being compressed has some dimension in the direction of the compressive force—as in the case of a column—there are other internal stress conditions that may well be the source of structural failure under the compressive force. Direct compressive force produces a three-dimensional deformation that includes a pushing out of the material at right angles to the force, actually producing tension stress in that direction, as shown in Figure 6.42*b*. In a tension-weak material, this action may produce a lateral bursting effect.

Because concrete is also weak in shear, another possible failure is along the internal planes where maximum shear stress is developed. This occurs at a 45° angle with respect to the direction of the applied force, as shown in Figure 6.42*c*. These effects are indeed the usual sources of failure in unreinforced concrete elements.

In concrete members, other than flat bricks, it is generally necessary to provide for all three stress responses shown in Figure 6.42. In fact, additional conditions can occur if the structural member is subjected to bending or torsion. Each case must be investigated for all the individual actions and the combinations in which they occur. This is the broad view of the behavior of the concrete column.

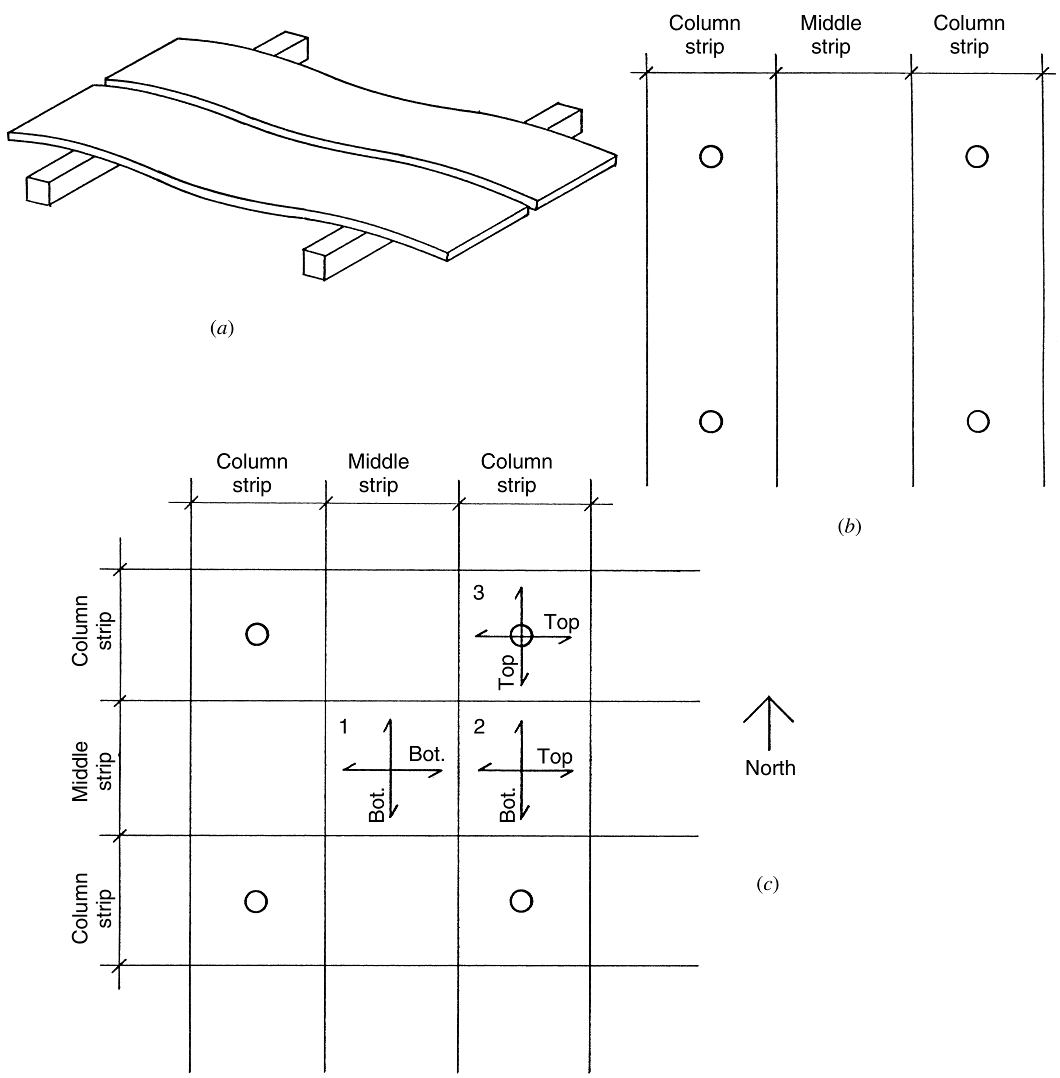


Figure 6.40 Strip analysis of the flat slab.

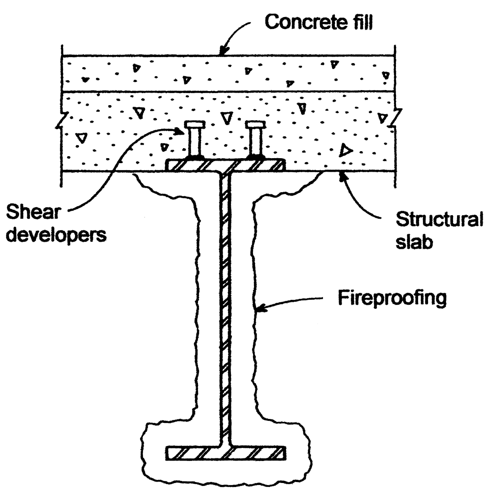


Figure 6.41 Composite construction with steel beams and a sitecast concrete slab.

Reinforcement for Columns

Column reinforcement takes various forms and serves various purposes, the essential consideration being to enhance the structural performance of the column. Considering the three basic forms of column stress failure shown in Figure 6.42, it is possible to visualize basic forms of reinforcement for each condition. This is done in Figure 6.43.

To assist the basic compression function (see Figure 6.43*a*), vertical steel bars are added. Although these displace some concrete, their superior strength and stiffness make them a significant improvement.

To assist in resistance to lateral bursting (Figure 6.43*b*), a critical function is to hold the concrete from moving out laterally, which may be achieved by so-called *containment* of the concrete mass, similar to the action of a piston chamber containing air or hydraulic fluid.

If compression resistance can be obtained from air that is contained, surely it can be more significantly obtained from contained concrete. This is a basic reason for the extra strength

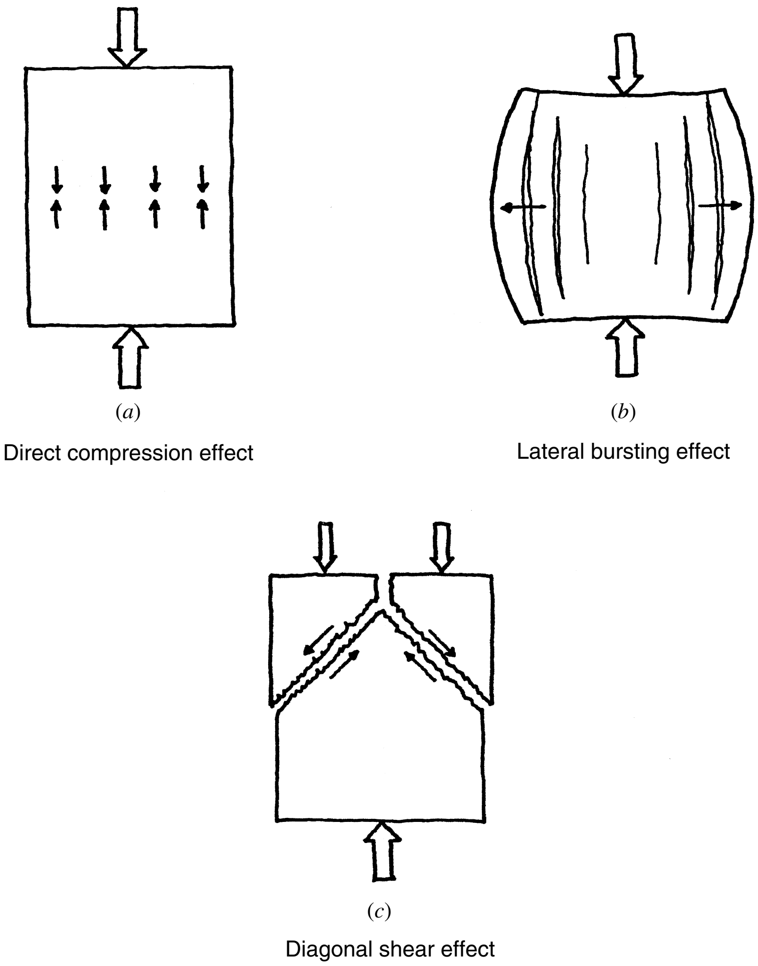


Figure 6.42 Fundamental failure modes of tension-weak concrete.

of the traditional helix-wrapped spiral column. Although not quite as effective, the wrapping ties shown in Figure 6.43*f* also provide a form of containment.

Natural shear resistance is obtained from the combination of the vertical bars and the lateral ties, as shown in Figure 6.43*c*. This resistance can be increased by closer spacing of the ties and by using a larger number of vertical bars spread out around the column perimeter.

When used as parts of concrete frameworks, columns are also typically subjected to torsion and bending, as shown in Figures 6.43*d* and *e*. Torsional twisting tends to produce a combination of longitudinal tension and lateral shear; thus, the combination of the full-perimeter ties and the perimeter vertical bars provide for this combined effect in most cases.

Types of Columns

Concrete columns occur most often as the vertical support elements in a structure generally built of sitecast concrete. Very short columns, called *pedestals*, are sometimes used as transitional devices between columns and their foundations; these are discussed in Chapter 8. The sitecast concrete column usually falls into one of the following categories:

Square columns with tied reinforcement
Oblong columns with tied reinforcement
Round columns with tied reinforcement
Round columns with spiral-bound reinforcement
Square columns with spiral-bound reinforcement
Columns of other geometries (L shaped, T shaped, octagonal, etc.) with either tied or spiral-bound reinforcement

Obviously, the choice of column cross-sectional shape is an architectural as well as a structural decision. However, forming methods and costs, arrangement and installation of reinforcement, and relations of the form and dimensions of columns to other parts of the structural system must also be dealt with.

In tied columns, the longitudinal (vertical) reinforcement is wrapped by loop ties made of small-diameter reinforcing bars, commonly No. 3 or No. 4 bars. Such a column is represented by the square section shown in Figure 6.44*a*. This type of reinforcement can quite readily accommodate other geometries as well as the square.

Spiral columns are those in which the longitudinal reinforcing is placed in a circle, with the whole group of bars enclosed by a continuous cylindrical spiral made from steel rod or large-diameter wire. Although this reinforcement

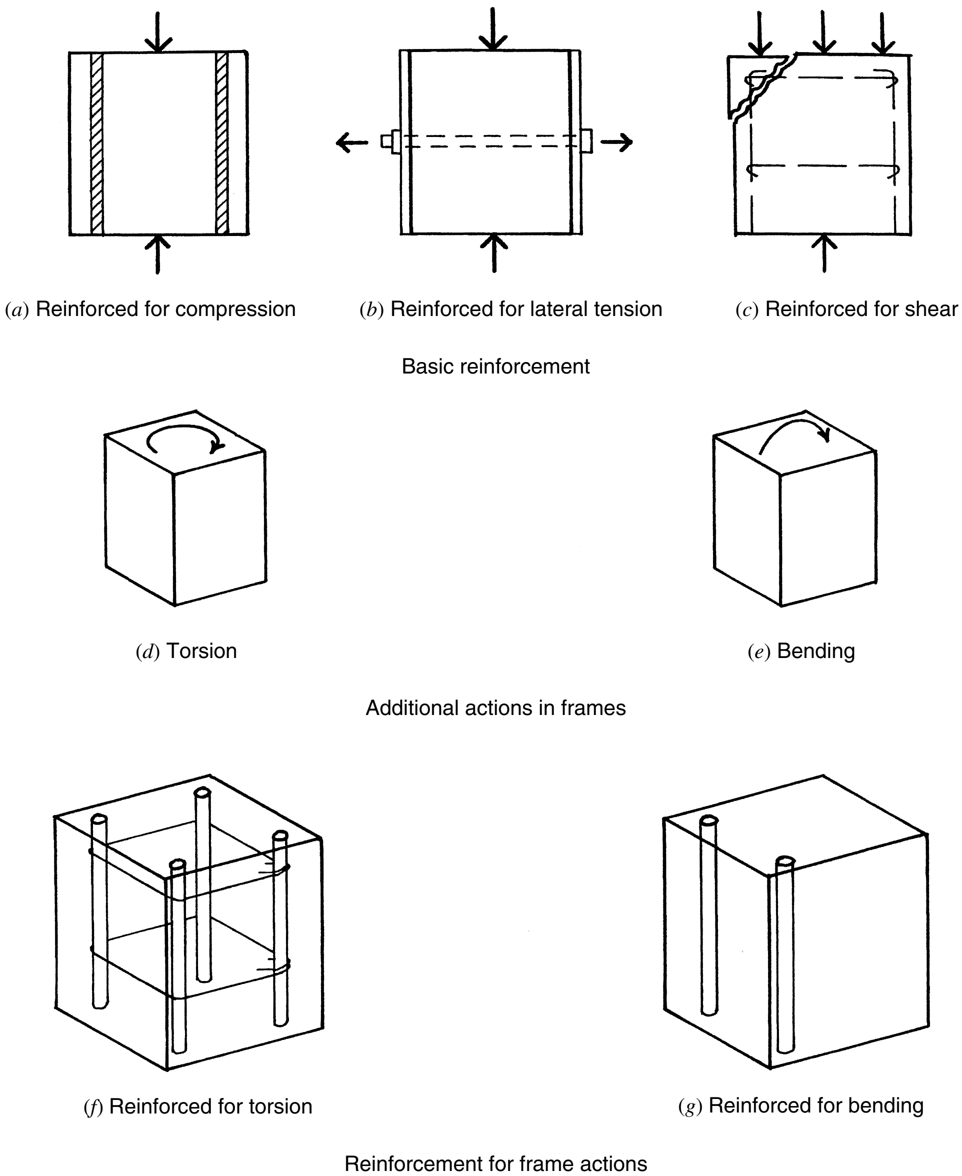


Figure 6.43 Forms and functions of column reinforcement.

system obviously works most logically with a round column cross section, it can be used with other shapes. A round column of this type is shown in Figure 6.44*b*.

Experience and testing have shown the spiral column to be slightly stronger than the tied column, mostly because of the more effective containment of the concrete core inside the perimeter of vertical bars. For this reason, code provisions allow slightly more load on spiral columns with the same concrete section and vertical reinforcement as that of a tied column. However, spirals are costly, and the round-bar pattern and round-column shape do not mesh well with other construction details; thus, tied columns are often favored where structural considerations permit their use.

For columns subjected to shear (as with lateral wind or seismic forces) favored column forms are those with spirals or with very closely spaced ties that simulate the nature of the spiral.

General Requirements for Columns

Code provisions and practical construction considerations place a number of restrictions on column dimensions and choice of reinforcement:

Column Size. The current code does not place limits for column dimensions. For practical reasons, the following limits are recommended. Rectangular tied columns should be limited to a minimum area of 100 in.^{2} and a minimum side dimension of 8 in. if rectangular and oblong. Spiral columns should be limited to a minimum diameter of 12 in. if round and 12 in. side dimension if square.

Reinforcement. Minimum bar size is No. 5. Minimum number of bars is four for tied columns, five for spiral columns. Minimum amount of steel is 1% of the gross section; recommended maximum is 4%, although 8%

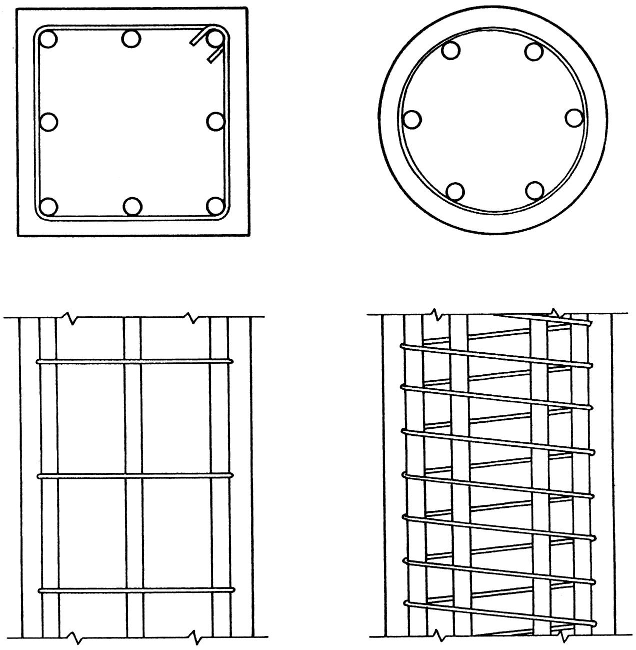


Figure 6.44 Primary forms of column reinforcement: (*a*) rectangular layout of vertical bars with lateral ties; (*b*) circular layout of vertical bars with a continuous helix (spiral) wrap.

may be used if bar spacing permits it. For columns with areas larger than required by computation, a reduced effective area not less than one-half the total area may be used to determine minimum reinforcement and design strength. Vertical bars are ordinarily all of a single size in each column. Higher grades of steel with greater yield strength are often used to reduce sizes of heavily loaded columns in tall buildings.

Ties. Ties should be at least No. 3 for bars that are No. 10 and smaller. No. 4 bars should be used for bars that are No. 11 and larger. Vertical spacing of ties should be not more than 16 times the vertical bar diameter, 48 times the tie diameter, or the least dimension of the column section. Ties should be arranged so that every corner and alternate bar is held by the corner of a tie with an included angle of not greater than 135°, and no bar should be farther than 6 in. clear from such a supported bar. Complete circular ties may be used for bars placed in a circular pattern.

Concrete Cover. A minimum of 1.5-in. cover is needed when the column surface is not exposed to weather and is not in contact with the ground. Cover of 2 in. should be used for formed surfaces exposed to the weather or in contact with the ground. Cover of 3 in. should be used if the concrete is cast directly against earth without forming, such as occurs on the bottom of footings.

Spacing of Bars. Clear distance between bars should be not less than 1.5 times the bar diameter, 1.33 times the maximum specified size for the coarse aggregate, or 1.5 in.

Combined Compression and Bending

Because of the nature of most concrete structures, design practices generally do not consider the possibility of a concrete column with axial compression alone. This is to say, the existence of some bending moment is always considered together with the axial force. Figure 6.45 illustrates the nature of the so-called *interaction response* for a column, with a range of combinations of axial load plus bending moment. In general, there are three basic parts of the range of this behavior, as follows (see the dashed lines in Figure 6.45):

Large Axial Force, Minor Moment. For this case, the moment has little effect, and the resistance to axial force is only negligibly reduced.

Significant Values for Both Axial Force and Moment. For this case, the analysis for design must include the full combined force effects, that is, the interaction of the axial force and the bending moment.

Large Bending Moment, Minor Axial Force. For this case, the column behaves essentially as a double reinforced bending member, with its capacity for moment resistance affected only slightly by the axial force. Since design of bending members is controlled to affect a tension failure, the axial compression actually partly prestresses the tension reinforcement, adding to its tension capability.

In Figure 6.45, the solid line on the graph represents the nature of the true reposnse of the column—a form of behavior verified by many laboratory tests. The dashed line represents the generalization of the three types of response just described.

The terminal points of the interaction response—pure axial compression or pure bending moment—may be reasonably easily established (P_o and M_o in Figure 6.45). the interaction responses between these two limits are established by empirically determined analyses derived from laboratory tests. These analyses are beyond the scope of this book.

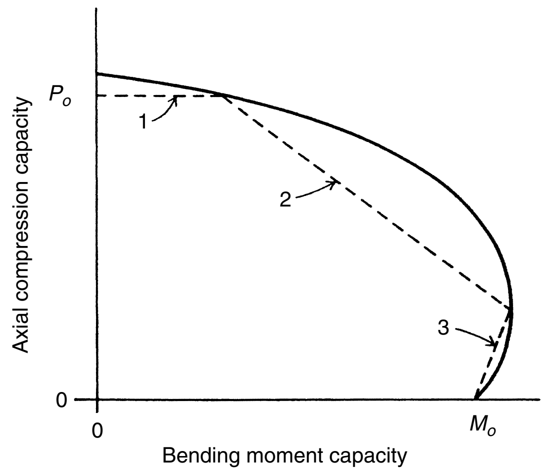


Figure 6.45 Interaction of axial compression and bending in a reinforced concrete column.

A special type of bending is generated when a relatively slender compression member develops some significant curvature due to bending. In this case, the center portion of the member literally moves sideways; this sideways deflection is called *delta*. When this occurs, a bending moment is induced consisting of the product of the compression force *P* and the deflected dimension delta. As this effect produces additional bending, and thus additional deflection, it may become a progressive failure, called the *P–delta effect*. Because concrete columns are not usually very slender, this effect is usually of less concern than it is for columns of wood and steel.

Considerations for Column Shape and Layout of Reinforcement

Usually, a number of possible combinations of reinforcing bars may be assembled to satisfy the steel area requirement for a given column. Aside from providing for the required cross-sectional area, the number of bars must also work reasonably in the layout of the column. Figure 6.46 shows a number of columns with various numbers of bars.

When a square tied column is small, the preferred choice is usually for the simple four-bar layout, with a bar in each corner and a single perimeter tie. As the column gets larger, the distance between the corner bars gets larger, and it is best to use more bars so that the reinforcement is spread out around the column perimeter.

For a symmetrical layout, and the simplest of tie layouts, the best choice is for numbers that are multiples of four, as shown in Figure 6.46*a*. The number of additional ties required for these layouts depends on the size of the column and the basic requirements for ties, as discussed previously.

An unsymmetrical bar arrangement (Figure 6.46*b*) is not necessarily bad, even though the column and its construction details are otherwise not oriented differently on the two axes. In a situation where there is a considerable moment on one axis, the unsymmetrical layout is actually preferred; in fact, the column shape will also be more effective if it is unsymmetrical, as shown for the oblong shapes in Figure 6.46*c*.

Figures 6.46*d* through *g* show some special shapes developed as tied columns. Spirals could be used here, but the use of ties permits a greater flexibility and simplicity for construction. One reason for using ties for narrow shapes may be the practical limit of a 12-in. width for a spiral column.

Round columns may be formed as shown in Figure 6.46*h* if built as tied columns. This allows for a minimum reinforcement with four bars. If a round-bar layout is used (as it must be for a spiral column), the usual preferred limit is for six bars, as shown in Figure 6.46*i*. Spacing of bars is much more critical in spiral columns, making it more difficult to use a high percentage of steel. For very large diameter round columns it is possible to use sets of concentric spirals, as shown in Figure 6.46*j*.

For sitecast columns, a concern that must be dealt with is the vertical splicing of the steel bars. Two places where this commonly occurs are at the top of the foundations and at floors where a multistory column continues upward. At these points, there are three ways to achieve the vertical continuity (splicing) of the bars, any of which may be appropriate for a given situation:

- Bars may be lapped the required distance for the development, as illustrated in Section 6.2. For bars of smaller diameter and lower yield strengths, this is usually the desired method.
- Bars may have milled square-cut ends butted together with a grasping device to prevent separation in a horizontal direction.
- Bars may be welded with full-penetration butt welds or by welding the grasping device described for the milled end bars.

The choice of splicing methods is basically a matter of cost comparison but is also affected by the size of the bars, by the degree of concern for bar spacing, and possibly by a need for development of tension through the splice if uplift or high magnitude of moment exists. This may also be left to the option of the bar fabricators and installers.

If lapped splicing is used, a problem that must be considered is the bar layout at the location of the splice, at which point there will be twice the usual number of bars. The lapped bars may be adjacent to each other, but the usual spacing between bars must be considered. If spacing is not critical, the arrangement shown in Figure 6.46*k* is usually chosen, with the spliced sets of bars next to each other at the column perimeter. If spacing limits prevent the arrangement shown in Figure 6.46*k*, that shown in Figure 6.46*l* may be used, with the lapped sets in concentric patterns. The latter arrangement is often used with spiral columns, where spacing is typically critical.

Bending of steel bars involves the development of yield stress to achieve plastic deformation (the residual bend). As bars get larger in diameter, it is more difficult to bend them. Also, as the yield strength increases, the bending effort gets greater. It is questionable to try to bend bars larger than No. 11 in any grade, and it is also advised not to bend bars with yield strength greater than 75 ksi.

Multistory Columns

Concrete columns occur frequently in multistory structures. In the sitecast structure, separate stories are typically cast in separate pours, with a cold joint (called a construction joint) between successive pours. Although this makes for a form of discontinuity, it does not significantly reduce the effective monolithic nature of the framed structure. Compression is made vertically continuous by the simple stacking of the levels of the heavy structure, and splicing of the reinforcement develops a form of tension continuity, permitting development of net bending moments at the cold joints.

The typical arrangement of reinforcement in multistory columns is shown in Figure 6.32, which illustrates the form of bar development required to achieve the splicing of the reinforcing bars. This is essentially compressive

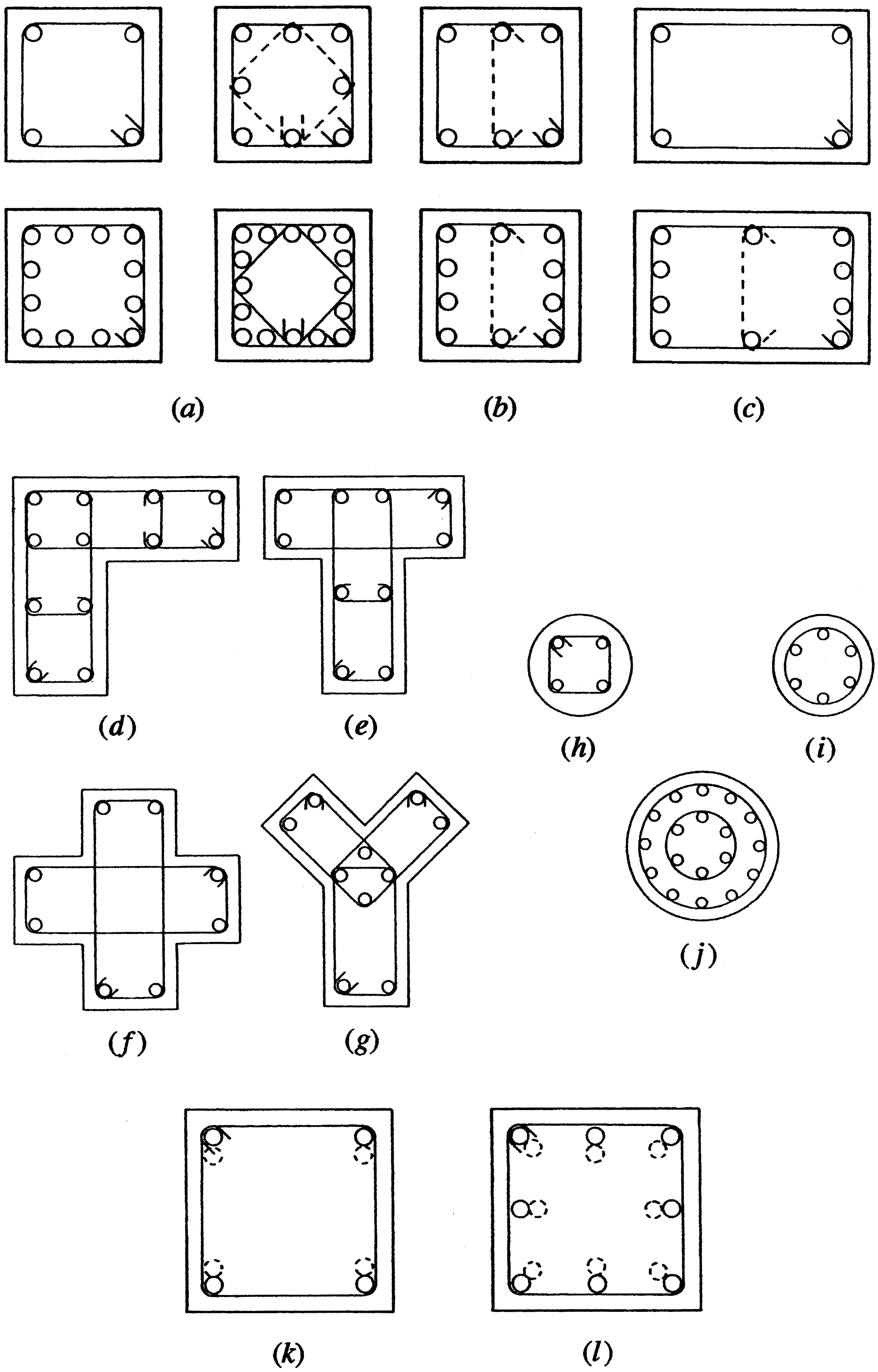


Figure 6.46 Considerations for reinforcement layout in concrete columns.

reinforcement, so its development is viewed in those terms. However, an important practical function of the column bars is simply to tie the structure together through the discontinuous construction joints.

Load conditions change in successive stories of the multistory structure. It is therefore common practice to change both the column size and the reinforcement to relate to this change. Design considerations for this are discussed for the example building in Section 10.8.

In very tall structures, the magnitude of compression in lower stories requires columns with very high resistance. There is often some practical limit to column sizes, so that all

efforts are made to obtain strength increases by means other than simply increasing the mass of concrete. The three basic means of achieving this are:

Increase the amount of reinforcement, packing columns with the maximum amount that is feasible and is allowed by codes.
Increase the yield strength of the steel using as much as twice the strength of ordinary bars.
Increase the strength of the concrete.

The superstrength column is a clear case for use of the highest achievable concrete strengths and is indeed the application that has resulted in spiraling high values for design strength. Strengths exceeding 20,000 psi have been achieved, resulting in the pushing up of height limits for high-rise concrete structures.

Design Methods and Aids

At present, design of concrete columns is mostly achieved by using either tabulations from handbooks or computer-aided procedures. Using the code formulas and requirements to design by "hand operation," with both axial compression and bending present, is prohibitively laborious. The number of variables present (column shape and size, f_c', f_y, number and size of bars, arrangement of bars, etc.) adds to the usual problems of column design. This makes for a situation much more complex than that for wood or steel columns.

The large number of variables also works against the efficiency of handbook tables. Even if a single concrete strength and single steel yield strength are used, tables would be very extensive if the rest of the variables are incorporated. Available tables, nevertheless, often are quite useful for preliminary design estimation of column sizes.

The obvious preference, when relationships and data are complex, is for the use of some computer-aided procedure for investigation and design. The software for such systems is readily available and is routinely used by most professional engineering offices.

As in many other situations, the common practices at any given time tend to narrow down to a limited usage of any type of construction, even though the potential for variation is extensive. It is thus possible to use some very limited but easy-to-use design aids to make early decisions and selections for preliminary cost estimates.

Approximate Design of Tied Columns

Tied columns are much preferred to other columns (essentially spiral columns) due to the relative simplicity of their construction and their lower cost. They are also very adaptable to column shapes, including square, round, oblong, L shaped, and T shaped. Round columns—most naturally formed with spiral-bound reinforcement—are often made with ties instead when the structural demands are modest.

The column with moment is often designed using the equivalent load method. Using this method, Figures 6.47 through 6.53 yield capacities for selected column sizes and shapes with varying amounts of reinforcement. Allowable axial compression loads are given for various degrees of eccentricity. Data for the curves were computed by strength design methods, as currently required by the ACI code.

The most widely used shape is the square tied column. However, when moment is high on one axis, the oblong shape is more efficient; curves for this shape are presented in Figures 6.51 and 6.52. In the oblong columns, the reinforcement shown is arranged into two sets, one at each narrow side, thus generating maximum moment capacity.

The following examples illustrate the use of the column design curves.

Example 12. A square tied column with $f_c' = 5$ ksi [34.5 MPa] and steel with $f_y = 60$ ksi [414 MPa] sustains an axial compression load of 150 kips [667 kN] dead load and 250 kips [1110 kN] live load, with no computed bending moment. Find the minimum practical column size if reinforcement is a maximum of 4% and the maximum size if reinforcement is a minimum of 1%.

Solution. As in all problems, this one begins with the determination of the factored ultimate axial load; thus,

$$P_u = 1.2(\text{DL}) + 1.6(\text{LL})$$
$$= 1.2(150) + 1.6(250) = 580\,\text{kips}\ [2580\,\text{kN}]$$

Since the curves in the figures are plotted with the factored resistance of the columns, no adjustment is made of the computed ultimate load.

With no consideration for bending moment, the maximum axial load capacity is determined from the graphs by simply reading up the left edge of the figure, with the eccentricity equal to zero.

Using Figure 6.47, the minimum size is a 14-in. square column with four No. 9 bars, for which the graph yields a maximum capacity of approximately 590 kips with a steel percentage of only 2.04%. Since this is not the limit for steel percentage, it is possible that a 13-in. square column may be possible, although the graph does not include this size.

What constitutes the maximum size is subject to some judgment. Any column with a curve above that of the chosen minimum column will work. It becomes a matter of increasing redundancy of capacity. However, there are often other design considerations involved in developing whole structural decisions, so these examples are quite academic. For this example, with the 14-in. square chosen as the minimum size, going up to a 15-in. or 16-in. size will reduce the reinforcement. From the limited choices in Figure 6.50, a maximum size is 16-in. square with four No. 8 bars, having a capacity of 705 kips and a steel area of 1.23%. Larger size columns may be used safely but will have increasing redundant strength.

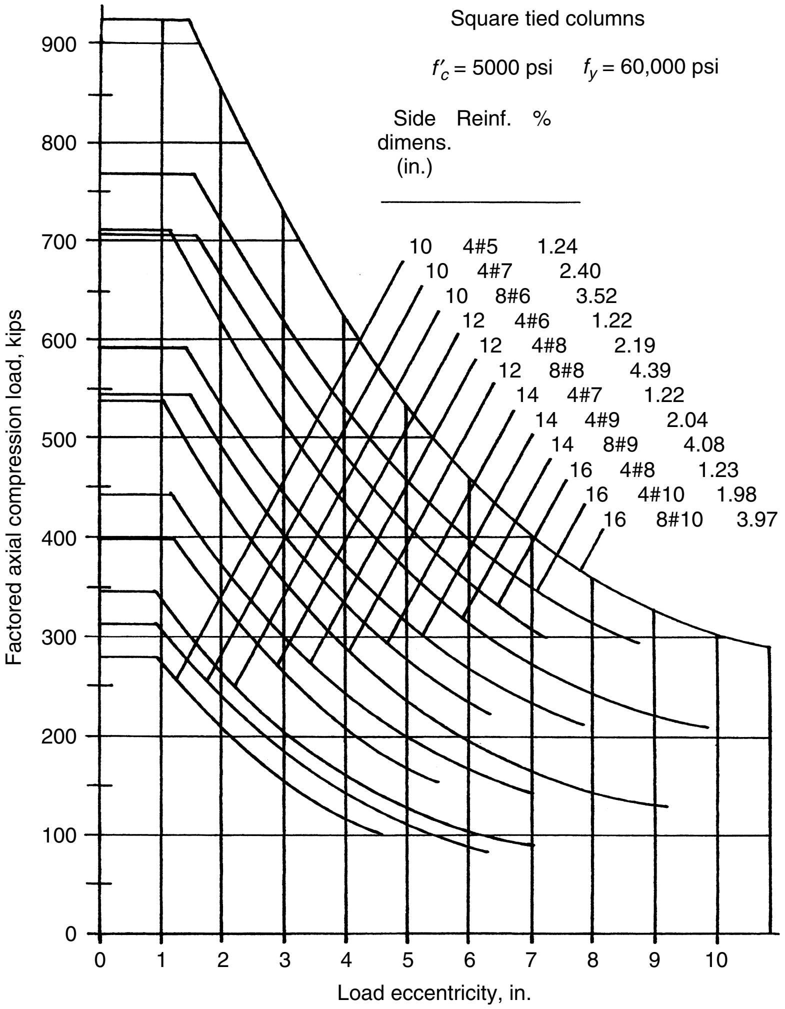


Figure 6.47 Factored axial compression capacity for selected square tied columns.

This example has dealt with a column without a stated bending condition. This may be virtually true in some real situations, the closest example being when the moment is relatively insignificant in comparison with the magnitude of the compression force. In fact, the design process created by the code limitations actually defines a minimum bending condition by eliminating an actual zero eccentricity. Inspection of the curves in the design graphs shows that they stop not at the edge of the graph but at a distance from the edge.

The following example illustrates the design procedure when a significant bending moment is combined with the axial compression load.

Example 13. A square tied column with $f'_c = 5$ ksi [34.5 MPa] and steel with $f_y = 60$ ksi [414 MPa] sustains axial loads of 150 kips [667 kN] dead load plus 250 kips [1110 kN] live load and bending moments of 75 kip-ft [112 kN-m] dead load plus 125 kip-ft [170 kN-m] live load. Determine the minimum-size column and its reinforcement.

Solution. First, determine the ultimate axial load and ultimate bending moment. The axial loads are the same as in Example 12, from which $P_u = 580$ kips [2580 kN]:

$$M_u = 1.2(75) + 1.6(125) = 290 \text{ kip-ft } [393 \text{ kN-m}]$$

Next, determine the equivalent eccentricity; thus,

$$e = \frac{M_u}{P_u} = \frac{290 \times 12}{580} = 6 \text{ in. } [152 \text{ mm}]$$

Then, from Figure 6.48, the minimum size is an 18-in. square with eight No. 11 bars and a steel amount of 3.85%. This column has a capacity, at a 6-in. eccentricity, of approximately 650 kips. If this is considered to be too high a percentage of steel, use a 20-in. square column with four No. 10 bars with a capacity of approximately 675 kips. The 20-in. column has only 1.27% steel.

The following example illustrates the process for design of a tied column with an oblong cross section.

Example 14. Select the minimum-size oblong-shaped (rectangular) column for the same data as used in Example 13.

Solution. With the factored axial load of 580 kips and the eccentricity of 6 in., Figure 6.51 yields the following: a $14 \times 24 -$ in. column with six No. 10 bars having an axial load capacity of approximately 730 kips.

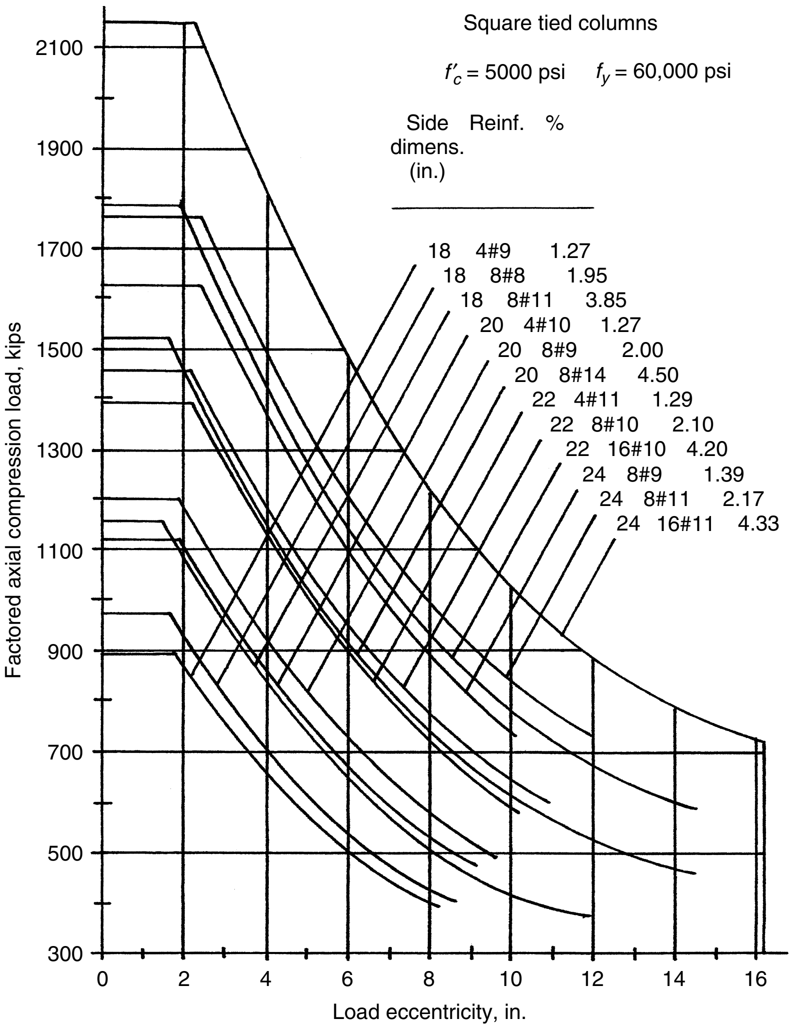


Figure 6.48 Factored axial compression capacity for selected square tied columns.

As previously described, the reinforcement in the rectangular columns is assumed to be placed with one-half the bars in each of the narrow ends of the section.

Round Columns

Round columns, as discussed previously, may be designed and built as spiral columns or they may be developed as tied columns with bars in a rectangular layout or with the bars placed in a circle and held by a series of round circumferential ties. Because of the cost of spirals, it is usually more economical to use the tied column, so it is often selected unless the additional strength of the spiral column is required.

Figure 6.53 gives capacities for round columns that are designed as tied columns. As for the square and oblong columns in Figures 6.47 through 6.52, load capacity has been determined by strength design methods, and use is similar to that illustrated in the preceding examples.

In general, square and rectangular columns are much easier to integrate with walls and other elements of the construction. Round shapes, on the other hand, generally appear less massive and are less intrusive when occurring in freestanding situations within occupied spaces.

6.4 CONCRETE FOUNDATIONS

Almost every building utilizes some concrete construction that is built directly in contact with the ground. Such elements include the following:

- Shallow bearing footings, consisting of concrete pads that are used to spread the vertical loads of the building onto the supporting soil materials
- Concrete piles, pile caps, or concrete-filled excavated shafts used to develop deep foundations
- Concrete walls, enclosing below-grade spaces or forming grade beams
- Concrete pavements for driveways, walks, parking lots, patios, or building floors placed directly on the ground
- Retaining walls, used to achieve sudden changes of the site profile.
- Underground tunnels and vaults for service systems
- Bases for signs, exterior lighting, elevators, and other equipment

This construction constitutes a major use of concrete. Its low bulk cost, general nondecaying nature, rocklike

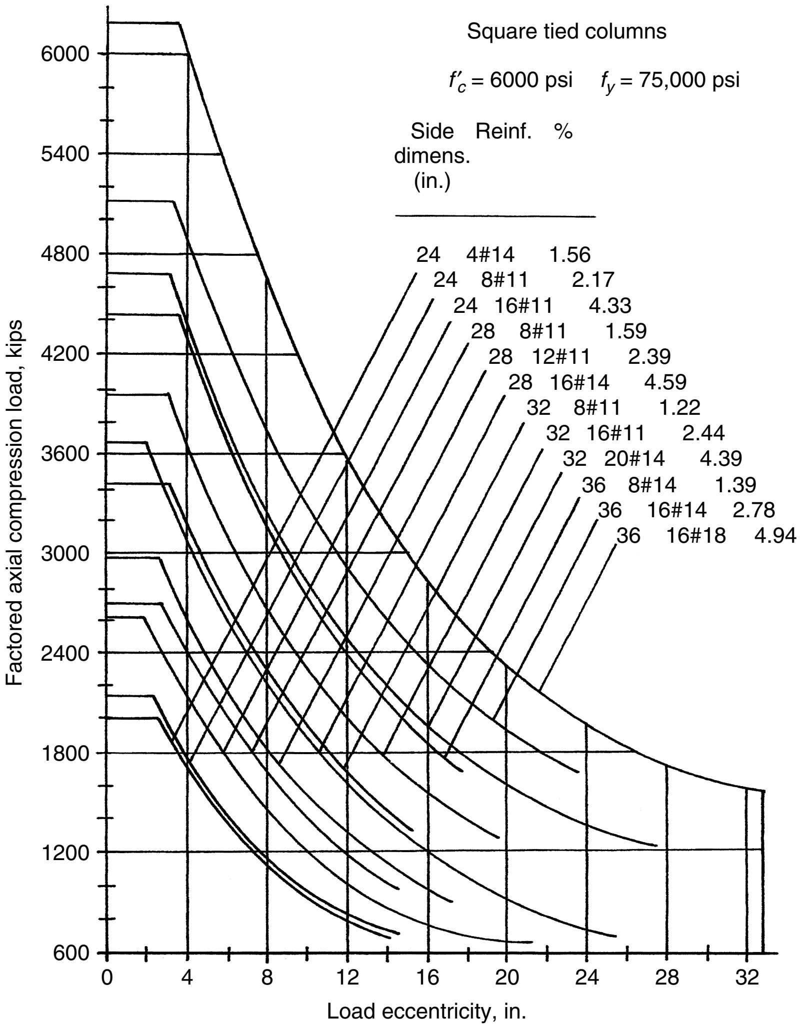


Figure 6.49 Factored axial compression capacity for selected square tied columns.

character, general overall stiffness of thick elements, and ease of placement in the ground-level work make it the natural choice for these applications.

In times past, much of this construction was achieved with masonry materials. This is still possible but is limited mostly to use of concrete block masonry, now referred to as CMU, for *concrete masonry unit,* construction.

There are two topics involved in foundation design. The first has to do with the nature and behavior of soils and their usage for construction. The second topic is the structural design of the foundation elements—mostly consisting of members of reinforced concrete. The first topic is treated in Chapter 8; the design of simple elements of column and wall footings is discussed here.

Shallow Bearing Foundations

The most common foundation consists of pads of concrete placed directly beneath the building. Because most buildings have relatively shallow penetration into the ground, these pads, called *footings*, are generally classified as *shallow bearing foundations.* For simple economic reasons, shallow foundations are generally preferred. However, when adequate soil does not exist at a shallow distance below grade, driven piles or excavated piers (called caissons)—which extend some distance below grade—must be used; these are called *deep foundations.*

The two common footings are the wall footing and the column footing. Wall footings occur in strip form, usually placed symmetrically beneath the supported wall. Column footings are most often simple square pads supporting a single column. When columns are very close together or at the edge of the building, special footings that carry more than a single column may be used. A critical design factor for footings is the permissible soil pressure for support of the footings.

Two other foundation construction elements that occur frequently are foundation walls and pedestals. Foundation walls may be used as basement walls or merely as transition between deeply placed footings and the above-ground building construction. For construction with wood or steel, the foundations must extend a sufficient distance above grade level to protect the supported construction. When spanning between isolated column footings or weak soils, walls may serve as grade beams.

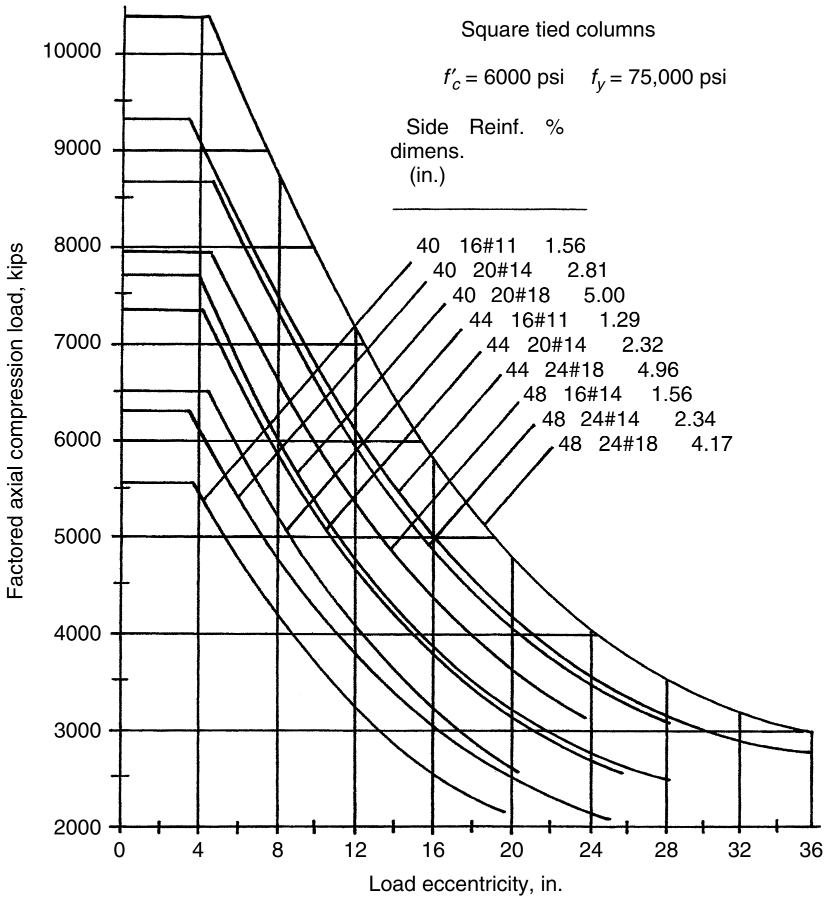


Figure 6.50 Factored axial compression capacity for selected square tied columns.

Pedestals are actually short columns used as transitions between building columns and their foundations. These may also be used to keep wood or steel columns above contact with the ground. Considerations for design of pedestals are treated in Chapter 8.

Column Footings

The great majority of independent or isolated column footings are square in plan, with reinforcement consisting of two equal sets of bars at right angles to each other. The column may be placed directly on the footing or it may be supported directly by a pedestal, wider than the column, which in turn is supported by the footing. The design of a column footing is based on the following considerations:

Maximum Soil Pressure. The sum of the unfactored, superimposed load on the footing and the unfactored weight of the footing must not exceed the limit for bearing pressure on the supporting soil materials. The required total plan area of the footing is derived on this basis.

Design Soil Pressure. By itself, simply resting on the soil, the footing does not generate shear or bending stresses. These are developed only by the superimposed loads. Thus, the pressure to be used for design of the footing is determined by dividing the factored, superimposed load by the actual plan area of the chosen footing.

Control of Settlement. Where buildings rest on highly compressible soil, it may be necessary to select footing plan areas that ensure uniform settlement of all the building foundations. For some soils (notably ones with a high clay content), long-term settlement under dead load only may be more critical in this regard and must be considered, along with maximum soil pressure.

Size of the Column. The larger the column, the less will be the shear and bending effects in the footing because these are developed by the cantilever effect of the footing projection beyond the edges of the column.

Shear Capacity Limit for the Concrete. For square-plan footings, this is usually the critical stress in the footing. To achieve an economical design, the footing thickness is usually chosen to reduce the need for reinforcement to a minimum. Although small in volume, the steel reinforcement is a major cost factor; thus the thickened footing reduces the area of steel and also the compression bending stress in the concrete.

Flexural Tension Stress and Development for the Bars. These are the main concerns for the steel bars, on the basis of the cantilever bending action. It is also to control the number of bars to keep spacing within some limits.

Development Length for Column Bars. When the column is directly supported, the vertical column bars must

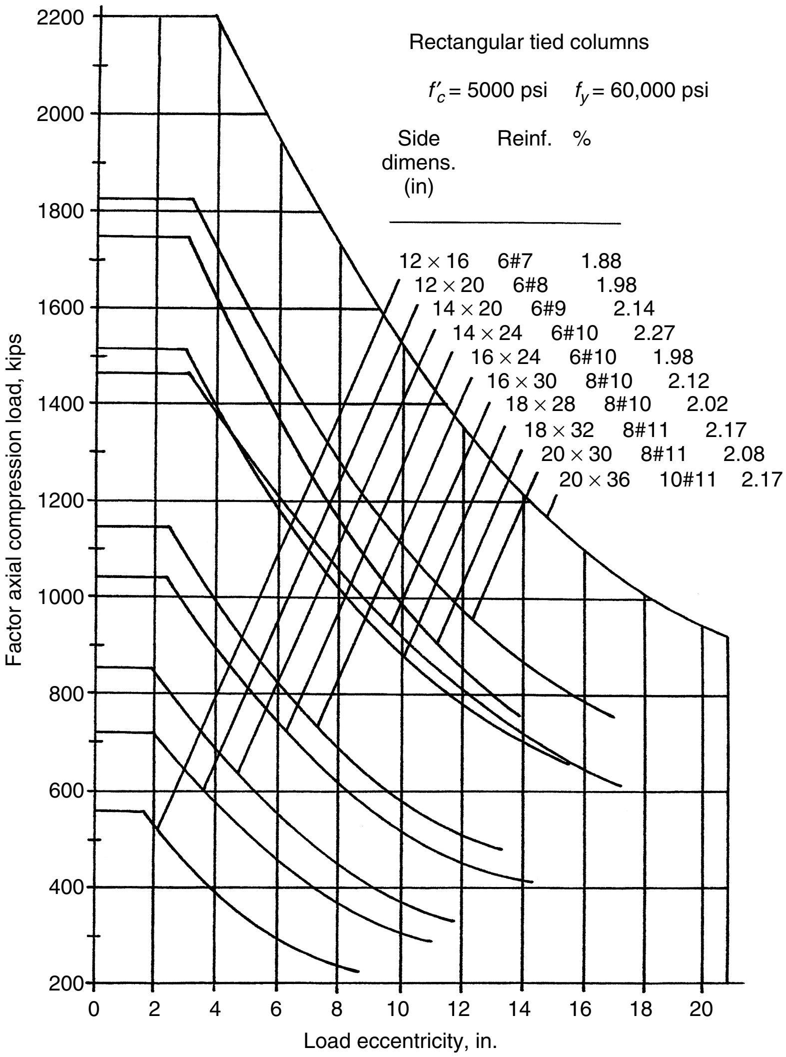


Figure 6.51 Factored axial compression capacity for selected rectangular tied columns.

be developed in the footing by dowelling action. The thickness of the footing must be adequate for this purpose.

The following example illustrates the design process for a simple square column footing.

Example 15. Design a square column footing for the following data:

Column load = 200 kips [890 kN] dead load and 300 kips [1334 kN] live load
Column size = 15 in. [380 mm]
Allowable soil pressure = 4000 psf [191 kPa]
Concrete design strength = 3 ksi [20.7 MPa]
Yield strength of steel = 40 ksi [276 MPa]

Solution. A quick guess for the footing size is to divide the total service load by the allowable soil pressure; thus,

$$A = \frac{500}{4} = 125\ \text{ft}^2, \quad w = \sqrt{125} = 11.2\ \text{ft}$$

This does not allow for the footing weight, so the actual size required will be slightly larger. However, it moves the guessing into the approximate range. For a footing this large, the first guess for a thickness is a shot in the dark unless a reference is used for footings in this approximate range. Try $h = 31$ in.; then footing weight is

$$w_f = \frac{31}{12}(150) = 388\ \text{psf}$$

and the net usable pressure is

$$4000 - 388 = 3612\ \text{psf}$$

The required plan area of the footing is thus

$$A = \frac{500}{3.612} = 138.4\ \text{ft}^2$$

and the required width is

$$w = \sqrt{138.4} = 11.76\ \text{ft}$$

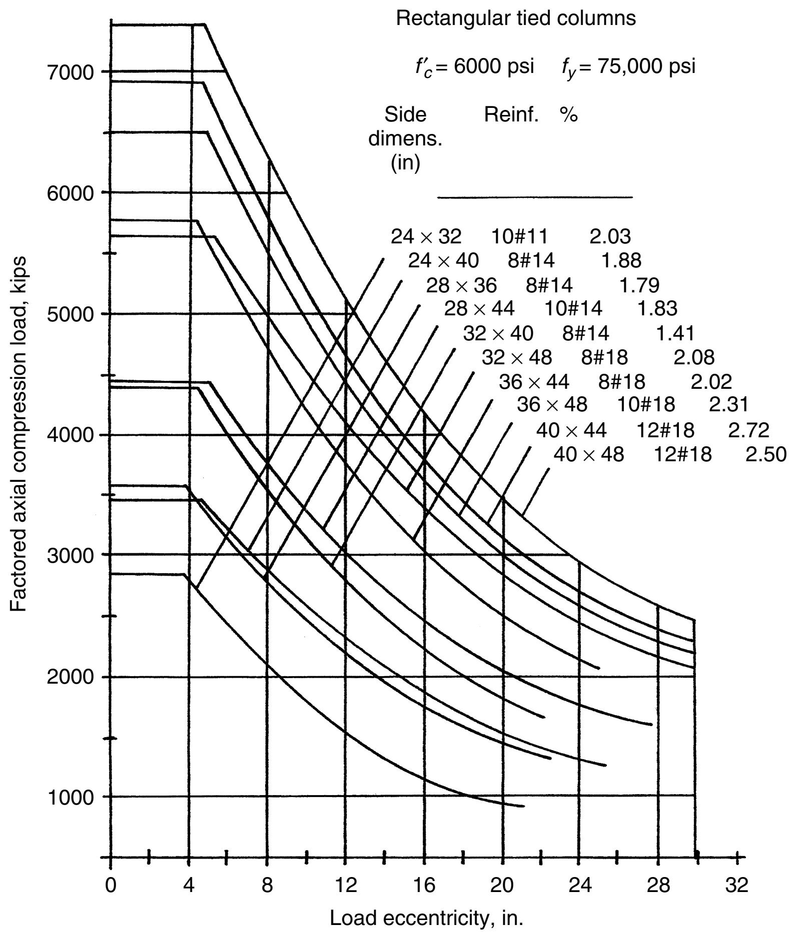


Figure 6.52 Factored axial compression capacity for selected rectangular tied columns.

Try $w = 11\,ft$ 9 in., or 11.75 ft. Then design soil pressure is

$$q = \frac{500}{(11.75)^2} = 3.622\,\text{ksf}$$

For determining footing thickness and reinforcement, a factored soil pressure is needed; thus,

$$P_u = 1.2(200) + 1.6(300) = 720\,\text{kips}$$

$$q_u = \frac{720}{(11.75)^2} = 5.22\ \text{ksf, or } 5220\,\text{psf}$$

Determination of the bending force and moment is as follows (see Figure 6.54):

$$\text{Bending force } F = 5.22 \times \frac{63}{12} \times 11.75 = 322 \text{ kips}$$

$$\text{Bending moment } M_u = 322 \times \frac{63}{12} \times \frac{1}{2} = 845 \text{ kip-ft}$$

$$\text{For design, } M_r = \frac{M_u}{\phi_b} = \frac{845}{0.9} = 939 \text{ kip-ft}$$

This bending moment is assumed to operate in both directions on the footing and is provided for with similar reinforcement in each direction. However, it is necessary to place one set of bars on top of the other, perpendicular, set, as shown in Figure 6.55, and there are thus different effective depths in the two directions. A practical procedure is to use the average of these two depths, that is, a depth equal to the footing thickness minus the 3-in. cover and one bar diameter. This theoretically results in a minor overstress in one direction, which is compensated for by a minor understress in the other direction.

To proceed it is necessary to assume a bar size, which is a guess until a choice is actually made. Assuming a No. 9 bar for the footing, the effective depth thus becomes

$$d = h - 3 - 1.13 = 31 - 3 - 1.13 = 26.9\,\text{in.}$$

The concrete section resisting the moment is thus one that is 141 in. wide and has an effective depth of 26.9 in. Using the resistance factor for a balanced section from Table 6.2, the balanced moment capacity of this section is

$$M_r = Rbd^2 = \frac{1.149 \times 141 \times (26.9)^2}{12} = 9{,}770 \text{ kip-ft}$$

As this is more than 10 times the required moment, the compressive bending stress in the concrete is clearly not critical. This leaves the consideration for shear as the potentially critical concern for the footing thickness.

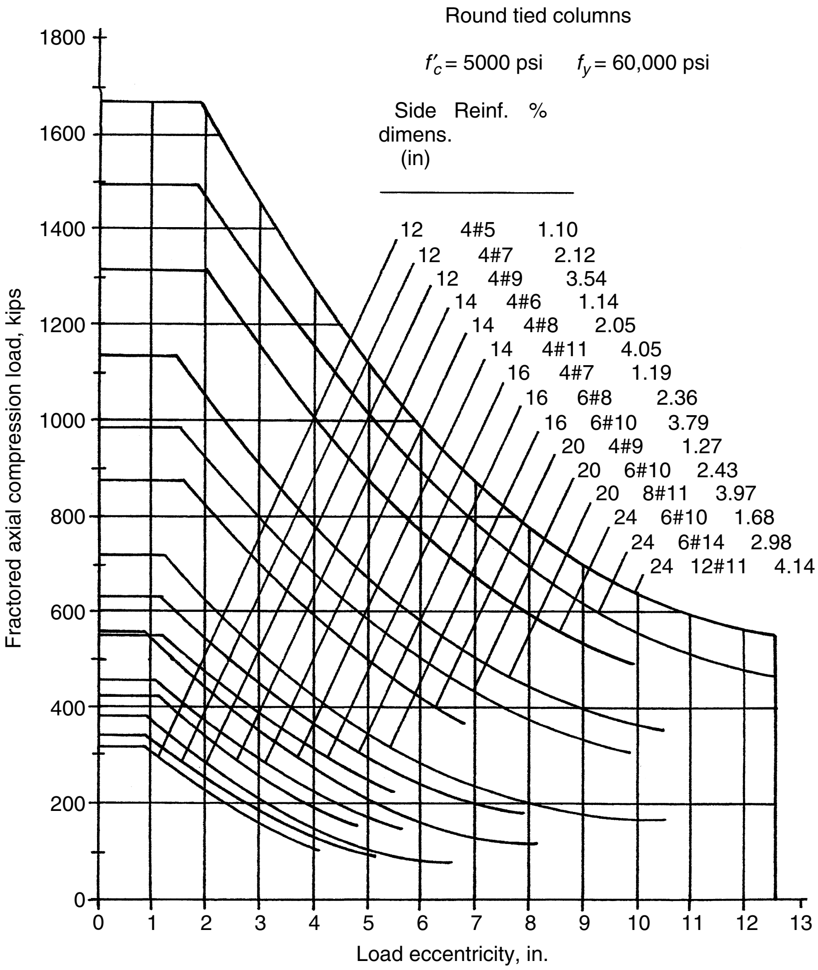


Figure 6.53 Factored axial compression capacity for selected round tied columns.

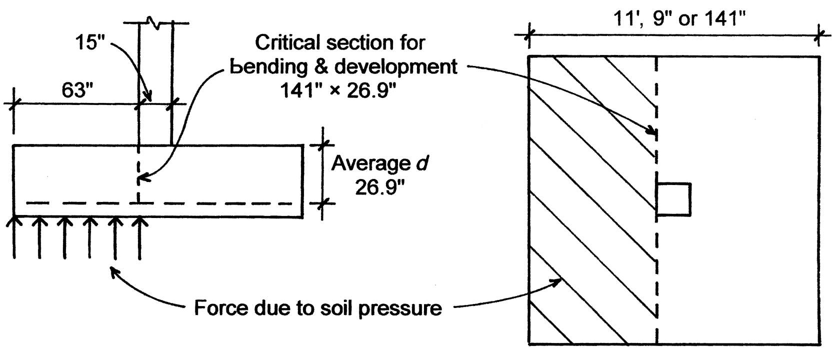


Figure 6.54 Consideration of bending and bar development in the column footing.

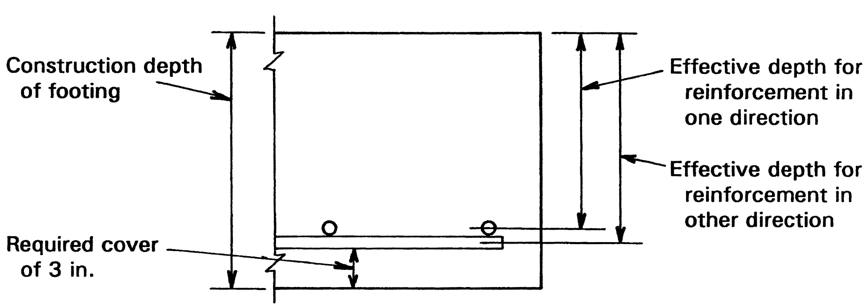


Figure 6.55 Consideration for effective depth in the column footing.

The two conditions for investigation of shear are shown in Figure 6.56. For beam-type shear (Figure 6.56*a*)

$$V_u = 5220 \times 11.75 \times \frac{36.1}{12} = 185{,}000 \text{ lb}$$

For the shear capacity of the concrete

$$V_c = 2\sqrt{f_c'}(bd) = 2\sqrt{3000}(141 \times 26.9) = 415{,}000 \text{ lb}$$

$$\phi_v V_c = 0.75(415{,}000) = 311{,}000 \text{ lb}$$

Beam shear is thus not critical.

For punching shear (Figure 6.56*b*)

$$V_u = 5220\left[(11.75)^2 - \left(\frac{41.9}{12}\right)^2\right] = 657{,}000 \text{ lb}$$

The shear capacity of the concrete for punching shear is

$$V_c = 4\sqrt{f_c'}(bd) = 4\sqrt{3000}(4 \times 41.9)(26.9) = 988{,}000 \text{ lb}$$

$$\phi_v V_c = 0.75 \times 988{,}000 = 741{,}000 \text{ lb}$$

The punching shear capacity is only slightly larger than that required, so the 31-in. thickness is probably the minimum full-inch-thickness dimension required.

Because the section is considerably underreinforced, the value for a/d will be much smaller than that for a balanced section (1.149 from Table 6.2). Assuming a value of 0.4 for a/d, $a = 0.4(26.9) = 5.38$ in.; then

$$A_s = \frac{M_r}{f_y(d - a/2)} = \frac{939{,}000 \times 12}{40{,}000 \times (26.9 - 5.38/2)} = 11.64 \text{ in.}^2$$

Using Table 6.1 for bar areas and Table 6.8 for development lengths, possible selections for the footing reinforcement are shown in Table 6.12. Also shown in the table is the center-to-center spacing of the bars. Any of these choices is acceptable, although lower cost is usually obtained with the fewest number of bars.

Table 6.12 Reinforcement Options for the Footing

Number and Size of Bars	Area of Steel Provided (Required = 12.7 in.^{2}) in.^{2}	mm^2	Required Development Length[a] in.	mm	Center-to-Center Spacing in.	mm
20 No. 7	12.0	7742	32	813	7.0	178
15 No. 8	11.85	7646	37	940	9.5	241
12 No. 9	12.0	7742	42	1067	12.1	307
10 No. 10	12.7	8194	47	1194	14.7	373
8 No. 11	12.48	8052	52	1321	19.0	483

[a]From Table 6.8, values for "other bars," $f_y = 40$ ksi, $f_c' = 3$ ksi.

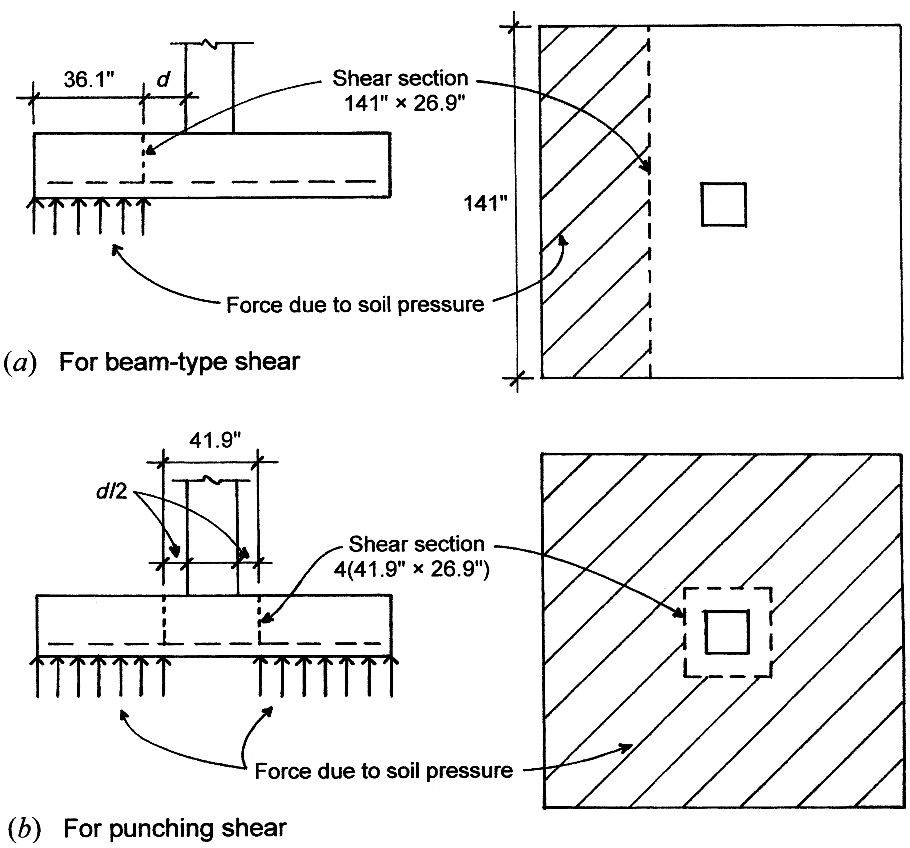


Figure 6.56 Considerations for shear in the column footing.

Although computations have shown the 31-in. dimension as the least possible thickness, it may be more economical to use a thicker footing with less reinforcement, in light of the usual costs of steel bars versus concrete dumped in a hole. True economy will be obtained with the lowest combined cost for excavation, forming, concrete, casting, and steel.

One possible limitation for the footing reinforcement is the total percentage of steel. If this is excessively low, the concrete section is hardly being reinforced. The ACI code stipulates that the minimum reinforcement be the same as that for temperature reinforcement in slabs, a percentage of 0.002 of the gross section (footing thickness times width) for bars with yield of 40 ksi and 0.0015 of the gross section for bars with yield of 60 ksi. For this footing cross section of 141 × 31 in., with 40 ksi yield, this means an area of

$$A_s = 0.002(141 \times 32) = 8.74\,\text{in.}^2$$

As this is less than the computed required area, the limitation is not critical.

Table 6.13 yields the allowable superimposed service load (not factored) for a range of predesigned column footings and soil pressures. This material has been adapted from more extensive data in *Simplified Design of Building Foundations* (Ref. 20). Designs are given for footings using a concrete strength of 3000 psi and steel yield strength of 40 ksi. Figure 6.57 indicates the symbols used for dimensions in the table.

Table 6.13 Safe Service Loads for Square Column Footings[a]

Maximum Soil Pressure (psf)	Minimum Column Width, t (in.)	Service Load on Footing (kips)	Dimensions h (in.)	Dimensions w (ft)	Reinforcement Each Way
1000	8	7	10	3	3 No. 2
	8	10	10	3.5	3 No. 3
	8	14	10	4	4 No. 3
	8	17	10	4.5	4 No. 4
	8	21	10	5	4 No. 5
	8	31	10	6	4 No. 6
	8	42	11	7	6 No. 6
1500	8	12	10	3	3 No. 3
	8	16	10	3.5	3 No. 4
	8	22	10	4	4 No. 4
	8	27	10	4.5	4 No. 5
	8	34	10	5	5 No. 5
	8	49	12	6	5 No. 6
	8	65	13	7	5 No. 7
	8	84	15	8	7 No. 7
	8	105	17	9	8 No. 7
2000	8	16	10	3	3 No. 3
	8	23	10	3.5	3 No. 4
	8	30	10	4	5 No. 4
	8	38	10	4.5	5 No. 5
	8	46	11	5	4 No. 6
	8	66	13	6	6 No. 6
	8	89	15	7	6 No. 7
	8	114	17	8	8 No. 7
	8	143	19	9	7 No. 8
	10	175	20	10	9 No. 8
3000	8	25	10	3	3 No. 4
	8	35	10	3.5	3 No. 5
	8	45	11	4	4 No. 5
	8	57	12	4.5	4 No. 6
	8	71	13	5	5 No. 6
	8	101	15	6	7 No. 6
	10	136	17	7	7 No. 7
	10	177	20	8	7 No. 8
	12	222	21	9	9 No. 8
	12	272	24	10	9 No. 9
	12	324	26	11	10 No. 9
	14	383	28	12	10 No. 10
4000	8	34	10	3	4 No. 4
	8	47	11	3.5	4 No. 5
	8	61	12	4	5 No. 5
	8	77	13	4.5	5 No. 6
	8	95	15	5	5 No. 6
	8	136	18	6	6 No. 7

Wall Footings

Wall footings consist of concrete strips placed under walls. The most common type is that shown in Figure 6.58, consisting of a strip with a rectangular cross section placed in a symmetrical position with respect to the supported wall and projecting an equal distance as a cantilever from both faces of the wall. For soil pressure, the critical dimension of the footing is its width as measured perpendicular to the wall. Transverse reinforcement (if required) is designed for the cantilever shear and bending. Reinforcement is also used in the longitudinal direction for shrinkage stresses and to allow the footing to function as a beam over variations in soil.

Wall footings function as construction platforms for the walls they support. Thus, a minimum width is established by the wall thickness plus a few inches on each side. The extra width is necessary because of the crude form of foundation construction, but it also may be required for support of forms for concrete walls. A minimum projection of 2 in. is recommended for masonry walls and 3 in. for sitecast concrete walls.

With relatively light vertical loads, the minimum construction width (wall thickness + 4 in. or more) may be adequate for soil-bearing capacity. With the minimum recommended footing width, the cantilever bending and shear will be negligible, so no transverse reinforcement is required. This pretty much holds true until the cantilevered distance exceeds the footing thickness. As the footing gets wider,

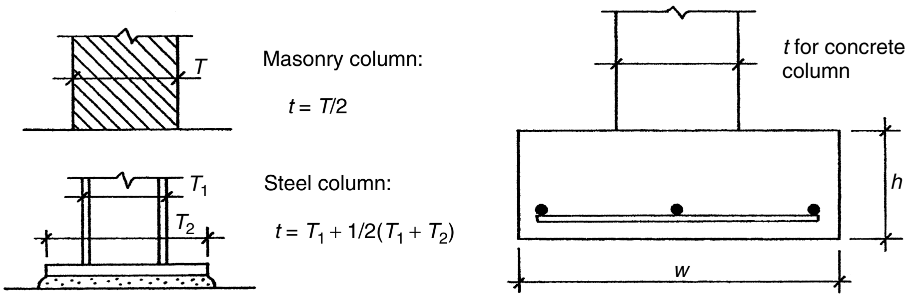


Figure 6.57 Reference for dimensions in Table 6.13.

Table 6.13 (*continued*)

Maximum Soil Pressure (psf)	Minimum Column Width, t (in.)	Service Load on Footing (kips)	Dimensions h (in.)	Dimensions w (ft)	Reinforcement Each Way
	10	184	20	7	8 No. 7
	10	238	23	8	8 No. 8
	12	300	25	9	8 No. 9
	12	367	27	10	10 No. 9
	14	441	29	11	10 No. 10
	14	522	32	12	11 No. 10
	16	608	34	13	13 No. 10
	16	698	37	14	13 No. 11
	18	796	39	15	14 No. 11

[a]Service loads do not include the weight of the footing, which has been deducted from the total bearing capacity. Criteria: $f'_c = 2000$ psi, grade 40 bars, $v_c = 1.1\sqrt{f'_c}$.

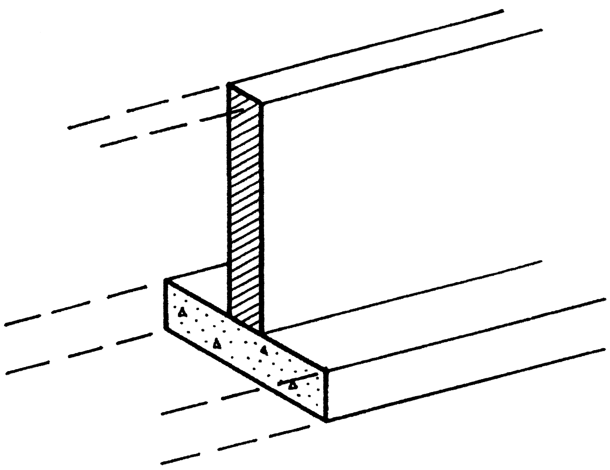


Figure 6.58 Typical form of a strip wall footing.

more thickness is required and reinforcement is needed. For all wall footings, however, some longitudinal reinforcement is recommended.

Determination of Footing Width

Footing width is mostly determined by allowable bearing pressure. A common procedure is to assume a footing thickness, design for the total load—including the weight of the footing—and verify that the thickness is adequate for shear and bending. In this work, design for soil pressure is done with the service (unfactored) loading, while design for shear and bending is done by the strength method using factored loads.

Determination of the Footing Thickness

If the footing has no transverse reinforcement, the required thickness is determined by the tension stress limit of the concrete in either flexural stress or diagonal stress due to shear. Transverse reinforcement is not required until the footing width exceeds the wall thickness by some significant amount, usually 2 ft or so. A good rule of thumb is to provide transverse reinforcement only if the cantilever edge distance exceeds the footing thickness. For average conditions, this means transverse reinforcement for footings of about 3-ft width or greater.

If transverse reinforcement is used, the critical concerns become for the shear in the concrete and the tension in the reinforcement. Thicknesses determined by shear will usually ensure a low bending stress in the concrete.

Minimum thicknesses are a matter of design judgment, unless limited by building codes. The ACI code recommends limits of 8 in. for unreinforced footings and 10 in. for footings with transverse reinforcement. Another possible consideration for the minimum footing thickness is the necessity for development of dowels for wall reinforcement.

Selection of Reinforcement

Transverse reinforcement is determined on the basis of flexural tension and development length due to the cantilever action. Longitudinal reinforcement is usually selected on the basis of providing minimum shrinkage reinforcement. A reasonable value for the latter is a minimum of 0.0015 times the gross concrete section area.

Cover requirements are for 2 in. from formed edges and 3 in. from surfaces generated without forming (such as the footing bottom).

For practical purposes, it may be desirable to coordinate the spacing of the transverse reinforcement with that of any dowels for wall reinforcement.

Data for predesigned wall footings are given in Table 6.14. Figure 6.59 provides an explanation of the listed dimensions

Table 6.14 Safe Service Loads for Wall Footings

Maximum Soil Pressure (lb/ft^2)	Minimum Wall Thickness, t(in.) Concrete	Masonry	Allowable load on Footing[a] (lb/ft)	Footing Dimensions (in.) h	w	Reinforcement Long Direction	Short Direction
1000	4	8	2,625	10	36	3 No. 4	No. 3 at 17
	4	8	3,062	10	42	2 No. 5	No. 3 at 12
	6	12	3,500	10	48	4 No. 4	No. 4 at 18
	6	12	3,938	10	54	3 No. 5	No. 4 at 13
	6	12	4,375	10	60	3 No. 5	No. 4 at 10
	6	12	4,812	10	66	5 No. 4	No. 5 at 13
	6	12	5,250	10	72	4 No. 5	No. 5 at 11
1500	4	8	4,125	10	36	3 No. 4	No. 3 at 11
	4	8	4,812	10	42	2 No. 5	No. 4 at 14
	6	12	5,500	10	48	4 No. 4	No. 4 at 11
	6	12	6,131	11	54	3 No. 5	No. 5 at 16
	6	12	6,812	11	60	5 No. 4	No. 5 at 12
	6	12	7,425	12	66	4 No. 5	No. 5 at 11
	8	16	8,100	12	72	5 No. 5	No. 5 at 10
2000	4	8	5,625	10	36	3 No. 4	No. 4 at 15
	6	12	6,562	10	42	2 No. 5	No. 4 at 12
	6	12	7,500	10	48	4 No. 4	No. 5 at 13
	6	12	8,381	11	54	3 No. 5	No. 5 at 12
	6	12	9,520	12	60	4 No. 5	No. 5 at 10
	8	16	10,106	13	66	4 No. 5	No. 5 at 10
	8	16	10,875	15	72	6 No. 5	No. 5 at 10
3000	6	12	8,625	10	36	3 No. 4	No. 4 at 11
	6	12	10,019	11	42	4 No. 4	No. 5 at 14
	6	12	11,400	12	48	3 No. 5	No. 5 at 11
	6	12	12,712	14	54	6 No. 4	No. 5 at 11
	8	16	14,062	15	60	5 No. 5	No. 5 at 10
	8	16	15,400	16	66	5 No. 5	No. 6 at 13
	8	16	16,725	17	72	6 No. 5	No. 6 at 11

[a]Allowable loads do not include the weight of the footing, which has been deducted from the total bearing capacity. Criteria: $f'_c = 2000$ psi, grade 40 bars, $v_c = 1.1\sqrt{f'_c}$.

in the table. The following example illustrates the design procedure for a wall footing.

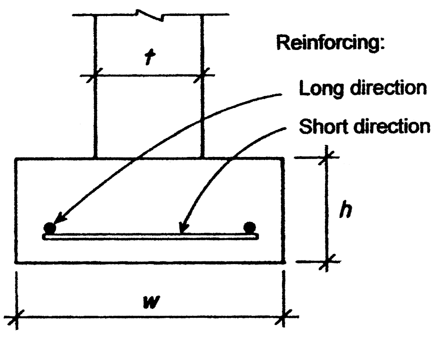


Figure 6.59 Reference for Table 16.14.

Example 16. Design a wall footing with transverse reinforcement for the following data:

Footing service load = 3750 lb/ft dead load and 5000 lb/ft live load
Wall thickness = 6 in.
Maximum soil pressure = 2000 psf
Concrete design strength = 2000 psi
Steel yield strength = 40 ksi

Note: Concrete strength of 2000 psi is sometimes used for smaller buildings because most building codes will permit its use without extensive field testing. However, it is best to specify a higher strength to assure reasonable quality of the material.

Solution. The usual design procedure is to assume a footing thickness, determine the required footing width, and check for shear in the footing to verify the adequacy of the thickness.

Try $h = 12$ in.; then footing weight is 150 psf, and the net usable soil pressure is $2000 - 150 = 1850$ psf. The footing design load is unfactored when determining footing width; therefore, the total design load is 8750 lb/ft and the required width is

$$w = \frac{8750}{1850} = 4.73\text{ ft}, \quad \text{say 4 ft 9 in.,} \quad \text{or 54 in.}$$

With 3 in. of cover on the footing bottom and assuming a No. 6 bar ($\frac{3}{4}$ in. diameter), the effective footing depth for strength computations is 8.625 in.

Next we need to determine how much the soil is pushing up on the footing using the factored loads and the actual footing width; thus

$$w_u = 1.2(3750) + 1.6(5000) = 12{,}500\text{ lb}$$

$$q = \frac{12{,}500}{4.75} = 2632\text{ psf}$$

The shear capacity of the concrete is

$$\phi_v V_c = 0.75\sqrt{f_c'}bd = 0.75\sqrt{2000}(12 \times 8.6) = 6923\text{ lb}$$

The critical section for shear stress is taken at a distance of d from the face of the wall. As shown in Figure 6.60*a*, this places the shear section at 16.9 in. from the edge of the footing. At this location, the shear force generated by soil pressure is

$$V_u = (2632)\left(1 \times \frac{16.9}{12}\right) = 3707\text{ lb}$$

Since the footing is overstrong for shear, it is possible to reduce the thickness. However, as discussed previously, cost effectiveness is usually achieved by reducing the steel to a minimum. If h is changed to 11 in., the shear capacity decreases only slightly, and the required footing width will remain effectively the same. A new effective depth $d = 7.6$ will be used, but the design soil pressure for strength analysis will remain the same because it relates only to the width of the footing.

The bending moment to be used for determination of the transverse reinforcement is taken at the face of the wall and is computed as follows (see Figure 6.60*a*):

$$F = 2632\left(1 \times \frac{25.5}{12}\right) = 5593\text{ lb}$$

$$M_u = 5593 \times \frac{25.5}{2} = 71{,}310\text{ in.-lb}$$

$$M_r = \frac{M_u}{\phi_b} = \frac{71{,}310}{0.9} = 79{,}233\text{ in.-lb}$$

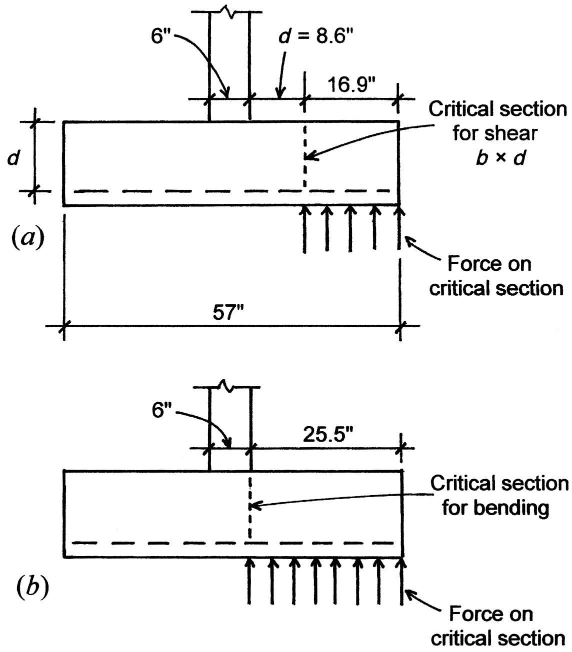


Figure 6.60 Shear and bending considerations for the wall footing.

Assuming the flexural section to be considerably underreinforced, with an approximate *a/d* of 0.2, the required steel area per foot of wall length is

$$A_s = \frac{M_r}{f_y(d - a/2)} = \frac{79{,}233}{40{,}000(0.9 \times 7.6)} = 0.290\text{ in.}^2/\text{ft}$$

The spacing required for a given size bar to satisfy this requirement can be determined as follows:

$$\text{Required spacing} = (\text{area of bar}) \times \frac{12}{\text{required area/ft}}$$

For a no. 3 bar

$$s = 0.11 \times \frac{12}{0.290} = 4.6\text{ in.}$$

Using this procedure, required bar spacing for bar sizes 3 through 6 are as shown in Table 6.15.

Bar sizes and spacings can be most easily selected using handbook tables that yield average steel areas for various combinations of bar size and spacing. One such table is Table 6.5, from which the spacing selections in the last column of Table 6.15 were taken. Selection of bar size and spacing is a matter of design judgment, for which some considerations are as follows:

Maximum recommended spacing is 18 in.

Minimum recommended spacing is 6 in. to reduce the number of bars and make placing concrete easier.

Preference is for smaller bars as long as spacing is not too close.

Table 6.15 Options for the Reinforcement for Example 16

Bar Size	Area of Bar ($in.^2$)	Area Required for Bending ($in.^2$)	Bar Spacing Required (in.)	Bar Spacing Selected (in.)
3	0.11	0.290	4.6	4.5
4	0.20	0.290	8.3	8.0
5	0.31	0.290	12.8	12.5
6	0.44	0.290	18.2	18.0

A practical spacing may be that of the spacing of vertical reinforcement in the supported wall, or some full number multiple or division of the wall bar spacing.

With these considerations in mind, plus the data from Table 6.15, choice may be made of the No. 4, 5, or 6 bars. For correlation, the closest size footing in Table 6.14 is 11 in. thick and is reinforced with No. 5 bars at 12 in.

The transverse reinforcement must be checked for development, in this case extending out from the face of the wall to 2 in. from the footing edge—a distance of 23.5 in. Inspection of Table 6.8 will show that this is an adequate length for all the bar sizes in Table 6.15. Data in Table 6.8 do not include the concrete strength of 2000 psi, however, so choice of the No. 6 bars may be marginally acceptable. The formulas that were used to generate the data in Table 6.8 can be used to find development length for any combination of concrete and steel strengths.

Another consideration that must be made for the footing is the choice for the longitudinal reinforcement, for which the minimum total area of steel is

$$A_s = (0.0015)(11 \times 57) = 0.94\ \text{in.}^2$$

Using three No. 5 bars yields

$$A_s = 3 \times 0.31 = 0.93\ \text{in.}^2$$

CHAPTER

7

Masonry Structures

This chapter deals with aspects of the use of masonry construction for structures. Most of the masonry seen as a finished surface in new construction these days is either nonstructural or of a few limited types of structural masonry. (See Figure 7.1) Much of what must be dealt with in using masonry falls in the general category of building construction rather than with strictly structural design considerations. One must learn a great deal about materials and processes of masonry construction to become generally capable of using it. For that large body of information we refer the reader to the various sources, including textbooks, handbooks, and industry standards. The discussion in this chapter is limited to the most common uses of structural masonry and the general design considerations relating to building structures.

7.1 GENERAL CONCERNS FOR MASONRY

There are many types of masonry and many factors that must be dealt with in producing good masonry structures.

Units

Masonry consists of a solid mass produced by bonding separate units. The traditional bonding material is mortar. The units include a range of materials, the common ones being stone, brick, and concrete blocks (now called CMUs, for concrete masonry units). Many other units are used for special construction, but these three types are used for most structural masonry.

The structural character of masonry depends greatly on the material and form of the units. From a material point of view, the high-fired clay products (brick and tile) are the strongest, producing very strong construction when achieved with the proper mortar, a good arrangement of units, and high-quality construction craft and work in general. This is particularly important if the general class of the masonry construction is the traditional, unreinforced variety.

Mortar

Mortar is usually composed of water, cement, and sand, with some materials added to make the wet mortar stickier (so that it adheres to the units during the laying up of the construction), is faster setting (hardening), and is generally more workable during the work. Building codes establish various requirements for the mortar, including the classification of the mortar and details of its use during construction.

The quality of the mortar is obviously important to the structural integrity of the masonry, both as a structural material in its own right and as a bonding agent that holds the units together. While the integrity of the units is dependent on the manufacturer or natural source, the quality of the finished mortar work is dependent primarily on the skill and care of the persons who produce the work.

There are several classes of mortar established by codes. The highest grades are required for uses involving major structural tasks for bearing walls, shear walls, columns, and heavy-load bearing supports. Specifications for the materials and required properties determined by tests are spelled out in detail. Still, the finished product is highly skill dependent.

Reinforcement

Although some joint reinforcing is typical in all structural masonry these days, the term *reinforced masonry* is reserved for a class of construction in which major vertical and horizontal reinforcement is used, making the finished work quite analogous to reinforced concrete construction.

(*a*)

(*b*)

Figure 7.1 Are they or aren't they? Real masonry structures that is. On the left are two views of the last of the tall brick masonry structures in Chicago from the late nineteenth century: the Monadnock Building, with the 16-story walls thickened to several feet at street level. On the upper right is Louis Sullivan's Auditorium Block, also in Chicago about the same time, with heavily expressed exterior masonry somewhat denying the internal steel frame. At lower right is a building strongly expressing the vintage masonry forms with arched openings; it is, however, of quite recent origin. Today, most of what looks like masonry structures really are not—just masonry-covered frames.

(*c*)

(*d*)

Figure 7.1 (*continued*)

Basic Construction and Terminology

Figure 7.2 shows some of the basic elements of masonry construction. The terminology and details shown apply mostly to construction with bricks or concrete blocks.

Units are usually laid up in horizontal rows, called *courses*, and in vertical planes, called *wythes*. Very thick walls have several wythes, but most often walls of brick have two wythes and walls of concrete blocks are single wythe. If wythes are connected directly, the construction is called *solid*. If a space is left between wythes, as shown in the illustration, the wall is called a *cavity wall.* If the cavity is filled with concrete, it is called a *grouted cavity wall.*

The multiple wythe must have the separate wythes bonded together in some fashion. If this is done with the masonry units, the overlapping unit is called a *header.* Various patterns of headers have produced some classic forms of arrangement of bricks in traditional masonry construction. For cavity walls, bonding is often achieved with metal ties, using single ties at intervals, or a continuous wire truss element that provides both the tying of the wythes and some minimal horizontal reinforcement.

The continuous element labeled *joint reinforcing* in Figure 7.2 is commonly used in both brick and concrete block construction that is code classified as unreinforced. For seriously reinforced masonry, vertical and horizontal rods are placed at intervals and encased in concrete poured into the wall cavity.

7.2 STRUCTURAL MASONRY

Masonry intended for serious structural purpose includes that used for bearing walls, shear walls, retaining walls, and spanning walls of various types. The forms of masonry most used for these purposes are the following:

Solid Brick Masonry. This is usually unreinforced masonry with the wythes directly connected (no cavity) and the whole consisting of a solid mass of bricks and mortar.

Grouted Brick Masonry. This is usually a two-wythe wall with a cavity filled with concrete. If unreinforced, it will have continuous joint reinforcing. If reinforced, steel rods are placed in the filled cavity.

Unreinforced Concrete Block Masonry. (See Figure 7.3*a*.) This is typically a single-wythe wall with vertical mortar joints staggered. Some reinforcement and filled cavities may be used, but this is not the form of block used for reinforced construction. Block walls are usually thick and the wall strength derives primarily from the strength of the units and the mortar joints.

Reinforced Concrete Block Masonry. (See Figure 7.3*b*.) This is produced with the type of unit shown in Figure 7.3*b*, with large hollow cores that permit the incorporation of considerable concrete filling in cores and large vertical reinforcing bars. Horizontal bars may be placed in courses with the modified block shown in Figure 7.3*c* and lintels over openings may use the block shown in Figure 7.3*d*.

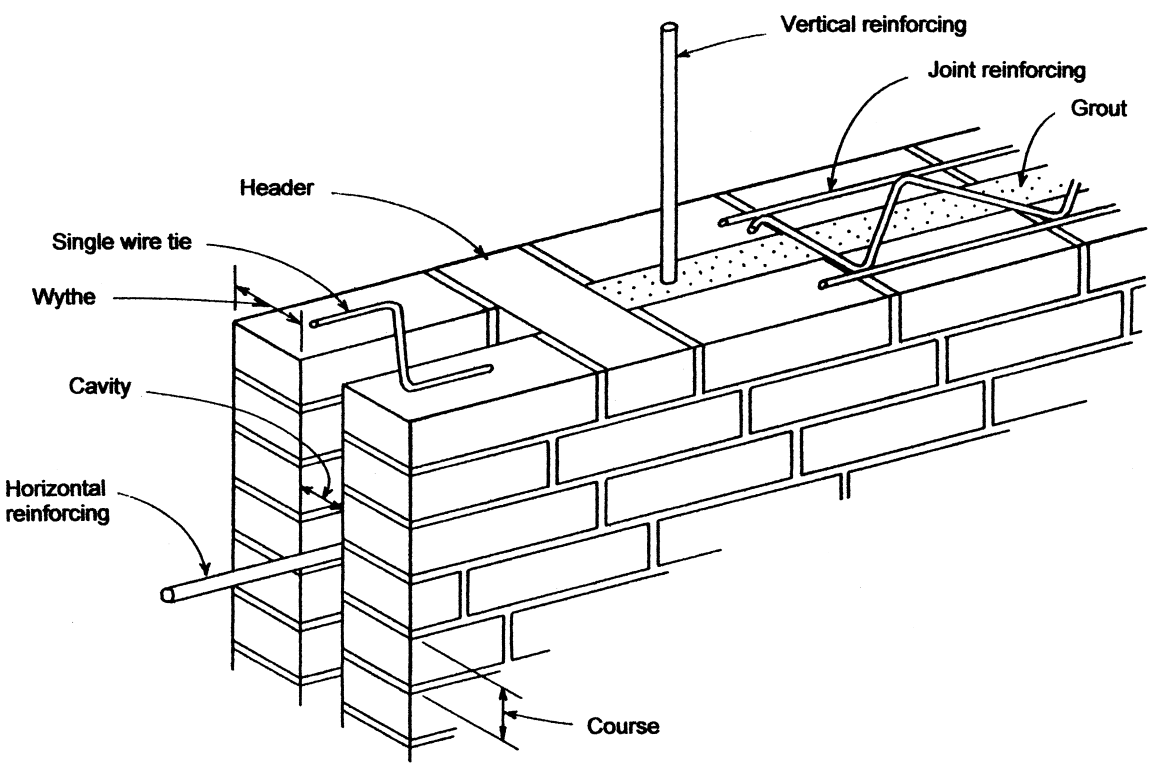


Figure 7.2 Elements of masonry construction.

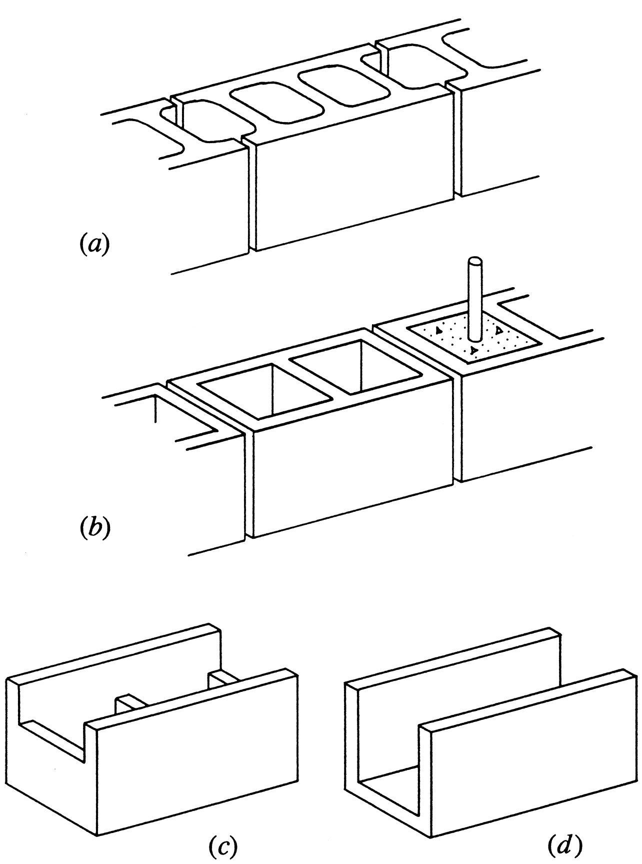


Figure 7.3 Construction with CMUs.

Walls that serve structural purposes and have a finished surface of masonry may take various forms. All of the walls just described may use the structural masonry unit as the finished wall face, although surfaces exposed to view may also be specially treated in various ways. For the brick walls, the single brick face that is exposed may have a special treatment: colored, glazed, textured, and so on. It is also possible, of course, to use the masonry strictly for structural purposes and to finish the wall surface with some other material, such as stucco or tile.

Another use of masonry consists of providing a finish of masonry on the surface of a structural wall of other construction. The applied finish may be achieved with real masonry units or may consist of thin masonry elements adhered to the structure. This method of achieving the appearance of traditional masonry without actual masonry structures is quite widespread.

Unreinforced Masonry

Many structures of unreinforced masonry have endured for centuries, and this form of construction is still widely used, although increasingly less so in the United States. (See Figure 7.4.) If masonry is essentially unreinforced, the character and structural integrity of the construction are highly dependent on the details and the quality of the masonry work. Strength and form of units, arrangement of units, quality of the mortar, and the form and details of the general construction are all important. Thus the degree of attention paid to design, to writing of specifications, to detailing of the construction, and to careful inspection during the construction work must be adequate to ensure good finished construction.

There are a limited number of structural applications for unreinforced masonry that are permitted by present codes. In many instances forms of construction are tolerated simply because they have been used with success for many years on a local basis. Success is hard to argue with when the evidence is at hand.

Minimal Construction

As in most types of construction, there is a minimal form of construction that results from satisfaction of various general requirements. This results in most cases in a basic minimum form of construction that is adequate for many ordinary functions, which is in fact usually the intent of building codes. Applications of minimal construction for various situations judged to be proper are thus permitted in many cases with little or no engineering design and computations—only adherence to a limited set of code requirements.

Reinforcement or Enhancement

The strength of a masonry structure can be improved by various means, including the insertion of steel reinforcing as is done to produce reinforced masonry. Steel rods or wire can be used on a partial basis with unreinforced masonry for assistance with localized stress conditions. Horizontal wire units are routinely used, mostly for resistance of shrinkage and temperature change effects.

Another way to achieve reinforcement is through form variation, examples of which are shown in Figure 7.5. Turning a corner at the end of a wall (Figure 7.5*a*) adds stability and strength to the discontinuous edge of the wall. Pilaster columns (Figure 7.5*b*) can be used to strengthen the end of a wall or to add strength at points along the wall. Curving a wall in plan (Figure 7.5*c*) is another means of improving the stability of a wall.

Building corners and openings for doors and windows are other locations where enhancement is sometimes required. Figure 7.5*d* shows the use of a thickened portion around a door opening. If the top of the opening is arched, the arch rib may be extended down through the edge of the opening. A change to stronger masonry units may be made to strengthen openings (Figure 7.5*f*) or building corners (Figure 7.5*g*). For wide openings with flat tops, lintels have been used, consisting of large cut stones in ancient times but usually of steel elements today.

Reinforced Masonry

As we use it here, the term *reinforced masonry* designates a type of masonry construction specifically classified by building code definitions. The chief function of the steel

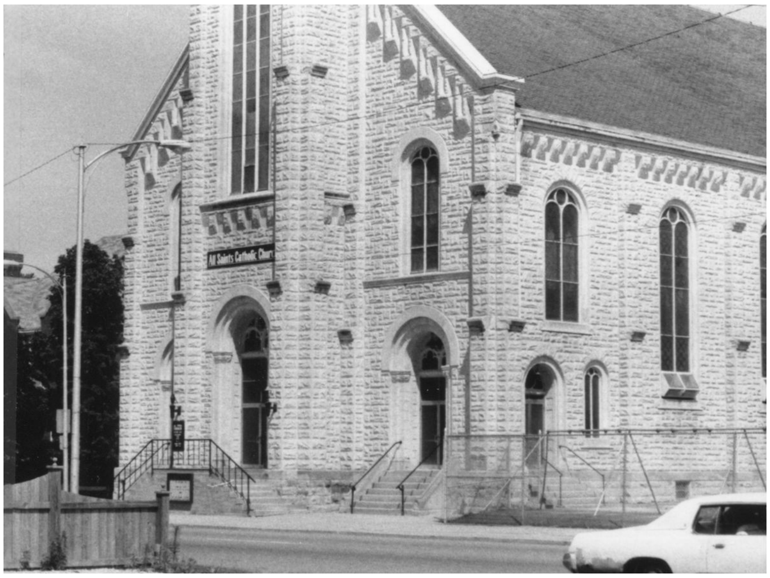


Figure 7.4 Concrete block structure for a church built in the early twentieth century in rural Indiana.

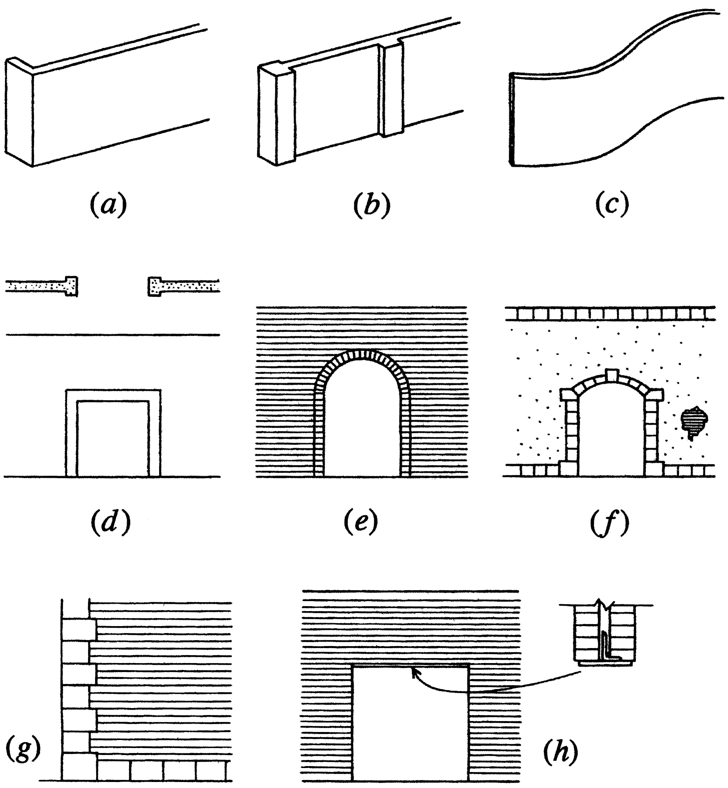


Figure 7.5 Reinforcement in masonry construction.

reinforcement used for this construction is to absorb tensile stresses in the masonry. This makes the masonry basically analogous to reinforced concrete.

Reinforced Brick Masonry

Reinforced brick walls typically consist of two wythes with a cavity, with concrete fill and steel rods in the cavity. General requirements and design procedures for the reinforced brick wall are similar to those for concrete walls. There are stipulations for minimum reinforcement and provisions for stress limits for compression and shear in the masonry. Reinforcement design is similar to that for reinforced concrete.

Despite the presence of the reinforcement, this type of construction is still essentially a masonry structure, with structural integrity highly dependent on the quality of the masonry work. The grouted, reinforced cavity is considered to be a third wythe, constituted as a very thin reinforced concrete panel.

Solid brick walls, especially reinforced ones, are very labor intensive and expensive and thus not very common.

Reinforced Hollow-Unit Masonry

This type of construction most often consists of single-wythe walls in which cavities are vertically aligned so that reinforced concrete columns can be formed within them, as shown in Figure 7.6. At some interval, horizontal courses are also used to form reinforced concrete members. The intersecting vertical and horizontal concrete members thus constitute a rigid frame bent inside the wall. This frame is the major structural component of the construction. Besides providing forming, the concrete blocks serve to brace the frame, to provide protection for the reinforcement, and to interact in composite action with the rigid frame.

If all void spaces are filled with concrete, the construction is fully solid. This is usually required for structures such as shear walls, basement walls, and retaining walls. Finally, if reinforcement is placed in more cavities, the wall strength is greatly enhanced. There is thus a wide range of strength variation which can be achieved without change in the external appearance of the wall. Multistory walls may thus be achieved with considerable height developed by a single thickness of the wall.

Most reinforced masonry construction is achieved with concrete units and design requirements are extensive for this form of construction.

Usage Considerations

Utilization of masonry construction for structural functions requires the consideration of a number of factors that relate to the structural design and to the proper details and specifications for construction. The following are some major concerns that must ordinarily be dealt with.

Units

The material, form, and specific dimensions of the units must be determined. Where code classifications exist, the specific grade or type must be defined. Type and grade of unit, as well as usage conditions, usually set the requirements for the type of mortar required.

Unit dimensions may be set by the designer, but the sizes of industrial products such as bricks and concrete blocks are often controlled by industry standard practices. As shown in Figure 7.7*a*, the three dimensions of a brick are the height and length of the exposed face and the width that produces the thickness of a single wythe. There is no single standard brick size, but most fall in a range close to that shown in the illustration.

Concrete blocks are produced in families of modular sizes. The size of block shown in Figure 7.7*b* is one that is equivalent to the 2-by-4 in wood—not the only size, but the most common. Concrete blocks have both nominal and actual dimensions. The nominal dimensions are used for designating the blocks and relate to modular layouts of building dimensions. Actual dimensions are based on the assumption of a mortar joint thickness of $\frac{3}{8}$ to $\frac{1}{2}$ in.; the actual dimensions shown in Figure 7.7*b* are based on the use of $\frac{3}{8}$-in. joints.

A construction sometimes used in unreinforced masonry is that of a single wythe of brick bonded to a single wythe of concrete block, as shown in Figure 7.7*c*. In order to install the metal ties in the joints that achieve bonding, as well as to have the bricks and blocks come out evenly at openings and the tops of walls, a special-height brick is used based on either two or three bricks to one block.

Figure 7.6 Common form of reinforced CMU construction.

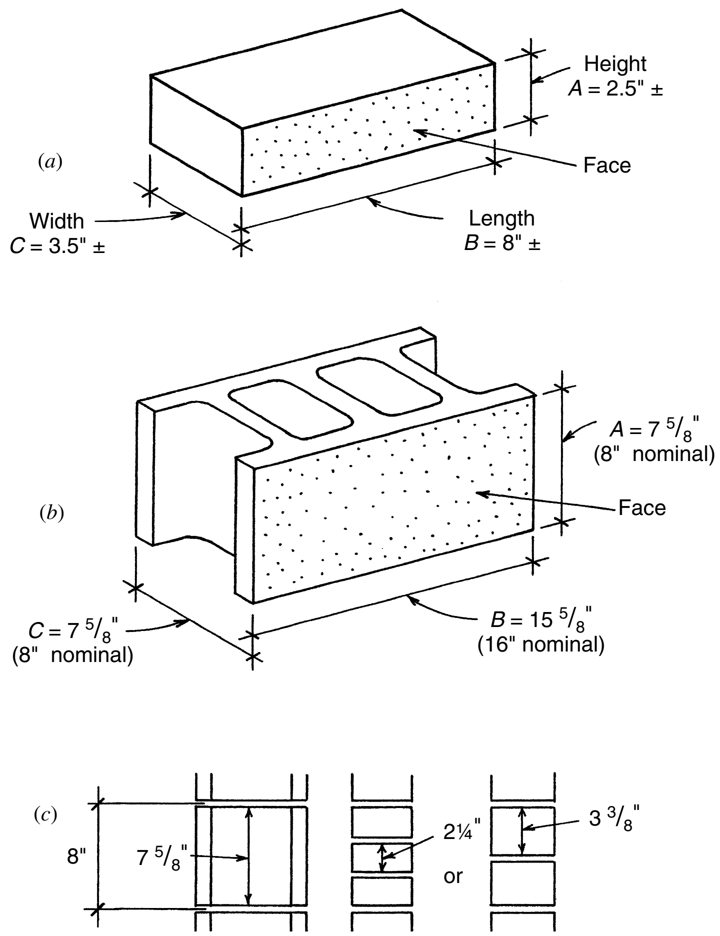


Figure 7.7 Dimensional considerations with masonry units.

Unit Layout Pattern

When exposed to view, masonry units present two concerns: for the face of the unit and for the pattern of layout of the units on the wall face. Patterns derive from unit shape and the need for bonding if unit-bonded construction is used.

Classic patterns were developed on the basis of bonding with the masonry units, but the present use of metal ties frees the choice for patterns from this concern. Nevertheless, classic forms are still widely used.

For reinforced construction with concrete blocks, units must be arranged to align the voids vertically, which allows for two possibilities with the two-void block: units a half block offset in alternating courses or units stacked vertically with all vertical face joints aligned.

Structural Functions

Masonry walls vary from those that are essentially nonstructural to those that serve major, and often multiple, structural tasks. The type of unit, grade of mortar, amount of reinforcement, and so on, may depend on the degree of structural demands. Wall thickness may relate to stress levels as well as to construction considerations. Most structural tasks involve force transfers—from supported structures, from other walls, and to supporting foundations. Need for brackets, pilasters, vertically tapered or stepped profile, or other form variations may relate to force transfers, wall stability, or other structural concerns.

Wall forms and details may also relate to the predominant structural functions required. The most common function is for resolution of vertical compression, but forces induced by wind and earthquakes may require special consideration.

Reinforcement

In the broad sense, reinforcement means anything that is added to help with the structural tasks. Structural reinforcement thus includes the use of pilasters, buttresses, tapered form, and other devices as well as the usual added steel reinforcement. Reinforcement may be generally dispersed or may be provided at critical points, such as at wall ends, tops, edges of openings, and locations of heavy concentrated loads.

Reinforcement is required mostly when masonry structures must perform multiple tasks, such as being simultaneously bearing walls, shear walls, and the general load-bearing structure for a building. This complete use of masonry structures is increasingly rare, so that the use of reinforcement is most often in response to limited functions.

Control Joints

Shrinkage of the mortar, temperature variations, and movements due to seismic actions or to settlement of foundations

are all sources for concern for cracking in masonry. Stress concentrations and cracking can be controlled to some extent by steel reinforcement. However, it is also common to provide some control joints (literally, preestablished cracks) to alleviate these effects. Planning and detailing of control joints are complex problems and must be studied carefully as structural and architectural design issues. Code requirements, industry recommendations, and construction practices on a local basis will provide guides for this work.

Attachment and Embedment

Attachment of elements of the construction to masonry is somewhat similar to that required with concrete. Where the nature and exact location of attached elements can be predicted, it is best to provide some built-in device, such as an anchor bolt, threaded sleeve, and so on. Accommodation of adjustment of such attachments must be provided as precision of the construction is limited.

Attachment can also be effected with drilled-in anchors or adhesives. These tend to be less constrained by the problem of precise location.

In some cases, items may need to be embedded in the masonry as it is laid up. Piping and conduits for wiring may be such items.

These matters involve some visualization of the complete construction and of the general problem of integrating the structure into the whole building. It is simply somewhat more critical for masonry and concrete structures, as simple use of nails, screws, or welding is not possible in the direct way that it is with structures of wood or steel.

7.3 MASONRY WITH CONCRETE UNITS

Most structural masonry is now achieved with precast CMUs, principally because of the low cost of units and the lower labor costs. This section presents discussions related to development of this form of construction.

Unreinforced Construction with CMUs

The standard form of unit used most often for unreinforced forms of construction is that shown in Figure 7.8*a*. This is called a *three-celled unit*, although it also has two half-cells at each end; thus the module of repetition is four cells per block. The actual dimensions shown here relate to the accommodation of $\frac{3}{8}$-in. mortar joints; the block shown in Figure 7.8*a* is therefore described as an 8 × 8 × 16-in. block.

Staying within this standard unit module, there are typically some special units provided for achieving common elements of the construction. As shown in Figure 7.8, these include the following:

End and Corner Units (Figures 7.8*b* and *c*). These are used at the ends and corners of walls, where the ends of the units are exposed to view. The half-blocks (Figure 7.8*c*) are also used to achieve a wall face pattern with vertical mortar joints offset in alternating courses.

Lintel or Bond Beam Units (Figure 7.8*d*). These are used at the top and bottom of openings or at the top of walls to create cast-in-place concrete beams with reinforcement and concrete fill.

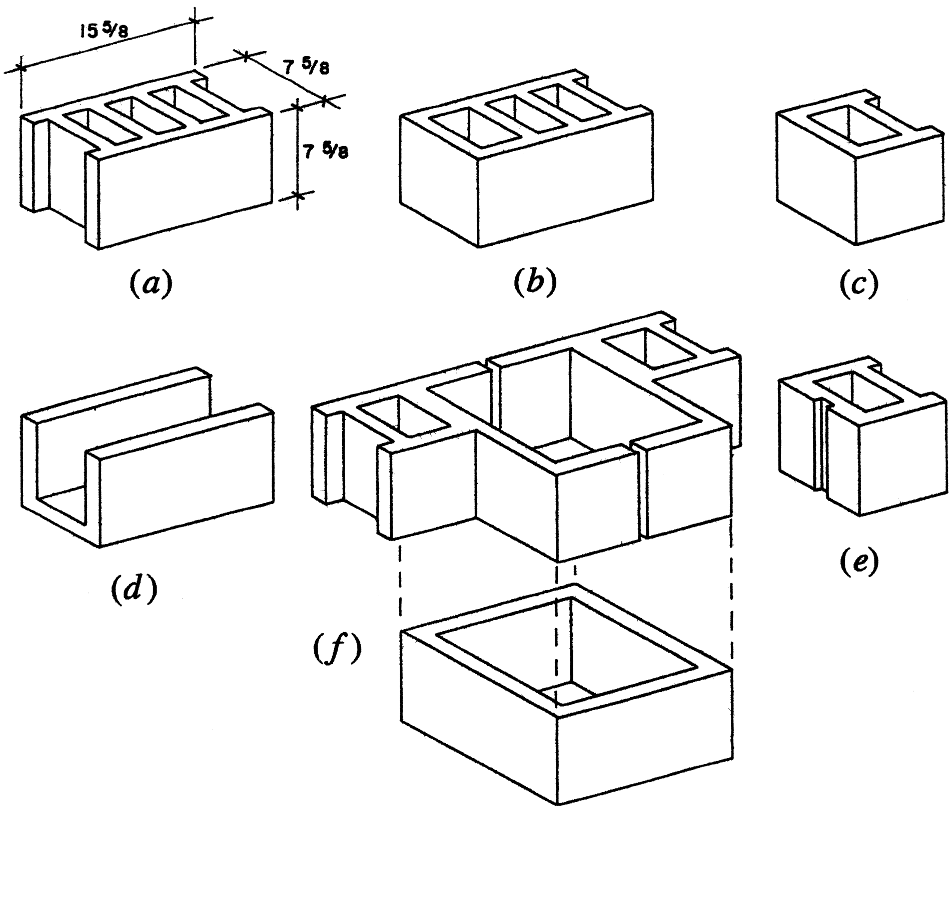


Figure 7.8 Forms of CMUs for unreinforced construction.

Sash Units (Figure 7.8*e*). These are units with squared ends and a groove to facilitate the anchoring of window and door frames at the vertical edges of openings.

Pilaster Units (Figure 7.8*f*). These are used to form pilaster columns within the block module system. The two types of units shown can be alternated in vertically stacked courses to match the offset vertical mortar joint pattern. Units of various sizes can be obtained to create different sizes of pilasters. The units shown here are typically filled with vertical rods and concrete fill to create a reinforced concrete column within the block system.

For construction exposed to view, units can be obtained with colored concrete or with various textured face forms.

Although the typical unreinforced class of construction is considered as a simple homogeneous material, permitting simple stress computations for most loadings, investigation is somewhat more complex when some voids are filled with concrete and possibly with steel rods. Use of horizontal wire joint reinforcement is quite common and adds considerably to the general integrity of the construction for resistance to effects of mortar shrinkage and temperature expansion and contraction.

Some procedures are generally followed in all construction with hollow concrete units, whether the work is intended for structural purposes or not. Horizontal joint reinforcement is commonly used, as are bond beams (concrete filled and reinforced) at the tops of walls. These practices simply ensure better construction. Other enhancements may relate to direct concerns for structural actions.

Reinforcement for Unreinforced Construction

Taken in its broader context, reinforcement means anything that adds strength to the otherwise unadorned (minimal) construction. As discussed in other situations in this book, this includes the use of steel elements in horizontal mortar joints or in concrete-filled cavities, but it may also refer to the use of pilasters and stronger masonry units at strategic locations. Without transforming the class of this construction to that of reinforced masonry, some means for improving strength of hollow-unit masonry are the following:

Use denser concrete for the units. This generally results in stronger units, with strength gain usually being proportional to density (weight) gain. This may be done together with the next measure.

Use units with thicker walls. This is usually done for all units intended for major structural use.

Fill all voids with concrete. This usually approximately doubles the total solid mass of the wall.

Use larger and/or stronger units at strategic locations. For example, pilasters that receive major loads may be made of greater strength than the general construction of the walls in which they occur.

Use steel elements (wire or rods) in mortar joints or in concrete-filled cavities at strategic locations. This may be a means of giving localized enhancement to the structure—as with the pilasters—without necessarily strengthening the entire wall.

Reinforced CMU Construction

Hollow-unit masonry that qualifies for the building codes' definition of reinforced is typically made with standard units (CMUs) that provide larger voids. The standard unit used is the two-void block, which produces vertically aligned void spaces of approximately twice the size as the unit used for unreinforced construction. The denser units used for unreinforced construction are intended to provide greater wall strength on their own; the units for reinforced construction are intended to maximize the space allowed for the reinforced concrete frame to be cast in the voids (see Figure 7.9).

The internally contained frame does the double job of tying the masonry together into an integral mass and developing its own independent strength as a reinforced concrete structure. Building code requirements ensure the presence of the vertical and horizontal reinforced elements at least every 4 ft. Additional elements are provided at wall tops and ends, at corners, at wall intersections, and at all sides of any openings in the wall. Dependence on the integrity of the masonry itself is minimal for this construction, although its presence and its interaction with the concrete frame are highly useful for some actions—most notably for resistance to earthquakes.

Minimum Construction

As with many other types of construction, building code requirements and industry standards result in a certain minimum form of construction. The structural capacity of this minimum construction represents a threshold that may be sufficiently high to provide for many situations of structural demand. Such is frequently the case with reinforced CMU construction, so quite frequently, for buildings of modest size, the minimum construction is quite adequate for structural needs.

From this minimum construction, increased strength can be produced by filling more voids with concrete and reinforcement, all the way up to a completely filled wall if necessary. Steel reinforcement can be increased with larger rods and more of them—up to a major increase over the minimum amount.

As with unreinforced construction, walls for some purposes are typically completely filled with concrete in the voids. This is usually done for basement walls, shear walls, and retaining walls.

Wall thickness may be varied, with a 6-in. thickness usually a lower limit for nonstructural walls and an 8-in. thickness for structural walls. Thicker walls may be obtained with units up to 16 in. thick.

Bearing Walls

Walls made with CMUs are often used to support gravity loads from roofs, floors, or walls above in multistory buildings.

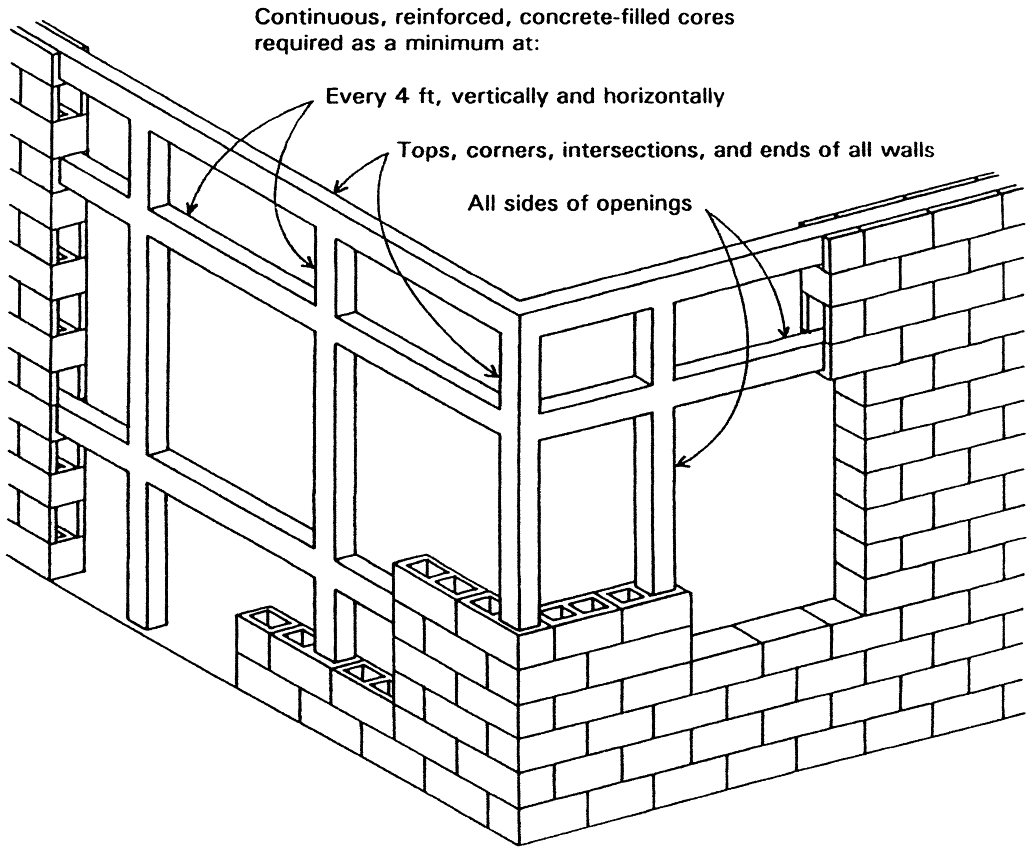


Figure 7.9 Form of the internal reinforced concrete structure in reinforced CMU construction.

Walls serving such functions are called *bearing walls.* The principal structural task is resistance to vertical compressive force.

The basic minimum construction described in building codes is based primarily on visualization of walls as bearing walls. The vertical loads on the walls are considered as uniformly distributed forces delivered to the wall top. Other loading conditions, such as heavy concentrated loads, loads eccentric to the wall center, lateral loads perpendicular to the wall, or shear forces in the wall plane, require considerations beyond those assumed for the minimum construction.

Unreinforced walls generally have limited capacity for loads that cause bending, shear, or concentrated force at a single point. Reinforced walls have greater potential for accommodation of these load effects.

For simple bearing tasks, the minimum construction will have a specific limited capacity. This capacity will often be adequate for ordinary tasks in low-rise buildings. Use of stronger units or the various means for reinforcement can increase simple bearing capacity.

Shear Walls

Structural masonry walls are quite often used as shear walls, forming part of the lateral-load-resisting system for a building. Various systems can be used for lateral bracing, but the most common one for low-rise buildings is the box system, typically formed with horizontal roof and floor diaphragms and vertical shear walls.

Figure 7.10 shows three common forms of shear walls. In Figure 7.10*a*, individual shear wall units are spaced at intervals along a wall, each acting independently as a bracing unit, although being linked together as a set to share the total lateral force on the building.

In Figure 7.10b, the entire wall is developed as a single shear wall, albeit with holes for windows. As the openings get larger, this wall begins to work as a very stiff rigid frame, and its construction will reflect the need for the frame resistance.

In Figure 7.10*c*, the wall is constructed as a continuous unit, with individual units between openings acting as cantilevered piers fixed to the continuous top strip of wall. This also represents a form of rigid-frame behavior.

Unreinforced masonry walls were used extensively in buildings for many centuries before steel reinforcement was developed. Many ancient buildings that are still standing testify to the strength of this construction. However, every major earthquake demonstrates the vulnerability of this construction; thus reinforced construction is now generally required wherever earthquake risk is high.

The general actions of shear walls and the forms of construction used to achieve them are discussed extensively in Chapter 9.

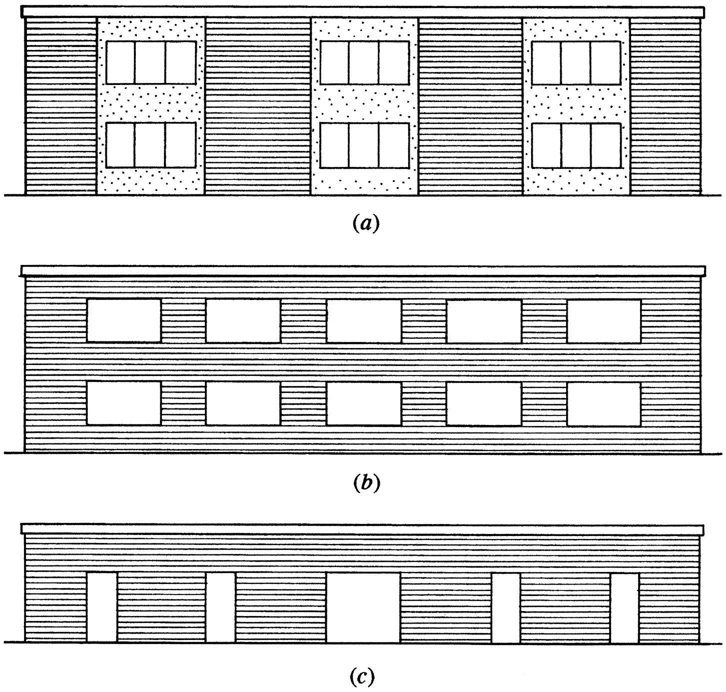


Figure 7.10 Form variations for masonry shear walls.

CMU Columns and Piers

Structural columns may be formed with CMU construction for use as entities or as parts of a general CMU structure. In light construction, column pedestals are commonly formed with CMUs, especially if no other sitecast concrete is being used, other than ground slabs and foundations.

Figure 7.11 shows several forms of CMU columns, commonly used with construction that is generally reinforced to qualify as structural masonry. Figure 7.11*a* shows the minimum column, formed with two block units in plan. The positions of the two blocks are ordinarily rotated 90° in alternating courses, as shown in the figure.

The two-block column is totally filled with concrete and usually reinforced with a vertical rod in each cavity. Horizontal ties—necessary for qualification as a structural column—must be placed in the mortar joints, which are ordinarily 1/2 in. thick for this construction.

Figure 7.11*b* shows the four-block column, forming in this case a small void area in the center of the column. The capacity of this column may be varied, with the minimum columns having concrete fill and steel rods in only the four corner block cavities. Additional fill and rods may be placed in the other block voids for a stronger column; finally, the center void may also be filled. Larger columns and oblong plan shapes may also be formed by this process.

It is also possible to form an ordinary reinforced concrete column by casting the concrete inside a boxlike shell made from CMU pieces that define a considerable void. The simplest form for this is shown in Figure 7.11*c* using a single square unit or two U-shaped units. Columns as small as 8 in. square can be formed this way, but the usual smallest size column is one with a 12-in. side, and the most common is one with a 16-in. side, producing an exterior that resembles that in Figure 7.11*a*.

The form of the column in Figure 7.11*c* is also frequently used to produce pilasters in continuous walls of CMU construction. This is typically done by using alternating courses of units, with one course being as shown in Figure 7.11*c* and the alternating course being as shown in Figure 7.11*d*.

7.4 FORM AND CLASSIFICATION OF COMPRESSION ELEMENTS

There are several types of construction elements used to resist vertical compression for building structures. The proportions of dimensions of the elements are used to differentiate between defined elements. Figure 7.12 shows four such elements, described as follows:

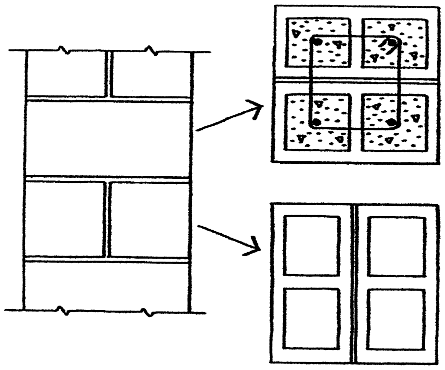
Wall. Concrete and masonry walls frequently work as bearing walls, resisting vertical compression loads. Walls may be quite extensive in length but are also sometimes built in relatively short plan segments.

Pier. When a segment of wall has a plan length that is less than six times the wall thickness, it is called a pier or, sometimes, specifically a wall pier.

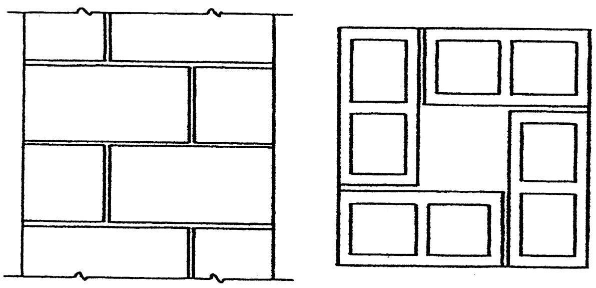
Column. Columns come in many shapes but generally have some extent of height in relation to dimensions of the column cross section. The usual limit for classification as a column is a minimum height of three times the column diameter (e.g., side dimension). A wall pier may serve as a column, so the distinction gets somewhat ambiguous.

Pedestal. A pedestal is really a short column, that is, a column with height not greater that three times its

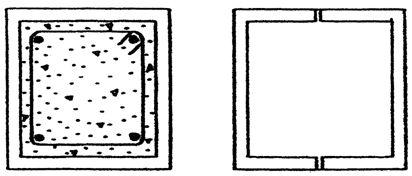
Figure 7.11 Forms of CMU columns.



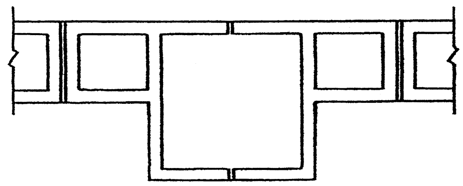
(*a*)
The two-block column.
Arrangement of blocks alternated in successive courses. Column ties placed in mortar joints.



(*b*)
The four-block column.
Also with alternating courses of blocks. Can be lightly or heavily reinforced, minimally or totally filled with concrete.



(*c*)
The box column.
Basically a reinforced concrete column cast in a masonry shell.



(*d*)
The box pilaster.
A column formed within a continuous wall of CMU construction.

thickness. As used in structures, this element is also sometimes called a pier—adding to the confusion of names. Pedestals are frequently used as transition elements between columns and their footings and are discussed in Chapter 8.

To add more confusion, large concrete and masonry support elements are sometimes called piers but also are called *abutments*. These supports may be used for bridges or for various large, long-span structures. Identity in this case is more a matter of overall size, rather than any specific proportions of dimensions.

Another use of the word pier is for description of a type of foundation element which is also sometimes called a *caisson*. This consists of a concrete column cast in a vertical shaft that is dug in the ground, forming a so-called *deep foundation.* Actually, caisson (French for "box") is more correctly used to describe the process for building large piers under water by sinking a bottomless box (caisson) into the muck.

When it is necessary to be specific with names, it is best to refer to legal documents—such as building codes.

7.5 BRICK MASONRY

Brick masonry has a long history and endures steadily as a popular visual element in architecture. In ancient times, brick construction was generally considered to be crude—suffering by comparison to cut stone or finely shaped plaster. Nevertheless, bricks were often used for massive structures, with stone being essentially a thin veneer finish. Early bricks were made of dried mud and the present form of high-fired, durable, strong elements were probably not discovered until some great fires burned the mud bricks. Now, bricks have stature, but the cost of construction is high.

Typical Elements of Brick Construction

Structural brick masonry, in the form of walls or columns, is often used for support of horizontal-spanning structures and possibly for support of other walls or columns above in multistory construction. A major consideration, therefore, is that for resistance to vertical gravity loads that induce compression stress in the walls or columns. In ancient, very

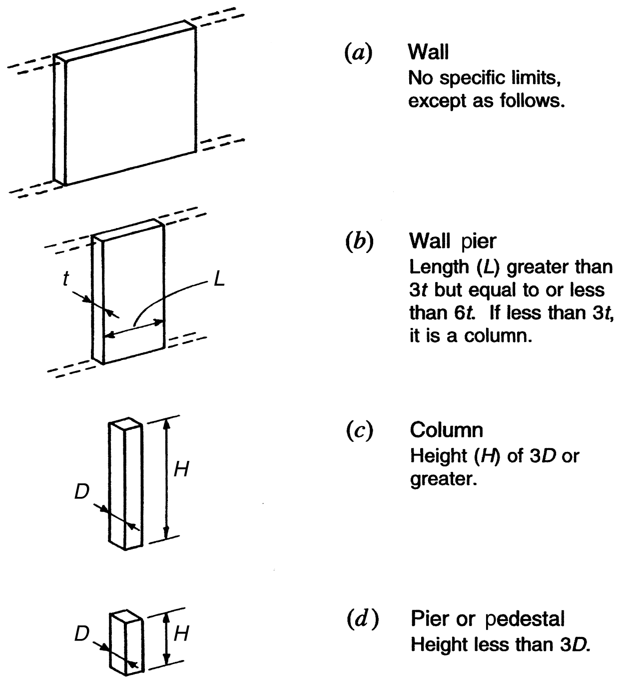


Figure 7.12 Classification of compression elements.

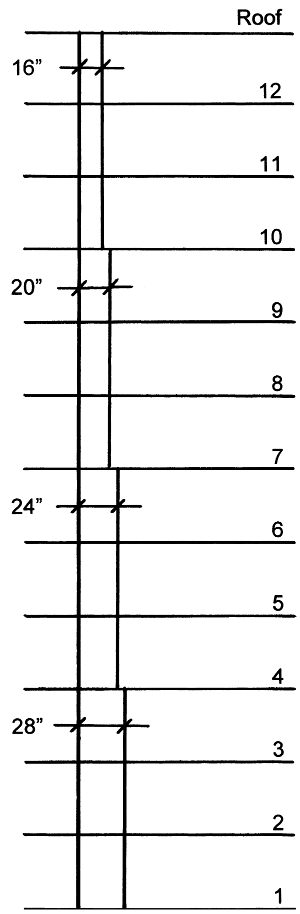


Figure 7.13 Required thickness of brick walls for a 12-story mercantile building, Chicago Building Code, 1932.

heavy masonry, a significant amount of the total vertical gravity load often came from the weight of the masonry itself.

Required thicknesses for walls and horizontal cross sections for columns were developed from experience. As larger and taller buildings were produced, various rules of thumb were developed and passed on, becoming part of the traditions and craft. This tradition-based design was dominant until a relatively short time ago. Figure 7.13 shows the requirements for the graduated thickness of a 12-story-high bearing wall of brick, as defined by the Chicago Building Code some 80 years ago. (See also Figure 7.1 for a tall brick building in Chicago.) These criteria were largely judgmental and based primarily on demonstrated success with years of construction.

Structural walls and columns in present-day masonry construction tend to be considerably slimmer and lighter than those in ancient buildings. Economic pressures and the availability of better, stronger materials partly account for this. A major factor, however, is the considerable development of the fields of materials testing and structural engineering, permitting reasonably precise and dependable mathematical modeling for structural behaviors and, as a result, some heightened confidence in predictions of safety against failure. This has somewhat reduced the dependency on experience alone as a guarantee of success, although confidence is still boosted significantly when experience is there to back up speculations about safety.

General Concerns for Brick Walls

Structural brick walls can be used for many purposes, and basic design concerns derive initially from the specific structural requirements. Compression stress from vertical loading is a predominant condition, deriving—if from nothing else—from the weight of the masonry itself. For simple bearing walls, this may be the only real concern, and a simple axial compression stress investigation may suffice for the structural design.

Additional structural functions for walls may include any of those illustrated in Figure 7.14. Exterior walls in buildings frequently must serve multiple purposes: as bearing walls, as shear walls, and as spanning walls resisting direct wind pressures. These situations produce multiple concerns for stress combinations and interactions as well as for the logical development of the wall form, for construction details, for attachment of supported elements, and so on. Structural walls are also themselves supported by other structures, the nature of which may affect the development of the walls.

Brick Bearing Walls

Brick bearing walls often occur as exterior walls. They thus quite frequently have other structural functions, in addition to that of resisting simple vertical compression. Three common additional functions are those shown in Figure 7.14*b* (spanning as a beam for wind pressure),

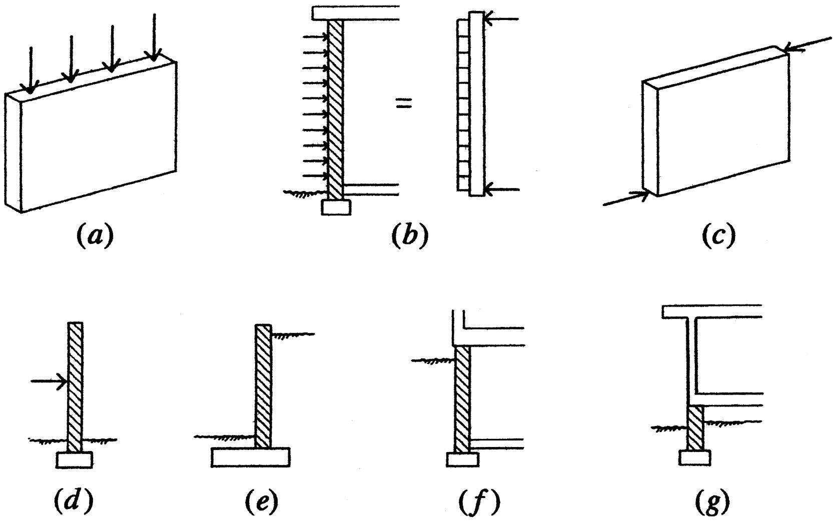


Figure 7.14 Structural uses for masonry walls.

Figure 7.14*c* (functioning as a shear wall as part of the building's lateral resistive system), and Figure 7.14*f* or *g* (as a basement or grade wall). The complete design of the wall must include provision for all of the necessary functions.

When walls sustain concentrated loads, it is necessary to investigate for two stress conditions. The first involves the direct bearing stress at the point of contact with the supported object. The second investigation involves a consideration of the effective portion of the wall that serves to resist the load in compression. The length of the wall for the latter situation is usually limited to six times the wall thickness, four times the wall thickness plus the width of bearing, or the center-to-center spacing of the concentrated loads.

Solid brick masonry is usually the strongest form of unreinforced masonry and is thus capable of considerable force resistance for situations involving only simple compression and bearing stresses. For any added bending effects, however, careful investigation must be made and a case for reinforced construction may well be made.

Beam ends may be supported by direct bearing on top of a wall. However, when a wall is continuous vertically—as in a multistory wall—other provisions must be made. A common one consists of a supporting member attached to the wall face, as shown in Figure 7.15*a*. In earlier times, it was common to use a recessed pocket in the wall to support the ends of beams or trusses (see Figure 7.15*b*). Additional support could also be provided by a bracket developed at the wall face by corbelled courses beneath the pocket (Figure 7.15*c*). To permit the beam to collapse in a fire without ripping up the wall, the ends were cut as shown.

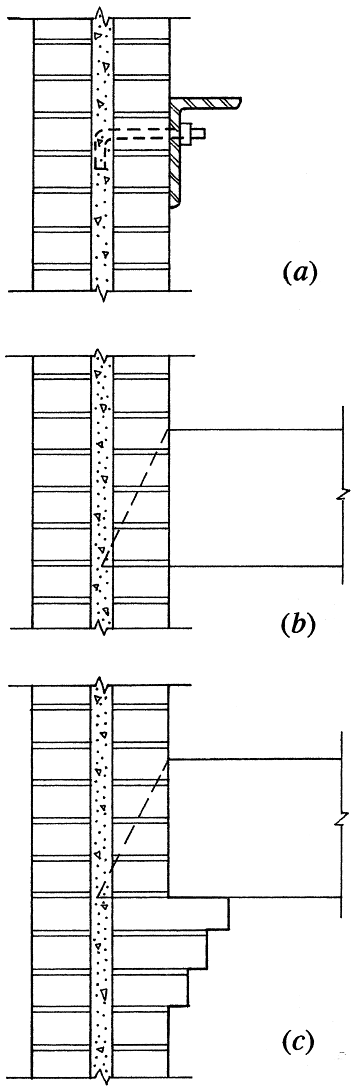


Figure 7.15 Beam end supports in brick walls.

Brick Masonry Shear Walls

Brick masonry structures were extensively created throughout the United States up until about the middle of the twentieth century. Around that time two factors combined to significantly reduce the use of this form of construction. First was the increasing cost of the masonry work in terms of the bricks and the extensive labor required for laying

them. Other forms of construction became more competitive, with CMU construction and precast concrete construction leading the way.

The second major factor was the extensive damage suffered by many unreinforced masonry structures in earthquakes. (See Figure 7.16) For some time, this form of construction has not been permitted in regions with high risk for earthquakes.

New techniques for creating reinforced brick structures have produced options for significant seismic resistance, but the added cost merely squeezes the brick construction even farther out of the picture in respect to competing construction systems.

In fact, there are still many older buildings that have survived so far with lateral bracing by unreinforced masonry, but their vulnerability is a major risk. Various methods have been developed for adding reinforcing elements to these buildings with some success (see Figure 7.16).

Reinforced Brick Masonry

Brick masonry is now sometimes used with steel reinforcement, emulating the nature of reinforced concrete or the

(*a*)

Figure 7.16 These two brick masonry buildings were built in Los Angeles in the early twentieth century of traditional unreinforced construction. They were later upgraded structurally with steel ties between the walls and the roof and floor structures. They were then OK until the Northridge earthquake of 1994, when the damage seen here occurred. Both buildings remained standing in spite of major loss of the masonry—a tribute of sorts to the strengthening efforts. Of special concern are the building corners, which are hard to protect. In the lower photo, the building also suffered loss of the brick wall piers between the second floor windows, another vulnerable location.

(*b*)

Figure 7.16 (*continued*)

more frequently used reinforced construction with CMUs. Reinforced walls typically consist of two wythe construction with an internal cavity between the wythes of sufficient width to permit two-way steel rods in the cavity space. The cavity is then filled with concrete.

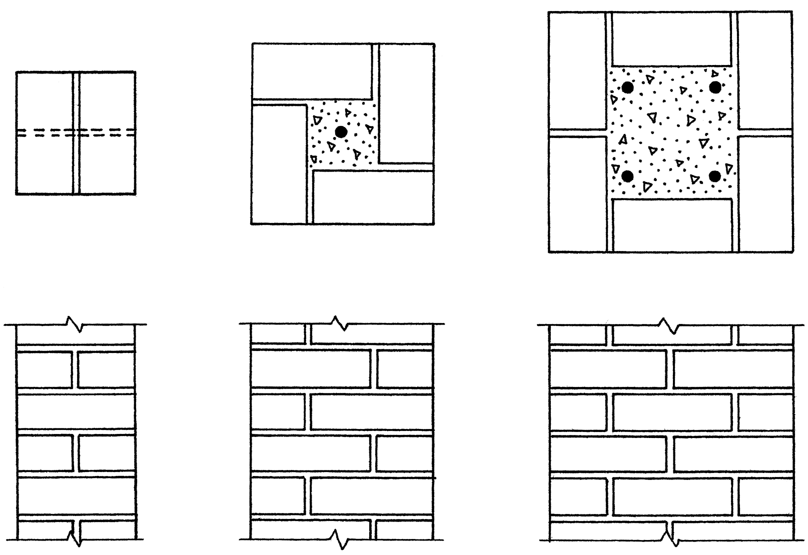
Various code requirements must be satisfied if the construction is to qualify as reinforced in the building codes' definitions of the class of construction. However, reinforcement may also be used simply for enhancement for local stress conditions, even with what is otherwise unreinforced construction.

Brick masonry is still used in regions of low risk for earthquakes, but in high-risk areas masonry structures consist mostly of reinforced CMU construction.

Brick Columns

Brick column sizes begin with the smallest, which consists of two bricks per course, as shown in Figure 7.17*a*. This layout requires a brick whose length is equal to two times its width plus the vertical mortar joint. There is no real opportunity to install vertical reinforcement, so this is strictly an unreinforced member and is not likely to be used for major loads or for any real structural purposes.

A pinwheel arrangement of the courses will produce the column shown in Figure 7.17*b*, resulting in a small cavity. A single vertical steel rod can be placed in this cavity, but to create a real reinforced member, it is usually necessary to use the arrangement shown in Figure 7.17*c*. The minimum,



(*a*) (*b*) (*c*)

Figure 7.17 Forms of brick columns.

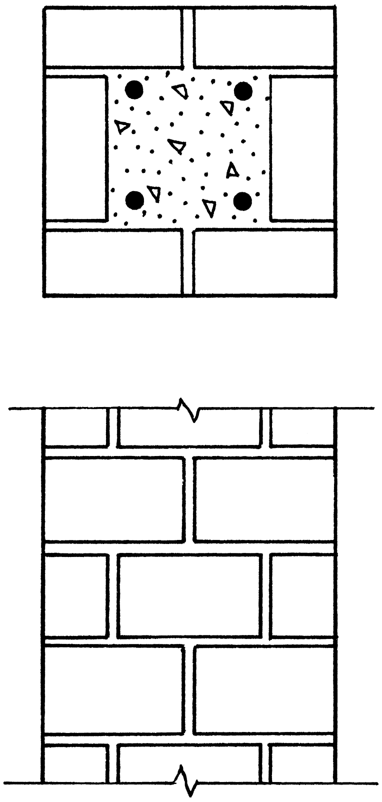


Figure 7.18 Column formed with the narrow face of the brick down.

officially reinforced column is therefore one that is about 16 in. wide. Oblong layouts can also be developed in this fashion to produce wall piers.

A slightly narrower reinforced column can be produced if the bricks are laid with the narrow side (face) down, as shown in Figure 7.18. This form may be structurally acceptable, but appearance may be questionable, since bricks for viewed exposure are usually produced and selected with the intent of having the narrow edge exposed, and their flat bottom sides may have a significantly cruder finished surface.

Very large brick columns are most likely to be produced with a brick shell that functions mostly as a finish surface and for the forming of a reinforced concrete column in the center cavity.

Masonry Arches

The arch was undoubtedly invented in construction with rough stone and mud bricks and eventually refined with cut-stone construction. Massive arches, vaults, and domes were eventually achieved in many cultures. The brick arch has seen major use in wall construction as a spanning device for window and door openings and arcades. Many basic forms were developed, a sampling of which is shown in Figure 7.19, which is reprinted from an early edition of *Architectural Graphic Standards*.

Long-span arches, vaults, and domes are now achieved almost entirely with other forms of construction, principally wood, steel, and reinforced concrete. Short-span arches for windows, doors, and arcades can be developed in brick masonry but are most likely to be done so only for decorative purposes. The flat span is now most common, and although a form is used in masonry, a reinforcement of some kind is usually built into the construction.

Two major concerns must be noted for arches. The first has to do with the horizontal, outward thrust at the base. This may not be a concern when the arch occurs within a large continuous wall but must be considered for other situations or when an opening is quite close to the end of a wall.

The other primary concern for arches is for the rise-to-span aspect. The rise should be as high as possible, preferably approaching that for a semicircle. Compression in the arch and outward thrust at the base are proportional to the rise-to-span ratio.

7.6 MISCELLANEOUS MASONRY CONSTRUCTION

Masonry structures have been, and still can be, produced with many different materials in various forms. The preceding treatments in this chapter have dealt with the two major forms of construction: CMUs and brick. We now consider some additional options.

Stone Masonry

In early times, stone was used as a major structural building material. Now it is either too expensive, too scarce, or too difficult to work with in comparison to alternative structural materials. However, real stone remains a popular material for building exteriors, so extensive use is made in the form of finishing materials. Here we consider some structural uses of stone—past and present.

Rubble and Fieldstone Construction

The earliest uses of stone were probably in the form of rock piles made with stones as found in nature. Craft was eventually developed in the task of piling stones and the forms of structures produced (see Figure 7.20). Even today, stone structures achieved without mortar, using as-found rocks (rubble) or lightly shaped ones, are best built with some attention to fundamental principles slowly developed over the years by skillful stackers of rocks. Some of these tricks, as shown in Figure 7.20, are as follows:

Most stones should be of a flat or angular form to minimize the tendency for upper stones to roll off of lower ones.

Vertical joints in successive layers (courses) should be offset rather than aligned. This helps to develop the overall horizontal continuity of the structure.

As with a pile of any loose material (soil, sand, pebbles), a vertically tapered profile with a wide base is preferred for stability. If a single-direction lateral force must be resisted (as for a retaining wall), the pile may be leaned slightly in opposition to the force. (see Figure 7.20*e*).

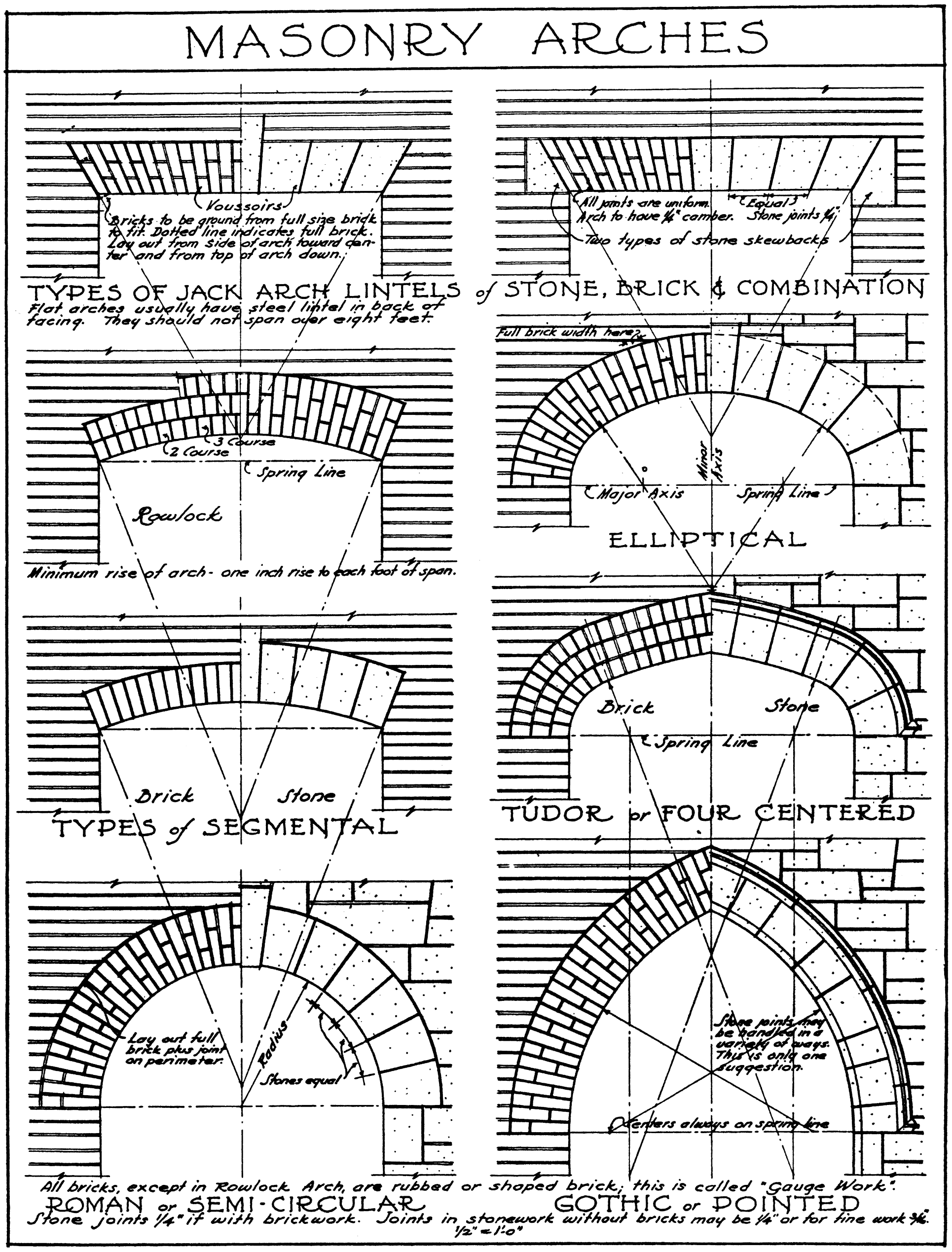


Figure 7.19 Forms of masonry arches in walls. Reprinted from *Architectural Graphic Standards,* 3rd edition, 1941, with permission of the publisher, John Wiley & Sons, New York.

If the stack is tall, the horizontal layers should be dished slightly (Figure 7.20*f*), so that the entire stack leans slightly to the inside.

Very wide stacks may be filled, with larger stones reserved for edges and topping. Filler should be a coarse, granular material (broken rock, gravel, coarse sand), but the top and sides may be filled with some clay materials to seal the stack (an ancient form of crude mortar or concrete).

As the ancient builders learned, the best rock piles are those that maintain a stable equilibrium without assistance. Mortar or concrete should be used primarily to fill voids after the rocks are settled in place; once it hardens, it may serve to further stabilize the stack, but it should not be used initially to prop up the rocks.

Stones may be used as found or they may be shaped. Depending on the source, natural forms may be rounded, angular, or flat. All forms may be used for piles, but the flat and angular shapes will produce more stable structures. Rounded shapes may be used to fill spaces between some stones but should not be used for primary development of the pile.

Shaping may be minor and crude (just breaking or chipping) or it may be done with some precision and accuracy. Construction with natural or minor shaped stones is called *rubble*. Work done with stones shaped to reasonably accurate rectangular forms is called *ashlar*.

If stones are laid in precise horizontal layers, the work is described as *coursed*. If there is no particular attempt to achieve layers within the mass of the pile, the work is described as *random*.

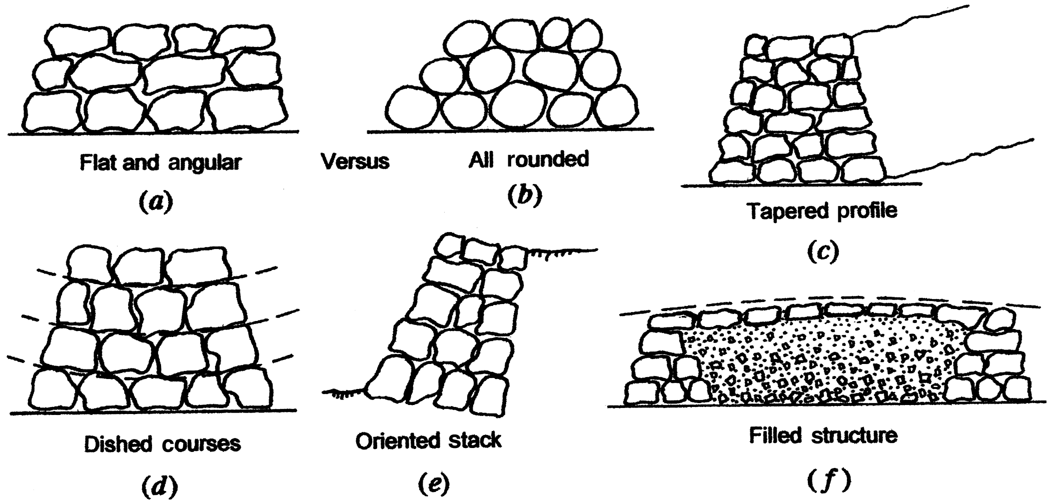


Figure 7.20 Good rock piles.

Unreinforced Construction

Stone structures for buildings should be built with the same care and requirements as those used for construction with bricks or CMUs. The resulting construction can be as structurally sound as other forms of unreinforced masonry.

Stone structures will be somewhat bulkier and heavier than those of brick or CMUs. Joining of the stone work with that of connected elements of the building construction must be achieved with details that allow for the somewhat rough character of the stone work. Anchor bolts or other attachment devices require some greater effort, especially with structures of random rubble form.

For long walls, some horizontal reinforcement should be used. If horizontal courses are achieved, ordinary wire joint reinforcement may be used. Otherwise, or as an alternative, concrete bond beams may be cast into the stonework at vertical intervals.

A bond beam may also be used for the top of a wall. If a stone top is desired, the bond beam may occur in the second course from the top. If the stone wall provides support for other structural elements, the wall top may be developed with a concrete member for this purpose.

Reinforced Construction

Stone walls that support major loads are best developed as reinforced structures. These may take various forms, but the two shown in Figure 7.21 are frequently used. The wall shown in Figure 7.21*a* is developed in the same general way as a two-wythe, fully grouted, reinforced brick wall. A significant cavity is formed in the center of the wall and regularly spaced vertical and horizontal bars are grouted into the cavity.

The wall in Figure 7.21*b* forms a wider cavity that is essentially filled with a sitecast reinforced concrete wall. The stone in this case is developed as faced construction, working as a composite structure with the reinforced concrete core. For a conservative design, such a structure may be considered simply as a concrete wall, ignoring the structural capacity of the stone.

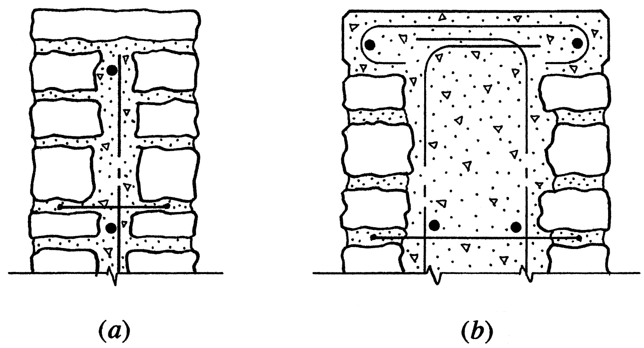


Figure 7.21 Reinforced stone walls: (*a*) as fully grouted (solid) masonry; (*b*) as a faced concrete wall.

For any form of construction, including that which is basically unreinforced, it is advisable to provide some steel reinforcement around any wall openings and at wall ends, corners, tops, and intersections.

Cut-Stone Construction

Construction of masonry structures with large cut stones (quarried granite, etc.) is now rarely done, except in restoration of old buildings. The source material is simply too expensive to use in this bulk form, and other means of creating structures are much more economical. Cut stone is now used almost exclusively for thin veneers, often with supporting structures of steel or reinforced concrete.

Construction similar to that shown in Figure 7.21*b* might be built with facing of cut stone. However, potential damage to the expensive stone facing during casting of the concrete makes this unlikely. The construction is much more likely to be produced by casting the concrete core and then attaching the cut stone as a veneer using the usual details for the full development of the veneered construction.

7.7 ADOBE CONSTRUCTION

Adobe is a term that refers to a masonry unit of sun-dried mud and to the forms of construction developed with the units—usually walls or large piers. It is a very old form of construction, probably the antecedent of all other forms of masonry construction. (See Figure 7.22.)

Adobe bricks are simply produced with soil materials that somewhat emulate a good concrete mix. There must be a binder (cement/clay) and preferably a graded aggregate (for adobe a mixture of silt and fine-to-coarse sand). This just happens to be the normal constituency of the surface soils in many temperate, arid regions—exactly where adobe construction thrives. So, if you live in Arizona and want to make adobe bricks, just go out into your backyard and scoop up some dirt, mix a little water with it, fill a box with it, set it out in the sun, and make adobe bricks.

To make a wall, lay out a row of bricks, fill the cracks between them with the same mud you used to make the bricks, spread a layer of mud on top of them, and lay another row of bricks on top of the mud. Keep this up and you will eventually have a wall.

Figure 7.23 is a reproduction from a 1941 reference presenting a modern version of the ages-old process with a few high-tech touches, such as steel industrial windows, flashing, anchor bolts, steel reinforcement, and some concrete elements for tying and trimming the structure. Also indicated are the basic techniques of protecting the soft, moisture-susceptible adobe with an exterior coating of stucco (cement plaster) and reinforcing with good old "chicken wire." Pragmatic references to "cheap" and "good" construction are also noted for communication with the practical builder.

A variation of adobe brick constructiuon is the use of *rammed earth*, in which the same basic soil material of sand–silt–clay is used in a manner similar to sitecast concrete. Forms are erected, and shallow layers of the soil are tamped (rammed) into place in the forms to create a dense mass. A variation on this is the use of *soil–cement*, consisting of the addition of cement to the soil mixture. Soil–cement construction can also be used to produce foundations for ordinary adobe buildings.

Adobe is fundamentally brick construction, and all of the ideas and tricks for improving and strengthening masonry in general can be used and, in fact, were probably largely learned originally with mud brick construction. Use of pilasters, lintels, general strengthening of openings, corners, and wall intersections, and bonding of multiple wythes apply equally with adobe construction.

Bricks for adobe construction are ordinarily made for use in single-wythe walls—as with CMU construction. The standard brick is therefore quite wide, usually 10 to 12 in. It is also usually made as large as possible to reduce the number required and increase the speed of the construction.

7.8 HOLLOW CLAY TILE

In early times, fired clay was used to produce large, voided units called *tiles* or *tile blocks*. Although now largely displaced

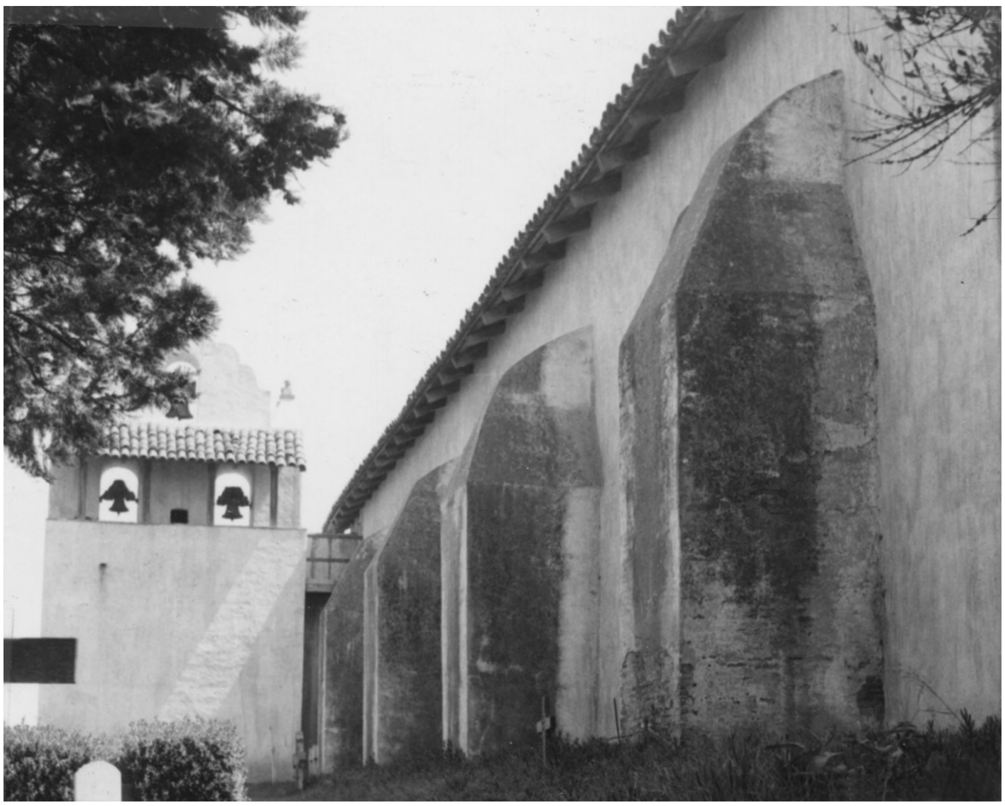


Figure 7.22 Adobe bricks form the thick walls and massive buttresses of this early nineteenth-century mission church in Solvang, California. The early Spanish explorers and clergy brought their architecture to the uncivilized lands of California. However, they had no previous experience with the devastating earthquakes of the region and no resource for the finely cut stones of Europe. So, they stacked up the local mud bricks and watched them fall, dropping the heavy roof on the heads of the worshippers. Eventually, they developed a reliance on very thick walls plus the lateral bracing of massive adobe piers. Still there after all these years and many earthquakes.

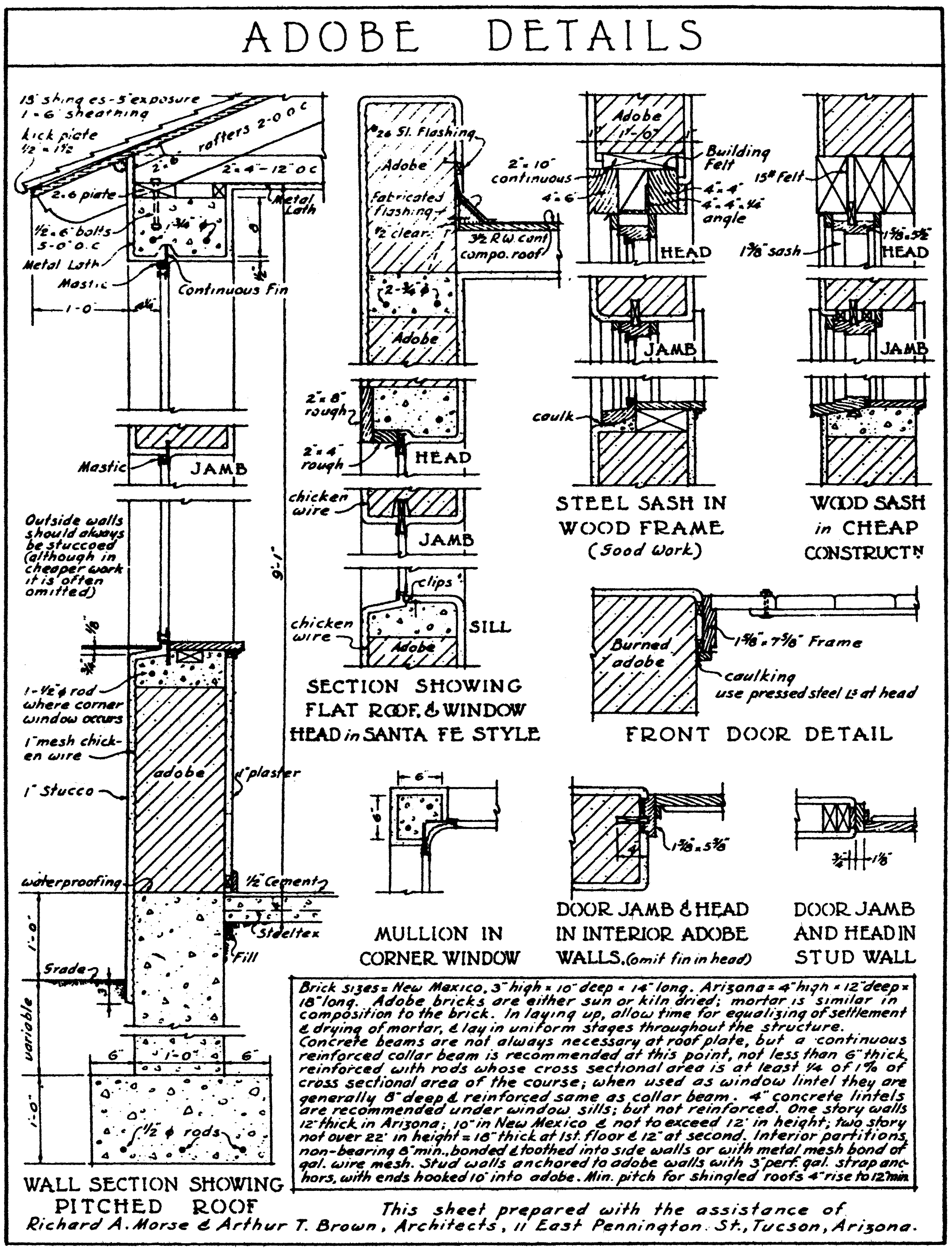


Figure 7.23 Details for adobe construction. Reproduced from *Architectural Graphic Standards*, 3rd edition, 1941, with permission of the publisher, John Wiley & Sons, New York.

by CMUs, the hollow clay units were forerunners of present CMUs, and much of the detail of present CMU construction was originally developed with clay tile units.

A special type of fired clay tile unit was referred to as *architectural terra cotta*, which consisted of units with glazed surfaces intended for exposure to view. These units were developed as economical imitations of cut stone and were widely used for elaborate cornices and other decorative features on buildings in the nineteenth and early twentieth centuries.

CHAPTER

8

Building Foundations and Site Structures

Almost all buildings, regardless of their size, shape, intended purpose, type of construction, or geographic location, share a common problem: They must rest on the ground. Thus the design of adequate foundations is a general problem in building design. Since each building site is unique in terms of specific geological conditions, each building foundation presents unique design problems.

Most buildings have relatively simple foundation systems. However, there are all kinds of problems that may occur at or below the ground surface. Modern geotechnical science and technology offer a vast array of special solutions to both ordinary and severe problems. Some of these measures are discussed here.

Much of the material presented in this chapter is derived from *Simplified Design of Building Foundations* (Ref. 20). For amplification of most of the topics developed here, reference may be made to that publication.

8.1 GENERAL CONSIDERATIONS

This section summarizes the general issues involved in foundation design, the properties and behavioral characteristics of foundation materials of significance for design work, and the problems of establishing useful design data and criteria.

Basic Problems in Foundation Design and Site Development

The design of the foundation for a building cannot be separated from the overall problems of the building structure and the building and site designs in general. It is useful, nevertheless, to consider the specific aspects of the foundation design that must typically be dealt with.

Site Exploration

For purposes of the foundation design, as well as for building and site development in general, it is necessary to know the actual site conditions in some detail. This investigation usually consists of two parts: determination of the ground surface conditions and of the subsurface conditions.

The surface conditions are determined by a site survey that establishes the three-dimensional geometry of the surface and the location of various objects and features on the site. Where they exist, the location of buried objects such as sewer lines, underground water supply lines, power and telephone lines, and so on, may also be shown on the site survey.

Unless they are known from previous explorations, the subsurface conditions must be determined by penetrating the surface to some depth to obtain samples of materials at various levels below the surface. Inspection and testing of these samples in the field, and possibly in a testing lab, are used to identify the materials and to establish a general description of the subsurface conditions.

Site Design

Site design includes the positioning of the building on the site and the general development, or redevelopment, of the site contours and features. The building must be both horizontally and vertically located. Recontouring of the site may involve both taking away of existing materials (called *cutting*) and building up to a new surface with materials brought to the site or borrowed from other locations on the site (called *filling*). Development of controlled site drainage for water runoff is an important aspect of the site design.

Selection of the Foundation Type

The first part of the foundation design is the determination of the type of foundation system to be used. This decision

cannot be made until subsurface conditions are known in some detail and the size, shape, and location of the building are determined. In many cases it is necessary to proceed with an approximate design of several possible foundation schemes so that results can be compared.

Design of Foundation Elements

With the building and site designs reasonably established, the site conditions known, and the type of foundation determined, work can proceed to the detailed design of individual structural elements of the foundation system.

Site Structures

Although they may have little direct relation to the building and its foundations, design of site structures must also be done using the same information base as that for the foundations. With unusual sites, such as those with waterfront locations or very steep slopes, site structures and building foundations may have some direct relationships and require a coordinated design.

Construction Planning

In many cases the construction of the foundation requires a lot of careful planning. Some possible problems include conditions requiring dewatering the site during construction, bracing the steep sides of excavations, underpinning of adjacent structures, excavating difficult objects such as large tree roots or existing construction, and working with difficult soils such as wet clay, quick sand, or soils with many large boulders. The feasibility of dealing with these problems may influence the foundation design as well as the positioning of the building on the site and the general site development.

Inspection and Testing

During the design and construction of the foundation there may be times when it is necessary to perform additional discovery and testing of site materials. This information may be required because of choices for the foundation design or because of inadequacy of the information obtained before design work was started. Testing may also be required if soil materials or conditions not revealed by other discoveries are encountered during construction. With very large sites, it is often not reasonable to do investigations on a widespread basis before any design work begins.

Soil Considerations Related to Foundation Design

The general character of soils as well as some specific physical properties must be established for planning and design of the building foundation and site structures. This information may be related to the structural design work or simply to the necessary planning of the site work in general. The general areas of concern include the following.

Structural Properties

The principal properties and behavior characteristics of soils that are of direct concern in foundation design are the following:

Strength. For bearing-type foundations the main concern is the resistance to vertical compression. Resistance to horizontal pressure and to friction are of concern when foundations must resist horizontal forces of wind and earthquakes or retained earth.

Strain Resistance. Deformation of soil under stress is of concern in designing for limitations of the movements of foundations, such as the vertical settlement of bearing foundations.

Stability. Frost action, fluctuations in water content, seismic shock, organic decomposition, and disturbance during construction are some of the sources that may produce changes in physical properties of soils. The degree of sensitivity of the soil to these actions is called its *relative stability*.

Properties Affecting Construction Activity

A number of possible factors may affect construction activity, including the following:

Relative ease of excavation

Ease of and possible effects of site dewatering during construction

Feasibility of using excavated materials for required site fill operations

Ability of the soil to stand on a near-vertical cut at the edge of an excavation

Effects of construction activities—notably the movement of workers and equipment—on unstable soils

Miscellaneous Conditions

In specific situations various factors may affect the foundation design or the problems to be dealt with during construction. Some examples are the following:

Location of the water table, affecting soil strength or stability, need for waterproofing basements, requirements for dewatering during construction, and so on

Nonuniform conditions on the site, such as pockets of poor soil and soil strata that are not horizontal

Local frost conditions, possibly producing heave (swelling) and settlement of foundations and paving

Deep excavation or dewatering operations, possibly affecting the stability of adjacent properties, buildings, streets, and so on

Foundation Design Criteria

For design of bearing-type foundations several structural properties of a soil must be established. The principal such values are the following:

Allowable Bearing Pressure. This is the maximum permissible value for vertical compression stress at the contact surface of the bearing elements. It is typically quantified in terms of pounds or kips per square foot of the contact surface.

Compressibility. This is the predicted amount of volumetric consolidation that determines the amount of vertical settlement of a foundation. Quantification is in terms of inches of vertical movement.

Active Lateral Pressure. This is the horizontal pressure exerted by a soil against a retaining structure (basement wall, retaining wall. etc.). In its simplest form, this is visualized as an equivalent hydraulic pressure.

Passive Lateral Pressure. This is the horizontal resistance offered by a soil to forces exerted against the soil mass. It is also visualized as an equivalent hydraulic pressure.

Friction Resistance. This is the horizontal resistance to sliding along the contact face of a bearing foundation element. It is quantified as a shear stress that is a constant value for clays but is proportional to the contact bearing force for sands.

Design values for these effects should be established from thorough investigation and testing of the soils. However, most building codes allow for the use of approximate values for ordinary situations; these are described as *presumptive values.*

8.2 SOIL PROPERTIES AND FOUNDATION BEHAVIOR

Information about the materials that constitute the earth's surface is forthcoming from a number of sources. Persons and agencies involved in fields such as agriculture, landscaping, highway and airport paving, waterway and dam construction, and the basic earth sciences of geology, mineralogy, hydrology, and seismology have generated research and experience that is useful to the field of foundation engineering design. This section present a brief summary of issues and data related to foundation design.

Soil Properties and Identification

A distinction is made between two basic materials: *soil* and *rock.* At the extreme the distinction is clear, loose sand versus solid granite, for example. A precise distinction is more difficult, since some highly compressed soils may be quite hard, while some types of rock are quite soft and easy to disintegrate.

For practical use in engineering, soil is generally defined as material consisting of discrete particles that are relatively easy to separate, while rock is any material that requires considerable brute force for excavation.

A typical soil mass is visualized as consisting of three parts, as shown in Figure 8.1. The total soil volume is taken up partly by solid particles and partly by the open spaces between the particles, called the *void*. The void is typically filled partly by liquid (usually water) and partly by gas (usually air). There are several soil properties that can be expressed in terms of this composition, such as the following:

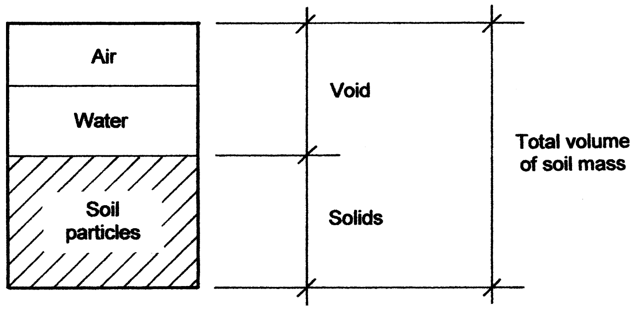


Figure 8.1 Three-part composition of a soil mass.

Soil Weight (γ, gamma). Most of the materials that constitute the solid particles in ordinary soils have a unit density that falls within a narrow range. Expressed as specific gravity, the range is from 2.60 to 2.75. Sands typically average about 2.65, clays about 2.70. Notable exceptions are soils containing large amounts of organic materials, which are mostly much lighter. Specific gravity refers to a ratio of the density of the material to that of water, usually considered to weigh 62.4 lb/ft^3. Thus for a dry soil sample the soil weight may be determined as follows:

$$\gamma = (\text{\% of solids})(\text{specific gravity of solids})(62.4)$$

For a sandy soil with a void of 30%, the dry weight may be approximated as follows:

$$\gamma = \frac{70}{100}(2.65)(62.4) = 116\ \text{lb/ft}^3$$

Void Ratio (e). Instead of expressing the void as a percentage, the term generally used is the void ratio, e, which is defined as

$$e = \frac{\text{volume of the void}}{\text{volume of the solids}}$$

Referring to the preceding example for determination of soil weight, another means for determining unit weight is simply to use the measured volume and measured dry weight of a sample; then,

$$\gamma = \frac{\text{dry weight of the sample}}{\text{volume of the sample}}$$

The unit weight thus determined can be compared with the unit weight of a sample with no void. Assuming a specific gravity (G_s) of 2.65 for the sandy soil and a measured dry weight of 116 kb/ft^3, the percentage of the volume of the solids can then be determined:

$$\text{\% of solids} = \frac{\text{dry weight}}{2.65(62.4)}(100) = \frac{116}{2.65(62.4)}(100) = 70\%$$

The percentage of the void is thus $100 - 70 = 30\%$ and the void ratio can be found as

$$e = \frac{\text{\% of the void}}{\text{\% of the solids}} = \frac{30}{70} = 0.43$$

Porosity (n). The actual percentage of the void is expressed as the porosity of the soil, which in coarse-grained soils (sands and gravels) is generally an indication of the rate at which water flows through or drains from the soil. Actual water flow is determined by standard tests, however, and is described as the relative *permeability* of the soil.

Water Content (w). The amount of water in a soil sample can be expressed in two ways: by the water content (w) and by the saturation (S). They are defined as follows:

$$w \text{ (in \%)} = \frac{\text{weight of water in the sample}}{\text{weight of solids in the sample}}(100)$$

The weight of the water is simply determined by weighing the wet sample and then drying it to find the dry weight. The saturation is expressed in a ratio, similar to the void ratio, as follows:

$$S = \frac{\text{volume of water}}{\text{volume of void}}$$

Full saturation ($S = 1.0$) occurs when the void is filled with water. Oversaturation ($S > 1$) is possible in some soils when the water literally floats some of the solid particles, thus increasing the void above that in the partly saturated soil mass.

In the preceding example, if the soil weight of the sample as taken at the site was found to be 125 lb/ft^3, the water content and saturation would be as follows:

$$\begin{aligned}\text{Weight of water} &= (\text{wet sample weight}) \\ &\qquad - (\text{dry sample weight}) \\ &= 125 - 116 = 9 \text{ lb}\end{aligned}$$

Then

$$w = \frac{9}{116}(100) = 7.76\ \%$$

The volume of water may be found as

$$\begin{aligned}V_w &= \frac{\text{weight of water in sample}}{\text{unit weight of water}} = \frac{9}{62.4} \\ &= 0.144 \text{ ft}^3, \quad \text{or} \quad 14.4\% \text{ of the total}\end{aligned}$$

Then the saturation is

$$S = \frac{14.4}{30} = 0.48$$

The size of the discrete particles that constitute the solids in a soil is significant with regard to the identification of the soil and the evaluation of many of its physical characteristics. Most soils have a range of particles of various sizes, so the full identification typically consists of determining the percentage of particles of particular size categories.

The two common means for measuring grain size are by sieve and sedimentation. The sieve method consists of passing the pulverized dry-soil sample through a series of sieves with increasingly smaller openings. The percentage of the total original sample retained on each sieve is recorded. The finest sieve is a No. 200, with openings of approximately 0.003 in. A broad distinction is made between the amount of solid particles that pass the No. 200 sieve and those that are retained on all the sieves. Those passing are called the *fines* and the total retained is called the *coarse fraction*.

The fine-grained soil particles are subjected to a sedimentation test. This consists of placing the dry soil in a container with water, shaking the container, and measuring the rate of settlement of the particles. The coarser particles will settle in a few minutes; the finest may take several days.

Figure 8.2 shows a graph used to record grain size for soils. The vertical scale of the graph indicates the percentage of particles passing the sieves or qualifying for distinction of size in the sedimentation test. Common soil names, based on grain size, are given at the top of the graph. These are approximations, since some overlap occurs at the boundaries, particularly for the fines.

The distinction between sand and gravel is specifically established by the No. 4 sieve with openings of $\frac{3}{16}$ in., although the actual materials that constitute the coarse fraction are sometimes the same across the grain size range. The curves shown on the graph are representative of some particularly characteristic soils, described as follows:

A well-graded soil consists of some significant percentages of a wide range of soil particle sizes.

A uniform soil has a major portion of the particles grouped in a small range of sizes.

A gap-graded soil has a wide range of sizes, but with some concentrations of single sizes and small percentages over some ranges.

These size-range characteristics are established by using some actual numeric values from the size-range graph. The three size values used are points at which the curve crosses the percent lines for 10, 30, and 60%. These values are interpreted as follows:

Major Size Range. This is established by the value of the grain size in millimeters at the 10% line, called D_{10}. This expresses the fact that 90% of the solids are above a certain grain size. The D_{10} is defined as the *effective grain size.*

Degree of Size Gradation. The distinction between uniform and well graded has to do with the slope of the major portion of the size-range curve. This is established by comparing the size value at the 10% line, D_{10}, with the size value at the 60% line, D_{60}. The relationship is expressed by the *uniformity coefficient*, C_u, which is defined as

$$C_u = \frac{D_{60}}{D_{10}}$$

The higher this number, the greater the degree of size gradation.

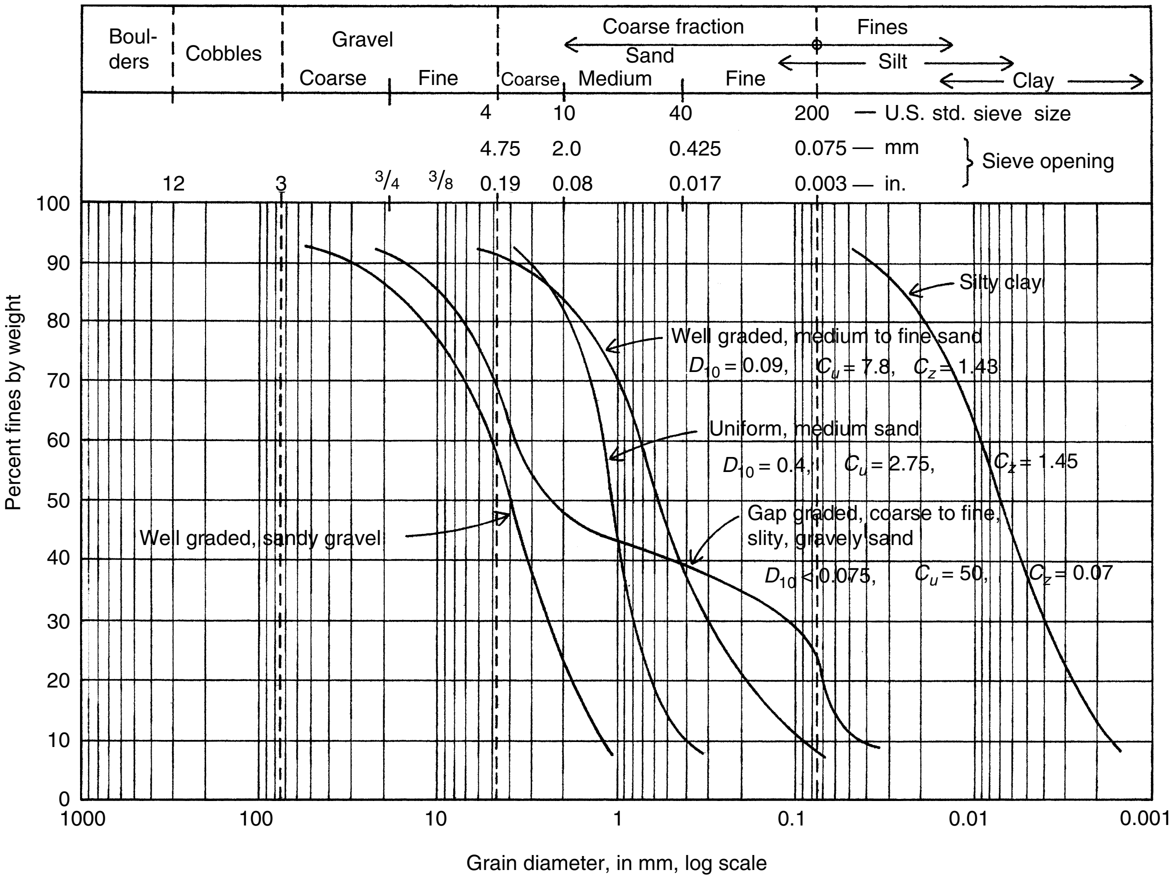


Figure 8.2 Grain size range for typical soils.

Continuity of Gradation. The value of C_u does not express the character of the graph between the 10 and 60% lines; that is, it does not establish whether the soil is gap graded or well graded, only that it is graded.

To establish the character of gradation, another property is defined, called the *coefficient of curvature* (C_z), which uses all three size values, D_{10}, D_{30}, and D_{60}, as follows:

$$C_z = \frac{(D_{30})^2}{(D_{10})(D_{60})}$$

These coefficients are used only for classification of the coarse-grained soils: sand and gravel. For well-graded gravel, C_u should be greater than 4 and C_z between 1 and 3. For well-graded sand, C_u should be greater than 6 and C_z between 1 and 3.

The shape of soil particles is also significant for some soil properties. The three main classes of shape are bulky, flaky, and needlelike, the latter being quite rare. Sand and gravel are typically bulky; further distinction is made with regard to the degree of roundness of the particle form. Bulky-grained soils are usually quite strong in resisting static loads, especially when the grain shape is quite angular, as opposed to well rounded. Unless a bulky-grained soil is well graded or contains some significant amount of fine-grained material, however, it tends to be subject to displacement and consolidation due to vibration or shock.

Flaky-grained soils tend to be easily deformable and highly compressible, similar to the action of randomly thrown loose sheets of paper or dry leaves in a container. A small percentage of flaky-grained particles can impart the character of a flaky soil to an entire soil mass.

Water has various effects on soils, depending on the proportion of water and on the particle shape, size, and chemical properties. A small amount of water tends to make sand particles stick together. As a result, the sand behaves differently than usual, no longer acting like a loose, flowing mass. When saturated, however, most sands behave like viscous fluids, moving easily under stress due to gravity or other sources. The effect of the variation of water content is generally more dramatic in fine-grained soils. These will change from rocklike solids when dry to virtual fluids when supersaturated.

Table 8.1 describes for fine-grained soils the Atterberg limits, which are the water content limits, or boundaries, between four stages of structural character of the soil. An important property of such soils is the *plasticity index*, I_p, which is the numeric difference between the liquid limit and plastic limit. A major distinction between clays and silts is the range of the plastic state. Clays have a considerable plastic range, and silts generally have almost none, going directly

Table 8.1 Atterberg Limits for Water Content in Fine-Grained Soils

Description of Structural Character of Soil Mass	Analagous Material and Behavior	Water Content Limit
Liquid	Thick soup; flows or is very easily deformed	
		Liquid limit: w_L
Plastic	Thick frosting or toothpaste; retains shape but is easily deformed without cracking	(Magnitude of this range is *plasticity index: I_p*)
		Plastic limit: w_p
Semisolid	Cheddar cheese or hard caramel candy; takes permanent deformation but cracks	
		Shrinkage limit: w_s
Solid	Hard cookie; crumbles if deformed	(Least volume attained upon drying out)

from the semisolid state to the liquid state. The plasticity chart, shown in Figure 8.3, is used to classify clays and silts on the basis of two properties: liquid limit and plasticity. The line on the chart is the classification boundary between the two soil types.

Another water-related property is the ease with which water flows through or can be drained from the soil mass. Coarse-grained soils tend to be rapid draining, or permeable. Fine-grained soils tend to be nondraining, or impervious, and may literally seal out flowing water.

Soil structure may be classified in many ways. A major distinction is made between soils that are considered to be *cohesive* and those considered *cohesionless*. Cohesionless soils are those consisting predominantly of sand and gravel with no significant bonding of the discrete soil particles.

The addition of a small amount of fine-grained material will cause a cohesionless soil to form a weakly bonded mass when dry, but the bonding will virtually disappear with a small percentage of moisture. As the percentage of fine materials is increased, the soil mass becomes progressively more cohesive, tending to retain some defined shape right up to the fully saturated, liquid consistency.

The extreme cases of cohesive and cohesionless soils are typically personified by a pure clay and a pure, or clean, sand respectively. Typical soil mixtures will range between these two extremes, so they are useful in establishing the boundaries for classification. For a clean sand the structural nature of the soil mass will be largely determined by three properties: the particle shape (well rounded versus angular), the nature of the size gradation (well graded, gap graded, or uniform), and the density or degree of compaction of the soil mass.

The density of a sand deposit is related to how closely the particles are fit together and is essentially measured by the void ratio. The actions of water, vibration and shock, and compressive forces will tend to pack the particles into tighter (more compact) arrangements. The same sand particles may thus produce strikingly different soil deposits as a result of density variations.

Table 8.2 gives the range of density classifications that are used in describing sand deposits, varying from very loose to very dense. The general character of the deposit and the typical range of usable bearing strength are shown as they relate to the density. Also of concern are particle shape and size gradation and absolute particle size, generally established by the D_{10} property, and the amount of water present, as measured by w or S. A minor amount of water will often tend to give a slight cohesiveness to the sand, as the surface tension in the water partially bonds the discrete sand particles. When fully saturated, however, the sand particles are subject to a buoyancy effect that can work to substantially reduce the stability of the soil.

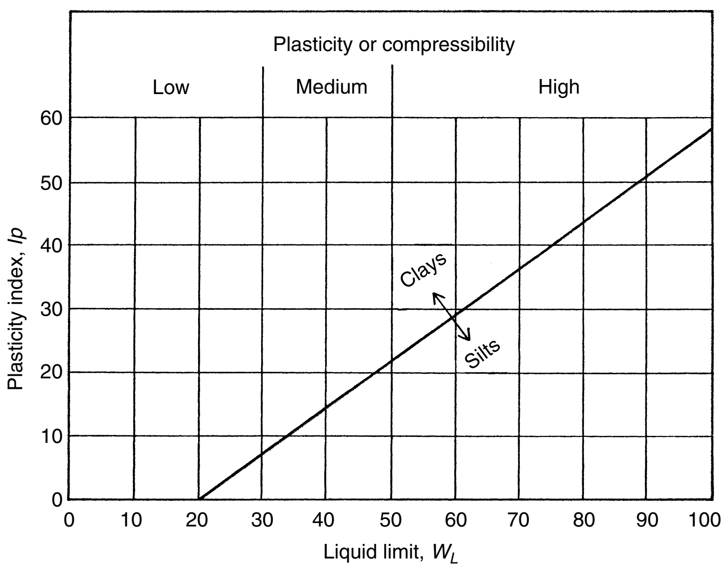


Figure 8.3 Plasticity chart using Atterberg limits.

Table 8.2 Average Properties of Cohesionless Soils.

Relative Density	Blow Count, N (blows:ft)	Void Ratio, e	Simple Field Test with $\frac{1}{2}$-in.-Diameter Rod	Usable Bearing Strength (kips/ft^2)
Loose	<10	0.65–0.85	Easily pushed in by hand	0–1.0
Medium	10–30	0.35–0.65	Easily driven in by hammer	1.0–2.0
Dense	30–50	0.25–0.50	Driven in by repeated blows with hammer	1.5–3.0
Very dense	>50	0.20–0.35	Barely penetrated by repeated hammer blows	2.5–4.0

Many physical and chemical properties affect the structural character of clays. Major considerations are the particle size, the particle shape, and whether the particles are organic or inorganic. The percentage of water in clay has a very significant effect on its structural nature, changing it from a rocklike material when dry to a viscous fluid when fully saturated. The property of a clay corresponding to the density of sand is its consistency, varying from very soft to very hard. The general nature of clays and their typical usable bearing strengths as they relate to consistency are shown in Table 8.3.

Another major structural property of fine-grained soils is relative plasticity. This was discussed in terms of the Atterberg limits and the classification was made using the plasticity chart shown in Figure 8.3. Most fine-grained soils contain both silt and clay, and the predominant character of the soil is evaluated in terms of various measured properties, most significant of which is the plasticity index. Thus an identification as "silty" usually indicates a lack of plasticity (crumbly, friable, etc.) while that of "claylike" or "clayey" usually indicates some significant degree of plasticity (moldable, even when only partly wet).

Various special soil structures are formed by actions that help produce the original soil deposit or work on the deposit after it is in place. Coarse-grained soils with a small percentage of fine-grained material may develop arched arrangements of the cemented coarse particles resulting in a soil structure that is called *honeycombed*. Organic decomposition, electrolytic action, or other factors can cause soils consisting of mixtures of bulky and flaky particles to form highly voided soils that are called *flocculent*. The nature of formation of these soils is shown in Figure 8.4. Water-deposited silts and sands, such as those found at the bottom of dry streams or ponds, should be suspected of this condition if the tested void ratio is found to be quite high.

Honeycombed and flocculent soils may have considerable static strength and be adequate for foundation purposes as long as no destabilizing effects are anticipated. A sudden increase in the water content or significant vibration or shock may disturb the fragile bonding, however, resulting in major consolidation of the soil. This can produce major settlement of ground surfaces or of foundations if the affected soil mass is large.

Behavior under stress is usually quite different for the two basic soil types: sand and clay. Sand has little resistance to stress unless it is confined. Consider the difference in behavior of a handful of dry sand and sand rammed into a strong container. Clay, on the other hand, has resistance to tension in its natural state all the way up to its liquid consistency. If hard, dry clay is pulverized; however, it becomes similar to loose sand until some water is added.

In summary, the basic nature of structural behavior and significant properties that affect it for the two soil types are as follows:

Sand. Little compression resistance without some confinement; the principal stress mechanism is shear resistance (interlocking particles grinding together); important properties are angle of internal friction, penetration resistance to a driven object, unit density

Table 8.3 Average Properties of Cohesive Soils

Consistency	Unconfined Compressive Strength (kips/ft^2)	Simple Field Test by Handling of Undisturbed Sample	Usable Bearing Strength (kips/ft^2)
Very soft	<0.5	Oozes between fingers when squeezed	0
Soft	0.5–1.0	Easily molded by fingers	0.5–1.0
Medium	1.0–2.0	Molded by moderately hard squeezing	1.0–1.5
Stiff	2.0–3.0	Barely molded by strong squeezing	1.0–2.0
Very stiff	3.0–4.0	Barely dented by very hard squeezing	1.5–3.0
Hard	4.0 or more	Dented only with a sharp instrument	3.0+

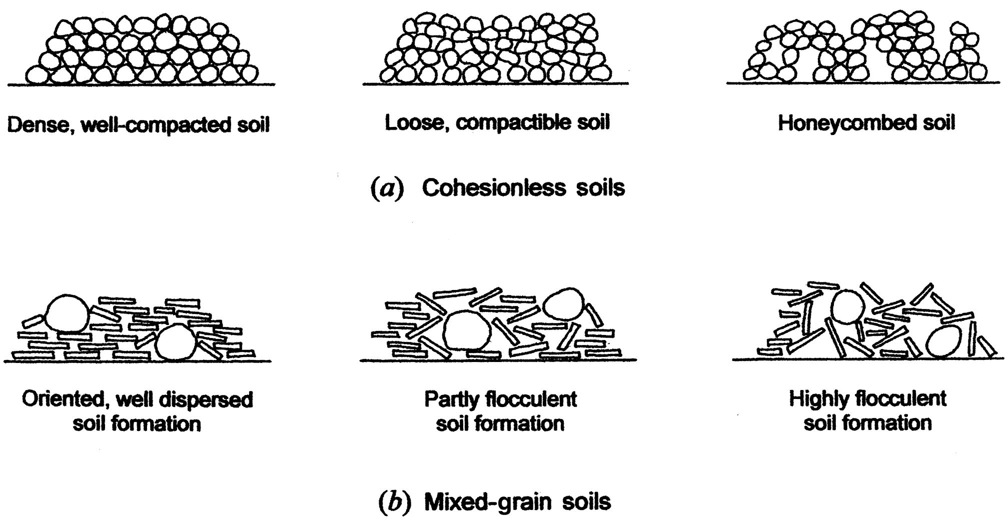


Figure 8.4 Typical soil structures.

in terms of weight or void ratio, grain shape, predominant grain size, and nature of size gradation. Some reduction in stress capacity occurs with high water content.

Clay. Principal stress resistance in tension; confinement is generally of concern only in soft, wet clays (to prevent flowing or oozing of the wet mass); important properties are the unconfined compressive strength of soil samples, liquid limit, plasticity index, and relative consistency (soft to hard).

We must remind the reader that these descriptions present the cases for pure clay and clean sand, which generally represent the outer limits for the range of soil types. Soil deposits typically contain some percentage of all three basic soil ingredients: sand, silt, and clay. Thus most soils are neither totally cohesionless nor totally cohesive and possess some of the characteristics of both of the extremes.

Soil classifications or identification must deal with a number of properties for precise categorization of a particular soil sample. Many systems exist and are used by various groups with different concerns. The three most widely used systems are the triangular textural system used by the U.S. Department of Agriculture; The AASHO system, named for its developer, the American Association of State Highway Officials; and the so-called unified system, which is primarily used in foundation engineering. Each of these systems reflects some of the primary concerns of the developers of the system.

The unified system relates to properties of major concern in stress and deformation behavior, excavation and dewatering problems, stability under use, and other issues of concern to foundation designers. The unified system is shown in Figure 8.5. It consists of categorizing the soil into one of 15 groups, each identified by a two-letter symbol. The primary data used are the grain size analysis, the liquid limit, and the plasticity index. It is not significantly superior to other systems in terms of its database, but it provides more distinct identification of the soil pertaining to significant considerations of structural behavior. This system is the one mostly used by building codes and design handbooks for the issues relating to the regulation of foundation design.

Behavior of Foundations

Most foundations consist of some elements of concrete, primarily because of the relative cost of the material and its high resistance to water, rot, insects, and various effects of being buried in the ground. The two fundamental types of foundations are shallow bearing foundations and deep foundations. This distinction has mostly to do with where the load transfer to the ground occurs. With shallow foundations it occurs near the bottom of the building; with deep foundations the load transfer involves soil strata at some distance below the building.

The most common types of shallow foundations are wall and column footings, consisting of concrete strips and pads cast directly on the ground and directly supporting structural elements of the building. The basic stress transfer between the footing and the ground is by direct contact bearing pressure, inducing general mechanisms of soil behavior. Occasionally several structural elements of the building may be supported by a single large footing. The ultimate extension of this is to turn the entire underside of the building into one large footing, simulating the action of the hull of a ship. This is actually done in rare cases; such a foundation does literally float on the soil and is called a *raft* or *mat* foundation.

If the soil at the bottom of the building is not adequate for the necessary load transfers, or possibly is underlain by weak materials, it becomes necessary to utilize the resistance of lower soil strata. This may require going all the way down to bedrock or merely to some more desirable soil layer. The usual technique used to accomplish this is to place the building on stilts, or tall legs, in the ground.

The two basic elements used to do this are piles and piers, described generally as *deep foundations*. Piles are elements that are driven into the ground, much the same as nails are driven into wood. Piers are shafts that are excavated and then filled with concrete.

Major Divisions			Group Symbols	Typical Names	Classification Criteria		
Coarse-Grained Soils More than 50% retained on No. 200 sieve	Gravels 50% or more of coarse fraction retained on No. 4 sieve	Clean Gravels	*GW*	Well-graded gravels and gravel–sand mixtures, little or no fines	Classification on basis of percentage of fines: Less than 5% Pass No. 200 sieve *GW, GP, SW, SP*; More than 12% Pass No. 200 sieve *GM, GC, SM, SC*; 5% to 12% Pass No. 200 sieve Borderline classification requiring use of dual symbols	$C_u = D_{60}/D_{10}$ Greater than 4 $C_z = \frac{(D_{30})^2}{D_{10} \times D_{60}}$ Between 1 and 3	
			GP	Poorly graded gravels and gravel–sand mixtures, little or no fines		Not meeting both criteria for *GW*	
		Gravels with Fines	*GM*	Silty gravels, gravel–sand–silt mixtures		Atterberg limits plot below "*A*" line or plasticity index less than 4	Atterberg limits plotting in hatched area are borderline classifications requiring use of dual symbols
			GC	Clayey gravels, gravel–sand–clay mixtures		Atterberg limits plot above "*A*" line and plasticity index greater than 7	
	Sands More than 50% of coarse fraction passes No. 4 sieve	Clean Sands	*SW*	Well-graded sands and gravelly sands, little or no fines		$C_u = D_{60}/D_{10}$ Greater than 6 $C_z = \frac{(D_{30})^2}{D_{10} \times D_{60}}$ Between 1 and 3	
			SP	Poorly graded sands and gravelly sands, little or no fines		Not meeting both criteria for *SW*	
		Sands with Fines	*SM*	Silty sands, sand–silt mixtures		Atterberg limits plot below "*A*" line or plasticity index less than 4	Atterberg limits plotting in hatched area are borderline classifications requiring use of dual symbols
			SC	Clayey sands, sand–clay mixtures		Atterberg limits plot above "*A*" line and plasticity index greater than 7	
Fine-Grained Soils 50% or more passes No. 200 sieve	Silts and Clays Liquid limit 50% or less		*ML*	Inorganic silts, very fine sands, rock flour, silty or clayey fine sands	Plasticity chart (see below)		
			CL	Inorganic clays of low to medium plasticity, gravelly clays, sandy clays, silty clays, lean clays			
			OL	Organic silts and organic silty clays of low plasticity			
	Silts and Clays Liquid limit greater than 50%		*MH*	Inorganic silts, micaceous or diatomaceous fine sands or silts, elastic silts			
			CH	Inorganic clays of high plasticity, fat clays			
			OH	Organic clays of medium to high plasticity			
Highly organic soils			*Pt*	Peat, muck and other highly organic soils	Visual–manual identification		

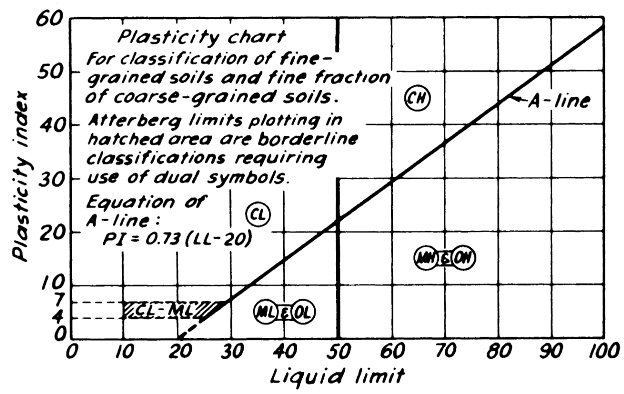


Figure 8.5 Unified System classification of soils for engineering purposes. Reproduced from *Foundation Engineering* (Ref. 21), with permission of the publisher, John Wiley & Sons, Inc., Hoboken, NJ.

When loads are applied to a bearing foundation, stresses are generated in the supporting soil mass. In order to visualize these stresses and the accompanying strains, it is necessary to consider the nature of the potential movement of the foundation and the resulting deformations of the supporting soil.

Figure 8.6 shows the typical failure mechanism for a simple bearing footing as it is pushed downward into a soil mass. Part of the vertical movement of the footing is accounted for by the consolidation, or squeezing, of the soil immediately beneath the footing. If any additional movement of the footing is to occur, it must be accomplished by pushing some of the soil out from under the footing, which then involves stresses and deformations in the soil mass adjacent to and even above the footing. These deformations and movements in the soil mass are all possible due to the relatively soft and easily deformed soil.

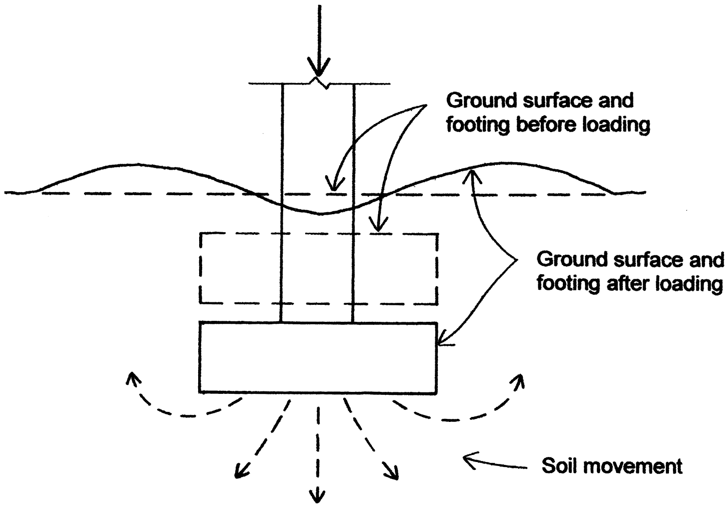


Figure 8.6 Failure mechanism for a bearing foundation.

Figure 8.7 shows the so-called bulbs of pressure that occur in a typical compressed soil beneath a bearing footing. The contour lines of pressure indicate both the direction of the pressure and the location of equal points of pressure magnitude in terms of percentages of the contact pressure, q, at the bottom of the footing. Although the foundation load is directed vertically downward, the net pressure is vertically downward only in the soil mass immediately beneath the center of the footing. Moving away from the center, the net pressure becomes increasingly horizontal. Adjacent to the footing and above the level of its bottom, the net pressure will actually be upward.

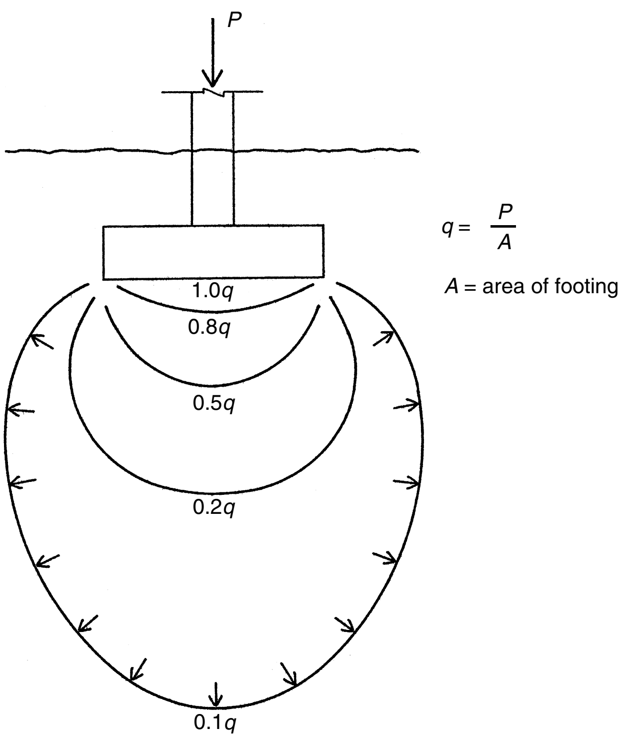


Figure 8.7 Locations of equal pressure under a bearing footing.

Figure 8.8 shows a typical subsurface profile with stratified layers of soils with different properties. If footings placed at the same level are considerably different in width, there will be significant pressure at greater depth below the larger footings. Thus the existence of a highly compressible soil in stratum 4 will have a negligible effect on the narrow footings but may produce some significant settlement of the wider footings.

Special Soil Problems

Expansive Soils

In climates with long dry periods, fine-grained soils often shrink to a minimum volume, sometimes producing vertical cracking in the soil masses that extends to considerable depths. When significant rainfall occurs, two phenomena occur that can produce problems for soil-supported structures. The first problem is the swelling of the surface ground mass as water is absorbed, which can produce considerable upwards or sideways pressures on structures. The second problem is the rapid seepage of water into lower soil strata through the vertical cracks, which may produce weakening of some lower soils.

Soil swelling can produce stresses and movements that affect site and foundation elements. Compensation for these effects depends on the details of the building construction. Local codes usually have provisions for dealing with these problems, relating to regional climate and geology. Removal and replacement or modification of problem soils may be required. Soil expansion may not be sufficient for raising heavily loaded building foundations, but they will affect site construction and paving as well as floor slabs in the building that are placed directly on the ground surface.

Collapsing Soils

Collapsing soils are in general soils with large voids. The collapsing mechamism is essentially one of rapid consolidation as whatever tends to maintain the soil structure in the large void condition is removed or altered. Very loose sands may display such behavior when they experience drastic changes

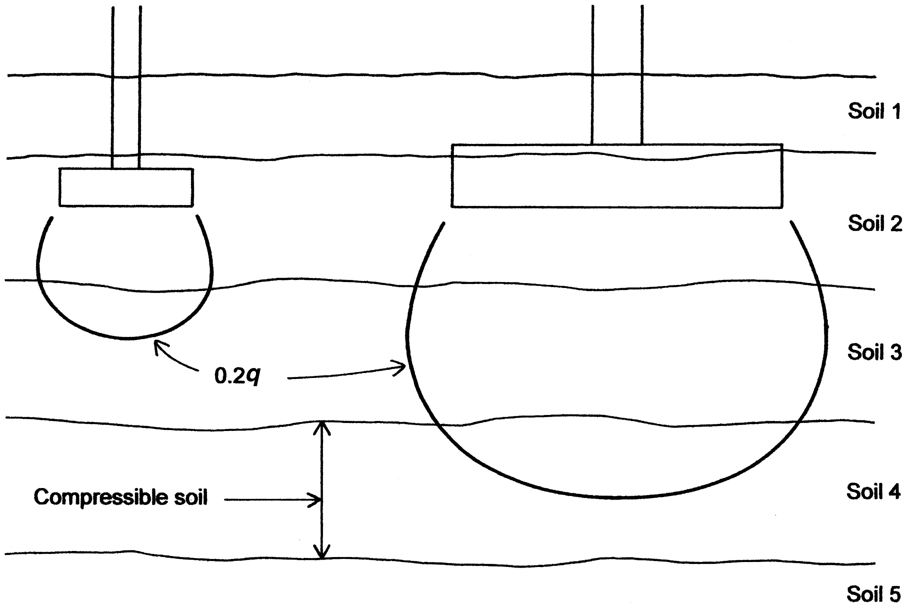


Figure 8.8 Differences in soil pressure effects under footings of different widths.

in water content or are subjected to shock or vibration. The more common cases, however, are those involving soil structures in which fine-grained materials achieve a bonding or molding of cellular voids. These soil structures may be relatively strong when dry but may rapidly collapse when the water content is significantly raised. The bonded structures may also be destroyed by excessive compressive stress.

This behavior is generally limited to a few types of soil and can usually be anticipated when such soils display high void ratios. The two ordinary methods for dealing with collapsing soils are to stabilize the soil by introducing materials to fill the void and reduce the potential degree of collapse or to use vibration, saturation, or other means to cause the collapse to occur prior to construction work.

When considerable site grading is to be done, it is sometimes possible to temporarily place soil at some height on the site, providing sufficient compression to cause significant deformation of the foundation-bearing materials.

Differential Settlements

It is generally desirable that the foundation of any building settles uniformly. If separate elements of the foundation system settle by significantly different amounts, there will be some distortion of the supported building structure. The seriousness of this situation depends upon the materials and type of the construction; critical cases are ones involving masonry, concrete, and plaster construction that tend to be quite rigid and subject to brittle cracking.

A number of situations can result in differential settlements. Some of these can be adjusted for by careful deign of the foundations, whereas others are more difficult to compensate for. The following are some of the situations that can cause this problem:

Nonuniform Subgrade Conditions. Pockets of poor soil and soil strata that are not horizontal can result in different settlement conditions for footings at different locations on the site. Any attempts to equalize settlements in this case require extensive information about the subsurface soil conditions at all points on the site where foundations occur.

Footings of Significantly Different Size. As discussed before, the vertical compressive stresses under large footings can reach great depths. This can produce greater settlements of the large footings, even though the contact bearing pressure is the same for all footings.

Footings Placed at Different Elevations. Footings at different elevations may bear on different soil strata with significantly different settlement resistance or—if close to each other—may influence each other's settlement.

Varying Ratios of Live and Dead Loads. Although the foundations must carry all the building loads, the effect of dead load is often more critical. One reason for this is that the dead load tends to be more "real," while the live load is often quite vaguely established. Another reason is that the dead load is permanent and may thus have more influence on settlements that are progressive with time. This problem relates to soils with high clay content and to soils susceptible to effects of fluctuations of water content or to freezing. This sometimes favors a design procedure for footings that seeks to achieve equal soil pressure under dead load for all the foundations.

Discovery of Information for Soils

The amount of information necessary for a good foundation design depends on a number of factors. For a small building located on a flat site with good soil conditions the necessary information may be minimal. For a large building on a difficult site considerable site exploration and extensive field

or laboratory testing may be required. The purpose here is not to explain how to do such investigations but to simply describe the usual processes, the form of information generally provided, and the relation in general of soil investigation to foundation design.

In some cases, persons with experience in soil exploration may be able to determine major aspects of the subsurface conditions without elaborate equipment or tests. Soil samples to some depth may be obtained with a posthole digger or a hand auger, and the general character as well as some significant properties may be reasonably determined from handling these samples. Color, odor, general texture, moisture content, density, and ease of excavation of the samples can be correlated to give a fair approximation of the classification and average structural properties of the soil. This type of investigation may be quite adequate for small projects where no unusual conditions exist.

A considerable amount of construction of modest buildings is achieved with very minimal soil exploration. Local experience with climate conditions and well known soils, plus a lot of previous construction (successful or not successful), can provide confidence in use of familiar forms and details of construction. For large buildings, however, or with unusual site conditions, it is usually necessary to have a thorough site exploration by a well-qualified soil-testing service.

An extensive soil exploration and testing program can be quite expensive, especially for large sites or when very deep sampling of the subgrade materials is required. Before such a program is undertaken, it is desirable that there be some information about the building size, location, and type of construction and preferably some idea about general subsurface conditions that can be anticipated. If such is the case, the desired locations for soil borings, the necessary depth to which they should be carried, and the extent of testing of soil samples required can be more intelligently planned.

If soil exploration is done on a large site with virtually unknown subgrade conditions and little idea about the location or form of construction that is planned, it is to be expected that a second stage of exploration and testing will be required to support the final foundation design work. And, indeed, additional sampling and testing may be required during excavation and construction if any conditions emerge that were not revealed by prior exploration. All of this work will add up to a considerable expense, not so critical for a large project, but possibly scarcely feasible for small projects.

A soil exploration and testing program intended to provide support for building foundation design work ordinarily consists of the following:

- Exploration of the subgrade conditions by some means to obtain samples of soil at various levels below the ground surface
- Field tests consisting of some observations made during the exploration as well as observations and some field tests on the soil samples obtained
- Laboratory tests on some samples, such as determination of dry weight and particle size gradation
- Interpretation of the information obtained and recommendations for criteria and some details of the foundation design

The extent of exploration, the type of equipment used, the type of tests performed, and the soil properties determined are all subject to considerable variation.

The soil-testing service will provide some analysis of the implications of the information discovered, and—if they were informed of any details of the proposed design work—may make some recommendations about the foundation design. However, the final responsibility for applying the information to actual design of the foundations must eventually be accepted by the engineers doing the design work.

Soil Properties for Foundation Design

For foundation design purposes, soil properties may be broadly separated into two groups. The first consists of those properties that are significant to the identity of the soil type. With reference to the Unified System, these are the properties required to establish the soil identity as one of the 15 types in the system chart (Figure 8.5) or, in some cases, as a soil with marginal properties placing it between two closely related types.

For sand and gravel, the significant properties for identification are the amount of fine-grained materials and the size gradation characteristics of the coarse fraction. There are several properties interpreted from these two pieces of information that fine tune the group identity. For silt and clay, major factors are those related to liquid limit, range of particle size, and the plasticity index. For those with some experience in soil identification, there are many additional observations—such as color, odor, and texture—that help to clarify the group identities.

The second group of soil properties includes those that relate directly to the structural character of the soil, involving stability, stress resistance, and deformation behavior. While some of these properties can be presumed in a general way on the basis of soil-type identification, there are some specific tests that provide more information, permitting more accurate predictions of structural performance.

For a sand, there are typically four items of information not included in the data used for classification by the Unified System that are significant to structural behavior:

- *Grain Shape.* Grain (soil particle) shape ranges from well rounded to angular.
- *Water Content.* Water content is expressed as measured in the natural state at the time of sampling but must also be considered with reference to seasonal changes.
- *Density.* Density ranges from loose to dense and indicates compressive strength as well as the potential for consolidation and the resulting settlement.

Penetration Resistance. Penetration resistance is quoted as the *N* value, which is the number of blows required to advance a particular type of soil sampler into the deposit. This correlates with density properties.

For a clay, the principal tested structural property is the unconfined compression strength, q_u. This may be approximated by some simple field tests but is most accurately established by a laboratory test on a carefully excavated, so-called undisturbed sample of the soil.

Many soil properties are interrelated or derivative; thus a crosscheck is possible when considerable information is available. For example, unit dry weight and penetration resistance are both related to density of sand. Similarly, unconfined compression strength, relative consistency, and plasticity are interrelated for clay.

The structural character of silts ranges considerably from that resembling a low plasticity clay to that resembling a fine sand. Thus it is sometimes necessary to use tested properties pertinent to both cohesive and cohesionless soils for complete evaluation of the structural character of silty soil.

For the foundation designer, information about soils and ground conditions in general has as a primary purpose the establishment of data useful for design applications. The standard notation, classification, and general format for presentation of such data usually follow the forms used in building codes or by local engineering and testing organizations.

8.3 SHALLOW BEARING FOUNDATIONS

Shallow foundation is the term used to describe the type of foundation that transfers vertical loads by direct bearing on soil strata close to the bottom of the building and a relatively short (shallow) distance below the ground surface. The form of construction used mostly for these structures is reinforced concrete; examples of design of a wall footing and a column footing are given in Chapter 6. Figure 8.9 illustrates a variety of elements ordinarily used in shallow bearing foundation systems.

Wall Footings

Wall footings serve to transfer loads from supported structural walls of the building but often also serve a primary function as a platform for the wall construction. If the supported

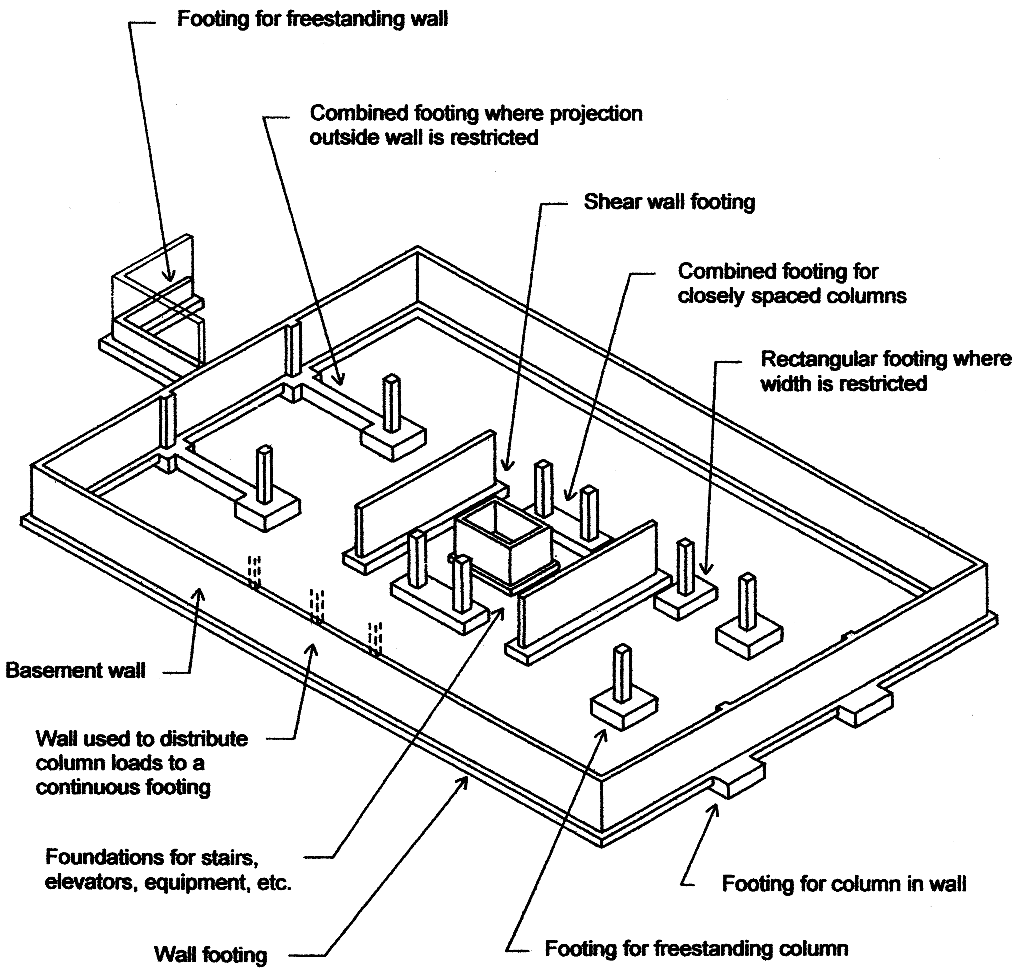


Figure 8.9 Elements of shallow bearing foundations.

wall is of concrete or masonry, dowels for the wall vertical reinforcement will be anchored in the footing. If the wall is constructed with wood or steel, and bearing is at some distance below grade, there will often be a foundation wall that serves to keep the vulnerable wall materials from contact with soil.

Column Footings

Footings for columns must relate to the structural actions of the columns they support. Action is often limited to simple, direct, vertical compression but may be combined with bending moment and lateral shear. For simple compression, the most common footing is the square concrete pad with reinforcement in two layers at right angles to each other. The design of this simple footing is described in Section 6.5.

As with wall footings, it is often necessary to consider factors other than the simple bearing of the footings. Provision must first be made for the bending and shear in the footing. Accommodation of supported columns may also require consideration, with required development of column vertical reinforcement or insertion of anchor bolts. Pedestals are frequently used to facilitate the transition from the columns to the footing. Additional considerations for the form of the footings are as follows.

Oblong Column Footings

Restraints of adjacent construction may prevent the use of the simple square footing, in which case a rectangular, oblong footing may be used. Although a square footing is also rectangular, the term *rectangular footing* is commonly used for this shape of footing.

A rectangular footing must be designed differently for the bending in two directions, called the *long* and *short* directions. The greater the difference between these two dimensions, the more the footing tends to act primarily in one-way bending as a simple double-cantilevered beam.

Combined Column Footing

When two or more columns are quite close together, it is sometimes desirable or necessary to use a single footing, as shown in Figure 8.10. If the columns and their loads are symmetrical, the footing is designed as a uniformly loaded simple-span beam with two cantilevered ends, with the resulting distribution of bending and shear as shown in Figure 8.10. If the column loads are not equal, a different layout of the footing is developed to produce a uniform bearing pressure.

Cantilever Footing

Occasionally, due to various problems, it is not possible to place the usual square footing under an exterior edge building column. One solution for this case is the so-called *cantilevered footing* or *strap footing*. This consists of doubling up the exterior column with an adjacent interior column and designing a footing with equal bearing pressure for the two column loads, as shown in Figure 8.11.

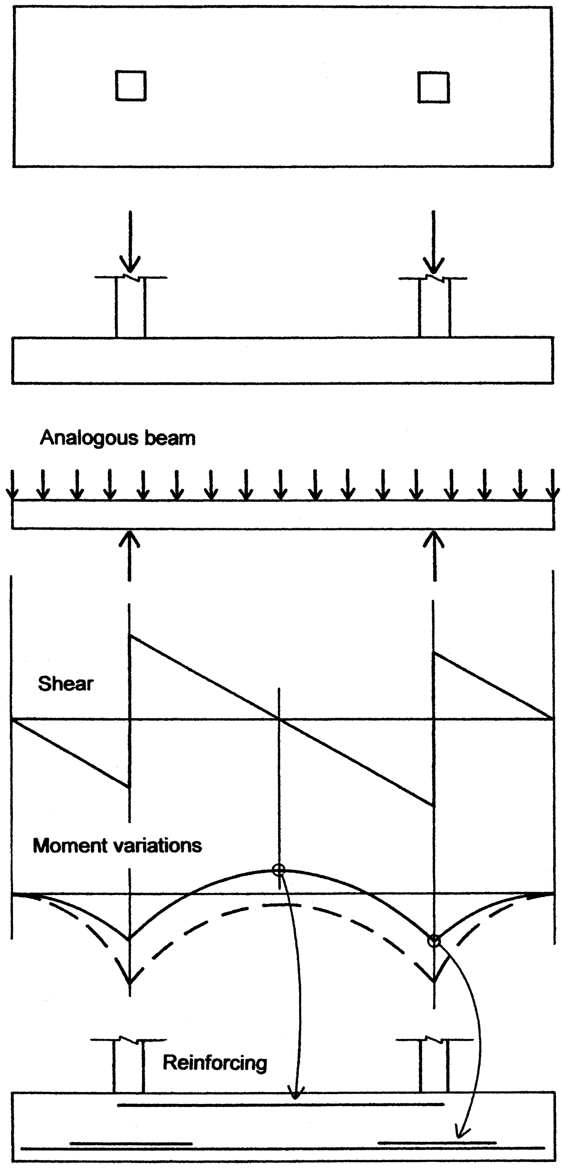


Figure 8.10 Actions of a combined footing.

Moment-Resistive Footings

Bearing footings must occasionally resist moments, in addition to some combination of vertical and horizontal loads. Some situations that produce this effect, as shown in Figure 8.12, are the following:

Freestanding Walls. When a wall is supported only at its base and must resist horizontal forces on the wall, it requires a moment-resistive foundation. Examples are exterior walls used as fences and interior walls that are not full story in height. The horizontal forces are usually due to wind or seismic effects.

Cantilever Retaining Walls. Cantilever retaining walls must sustain horizontal soil pressure from the high-grade-level side of the wall. The resulting overturning moment is usually the major concern for the wall footing.

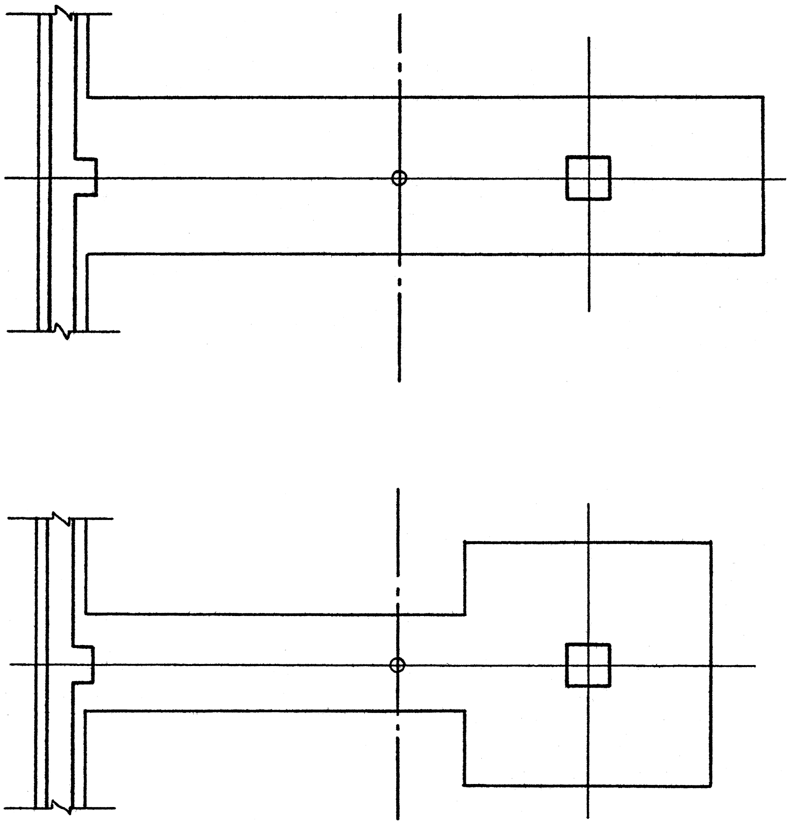


Figure 8.11 Common plan forms for the cantilever footing.

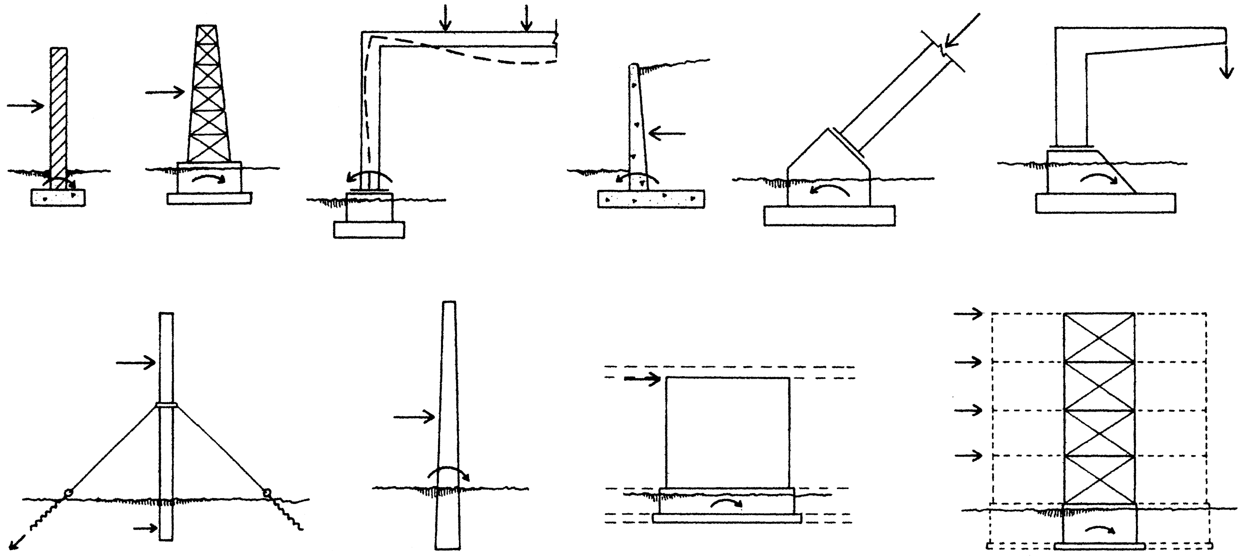


Figure 8.12 Structures with moment-resistive foundations.

Bases for Shear Walls and Freestanding Trussed Frames. When these structures are supported only by their bases, the lateral forces on the supported structure must be resisted by the bearing foundation.

Supports for Rigid Frames, Arches, Cable Structures, And So On The foundations for these structures must often sustain horizontal forces and bending moments, even when loads from the supported structure are due to gravity alone.

Bases for Chimneys, Signs, Towers, Flagpoles, And So On Any freestanding vertical element supported only at its base must have a moment-resistive foundation.

Moment-resistive footings must usually sustain combined bending and vertical compression. There are typically two concerns for such a footing: basic stability in terms of resistance to overturning effect and maximum soil pressure due to the combined load effects. For investigation of stress,

a special condition is the inability of the contact bearing surface to develop tension resistance. The stress combination situation is described in Section 3.3.

Foundations for Shear Walls

When shear walls rest on bearing foundations, the situation is usually one of the following:

The shear wall is part of a continuous wall and is supported by a foundation that extends beyond the shear wall ends.

The shear wall is a separate wall unit and is supported by its own foundation in the manner of a freestanding tower structure.

For the first case, the design of the foundation must be one that includes considerations for the entire wall structure, for which many different possibilities exist. In most cases the foundation will include a foundation wall, a grade beam, or a basement wall that serves as a continuous distributing element between the walls and the footings. For the second case, the design must include considerations for the following:

Anchorage of the Shear Wall. This consists of the attachment of the shear wall for resistance to overturning moment and to horizontal sliding.

Overturn Effects. Overturn is a two-part action; first is the attachment of the wall to its support; second is the interface of the footing and the soil.

Horizontal Sliding. Again, two issues: attachment of the wall to the support and prevention of horizontal sliding of the footing.

Maximum Soil Pressure and Its Distribution. This involves the consideration for the combined vertical compression and the flexural stress caused by the overturning moment.

Miscellaneous Problems of Shallow Foundations

A number of special problems occur in the general design of foundation systems that utilize shallow bearing footings. The following are some problems that are often shared by the several elements that constitute the complete foundation system for a building.

Equalizing of Settlements

It is usually desired that all of the elements of a foundation system settle the same amount. If part of a wall settles more than another, or if a column settles more than walls, there are a number of problems that can result, such as:

Cracking of walls, especially those constructed with rigid materials such as masonry, concrete, or plaster

Jamming of doors and of operable windows

Misalignment of elevator rails

Fracturing of piping or electrical conduits incorporated in the construction

Production of undesirable stress or stability conditions in structures that have some degree of continuity, such as multispan beams or rigid frames of steel or concrete

A technique sometimes used to reduce these problems is to design for so-called equalized settlements. This is a process in which the sizes of bearing elements are determined on the basis of developing equal vertical settlements, rather than producing a common maximum soil pressure. The ease of accomplishing this depends on several factors, including the specific soil conditions. The simplest case occurs when all footings are at the same approximate level and all bear on the same type of soil. In this case the technique most often used is to design for a relatively constant pressure under dead load, since this most often represents the critical loading condition for settlements.

Proximity of Foundation Elements

Building planning often results in situations in which separate parts of the building structure are located so that their foundations are close together. Some of the situations of this type and the design problems that result are the following:

Closely Spaced Columns. Columns are occasionally so closely spaced that it is not possible to use the usual square footings under each column. When this occurs, options are to place an oblong footing under one column, to place oblong footings under both columns, or to use a single oblong footing to support the two columns.

Closely Spaced Buildings. When new construction must be placed very close to an existing building, the excavation and foundation construction must be performed in a manner that does not cause settlement or collapse of the adjacent building. If the new footings must be lower than the existing ones, it will be most likely necessary to underpin the existing foundations.

Columns or Bearing Walls Close to Other Construction. There are often many opportunities for this that must be treated as individual situations for planning.

Adjacent Footings at Different Elevations

When individual footings are horizontally closely spaced but occur at different elevations, a number of potential problems may be created. As shown in Figure 8.13, some of these are as follows:

Disturbance of the Upper Footing. Excavation for the lower footing may result in vertical settlement or sideways movement of the upper footing if the upper footing is already in place and carrying some load. Even if the excavations for the two footings are performed at the same time, disturbance of the supporting soil for the upper footing may occur. There are a number of variables with this situation, but some limit must be established for the combined differences of horizontal separation and vertical difference in elevation.

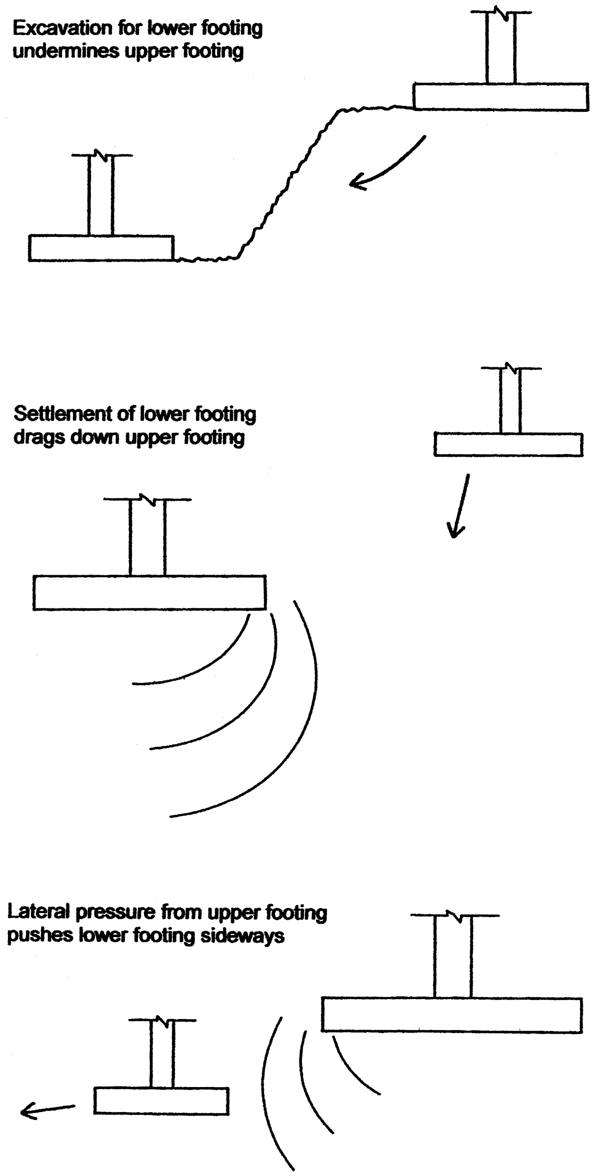


Figure 8.13 Problems with adjacent footings at different elevations.

Additional Settlement of the Upper Footing. If the lower footing is large, its pressure may spread sufficiently through the soil mass to cause some additional settlement of the upper footing.

Lateral Pressure Effect on the Lower Footing. If the upper footing is large, the horizontal spread of pressure may produce some lateral movement of the lower footing.

The critical design limit in these situations is the relation of the vertical and horizontal dimensions of the separation between the footings, as shown in Figure 8.14. This limit is a matter of judgment and cannot be generalized for all situations. Some building codes set limits for this situation, but the number of variables make them dubious. This is a place for some serious study by geotechnical experts, preferably those who perform the soil exploration and testing.

Although the ratio of the dimensions h and L is a critical concern with regard to the various effects illustrated in Figure 8.13, of equal concern is the actual value of L. When this distance is close to or less than the dimension of the larger footing, the soil stress may cause difficulties, even though the footings are at the same elevation. Thus a value of h/L of one-half or less does not necessarily mean the design is conservative. Conversely, with the same soil conditions, when L is several times the dimension of the larger footing, a considerable elevation difference can be tolerated.

Another problem involving differences in footing elevations occurs when the bottom of a continuous foundation wall must be lowered at some point, as shown in Figure 8.15*a*. One situation of this type occurs when a building has only a partial basement, requiring the foundation wall to be considerably taller in the area of the basement. The solution to this problem is to either slope the footing (Figure 8.15*b*) or step the footing in increments (Figure 8.15*c*).

There are three critical considerations for the design of the stepped footing:

Length of the Step. If the step length is too short, the individual steps will have questionable value as individual footings, and the footing is effectively the same as a sloped one. As shown in Figure 8.16, the toe portion of the step is essentially unusable for bearing due to unavoidable disturbance during excavation. Thus, if the step is very short, the length remaining for use as a bearing footing may be quite minor.

Height of the Step. The higher the step, the longer the unusable toe portion of the upper level. This has something to do with the soil type. In soils that cannot be excavated with a vertical cut, it may be necessary to slope the stepped cut, as shown in Figure 8.16.

Angle of the Step. The step angle—defined by the h/L ratio—is essentially the same problem as was discussed for adjacent footings and illustrated in Figure 8.14.

For a generally conservative design and in the absence of other design limitations, the following is recommended:

Limit the step length L to not less than 3 times the footing width.

Limit the step height h to not more than 1.5 times the footing width or 2 ft, whichever is smaller.

Limit the h/L ratio to $\frac{1}{3}$.

Footings on Fill

In general, it is desirable that footings be placed on undisturbed soil, meaning soil that has not been recently excavated. If this is not feasible, the choices are limited to the use of deep foundations or footings placed on fill materials. When footing loads are light, it may be reasonable to consider the latter option. Some such situations are the following:

When a thin layer of undesirable material is found at the level desired for the footings. In this event it may be possible

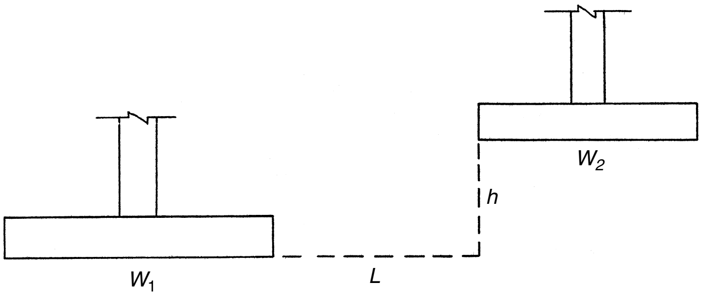


Figure 8.14 Dimensional relationships for footings at different elevations.

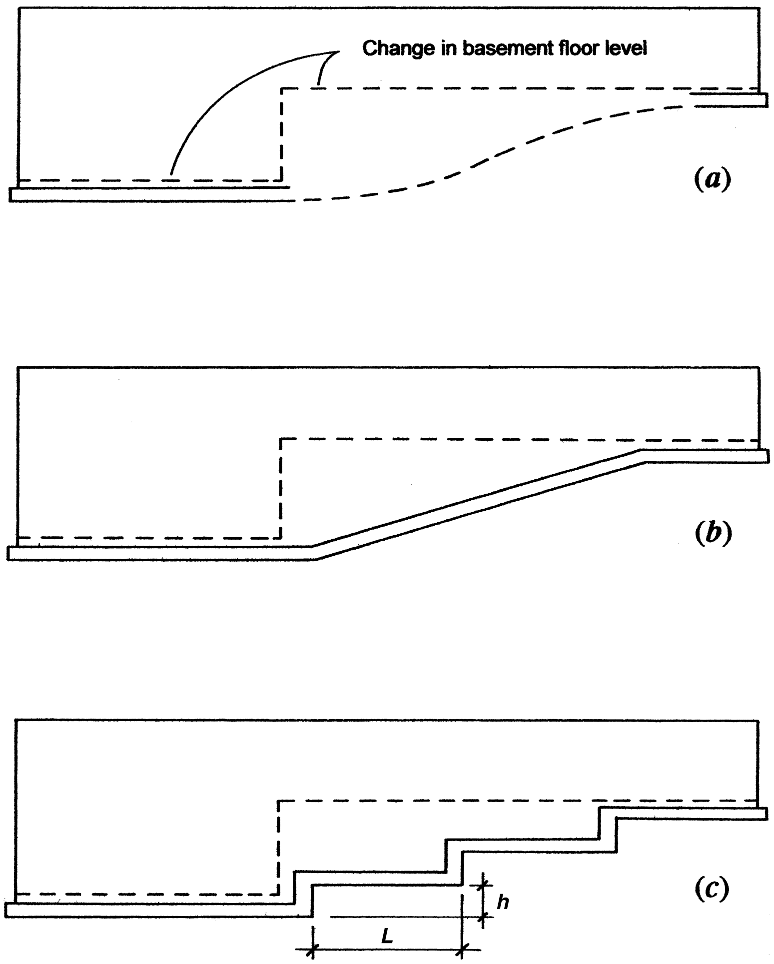


Figure 8.15 Change in footing elevation in a continuous foundation wall.

to deposit a thin layer of highly compacted material to replace the poor soil and to end up with a much improved bearing situation.

When the soil at the desired level is highly sensitive to disturbance by excavation or by foundation construction activities. This may be dealt with in a manner similar to the previous situation. If footings are small, the pressures developed some distance below the footings may be able to be sustained by the weaker materials.

When excavation of some buried object, such as a large tree root or boulder, leaves the usable level for bearing of a single footing considerably below that of adjacent footings. As in the previous cases, a small amount of compacted fill may be the best choice.

8.4 ELEMENTS OF FOUNDATION SYSTEMS

The complete foundation system for most buidings includes various elements in addition to the basic supporting objects (footings or deep foundations). This section treats various ordinary components of foundation systems that serve transitional functions between the building and the major foundation elements.

Foundation Walls

Foundation walls extend below the ground surface and are typically built of concrete or masonry due to the contact with soil and water. Their direct function is usually to support the exterior building walls, although support for floors and

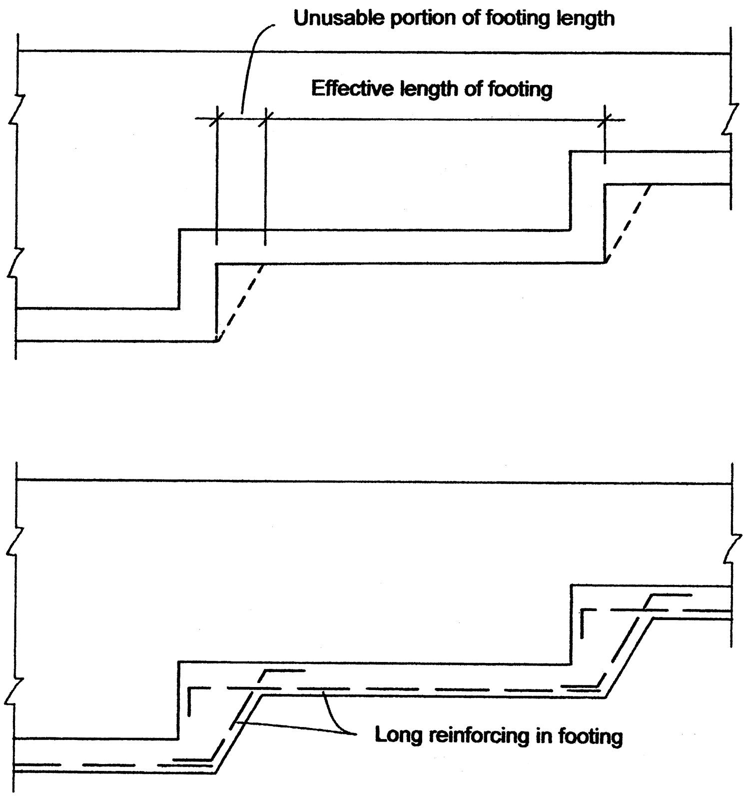


Figure 8.16 Considerations for a stepped footing.

roofs is also a factor. Structural and architectural functions of foundation walls vary considerably, depending on the type of foundation, the size (especially height) of the building, climate and soil conditions, and whether or not they are basement walls.

Figure 8.17*a* shows some typical situations for foundation walls in buildings without basements. These walls are not walls in the usual architectural sense. A major difference in this situation is whether the floor is a paving slab or a framed floor with some open space beneath it (called a crawl space). Another difference has to do with the distance of the footings below the ground surface. If this distance is great, the walls may be quite high. When the wall is very short and building loads are low, it is sometimes possible to use a construction described as a *grade beam*, which consists of combining the functions of foundation wall and wall footing into a single element that provides continuous support for the building construction.

When a basement is required, foundation walls will be quite high (see Figure 8.17*b*). An exception is the case of a half basement in which the basement floor is only a half story below ground and the wall portion above ground is of different construction. For multilevel basements, the foundation walls will be very heavy with wide footings required.

Lateral soil pressure must be dealt with for basement walls as well as the vertical loads due to gravity.

In addition to providing a ground-level edge for the building, walls often serve a variety of other functions; some of these are as follows:

Load Distributing or Equalizing. Walls of some length and height constitute stiff, beamlike elements. Their structural potential in these cases is sometimes utilized for load distributing or equalizing, as shown in Figure 8.18. A series of closely spaced, lightly loaded columns may be supported without column footings using the wall as a distributing element. Even the shallow-grade beam may be utilized in this manner—hence the derivation of its name.

Spanning as a Load-Carrying Beam. Walls may be used as a spanning member, carrying their own weight as well as some supported loads, as shown in Figure 8.18. This is often the case in buildings with deep foundations and a column structure. Deep-foundation elements are placed under the columns, and walls are used to span from column to column. This can also be the situation with bearing foundations when column footings are large; rather than bearing on its own narrow footing, the stiff walls tend to span between the larger column footings.

Distribution of Column Loads. When columns occur in the same plane as a foundation wall, many different relationships for the structural action of the walls, columns, and column foundations are possible. This issue is discussed in the following section.

Transfer of Building Lateral Loads to the Ground. There are many possible situations for the development of lateral resistive structural systems, but the total lateral load on the building must ultimately be transferred to the ground. The horizontal-force component is usually

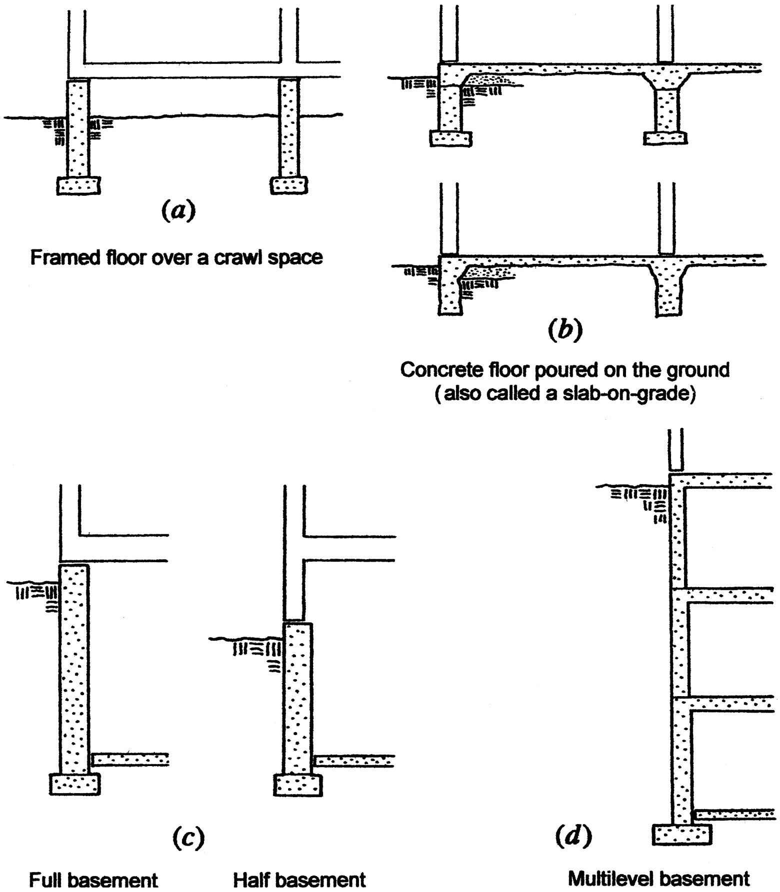


Figure 8.17 Foundation walls.

transferred through some combination of friction on the bottoms of the footings and the development of passive lateral soil pressure against the sides of the footings and foundation walls.

Ties, Struts, Collectors, And So On. Foundation walls may be used to push or pull horizontal forces between separate elements of the below-grade construction. For seismic design, it is required that the separate elements of the foundation system be adequately connected to permit them to act as a single mass; where they exist, foundation walls may help to serve this purpose.

Columns in Walls

A common problem in the design of building foundations is a foundation wall that shares its location with a row of building columns. This happens quite frequently along the exterior edge of buildings with framed structures. In the process of transferring loads to the ground there are various possibilities for the relationship between the columns, the wall, and the foundation elements. Some of the factors to consider in this situation are the following:

Magnitude of the Column Loads. It is one thing if the column loads are light, as in the case of a one-story building with light construction and short spans. It is quite another thing if the column loads are large, as in the case of a multistory building. By whatever means, heavy column loads will need to be carried directly to their supporting foundations.

Spacing of the Columns. If lightly loaded columns are closely spaced, the idea of using the wall as a spanning element will have more merit. If columns are quite far apart, it becomes less reasonable.

Height of the Wall. For spanning or load-distributing functions, the most critical dimension for the wall is its height. This relates to the relative efficiency of the wall as a beam and the classification of the wall as a spanning member. For the latter relationship, the spanning wall will fall into one of three categories of behavior as a function of the span-to-depth (height) ratio. As shown in Figure 8.19, these are:

An ordinary flexural member (beam) with beam shear, with flexure due to bending moment and with significant deflection, as they are usually considered to develop in a beam.

A deep beam with virtually no deflection due to flexure and with stiffness in proportion to its span that significantly affects the nature of distribution of stress and strain on a vertical section in the member.

As a member so stiff with respect to its span that there is essentially no flexure involved in its action.

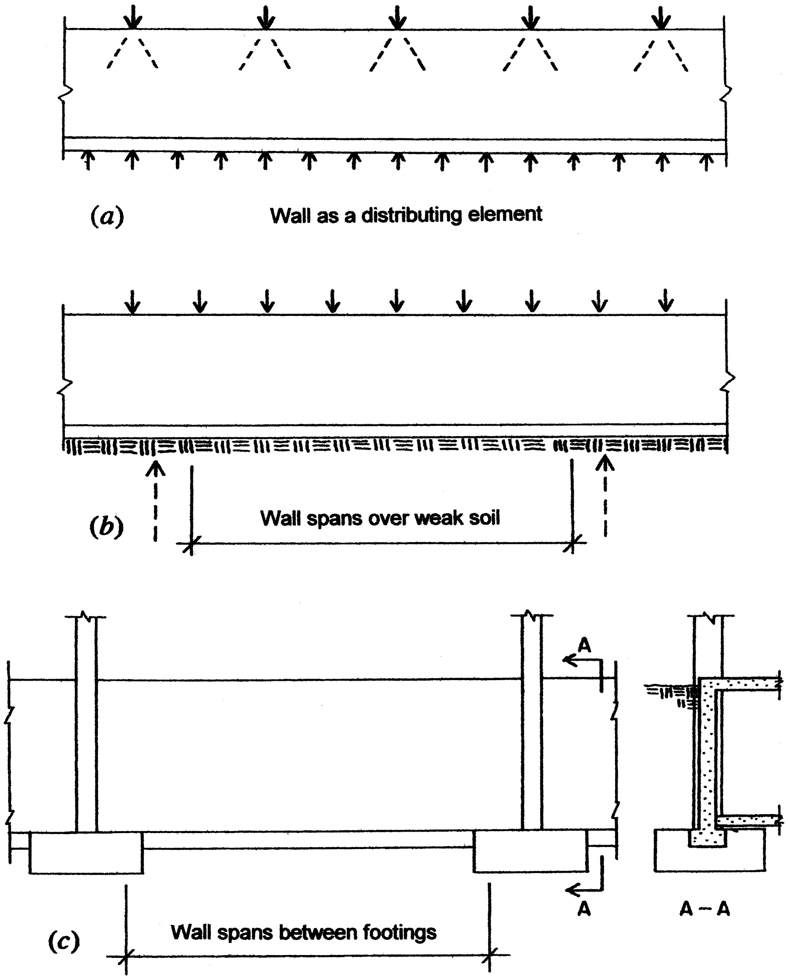


Figure 8.18 Spanning actions of foundation walls.

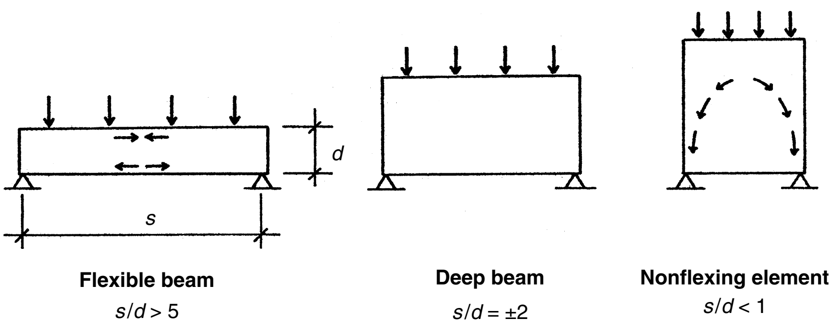


Figure 8.19 Effect of the span-to-depth ratio on the action of a spanning element.

Instead, it functions like a tied arch to bridge the space between supports.

The numerical values for the span-to-depth ratio as shown in Figure 8.19 are approximate limits for identifying these behavioral differences. As the wall changes in ratio, there is no sudden switch from one form of action to another, but rather there is a gradual shift.

Type of Foundation. If the foundation consists of deep elements—either excavated piers or groups of piles—the wall is most likely to be designed as a spanning element of one type or another. When the foundation consists of bearing elements, there may be several options for the column/wall/footing relationships.

Support Considerations

Objects that sit on foundations may be attached in a number of ways. As shown in Figure 8.20, the basic types of attachment are as follows:

Direct Bearing without Anchorage. Direct bearing is the usual case for unreinforced masonry construction. No uplift resistance and little lateral load transfer are possible with this attachment.

Dowelling of Reinforcing Bars. Dowels provided for vertical reinforcement in concrete or masonry construction

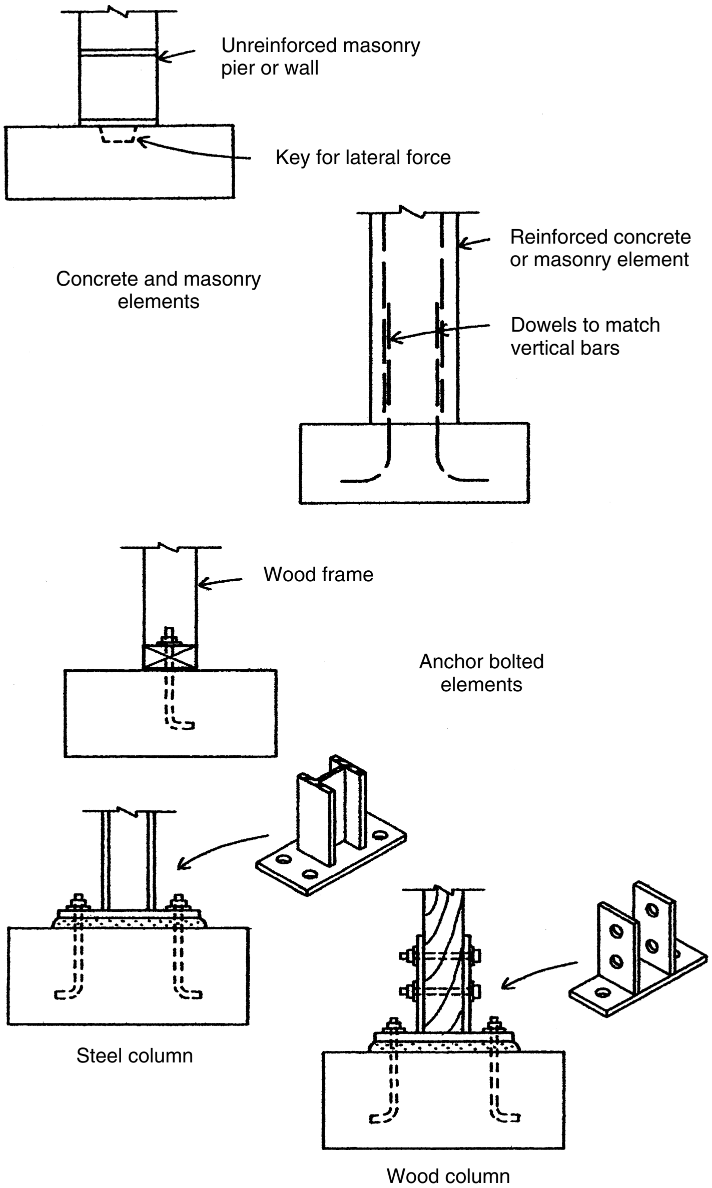


Figure 8.20 Common attachments to footings.

have the potential for providing uplift resistance and some lateral force resistance.

Anchor Bolts. Bolts are commonly used to attach elements of wood, metal, and precast concrete. In most cases the principal function of these bolts is simply to hold the supported elements in place during construction. However, as with dowels, there is a potential for development of resistance to uplift and lateral forces. For some forms of construction it may be possible to use drilled-in or pneumatically driven anchors for attachment, which eliminates the necessity for placing the cast-in anchor bolts during the pouring of the concrete for the footings.

Special Embedded Anchors. Embedded anchors may be patented devices or custom-designed elements for various purposes. A variety of elements are available for attachment of wood columns. One special attachment involves elements that provide for subsequent removal of supported elements. Another involves providing for future permanent attachment where addition to the structure is anticipated. Figure 8.21 shows a number of these types of attachment.

When a reinforced concrete column rests on a footing, it is usually necessary to develop the vertical compressive stress in the column reinforcement by doweling action into the supporting concrete construction. For reinforcing bars of large diameter and high yield stress, the distance required for this development can be considerable and may become a major consideration in determining required footing thicknesses.

A major construction detail problem with using anchor bolts and other embedded attachment devices is the accuracy of their placement. Foundation construction in general is typically quite crude and not capable of achieving high precision. It therefore becomes necessary to make some provision for this potential inaccuracy if elements to be attached

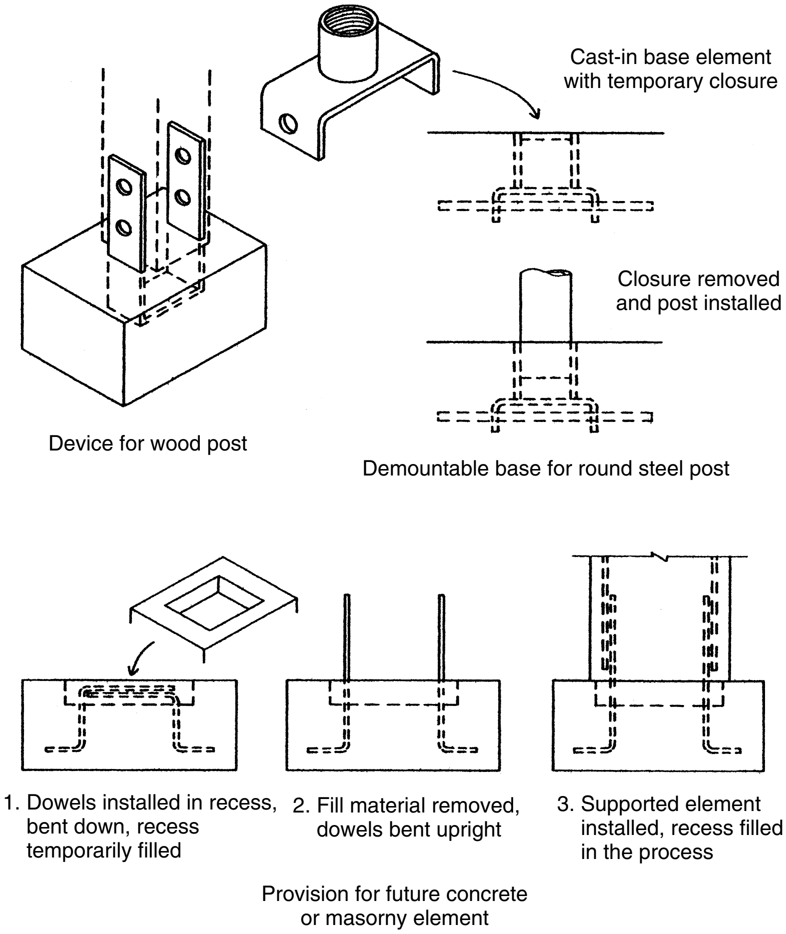


Figure 8.21 Special attachment devices for footings.

require relatively precise positioning. Leveling beds of grout under steel-bearing plates are used to provide accuracy of vertical dimensions in the attachment. Some tricks for compensating for horizontal adjustment are shown in Figure 8.22. It is, of course, sometimes possible to make adjustments to the supported elements to deal with this problem.

It is sometimes possible to use a transitional device that achieves a special form of attachment of supported construction. One use for such a device is that of keeping supported wood or metal construction free of contact with the ground. A common means of achieving this is with the use of a short concrete or masonry column on top of the footing, extended up to the point at which the supported construction can be attached.

The transitional column, usually called a *pedestal*, can also be used to achieve better dowelling of concrete column vertical bars. Rather than having to thicken the footing, a pedestal may be used of sufficient height for the dowelling of the column bars, with smaller bars completing the dowelling into the footing.

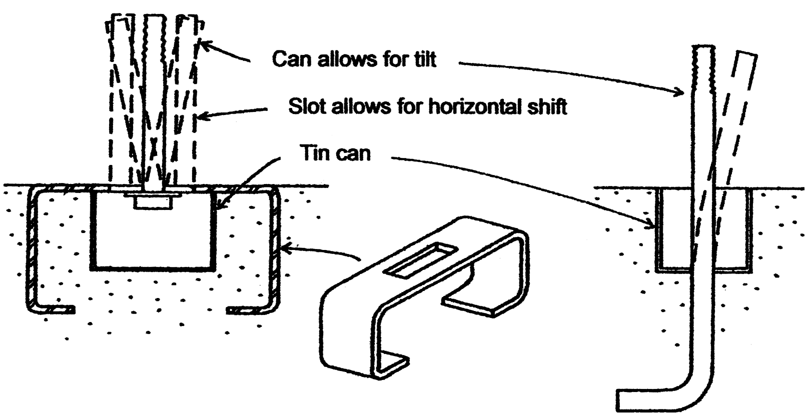


Figure 8.22 Means for anchor bolt attachment with adjustment.

8.5 DEEP FOUNDATIONS

In most cases deep foundations are utilized only where it is not possible to have shallow bearing foundations. The material in this section is provided to help the reader gain familiarity with the types of systems, their capabilities and limitations, and some of the problems involved in utilizing such systems for building foundations.

Need for Deep Foundations

The most common reasons for using deep foundations are the following:

Lack of Conditions That Favor Bearing Foundations. As shown in Figure 8.23, there are a number of conditions that may make it undesirable to place the usual bearing foundation elements near the bottom of the building. The deep foundation thus becomes a means for reaching a desirable bearing level at some distance from the bottom of the building.

Heavy Loads on the Foundations. In some cases the soil at upper levels may be sufficient for the use of footings for light loads but not for very high loads. High-rise buildings, long-span structures, and construction of massive elements of concrete or masonry are cases in which loads may become considerable and exceed the bearing capacity of ordinary soils.

Instability of Ground-Level Soil. In situations where ground-level soils are subject to erosion, subsidence, slippage, decomposition, or other forms of change in the soil structure, the strength and stability of deep foundations may be required. Hillside or waterfront locations may produce this action.

Buildings Sensitive to Settlement. Examples of this are buildings with stiff, rigid-frame structures and buildings housing precious, easily damaged objects or buildings with highly sensitive laboratory equipment. Bearing foundations will always have some settlement which is only approximately predictable in magnitude. Deep foundations tend to have very little settlement,

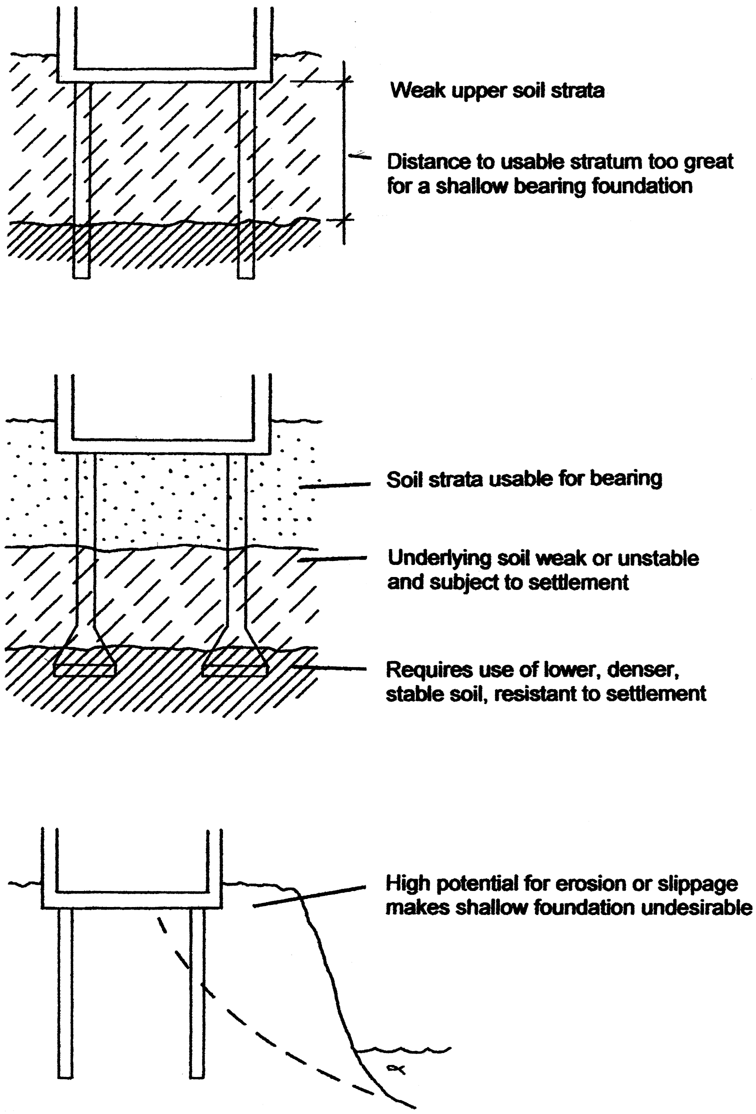


Figure 8.23 Situations requiring deep foundations.

especially when elements are carried down to rock or highly consolidated materials.

As stated previously, deep foundations are usually used only in situations in which shallow footings are not acceptable. The primary reason for this is cost. Where footings cannot be used, this cost may be borne, but price competition between the two methods of foundation construction is nonexistent.

A problem with deep foundations is the local availability of the contractors and equipment. Very heavy equipment must usually be used, so moving it to remote locations may be a significant cost factor.

Types of Deep Foundations

As shown in Figure 8.24, the common types of deep foundations are the following:

Friction Piles. These consist of shafts of timber, steel, or precast concrete that are forcibly inserted into the soil. Load-carrying capacity is developed by resistance to further penetration, its specific magnitude being measured by the actual effort to advance the pile.

End-Bearing Piles. These are elements similar to those used for friction piles, although in this case they are driven a specific distance to lodge their ends in some highly resistive soil stratum or in rock. While considerable skin friction may be developed during the advancement of the pile, the major load capacity is developed at the point of the pile.

Piers or Caissons. For various reasons it may be better to place an end-bearing element by excavating the soil down to the level at which bearing is desired and then backfilling the excavation with concrete. An advantage of this technique is that the material encountered at the bottom of the shaft can be tested before the concrete is poured.

Belled Piers. When piers bear on rock, they usually have an end-bearing capacity approximately equal to that of the concrete shaft in column action. When they bear on soil, however, they are usually enlarged at their ends in order to increase the bearing area. The usual conical form of this end yields the term *belled* to describe such an element.

Piles

Piles are generally driven in clustered groups. One reason for this is their limited individual capacity; another is the problem of precisely controlling their locations during the driving process. Even when supported loads are small, it is generally not feasible to place a concentrated column load on top of a single pile. The preferred minimum group for a concentrated load is three piles.

In order to transfer the load from a column to a group of piles, it is necessary to use a reinforced cap on top of the pile group. This cap functions very much like a bearing footing. When placed in groups, piles are ordinarily driven as close together as possible, primarily to reduce the bending in the pile cap. Pile layouts typically follow classical patterns, based on the number of piles in the group. Typical layouts are shown in Figure 8.25. Special layouts may be used for groups carrying bearing walls, shear walls, elevator towers, combined foundations for closely spaced columns, or other special situations.

If one of the piles is slightly mislocated during driving, it may be possible to relocate the rest of the piles in the group to compensate for this. The objective is to have an alignment of the column load and the centroid of the pile group.

With piles in a tight grouping, the driving of each pile generally causes some lateral movement of previously driven adjacent piles. If substantial movements occur, it may be necessary to add some piles to obtain a group that is successfully aligned with the column.

For both piles and piers limiting soil conditions are required. Driving piles where large boulders are present is usually not feasible. For piers, the excavation will be more successful with easily excavated soil materials.

Timber piles consist of straight, tapered tree trunks, similar to those used for utility poles, driven with the small

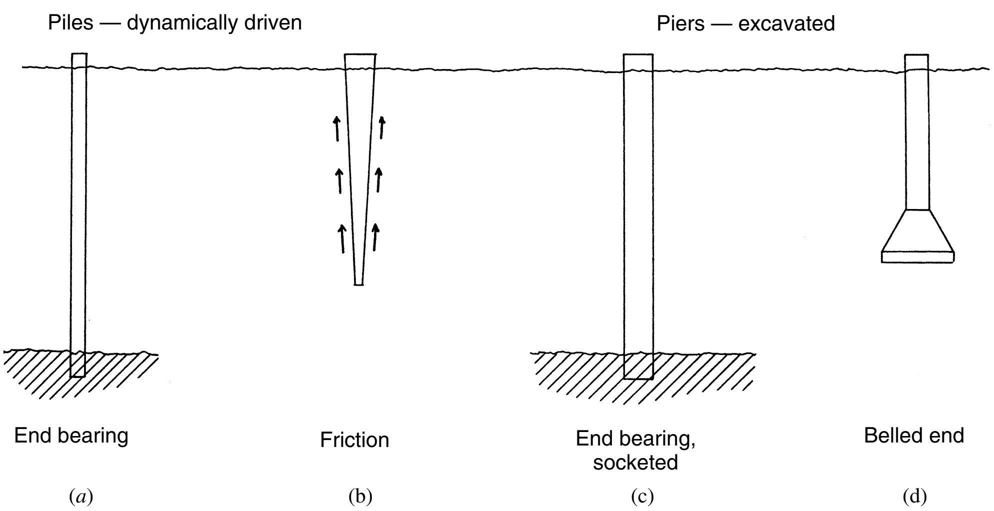


Figure 8.24 Types of deep foundations.

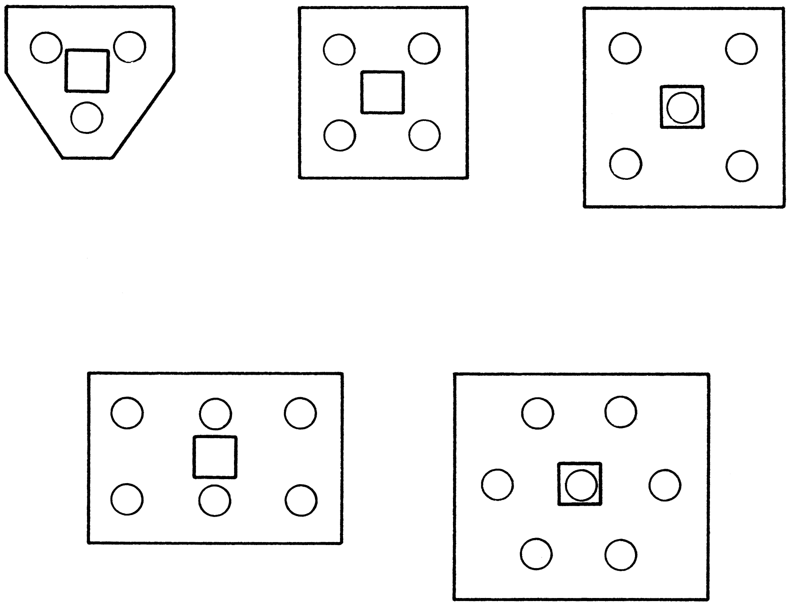


Figure 8.25 Typical pile group arrangements and cap forms.

end down, primarily as friction piles. Their length is limited to that obtainable from the species of tree available. The maximum driving force, and consequently the usable load, is limited by the problem of shattering of the driven end. Decay of the wood is a problem, especially where the tops of the piles are above the ground water line. Treatment of the wood is common to reduce decay and insect infestation, although, where they are available, wood species with natural resistance to these effects are used. For driving through difficult soils or to end bearing, wood piles are sometimes fitted with steel points.

Cast-in-place concrete piles are created by driving a steel shell and filling it with concrete. Various systems exist, most of which are proprietary products developed for particular companies that specialize in geotechnical services.

Precast concrete piles are frequently used at waterfront locations or within bodies of water. Some of the largest pile-form structures are produced in this manner using hollow, cylindrical precast units.

Steel pipes and H-shaped rolled shapes are widely used for piles. These may be used for piles of considerable length, with several pieces welded together end to end. Although the steel elements are quite expensive, load capacities are quite high, so the overall cost of the construction is usually competitive.

Piers

Piers can be produced in a large range of sizes. Large-diameter piers are hand dug with the shaft walls lined and braced as the excavation proceeds. Once the pier excavation is complete, the lining is removed as the concrete is deposited. Massive piers for tall buildings have been produced in this manner.

When loads are relatively light, the most common form of pier is the drilled-in pier, consisting of a vertical round shaft and a bell-shaped bottom, as shown in Figure 8.26. When soil conditions permit, the pier shaft is excavated with a large auger-type drill, similar to that used for postholes and water wells. When the shaft has reached the desired depth, the auger is withdrawn and an expansion element is inserted to achieve the form of the belled bottom. This form of foundation is usually used only when a usable bearing soil strata can be reached with a minimum-length pier.

Placing of piers generally permits more precise control over the location of the shaft than that possible with piles. For this reason piers are usually not placed in clusters, except for very large foundations for towers or other large structures. Because of the potential for greater accuracy of placement, single piers are frequently used for individual columns.

Although caps are not basically required for single piers, they are frequently used. Caps may be used to eliminate the need for high accuracy of placement of anchor bolts or column-reinforcing dowels at the time of the relatively rough foundation construction work. Caps may also be used like footing pedestals to achieve a transition from a column of high-strength concrete to a pier which usually has relatively low strength concrete.

Lateral, Uplift, and Moment Effects on Deep Doundations

Resistance to horizontal forces, to vertically directed upward forces, and to bending moments presents special concerns for deep foundation elements. Whereas a bearing footing has no potential for the development of tension between the supported structure and the ground, both piles and piers have considerable potential for uplift resistance. On the other hand, the sliding friction that constitutes a major resistance to horizontal force by a bearing footing is absent with deep foundation elements. The following discussion deals with some problems of designing deep foundations for force effects other than the primary one of vertically directed downward load:

Lateral Force Resistance. Resistance to horizontal force at the top of both piles and piers is very poor in

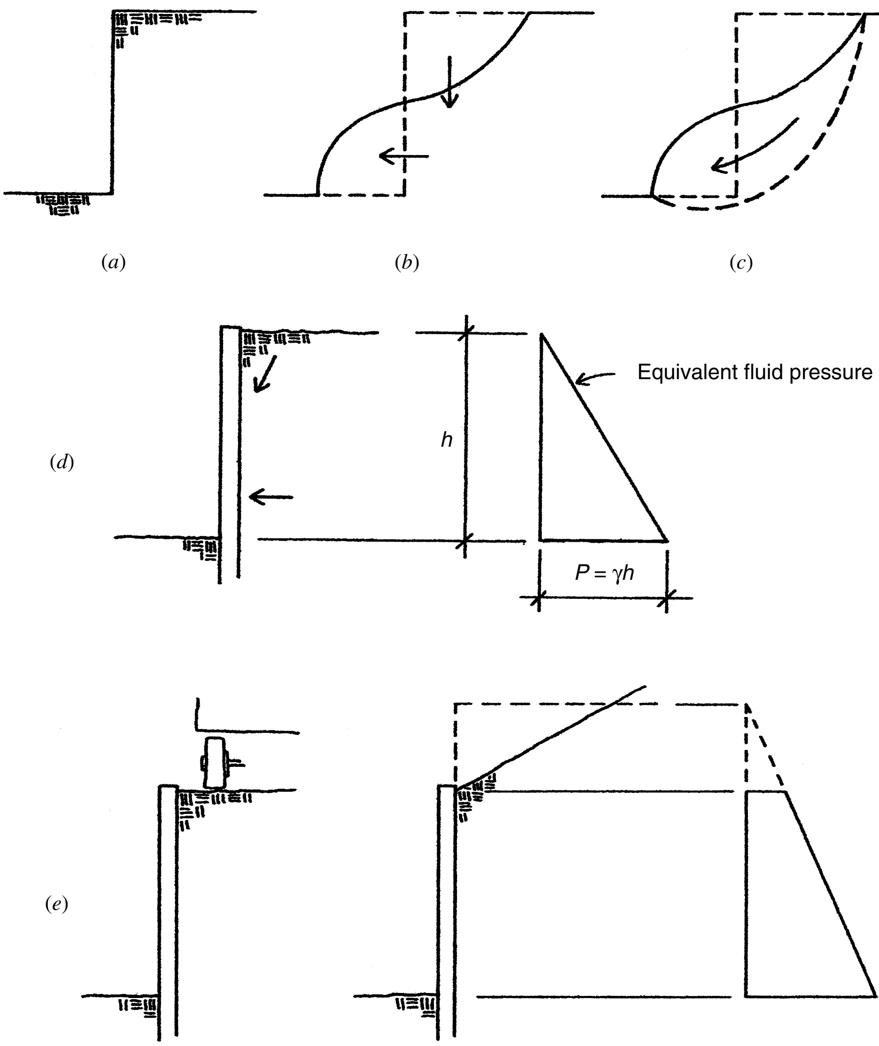


Figure 8.26 Development of active soil pressure.

most cases. The relatively narrow profile offers little contact surface for the development of passive soil pressure. In addition, the process of installation generally causes considerable disturbance of the soil around the top of the deep foundation elements. The usual method for resolving lateral forces is to transfer them to other parts of the building foundation system—primarily to grade or foundation walls that offer surfaces perpendicular to the direction of the horizontal forces. For large freestanding structures without grade walls, the horizontal-force resistance of pile groups is usually developed with some piles driven at an angle. These *battered* (tilted) piles are capable of some horizontal resistance.

Uplift Resistance. Friction piles and large piers have considerable resistance to upward forces. An exception is the pile with a tapered form, which is formed basically for resistance to downward forces. The combined weight of the shaft and bell of a drilled pier offers a considerable potential for resistance of upward force. In addition, if the bell is large, considerable soil pressure is developed on the bell top surface to resist upward movement of the bell. The shaft of the pier must be reinforced for the tension developed in these actions, although an alternate method is to bury a large steel plate at the bottom of the bell and attach it to a steel tension rod or cable encased in the pier shaft. Of course, large-diameter excavated piers of considerable height will have a significant dead weight to resist upward forces.

Moment Resistance. Piles and piers are seldom deliberately designed to develop bending moments. Although any element strong enough to function as a driven pile or a pier will inevitably possess some bending strength, it is generally desirable to design for an ideal condition of axial compression force only. However, the unavoidable inaccuracies inherent in the construction make it unlikely that perfect alignment of the pile or pier with a supported column will occur. Thus the true evaluation of any structural element that is primarily intended to carry axial load usually includes some consideration of the possibility of accidental moments produced by the eccentricity of the load.

The larger the diameter of a pier, the larger the eccentricity that can be tolerated with a significant load magnitude. If an eccentricity within the tolerable limit for a pier can be assured, there is some justification for use of a single pier for an individual concentrated load, such as that from a single column.

Piles, however, have less tolerance for error of alignment and are also much less subject to control of location from the driving actions. This is why the three-pile group is considered to be a minimum for a single point load.

8.6 SPECIAL PROBLEMS AND CONSTRUCTION

Horizontal Forces in Soils

There are a number of situations involving horizontal forces in soils. The three major ones of concern for foundations and site structures are the following:

Active Soil Pressure. Active soil pressure originates with the soil mass; that is, it is pressure exerted *by* the soil on something, such as a retaining wall or the outside surface of a basement wall.

Passive Soil Pressure. Passive soil pressure is exerted *on* the soil, for example, that developed on the side of a footing when horizontal forces push on the footing.

Friction. Friction is the sliding effect developed between the soil and the surface of some object in contact with the soil. To develop friction there must be some compressive pressure between the soil and the contact face of the object.

The development of all of these effects involves a number of different stress mechanisms and structural behaviors in soils.

Active Soil Pressure

The nature of active soil pressure can be visualized by considering the situation of an unrestrained vertical cut in a soil mass, as shown in Figure 8.26*a*. In most soils, such a cut will not stand for long. Under the action of various influences, primarily gravity, the soil mass at the cut face will tend to move to a form as shown in Figure 8.26*b*. There are two forces involved in this change. The soil near the top of the cut will drop due to gravity; the soil near the bottom of the cut will be squeezed and will bulge outward Another way to visualize this movement is in terms of a rotational effect that results in a slip plane along the heavy dashed line in Figure 8.26*c*. The slip plane failure may be avoided if the cut is made in a sloping form, rather than as a vertical form.

If a restraining structure is placed at the vertical cut face, the forces exerted on it will tend to be those that work to cause the slip plane failure. As shown in Figure 8.27*d*, the soil mass near the top of the cut will have combined vertical and horizontal effects. The horizontal component of this action will be minor, since the soil mass at this location will tend to move primarily downward. The soil mass near the bottom of the cut will exert primarily a horizontal force, very similar to that developed by a liquid in a tank. In fact, a common approach to design for this situation is to assume the soil to act as a fluid with a unit density of some percentage of the soil density and to consider a horizontal pressure that varies with the height of the cut, as shown in Figure 8.26*d*.

This simplified equivalent fluid pressure assumption is in general most valid when the retained mass is a well-drained, predominantly sandy soil. Building up of the water content in the retained soil will significantly increase the lateral pressure. In most cases—especially for basement walls—measures will be taken to assure good drainage of water in the retained soil mass.

In some situations it may be necessary to provide for a so-called *surcharge pressure* at the top of the restraining structure. As shown in Figure 8.26*e*, the two common ways that this occurs are when a direct gravity load is applied or the ground level behind the wall is sloped up at a reasonably steep pitch. When handled as fluid pressure, the surcharge is sometimes visualized as an increase in the assumed density of the equivalent fluid or as an addition of a certain height of the soil mass above the top of the retaining structure.

Passive Soil Pressure

Passive soil pressure is visualized by considering the effect of pushing an object through the soil mass. If this is done in relation to a vertical cut, as shown in Figure 8.27*a*, the soil mass will tend to move inward and upward, causing a bulging of the ground surface behind the cut. If the slip plane type of movement of the soil mass is assumed, the action is similar to that of active soil pressure, with the direction of the soil forces simply reversed. Since the gravity load of the upper soil mass is a useful force in this case, passive soil resistance will generally exceed active pressure for the same conditions.

If the analogy is made to the equivalent fluid pressure, the magnitude of the passive pressure is assumed to vary with depth below the ground surface. Thus, for structures whose tops are at ground level, the pressure variation is the usual simple triangular form as shown in the left-hand illustration in Figure 8.27*b*. If the structure is buried below the ground surface, as is the typical case with footings, the surcharge effect is assumed and the passive pressures are correspondingly increased.

As with active soil pressure, the type of soil and water content will have some bearing on development of passive pressure. This is usually accounted for by use of recommended values for specific soils to be used in the equivalent fluid pressure analysis.

Soil Friction

The potential force that is developed as friction to resist the horizontal sliding of an object resting on soil, such as the

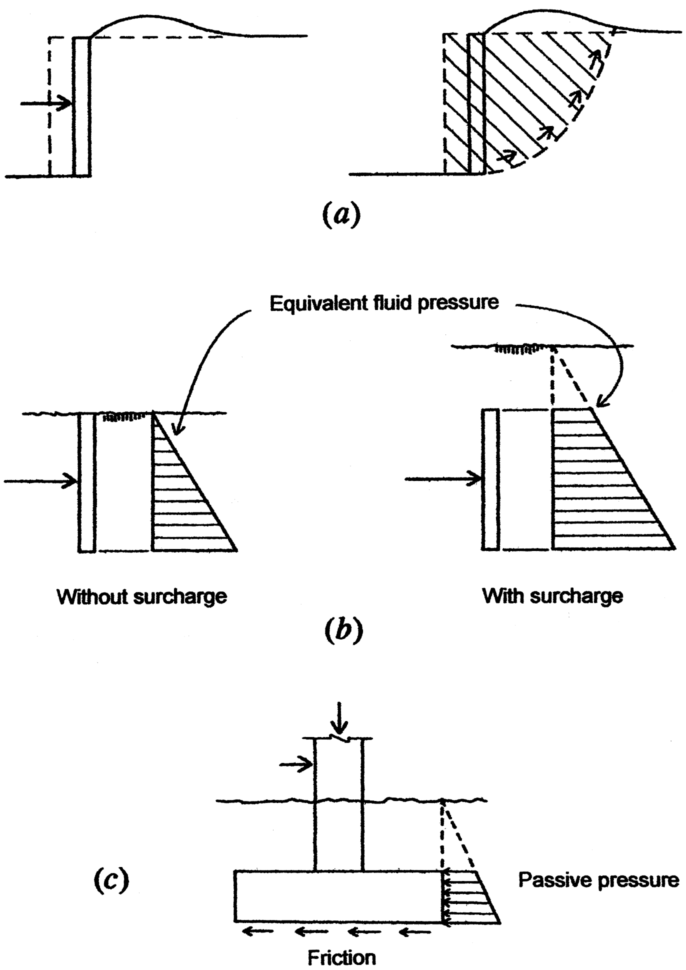


Figure 8.27 Development of passive soil pressure.

bottom of a footing as shown in Figure 8.27*c*, depends on a number of factors, including the following principal ones:

Form of the Contact Surface. If a smooth object is placed on the soil, there may be little resistance to its tendency to slide. However, for footings, the usual concern is for a surface formed by the direct pouring of concrete onto an excavated soil surface which tends to be quite rough and capable of developing a bonding action.

Type of Soil. The grain size, grain shape, density, and water content of the soil are all factors that will affect the development of soil friction. Well-graded, dense, angular sands and gravels will develop considerable friction. Loose, rounded, saturated, fine sand and soft clays will have relatively low friction resistance. For sand and gravel, the potential friction stress will be reasonably proportional to the compressive stress on the contact surface. For clays, the friction tends to be independent of the contact pressure, except for the minimum required to develop any friction.

Friction seldom acts alone as a horizontal resistive force. Bearing foundations are ordinarily buried with their bottoms some distance below the ground surface. Thus pushing the foundation horizontally will also result in the development of passive soil pressure, as shown in Figure 8.27*c*. Since friction and passive pressure are two totally different stress mechanisms, they will actually not develop simultaneously. Nevertheless, the usual practice is to simply assume that both forces combine to oppose the horizontal movement of the foundation.

In situations where simple sliding friction is not reliable or the resistance offered by friction and passive pressure is not adequate, a device called a *shear key* is sometimes used. This device is often used with retaining walls and is discussed later in this chapter.

Abutments

The support of some types of structures, such as arches, gables, and rigid frames, often requires the resolution of both horizontal and vertical forces. When this resolution is accomplished entirely by the supporting foundation elements, the element is described as an *abutment.* Figure 8.28*a* shows an abutment for an arch consisting of a rectangular footing and an inclined pier. The design of such a foundation has three primary concerns as follows:

Resolution of the Vertical Force. This consists of assuring that the vertical soil pressure does not exceed the maximum allowable value for the soil.

Resolution of the Horizontal Force. This consists of the development of an adequate combination of soil friction and passive soil pressure.

Resolution of the Moment Effect. The horizontal-force component at the top of the abutment has the potential for developing an overturning (toppling) effect on the foundation. The objective in determining the forms of the abutment and its footing is to resolve this effect into a vertical soil pressure that is uniformly distributed on the bottom of the footing. This is accomplished if the resultant of the forces at the bottom of the footing coincides with the centroid of the footing plan shape.

Figure 8.28*b* shows the various forces that act on the abutment shown in Figure 8.28*a*. The active forces consist of the load and the weights of the pier, the footing, and the soil mass above the footing. The reactive forces consist of the vertical soil pressure, the friction at the bottom of the footing, and the passive horizontal soil pressure on the end of the footing. The dashed line indicates the path of the resultant of the active forces. The condition shown is the ideal one, with the resultant at the bottom of the footing coinciding with the centroid of the footing bottom plan shape.

If the pier is tall and the load is large with respect to the pier weight or is inclined at a considerable angle from the vertical, it may be necessary to locate the footing centroid at a considerable distance horizontally from the load point at the top of the pier. This could result in a rectangular footing of considerable length. One method that is sometimes used to avoid this is to use a T shape, or other form, that results in a more favorable location of the footing centroid. Figure 8.28*c* shows the use of a T-shape footing for such a condition.

When the structure being supported is symmetrical, such as an arch with its supports at the same elevation, it may be possible to resolve the horizontal-force component at the support without relying on soil stresses. One method for accomplishing this is to tie the two opposite supports together, as shown in Figure 8.29*a*, so that the horizontal force is resolved internally (within the supported structure) instead of externally (by the ground). If the tie is attached at the point of contact between the structure and the pier, as shown in Figure 8.30*a*, the net load delivered to the pier is simply a vertical force, and the pier and footing can be developed for this limited action.

For practical reasons, it is often necessary to locate the tie, if one is used, below the support point for the structure. If the support point is above ground, as it usually is, the existence of the tie above ground is quite likely to interfere with the use of the structure. A possible solution to this problem is to move the tie down to the pier, as shown in Figure 8.29*b*. In this case the pier weight is added to the load to find the proper location for the footing centroid.

When the footing centroid must be moved a considerable distance horizontally from the load point, it is sometimes possible to add another element to the abutment system.

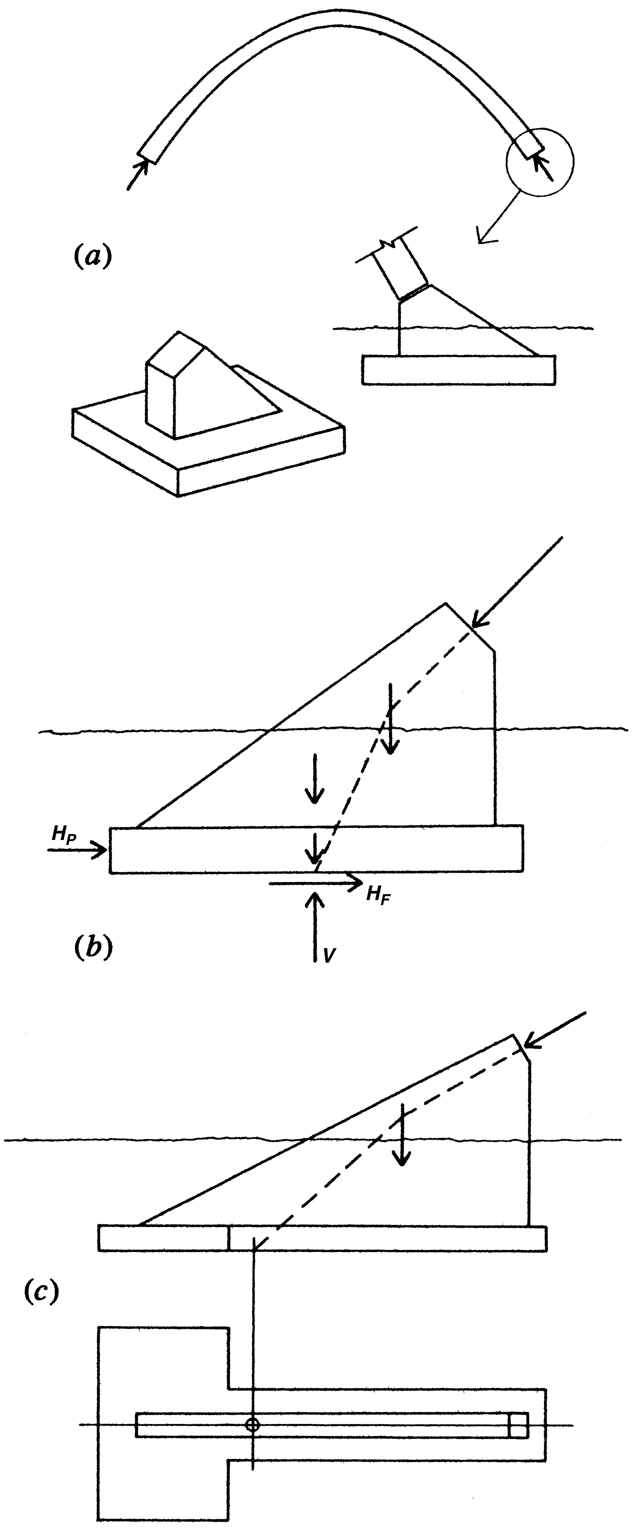


Figure 8.28 Abutments for arches.

Figure 8.29*c* shows a structure in which a large grade beam has been added between the pier and the footing. The main purpose of this element is to develop the large shear and bending resistance required by the long cantilever distance between the end of the pier and the end of the footing. In this example the tie is attached to the grade beam, so that the

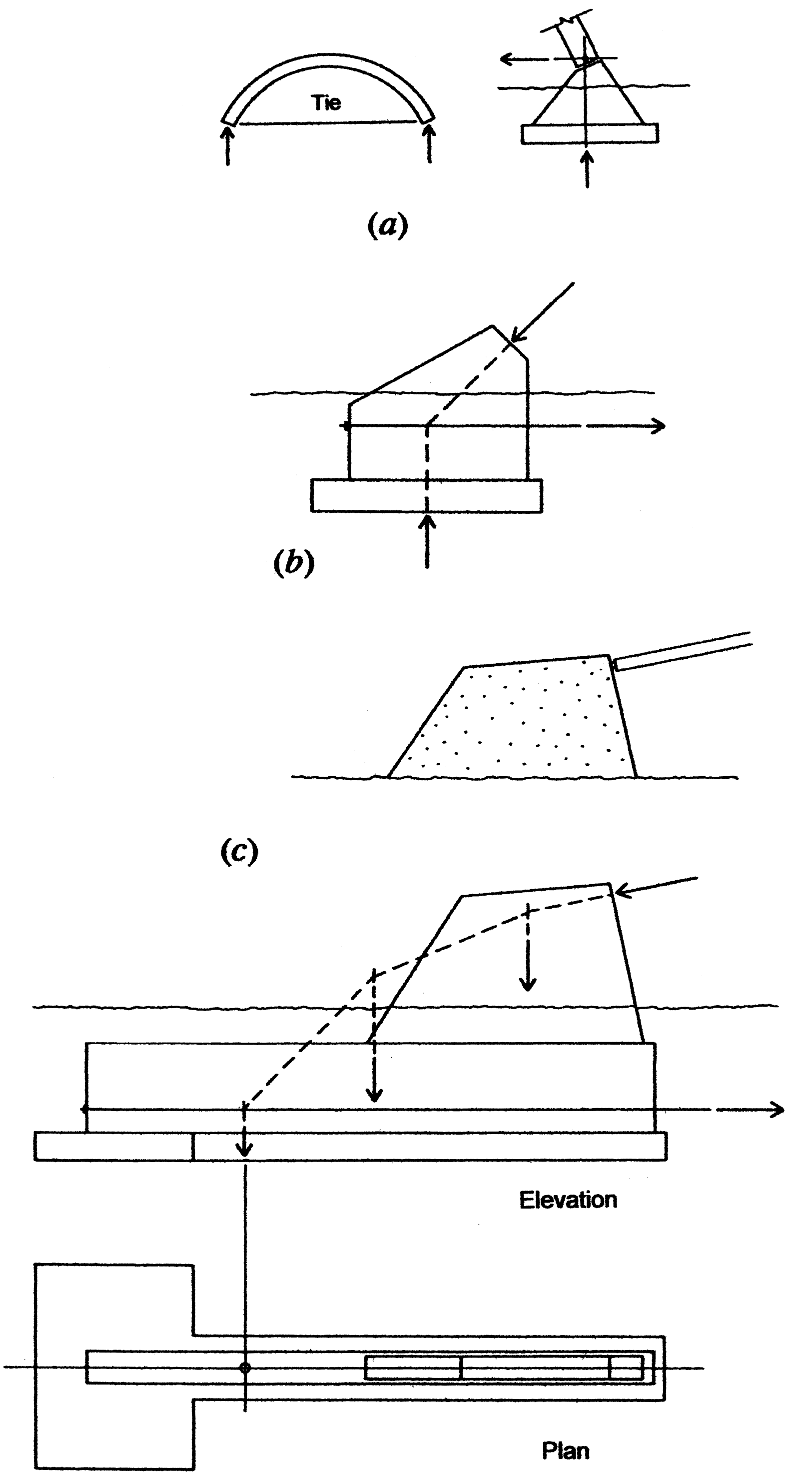


Figure 8.29 Abutments for tied arches.

weights of the pier and the grade beam are added to the load to find the location for the footing centroid. The T-shape footing is then determined so as to place its centroid at this location.

Unless they are resisted by other parts of the above-ground structure and the complete foundation system, the abutments may also need to resist the lateral loads due to wind or earthquakes. There are many variations of this situation that may affect the design of the abutments. In some cases, it may be possible to develop the abutments to resolve the combined gravity and lateral loads.

For buildings with basements, the abutment functions may be incorporated into the general subgrade construction development. Heavy concrete basement walls may serve abutment functions and also serve to resolve the building's lateral loads.

Paving Slabs

Concrete pavements are used for just about every building. Sidewalks, terraces, driveways, and parking lots are often extensive on the building site, outside the enclosed building. Inside the building, the first, lowest level, floor is likely to be a concrete slab on the ground; this may be a basement floor or, when no basement occurs, simply the floor of the lowest building level. Many different top-surface finishes may be created, depending on the function of the pavement and on whether the raw concrete is used or an applied finish is added.

Figure 8.30 shows the common form of construction for concrete paving slabs cast directly on the ground surface. While the basic construction process is simple, a number of factors must be considered in developing the details and specifications for a paving slab.

Thickness of the Slab

Pavings vary in thickness from a few inches (for most building floors) to a few feet (for airport landing strips). Although more strength is implied by a thicker slab, thickness alone does not guarantee a strong pavement. Of equal concern are the reinforcement and the integrity of the soil base on which the concrete is cast.

Minimum thickness commonly used for sidewalks and building floors is 3.5 in., derived partly from the ease of forming edges with wood 2 by 4s (real dimensions being 1.5×3.5 in.). Following the same logic, the next size jump would be to 5.5 in., formed with 2 by 6s. However, any thickness considered appropriate can be created.

Heavily loaded floors and exterior pavements subject to wheel loads should be thicker than the minimum 3.5 in. Some building walls, not heavily loaded for bearing, may be supported directly on the paving slab. However, columns and heavily loaded walls will need their own separate footings.

Reinforcement

Thin slabs are ordinarily reinforced with welded steel wire mesh, which is generally considered to provide only for temperature and shrinkage stresses. For thicker, heavily loaded slabs, crisscrossed reinforcing bars are used, as in

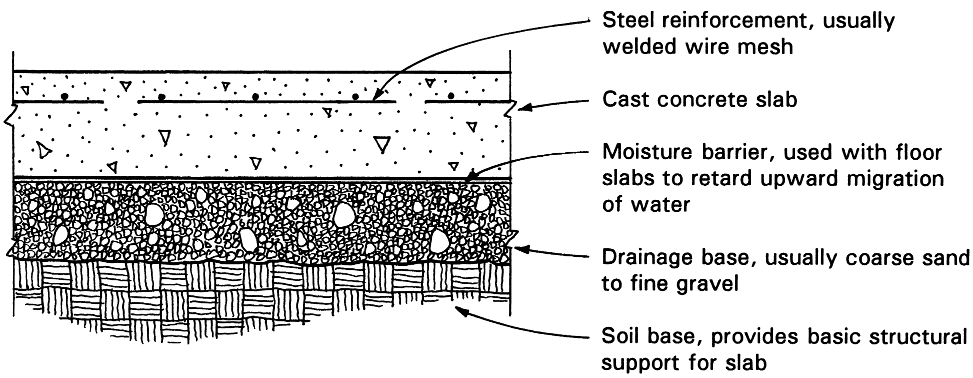


Figure 8.30 Basic form of construction for concrete slabs on ground.

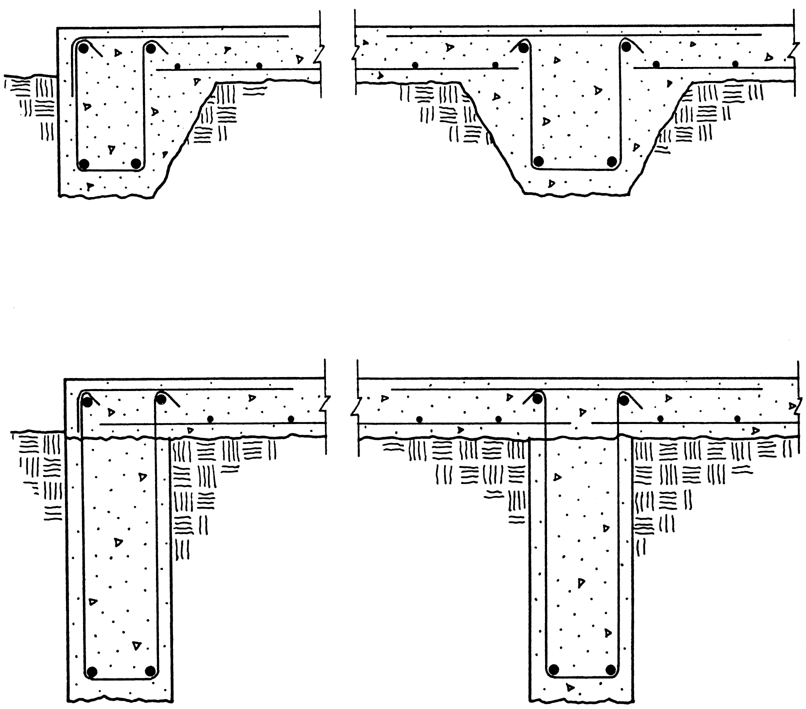


Figure 8.31 Concrete framed floors on grade.

spanning slabs. Reinforcement is usually positioned near the top of the slab, as cracking of the top surface is a greater concern.

A recent development is the use of fibrous materials added to the concrete mix, which results in a finished concrete with enhanced tension resistance.

Joints

Paving slabs are usually poured in relatively small units, the main purpose being to reduce cracking due to shrinkage, temperature change, and bending due to uneven settlement of the soil base. Joints may be formed as edges, with the cast unit allowed to harden and shrink before the next unit is cast. In some cases larger units may be cast with control joints created by tooling of the wet cast concrete or by shallow sawing of the hardened concrete. When it is critical to prevent separate settlement of the cast units (creating a bump), keys may be formed or steel dowels inserted in the cast edges.

Drainage Base

The ideal drainage base for floor slabs is a well-graded soil, ranging from fine gravel to coarse sand with a minimum of fine materials. This material can usually be compacted to a reasonable density to provide a good structural support. If the actual excavated soil base is quite dense and has considerable fine-grained materials, a layer of coarse-grained

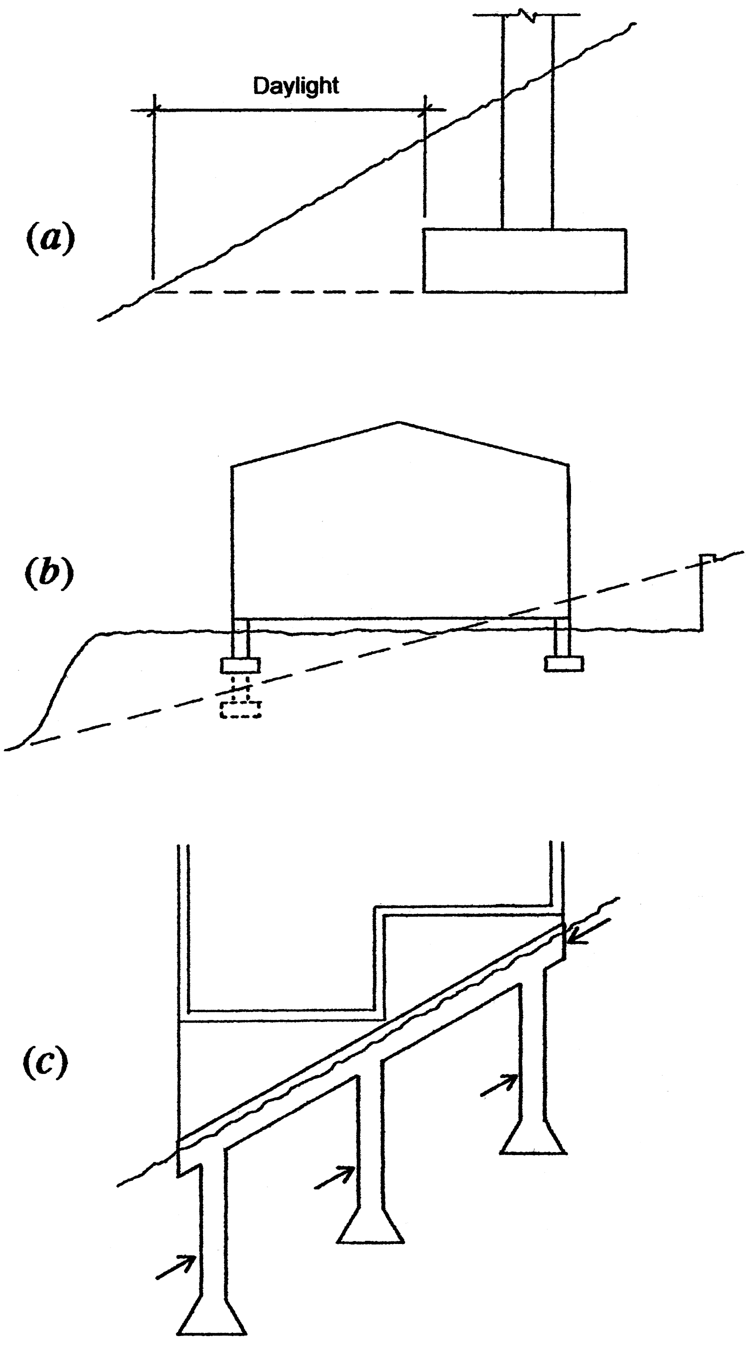


Figure 8.32 Hillside foundations.

material provides for drainage under the slab. If ground moisture is a problem, a layer of water-resistive membrane may be used between the drainage base and the concrete to inhibit the upward movement of moisture. The membrane will also reduce the effect of "bleeding," which is a tendency for the water and cement to be leeched out into the base during casting. Moisture will freely travel both upward and downward through outdoor slabs but should be prevented as much as possible for indoor slabs.

Soil Base

The excavated surface of the soil base provides the basic structural support for the paving slab. This should be a well-compacted soil, reasonably free of organic materials, and generally in a stable condition with minimal expectation for settlement.

Framed Floors on Grade

It is sometimes necessary to provide a concrete floor slab on the ground in a situation where a simple paving slab is not sufficient. The usual reason for this is the anticipation of settlement of the slab. Figure 8.31 illustrates two techniques that may be used to produce a slab-and-beam framed structure cast on the ground. Where spans are short and beam sizes not excessive, the beams may be formed simply by trenching, with the slab-and-beam system cast as one, as shown in the upper illustration in Figure 8.31. When larger beams are required, the beam stems may be formed and cast first, with the slab poured on top of fill placed between the beam stems, as shown in the lower illustration in Figure 8.31.

Hillside Foundations

Figure 8.32*a* shows the situation of a single footing in a hillside situation. The dimension labeled *daylight* in the figure must be sufficient to prevent pushing of the soil under the footing out into the face of the slope. Building codes sometimes require a minimum distance for this dimension. Obviously the type of soil and the angle of slope of the hill will affect the choice for the dimension as well as the magnitude of the load on the footing.

A common problem with hillsides is shown in Figure 8.32*b*, where the recontouring of the site places some footings on undisturbed soil and others on fill. This is not a good situation for equalized settlements, so the footings on fill should be lowered to also bear on undisturbed soil.

A special structure sometimes used for hillside and beach locations is shown in Figure 8.32*c*. This consists of a set of deep foundation elements—usually piles or drilled piers—that are embedded in a lower soil strata and form vertical cantilevers to hold the supported structure from sliding down the hill. A grade-level frame serves to tie the separate foundation elements together. The grade-level frame is cast with the deep elements to form a rigid frame. This system is described as a *downhill frame* and it is frequently used on hills and beaches where erosion of the site materials is highly likely.

The daylight dimension and the downhill frame are also important to resolution of lateral forces on the supported structures, especially those due to earthquakes. There are limits to their capabilities, however, and if mass sliding of the hillside or mass erosion of the beach occurs, the downhill frame may go with the soil movement.

Soil-Retaining Structures

Abrupt changes in the elevation of the ground surface usually require some form of structure to retain the soil on the high side of the cut. The critical dimension for this structure is the difference in elevation on the two sides of the cut. A variety of structures can be used for this purpose, including the rock piles described in Section 7.4. Also available are various systems using precast concrete units that interlock to produce a steeply banked surface. However, the two structures most often used are curbs and cantilever retaining walls.

Curbs

These are the shortest freestanding retaining structures. Two commonly used forms are shown in Figure 8.33*a*. The L-shaped curb is used to create a drainage gutter at the low side of the curb. Use of these structures is usually limited to grade-level changes of 2 ft or less.

Short Retaining Walls

Walls up to about 10 ft in height are usually built as shown in Figure 8.33*b*. These consist of a wall of masonry or concrete attached to a concrete footing. The wall thickness, footing width and thickness, vertical wall reinforcement, and transverse footing reinforcement are all designed for the lateral-load-induced moments plus the weight of the wall, footing, and soil fill behind the wall. When the bottom of the

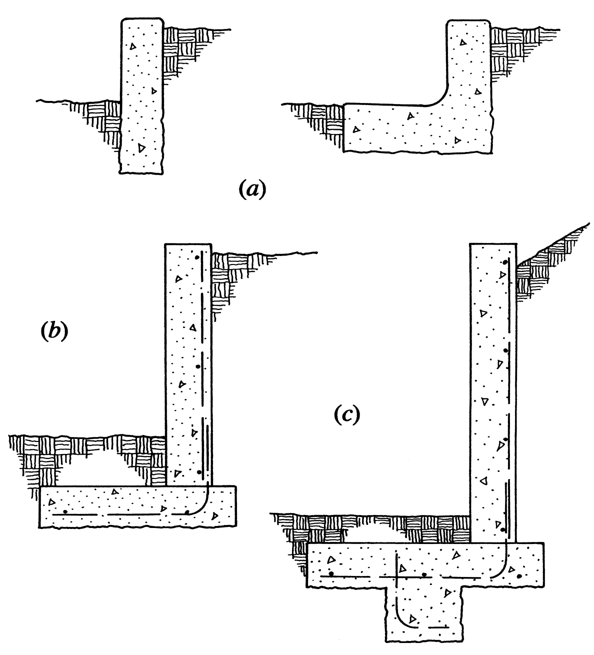


Figure 8.33 Typical forms for short retaining structures.

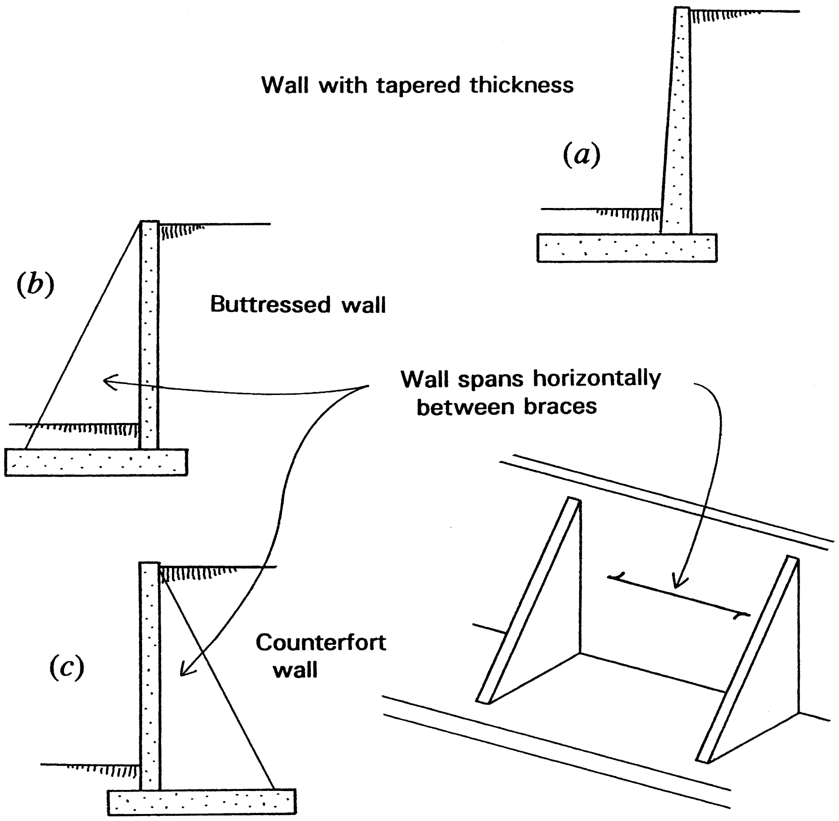


Figure 8.34 Typical forms for tall retaining walls.

footing is a short distance below grade on the low side of the wall and/or the lateral passive pressure resistance of the soil is low, it may be necessary to use a *shear key*, formed as shown in Figure 8.33*c*.

It is generally desired to avoid the buildup of water saturation behind the wall, as this greatly increases the active lateral pressure. Drainage of the soil mass may be achieved by using a fill of coarse cohesionless soil (sand and gravel) and/or a drainage surface device on the back of the wall. The drained water is then fed through the base of the wall with holes (called *weeps)*.

Tall Retaining Walls

As the wall height increases, it becomes less feasible to use the simple construction shown in Figure 8.33. The overturning moment increases sharply with increase in the wall height. For very tall walls, one modification is the use of a tapered form for the wall thickness, as shown in Figure 8.34*a*. This allows for the development of a strong cross section for the high bending moment at the base of the wall without an excessive increase in the concrete volume.

As the wall gets very tall, it is often necessary to brace it. Bracing may be achieved with buttresses on the face of the wall (see Figure 8.34*b*) or with counterforts on the back side of the wall (see Figure 8.34*c*). Bracing can also be achieved with tiebacks anchored in the soil behind the wall.

CHAPTER

9

Lateral-Force Effects

This chapter deals generally with the topic of horizontal-force effects in buildings. The term used for these effects is *lateral*, meaning sideways, which identifies them in relation to the major orientation of the effect of gravity as a vertical force. Conceptually, then, designing for lateral forces is typically viewed in terms of bracing a building against sideways collapse (see Figure 9.1). In truth, most load sources that produce lateral forces also generate some vertical effects and so it is of limited use to treat the horizontal-force effects in isolation. Even where this may be valid as an investigative technique, it should always be borne in mind that lateral effects always occur in some combination with some vertical effects, including those due to gravity. In the end it is the full combined effects that must be understood and dealt with.

In many situations, the elements of the building construction that work to achieve lateral bracing are not visible in the finished building. However, architects may choose to feature the lateral bracing in a major way, as shown in Figure 9.2. Visible or not, the bracing is there, and its planning and design are the topic of this chapter.

While the issues of lateral forces are generally treated in this chapter, lateral forces are included in the work in several other sections in this book. The problem of horizontal forces on foundations and sites is developed in Chapter 8. Some design examples of lateral resistive structures are treated in this chapter, but the building design examples in Chapter 10 present lateral-force design in the broader context of the whole building structure.

9.1 GENERAL CONSIDERATIONS FOR LATERAL EFFECTS

Sources of Lateral Loads

The principal sources for lateral-load effects in buildings are the following:

Wind. Wind is moving air. Air is a fluid, and some general knowledge of fluid mechanics is helpful for the understanding of the various effects of wind on buildings. Our primary concern here is for the effects of wind on the lateral bracing system for the building. As a net effect, this force is an aggregate of the various effects of the fluid flow of the air around the stationary object (the building) on the ground surface.

Earthquakes. Earthquakes—or *seismic activity* as it is called—produce various disastrous effects, including tidal waves, massive ruptures along earth faults, and violent vibratory motions. It is the last effect for which we design the lateral bracing systems for buildings, dealing mostly with the horizontal aspect of the ground motion. The force applied to the building is actually generated by the momentum of the building mass as it is impelled and rapidly reversed in direction. This activity cannot be fully understood in terms of static force alone, however, as dynamic aspects of both the ground motion and the building's response must be considered.

Soil Pressure. The problems of soil-restraining structures and the general action of soils under stress are discussed in Chapter 8. Wind and earthquake forces on the building must eventually be resolved by the building foundations and supporting soils. Some of these issues are discussed in this chapter, but also in Chapter 8.

Structural Actions. The natural action of various structures in resisting gravity loads may result in some horizontal forces on the supports of the structure even though the direction of the gravity load is vertical. Common examples of such structures are arches, gable roofs, cable structures, rigid frames, and pneumatic structures sustained by internal pressure.

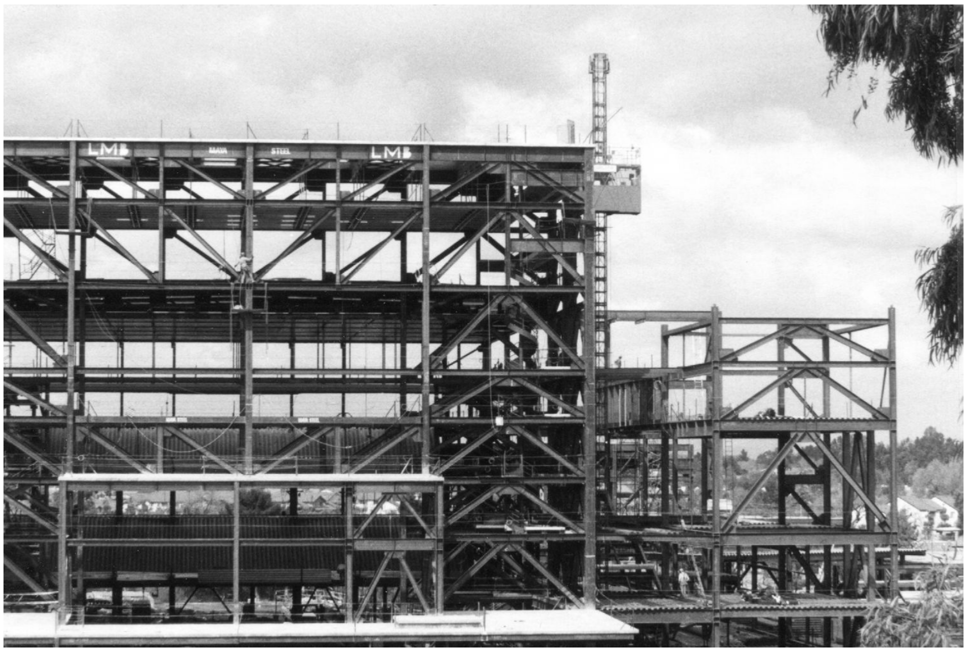


Figure 9.1 This is an erected steel frame for a multistory office building. The classic post-and-beam system is developed in direct response to the need for resistance of vertical gravity forces. In this example, the diagonal members develop lateral bracing by forming various trussed elements in conjunction with the vertical and horizontal members. For some buildings, the lateral bracing may be a major influence on the overall building form. Here the diagonal members will essentially be out of sight and unobtrusive in the finished building.

Volume Change: Thermal, Moisture, and Shrinkage. The actions of thermal expansion and contraction, moisture swelling and shrinkage, and the initial shrinkage of concrete, mortar, and plaster are all sources of dimensional change in the volume of building materials. Ideally, these effects are controlled through the use of expansion joints, deformable joint materials, or other means. However, the potential forces that they represent must be understood and must be provided for in terms of structural resistance in some instances.

Relation of Lateral to Gravity Effects

For the purpose of understanding the effects of lateral loads, there is some usefulness in investigations for lateral forces alone. Examples in this chapter deal with this type of investigation, since it is the principal purpose of the chapter. However, attention must be given to the complete design process, which treats the various combinations of loads and their aggregate effects on the whole structure. This is the purpose of the work in Chapter 10, where investigations of lateral bracing are performed within the full context of design.

Problems of Quantification

A major problem in designing for lateral effects is simply that of determining the magnitudes of the loads. This is probably easiest for structurally induced effects, although approximations in determining weights of construction and complex behaviors of highly indeterminate structures may make quantifications suspect in these situations as well. Translating the fluid flow effects of wind into so many pounds of force on a structure is a convoluted exercise in fantasy. Precise prediction of potential ground movements caused by some hypothetical earthquake at a specific site and estimating the building's response and any site–structure interaction are conjectures of ethereal proportions. Determining or controlling the conditions of a specific soil mass is a highly approximate exercise.

This should not be used as an argument for not being serious in designing for these effects. It is merely to make the point that what we do chiefly is provide the *kind* of structure that is likely to have the character suited for its task; its *exact* capacity is a very soft target. We do our best but cannot pretend to claim great precision.

Application of Wind and Earthquake Forces

In order to understand how a building resists the lateral-load effects of wind and earthquakes, it is necessary to consider the manner of application of the forces and then to visualize how these forces are transferred through the lateral resistive structural system and into the ground.

Wind Forces

The application of wind forces to a closed building is in the form of pressures applied normal (perpendicular) to the exterior surface of the building and surface shears applied to sides parallel to the wind direction. In one design method the total wind effect on the building is determined by considering the vertical profile, or silhouette, of the building as a single vertical plane surface at right angles to the wind direction. A direct horizontal pressure is assumed to act on this plane.

Figure 9.3 shows a simple rectangular building under the effect of wind normal to one of its flat sides. The lateral resistive structure that responds to this loading consists of the following:

Wall surface elements on the windward side are assumed to take the total wind pressure and are typically designed to span vertically between the roof and floor structures.

Figure 9.2 Exterior form of the John Hancock tower in Chicago. In this case, the architect chose to dramatically express the logical form of a truss-braced tower. The X-braced, tapered form echoes the many electrical transmission towers and old windmills that dot the prairie landscape of rural America. It also pays homage to the Eiffel Tower.

Roof and floor decks, considered as rigid planes (called diaphragms), receive the edge loading from the windward wall and distribute the load to the vertical bracing elements.

Vertical frames or shear walls, acting as vertical cantilevers, receive the lateral loads from the horizontal diaphragms and transfer them to the building foundations.

The *foundations* anchor the vertical bracing elements and transfer the lateral loads to the ground.

The propagation of the loads through the structure is shown in the left part of Figure 9.3, and the functions of the major elements of the lateral resistive system are shown in the right part of the figure. The exterior wall functions as a simple spanning element loaded by a uniformly distributed pressure normal to its surface and delivering reaction forces to its supports. In most cases, even though the wall may be continuous through several stories, it is considered as a simple span at each story level, thus delivering half of its load to each support. Referring to Figure 9.3, this means that the upper wall delivers half of its load to the roof edge and half to the edge of the second floor. The lower wall delivers half of its load to the second floor and half to the first floor.

This may be a somewhat simplistic view of the functions of the walls themselves, depending on their construction. If they are framed walls with windows or doors, there may be many internal load transfers within the wall. Usually, however, the external load delivery to the horizontal structure will be as described.

The roof and second-floor diaphragms function as spanning elements loaded by the edge forces from the exterior wall and spanning between the end shear walls.

The end shear walls act as vertical cantilevers anchored at their bottoms. The total shear force in the wall is delivered at its base. The bending caused by the lateral load produces an overturning effect at the base of the wall as well as tension and compression forces at the wall ends. The overturning effect is resisted by the stabilizing effect of gravity loads on the wall. If this stabilizing moment is not sufficient to prevent overturning, a tension tie must be made between the wall end and the support.

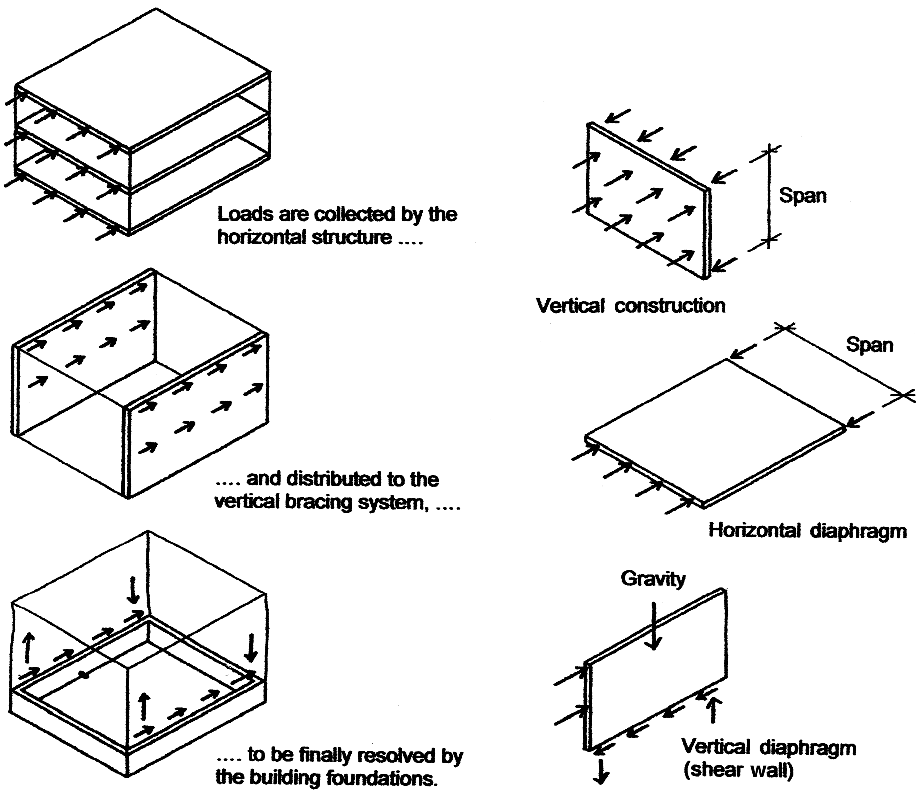


Figure 9.3 Propagation of wind force and basic functions of elements in a box building.

If the first floor is attached directly to the foundations, it will not function as a diaphragm, but rather will push its load directly to the leeward foundation. In this event, it may be seen that only three-fourths of the total wind load will be delivered to the shear walls.

This simple example illustrates the basic nature of the propagation of wind forces through the building structure, but there are many other possible variations with more complex building forms and other structures.

Seismic Forces

Seismic forces are generated by the dead weight of the building construction. In visualizing the application of seismic forces, we look at each part of the building and consider its weight as a horizontal force. The load propagation for the box-shaped building in Figure 9.3 will be quite similar for both wind and seismic forces.

If a wall is reasonably rigid in its own plane, it tends to act as a vertical cantilever for the seismic load in the direction parallel to its surface. Thus in the example building in Figure 9.3 the seismic load for the roof diaphragm would be considered to be caused by the weight of the roof and ceiling plus only those walls whose planes are normal to the direction being considered. These different functions of the walls are illustrated in Figure 9.4. If this assumption is made, it will be necessary to compute a separate seismic load in each direction for the building.

For determination of the seismic load, it is necessary to consider all elements that are permanently attached to the structure. Elevators, ductwork, piping, lighting and plumbing fixtures, supported equipment, signs, and so on, will add to the total dead weight for the seismic load. In buildings such as warehouses and parking garages it is also advisable to add some of the load of the building contents.

Types of Lateral Resistive Systems

The building in the previous example illustrates one type of lateral resistive system: the box, or panelized, system. As shown in Figure 9.5, the general types of systems are as follows:

The Box System. This system is as shown in the previous example, consisting of some combination of horizontal and vertical planar elements. Most buildings use roof and floor construction that qualifies as a horizontal diaphragm. The box also uses vertical diaphragms (shear walls), although planes of trusses may also qualify due to their relative stiffness.

Internally Braced Frames. The typical assemblage of post-and-beam elements is not inherently stable under lateral loading unless the frame is braced in some manner. Shear wall panels may be used to achieve this bracing, in which case the system functions as a box. It is also possible, however, to use diagonal frame members to achieve trussed panels, X-bracing, knee braces, and so on. The term *braced frame* is used to refer to a truss-braced frame.

Rigid Frames. Although commonly used, the term *rigid frame* is a misnomer since this technique often produces the most flexible lateral resistive system.

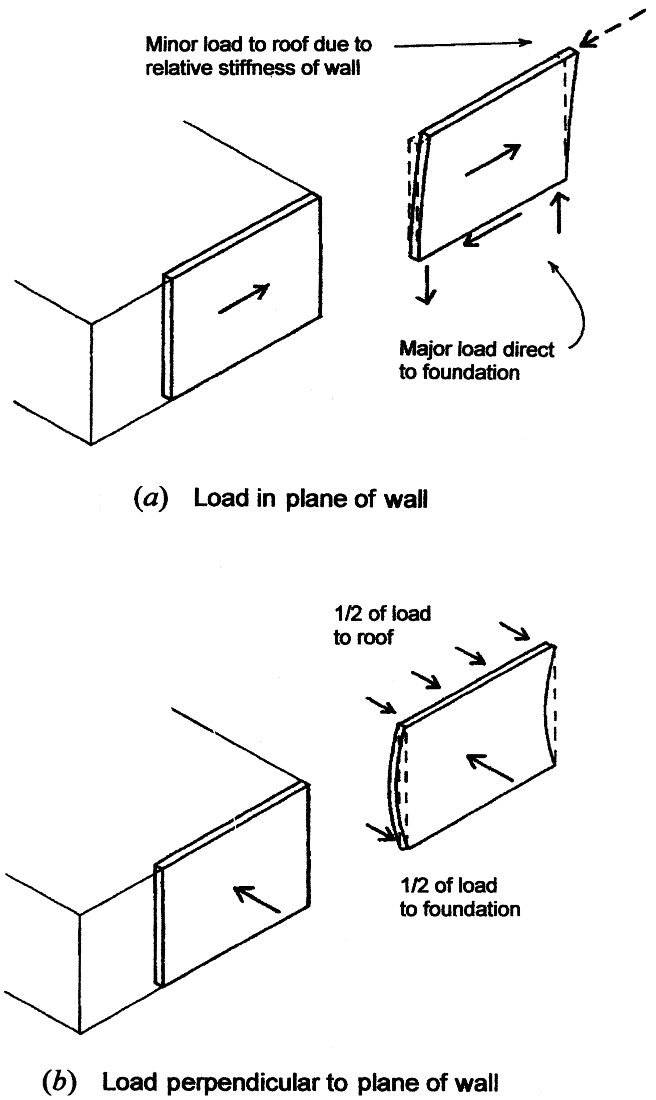


Figure 9.4 Seismic loads caused by wall weight.

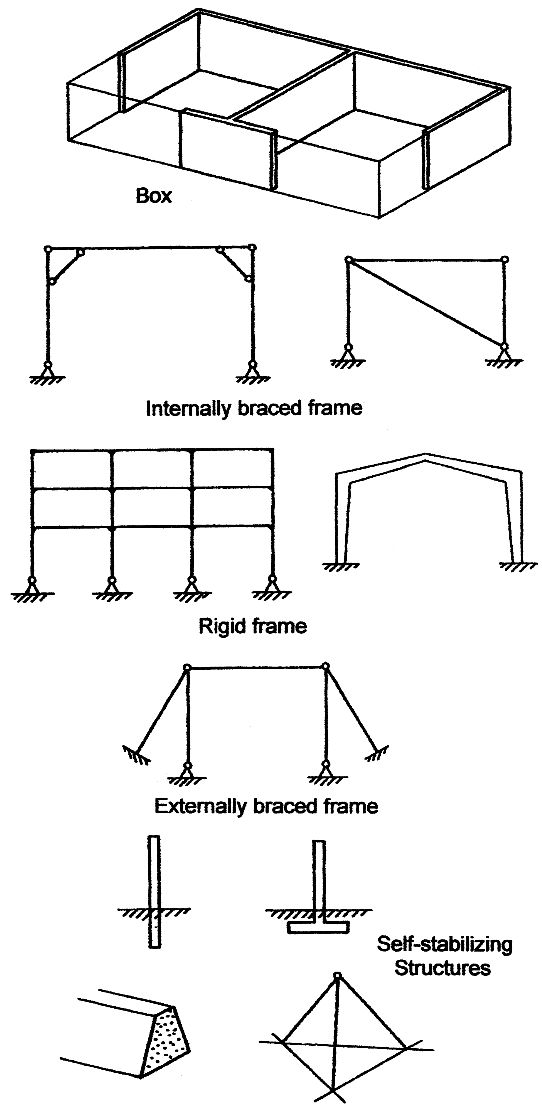


Figure 9.5 Types of lateral-load resistive systems.

The term derives from the use of moment-resistive joints between frame members and the description used officially now is *moment-resistive frame*, but the term most commonly used is still rigid frame.

Externally Braced Structure. Building systems lacking internal stability may be braced with external elements, such as guys, struts, buttresses, and so on.

Self-Stabilizing Elements and Systems. Retaining walls, flagpoles, pyramids, tripods, and so on, in which stability is achieved by the basic form of the structure, are examples of self-stabilizing elements and systems. These may be independently stabilized or may serve to brace an attached unstable system.

An important property of bracing systems is the relative stiffness or resistance to deformation. This is particularly important for evaluating energy capacity, which is especially critical to investigation for response to earthquake effects. A box system with vertical diaphragms of concrete or masonry is very rigid, whereas wood frames and moment-resistive steel frames are usually quite flexible. Stiffness affects the fundamental period of the structure and thus influences the percentage of the dead weight assumed as a horizontal force.

Vertical elements of the building construction developed for gravity load design, or for the general architectural design, may have a natural potential for functioning as part of the lateral bracing system. This is a typical starting point for design of the bracing system for small buildings, with ordinary construction elements often needing little modification for use for bracing.

Whether potential bracing elements can actually serve as such depends on their materials, construction details, and attachment to other parts of the building construction. They may be usable in unmodified form or may be relatively easily modified for improved response to lateral loads. Or, they may simply not be adequate for the tasks, regardless of modifications.

Many buildings consist of mixtures of the basic types of lateral resistive systems. Shear walls may be used to brace a building in one direction whereas a braced frame or rigid frame is used in a perpendicular direction. Multistory buildings occasionally have one type of bracing system, such as a rigid frame, for the upper stories and a different system, such as a rigid box system or heavy truss system, for the lower stories.

There is a possibility that some elements of the building construction that are not intended to function as bracing elements may actually end up taking some of the lateral load. In frame construction, surfacing materials of plaster, stucco, plywood, or masonry veneer may take some lateral load even though the frame is braced by other means. This is essentially a matter of relative stiffness, although connection for load transfer is also a consideration. What can happen in these cases is that the stiffer finish materials take the load first, and if they are not strong enough, they fail and the intended bracing system then goes to work. Although collapse may not occur, there can be considerable damage to the building construction as a result of the failure of the supposed nonstructural elements. See Figure 9.6.

In many cases it is neither necessary nor desirable to use every wall as a shear wall or to brace every bay of the building frame. The illustrations in Figure 9.7 show various situations in which the lateral bracing of the building is achieved by partial bracing of the system. This procedure does require that there be some load-distributing elements, such as the roof or floor diaphragm, horizontal struts, and so on, that serve to tie the nonstabilized portions of the building to the lateral resistive elements.

The choice of the type of lateral resistive system must be related to the loading conditions and to the behavior characteristics required. It must also, however, be coordinated with the design for gravity loads and with the architectural planning considerations. Many design situations allow for alternatives, although the choice may be limited by the size of the building, by code restrictions, by the magnitude of the lateral loads, by a desire for limited deformation, and so on.

Lateral Resistance of Ordinary Construction

Even when buildings are built with no consideration given to the design for lateral forces, they will have some natural capacity for lateral-force resistance. It is useful to have an understanding of the limits and capabilities of ordinary construction as a starting point for the consideration of designing for enhanced levels of lateral-force resistance.

Wood Frame Construction

Wood structures can be categorized broadly as light wood frame or heavy timber. Light wood frames, using mostly 2-in.-nominal-thickness lumber, account for a major portion of small low-rise buildings in the United States. The light wood frame uses roof and floor decks and wall sheathing of panel materials, most of which are rated for diaphragm capacity. See Figure 9.8. Thus, with little alteration of the basic frame, the structure can be made resistive to lateral loads as a box system with horizontal and vertical diaphragms.

Many of the ordinary elements of light wood frames can be utilized as parts of the lateral resistive system. Wall studs, posts, sills and plates, and roof and floor framing members, occurring naturally in the structure, are often able to be used for specific tasks. Alterations necessary to make the bracing system more functional are often limited to moderate increases in sizes or to the use of some added fastening or anchorage. When members are long and not able to be installed as a single piece (as with the top plate on a long wall), it may be necessary to use stronger splicing than is ordinarily required for gravity load resistance alone.

Figure 9.6 Typical diagonal cracking due to the back-and-forth movements during an earthquake. Stucco firmly attached to the wood frame is the stiffer bracing, whether you want it to be or not. When it cracks, the wood frame takes over.

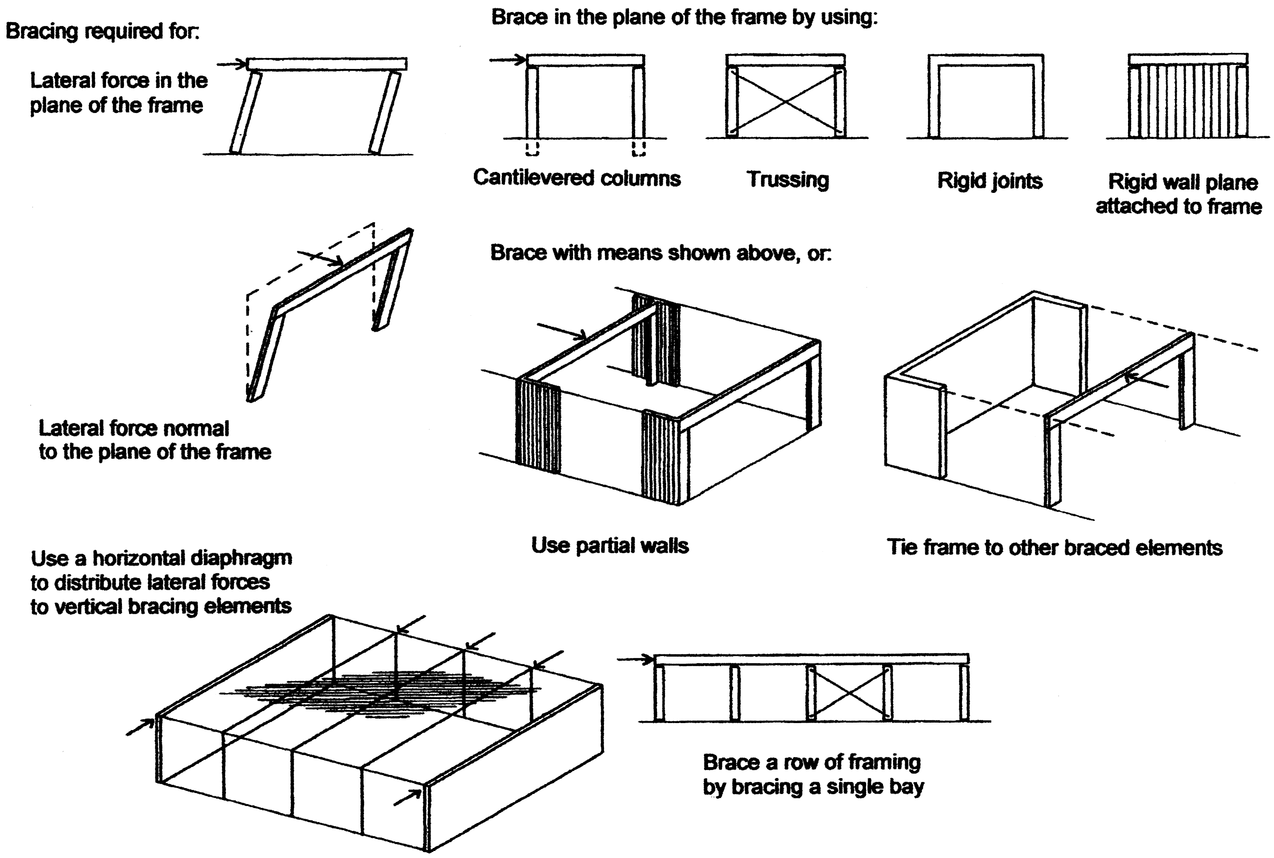


Figure 9.7 Bracing of framed structures for lateral loads.

In recent times there has been a trend toward the extensive use of sheet metal fastening devices for the assemblage of light wood frames. In general, these tend to increase resistances to lateral loads through more positive anchorage and the potential for load transfers through the connections.

Options for surfacing materials for the light wood frame are discussed in Chapter 4. There is a considerable range in the diaphragm shear capacity of various surfacing materials. Weaker materials with minimum attachment may be adequate for light loadings, but use of stronger material, such as plywood, and a larger number of fasteners can raise the capacity of shear walls and decks by a considerable amount.

When the same surfacing is applied to both sides of a wall, the capacities of the two materials can be added to obtain the capacity of the wall. This is common for interior walls and effectively doubles the capacity of construction with weaker materials. However, for exterior walls the two surfaces are seldom the same, and thus the stronger (usually the exterior) must be used alone for the wall capacity.

Some of the problems encountered in developing lateral resistance with light wood framing construction are as follows:

Lack of Adequate Solid Walls to Serve as Shear Walls. This may be due to the building planning, with walls insufficient at certain locations or in a particular direction. Walls may also simply be too broken up in short lengths by doors or windows. For multistory buildings there may be a problem where upper level walls do not occur above walls in lower levels.

Lack of Adequate Diaphragm Surfacing. The desired surfacing materials may not be strong enough for use in the lateral resistive system.

Lack of Continuity of Framing. Because it is often not required for gravity loads, horizontal framing may not be aligned in rows for use in the lateral resistive system. Any discontinuity in the regular order of the framing can present a problem—usually solvable, but requiring some attention for the structural design.

Wood structures can sometimes be made to function as braced frames or rigid frames, the latter being somewhat more difficult to achieve. Often, however, these structures occur in combination with wall construction usable for shear walls, so that the post-and-beam frame need not function for lateral bracing.

A problem with ordinary post-and-beam construction is often the lack of ability for transfer of horizontal loads between members required for the lateral resistive system. At present this is less often the case due to the increasing use of manufactured metal-framing devices which have been developed for multiple joint functions.

When heavy timber frames are exposed to view, a choice is sometimes made for timber deck whose underside is more presentable. Although such a deck has minimal diaphragm

Figure 9.8 Light wood frame construction

shear capacity, a simple solution is to nail a plywood surfacing to the top of the timber deck with the plywood and its nailing designed as the diaphragm.

Occasionally, buildings with light wood frames simply do not have sufficient walls—either interior or exterior—that can function as shear walls. This may be a general condition or one that exists for only one direction. One possible solution in these cases is to develop a steel braced frame or rigid frame that is integrated with the wood framing and supplies some or all of the bracing demands (see Figure 9.9).

The most common solution for this situation, however, is to use exterior wall construction of concrete or masonry with a potential for providing all the required functions for the vertical part of the lateral bracing system.

Structural Masonry

Masonry structures offer many advantages for the development of resistance to wind loads. As built today, they have a great potential for lateral shear resistance. They also are quite stiff and heavy, offering considerable resistance to lateral deformation and providing considerable resistance to horizontal and uplift effects of wind.

For regions of high risk for earthquakes the only masonry structural construction used is *reinforced masonry*. These comments are confined to structural masonry walls using hollow concrete blocks (now called *concrete masonry units*, or CMUs), with the voids partly or wholly reinforced and filled with concrete (see Figure 9.10).

Structural masonry walls have considerable potential for utilization as shear walls. However, a number of concerns must be addressed:

Increased Load. Due to their weight, stiffness, and brittle nature, masonry walls must be designed for higher levels of lateral seismic force.

Limited Stress Capacity. The unit strength and the mortar strength must be adequate for the required stress resistances. Both vertical and horizontal reinforcement are required. Voids are usually required to be totally filled for shear wall and foundation uses.

Figure 9.9 This low-rise office building is primarily constructed with light wood framing. However, the forming of exterior walls, added to a desire for a minimum of permanent interior walls, generated a plan with too little potential for development of wood-framed shear walls. The solution in this case was to develop a light steel frame in the center of the building with a few trussed bays that provide a major part of the lateral bracing for the building. This is, of course, the basic solution that was developed long ago for high-rise buildings, with strongly braced core structures.

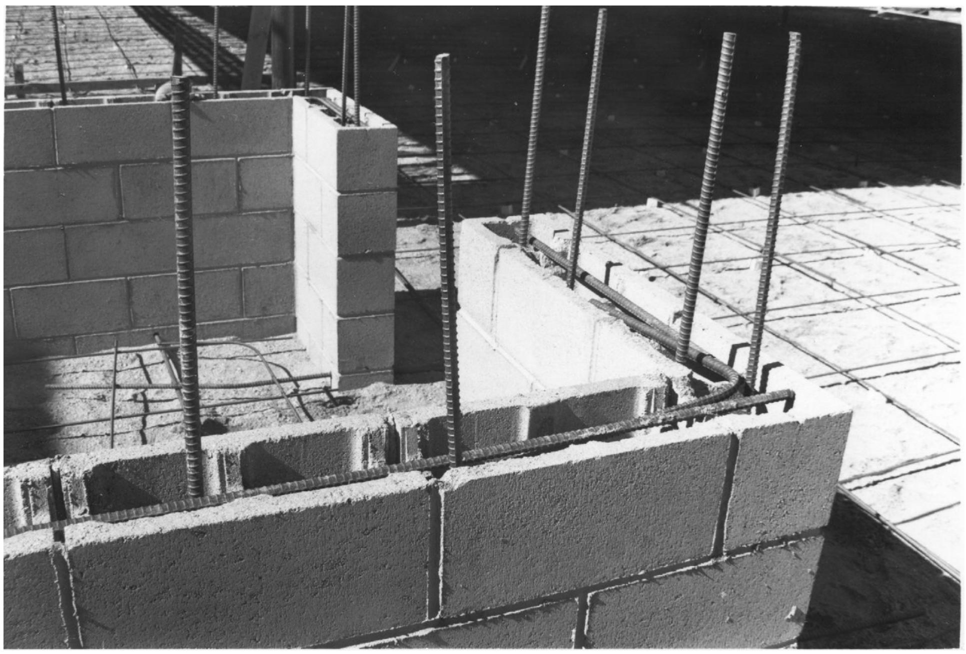


Figure 9.10 Reinforced masonry construction with CMUs (concrete blocks).

Cracks and Bonding Failures. If not reinforced as shown in Figure 9.10, the strength and bonding of mortar are critical to the wall strength.

Code specifications for concrete block walls result in a typical minimum construction that has a specific limit of shear wall capacity. This limit is generally beyond the limit for the strongest wood-framed wall with plywood on a single side, and so the change to a masonry wall is a significant step. Beyond the minimum value the load capacity can be increased by adding additional reinforcement and filling more voids in the wall with concrete. At its upper limits the reinforced masonry wall approaches the capacity of a reinforced concrete wall.

Anchorage of masonry walls to their supports is usually simply achieved. Resistance to vertical uplift and horizontal sliding can typically be developed by the usual doweling of the vertical wall reinforcement. The attachment of most horizontal diaphragms to masonry supporting walls for transfer of lateral forces is another matter, generally requiring more positive anchorage methods than those used for gravity loads alone.

Steel Frame Construction

Steel frame structures can often quite readily be made resistive to lateral loads by development of trussing or rigid frames. Steel has the advantage of having a high level of resistance to all types of stress and is thus not usually sensitive

to multidirectional stresses or to rapid stress reversals. In addition, the ductility of ordinary structural steel provides a toughness and a high level of energy absorption in the plastic behavior mode of failure.

Despite its high strength and very high stiffness (modulus of elasticity), steel is mostly used to produce structures with considerable deformation, especially with lateral loads. It thus ranks closer to the light wood structure than to structures of concrete or masonry. This problem is not insurmountable but does need attention for both gravity and lateral design.

The ordinary steel post-and-beam frame is essentially unstable under lateral loading. Frames must therefore either be made self-stable with trussing or moment-resistive connections or be braced by shear walls.

Steel frames in low-rise buildings are often braced by walls, with the steel structure serving only as the horizontal-spanning structure resisting gravity loads. Walls may consist of masonry, precast concrete, or frames of wood or steel with a shear-resisting surfacing.

The trussed steel frame is typically quite stiff, almost in a class with the wall-braced structure. This is an advantage in terms of reduction of deformations, but it means that the bracing structure must be designed for higher loads from earthquakes, as compared to the more deformable rigid frame. Of course, the rigid frame can be made truly rigid with very stout members if deformation needs to be controlled.

Planning of all framed buildings must accommodate the propagation of lateral loads through the bracing system. For buildings of complex form, or with many large openings in the horizontal structure, it may be difficult to develop vertical planar bents or the linear transfer of force through the system. This is an issue that needs to be carefully studied in three dimensions in the building planning.

Sitecast Concrete Construction

Sitecast concrete elements for most building structures are ordinarily quite extensively reinforced, thus providing some significant compensation for the vulnerability of the tension-weak concrete. Even where structural demands are not severe, a minimum amount of reinforcement is required so the ordinary form of the construction has considerable natural potential for lateral-force resistance.

Subgrade building construction most often consists of thick concrete walls. The typical result is a highly rigid, boxlike structure. Shears in the planes of the walls can be developed to considerable stress levels with minimum reinforcement.

Above-grade structures of sitecast concrete consisting of columns and some form of horizontal spanning system require careful study for the development of lateral-force resistance as rigid frames. The following are some potential concerns:

Weight of the Structure. This is ordinarily considerably greater than that for wood or steel structures, with the resulting increase in both gravity load and lateral seismic forces.

Ductile Yielding of Reinforcement. This is the desired first mode of failure, even for gravity loads. With proper design it is a means for developing a yield failure character for the otherwise brittle, tension-weak material.

Adequate Reinforcement for Seismic Effects. Of particular concern are the shears and torsions developed in the framing elements and the need for continuity of the reinforcement and anchorage at the intersections of elements.

Detailing of Reinforcement. Continuity at splices and adequate anchorage at member intersections must be assured by careful layout of the installation of the reinforcement.

Tying of Compression Reinforcement. Column bars are routinely tied, but beam reinforcement with compression stress must also be tied to prevent buckling.

When sitecast concrete walls are used in conjunction with concrete frames, the result is often similar to that for the panel-covered wood frame; the walls will absorb the major portion of lateral forces due to their relative stiffness. At the least, however, the frame members function as chords, collectors, and ties, interacting with the walls.

As with masonry structures, considerable cracking is normal in sitecast concrete construction. Most of this is due to shrinkage, temperature change, settlement of supports, and the normal form of development of internal tension forces. In addition, cracks are created at the joints that are formed between successive pours during the usual construction process for large buildings.

Under the back-and-forth swaying actions of an earthquake, cracks will be magnified and a grinding action may occur as stresses reverse. The grinding action can be a major source of energy absorption but can also result in progressive failure or simply some pulverizing and flaking off of the concrete. If reinforcement is adequate, the structure may remain safe, but the appearance is sure to be affected.

It is virtually impossible to eliminate all cracking from masonry and concrete building construction. Good design, careful construction detailing, and quality construction work can reduce the cracking to a minimum and possibly eliminate some types of cracking.

Precast Concrete Structures

Precast concrete structures present unique problems in terms of lateral bracing and various responses to earthquake effects. Although they generally share many characteristics with sitecast concrete structures, they lack the member-to-member continuity that provides natural lateral stability. They must be dealt with in a manner similar to that for post-and-beam structures of wood and steel.

Separate precast concrete members are usually attached to each other by means of steel devices cast into the members. The assemblage of the structure thus becomes a steel-to-steel connection problem. Where load transfer for gravity

resistance is limited to simple bearing, connections may have no significant stress functions and merely serve to hold members in place. Under lateral loading, however, all connections will likely be required to transfer shear, tension, bending, and torsion effects. Thus for lateral-force resistance many of the connections used for gravity resistance alone will be inadequate.

Because of their considerable weight, large precast concrete spanning members may experience special problems due to vertical seismic movements. When not sufficiently held down, members may be bounced off their supports by the simultaneous actions of vertical and horizontal movements (a failure described as *dancing*).

Foundations

Where considerable below-grade construction occurs, the below-grade structure as a whole usually furnishes a solid base for the above-grade building. Little additional provision may be required for the functions of lateral-force resistance. For seismic resistance the entire system must be securely tied together to function as a whole. Buildings without basements and those supported on deep foundations (piles and caissons) may present some additional problems for resolution of the lateral forces into the ground.

Freestanding Structures

Freestanding structures include site walls used as fences as well as large signs, water towers, and detached stair towers. The principal structural problem is often the large overturning effect. Rocking and permanent soil deformations that result in vertical tilting must be considered. It is generally advisable to be very conservative in the design for soil pressures due to lateral-load effects. When weight is concentrated at the top—as in the case of large signs or water towers—the dynamic rotational effect due to seismic force is a major factor. These issues also apply to objects placed on the roof of a building.

General Building Design Problems

When the need to develop resistance to lateral forces is kept in mind throughout the entire building design process, it will have bearing on many areas of the design development. When lateral concerns are dealt with as an afterthought—rather than being borne in mind in the earliest design decisions—it is quite likely that optimal conditions will not be obtained. Some of the major issues that should be kept in mind in the early planning stages are the following:

Need for a Lateral Bracing System. In some cases, because of the building form or size or the decision to use a particular type of construction, choices may be limited. In other situations there may be several options, with each having different required features, such as alignment of columns, incorporation of solid walls, and so on. The particular system to be used should be established early.

Implications of Architectural Design Decisions. When certain features are desired, it should be clearly understood that there may be some consequences in terms of problems with the design of the lateral bracing system. Some situations that commonly cause problems are as follows:

General complexity and lack of symmetrical form

Random arrangement of vertical elements (walls and columns)

Lack of continuity in horizontal structures due to openings, multiplane roofs, split-level forms, or open spaces within the building

Building complexes consisting of multiple, semidetached units that require consideration for linking or separation for response to lateral effects

Special forms—curved walls, sloping roofs, and so on

Large spans or tall walls that result in concentration of load effects on supports

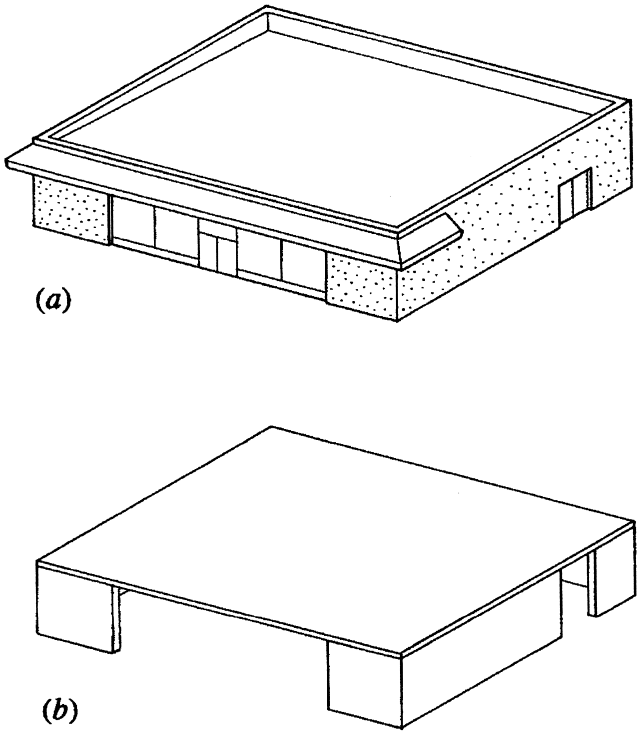
Use of nonstructural materials and construction details that produce vulnerability to damage caused by deformations due to lateral effects

Design Styles Not Developed for Lateral Loads. In many cases popular architectural design styles or features are initially developed in regions where windstorms or seismic effects are not of concern. When these are imported to other areas with high risk for lateral effects, a mismatch often occurs.

Most buildings are complex in form. They have plans defined by walls that are arranged in complex patterns. They have wings, porches, balconies, towers, and roof overhangs. They are divided vertical by multilevel floors. They have sloping roofs, arched roofs, and multiplane roofs. Walls are pierced by openings for doors and windows. Floors are pierced by stairways, elevator shafts, ducts, and piping. Roofs are pierced by skylights, vents, and chimneys. The overall response of the building to lateral forces can thus be complicated and is difficult to visualize, let alone to quantitatively determine.

Despite this complexity, investigation for lateral response may be simplified by the fact that we deal mostly with those elements of the building that are directly involved in the resistance to lateral forces, what we call the *lateral resistive system*. Thus most of the building construction, including parts of the structure that function only to resist gravity loads, may basically be along for the ride but have little involvement in the structural response to lateral effects. These nonstructural elements contribute to the building weight and may have a damping effect on movements, but they do not directly affect the actions of the bracing system.

A discussion of the issues relating to building form and lateral-force response must include consideration of two separate situations: the form of the building as a whole and the form of the lateral resistive system. Figure 9.11*a* shows a simple one-story building with the general exterior form illustrated. Figure 9.11*b* shows the same building with the parapet, canopy, window wall, and other elements removed,



(a)

(b)

Figure 9.11 Building form versus form of lateral bracing system.

leaving the essential parts of the lateral resistive system. This system consists of the horizontal roof surface and the portions of the exterior walls that function as shear walls. The whole building must be considered for determination of the lateral forces, but the stripped-down structure must be visualized in order to investigate the effects of the lateral forces.

In developing building plans and the building form in general, architectural designers must give consideration to many issues. Lateral response has to take its place in line with the needs for functional interior spaces, control of traffic, creation of acoustic privacy, separation for security, energy efficiency, and general economic and technical feasibility. In this section we deal primarily with the problems of lateral response, but it must be kept in mind that the architect must also deal with all the other design concerns, each of which must be studied in isolation as we do here but which must eventually be coordinated.

Development of a reasonable lateral resistive structural system within a building may be easy or difficult and for some proposed plan arrangements may be next to impossible. Figure 9.12 shows a building plan in the upper figure for which the potentiality for the development of shear walls in the north–south direction is quite reasonable but in the east–west direction is not so good as there is no possibility for shear walls on the south side. If the modification shown in the middle figure is acceptable, the building can be adequately braced in both directions. If the open south wall is really essential, it may be possible to brace this wall using a column-and-beam structure that is braced by trussing or by rigid connections, as shown in the lower figure.

In the plan shown in Figure 9.13*a* the column layout results in a limited number of column rows that permit the possible development of column/beam bents that may be developed as moment-resistive frames. In the north–south direction either the interior columns are offset from the exterior columns or the column rows are interrupted by the floor opening; thus the two end bents are the only ones usable. In the east–west direction the large opening interrupts two of the three interior column rows, leaving only three usable bents which are not disposed symmetrically in the plan. The modifications shown in Figure 9.13*b* represent an improvement in potential lateral response, with six usable bents in the north–south direction and four symmetrically placed bents in the east–west direction. This plan, however, has more interior columns, smaller open spaces, and a reduced size for the opening—all of which may present some drawbacks for architectural planning concerns.

Another possibility for the plan in Figure 9.13 is that shown as (*c*). This indicates the use of a *perimeter bent* system that is developed with only the exterior columns and edge framing beams. In this case, since the interior columns are not involved in the lateral resistive system, they may be arranged in any manner. Since the exterior columns do not interfere with planning of the interior space, it is possible to use extra columns, which might be beneficial for a tall building.

In addition to planning concerns the vertical massing of the building has various implications on its lateral response. The three building profiles shown in Figures 9.14*a–c* represent a range of potential responses with regard to the response to both wind and seismic effects. The short, stiff building in Figure 9.14*a* is probably more critical with regard to wind uplift on its roof surface than for lateral pressure on the short walls. Its relative stiffness (resistance to deflection and a very short fundamental period of harmonic vibration) makes it absorb a larger jolt from an earthquake. The tall building in Figure 9.14*c*, on the other hand, will be critical for the wind overturning effect at its base and will sustain higher wind pressures at its upper levels. It will have less stiffness and a longer period of vibration, so it will have a reduced impact from an earthquake. The building profile in Figure 9.14*b* will produce something between the two extremes: a moderate response to overturn and a seismic jolt somewhere between the other two examples.

The overall inherent stability of a building may be implicit in its vertical massing or profile. The building shown in Figure 9.14*d* has considerable potential for stability with regard to lateral forces, whereas that shown in Figure 9.14*e* is highly questionable. Of special concern is the situation in which abrupt change in stiffness occurs in the vertical massing. The building in Figure 9.14*f* has an open form at its base with a fairly solid wall in the stories above. If this is the true character of the structure, the situation represents a *soft-story* or *weak-story* condition, and response to lateral force from an earthquake can be disastrous. This response will not be indicated by a static force analysis but will be quickly identified by a dynamic force analysis.

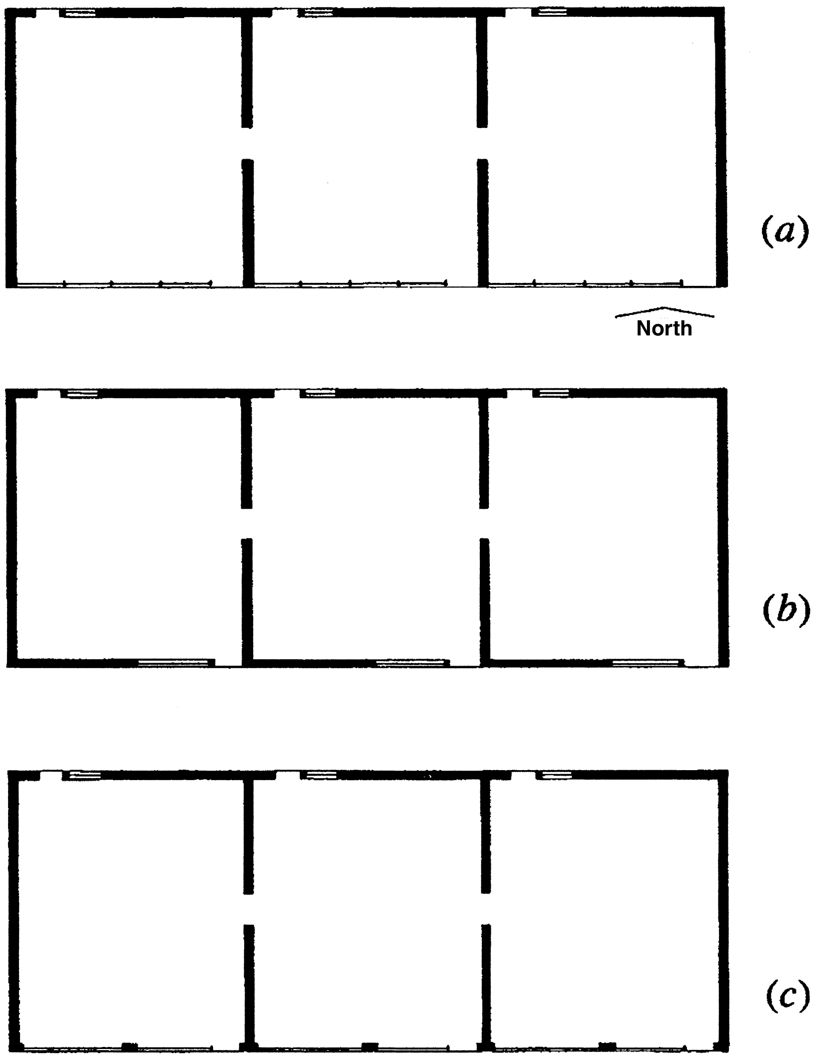


Figure 9.12 Options for bracing for a three-sided building.

As with the building plan, consideration of the vertical massing must include concerns for the form of the lateral resistive system as well as the form of the building as a whole. The illustration in Figure 9.14*g* shows a building whose overall profile is quite stout. However, if the building is braced by a set of shear walls, as shown in the section view, it is the profile of the shear walls that must be considered. In this case the shear wall is quite slender in profile and may have some critical concerns for overturn at its base—a condition not indicated by the relatively squat profile of the whole building.

Investigation of the seismic response of a complex building is, in the best of circumstances, a difficult problem. Anything done to simplify the investigation will not only make the analysis easier to perform but also tend to make the reliability of the results more certain. Thus, from a seismic design point of view, there is an advantage in obtaining some degree of symmetry in the building massing and in the disposition of the elements of the lateral resistive structure. Recent provisions of building codes promote this attempt by providing various penalties for irregular building shapes.

When symmetry does not exist, a building tends to experience twisting as well as the usual rocking back and forth under lateral forces. Twisting often has its greatest effects on the joints between elements of the bracing system. Most buildings are not symmetrical, being sometimes symmetrical on only one axis and sometimes on none. What is truly critical for lateral-load response is the alignment of the net effect of the lateral load with the center of resistance of the bracing structure. If there is an eccentricity between these, twisting will occur—the greater the eccentricity, the greater the twisting effect.

Figure 9.15 shows the case of the *three-sided building*, in which no bracing occurs on one side. While symmetry may occur in one direction, the center of resistance is centered in one exterior wall in the other direction, with a very high eccentricity. This type of structure is highly restricted for use in regions of high seismic risk.

Many buildings are multimassed rather than consisting of a single geometric form. The building shown in Figure 9.16 is multimassed, consisting of an L-shaped tower that sits above an extended lower portion. Under lateral load, the various

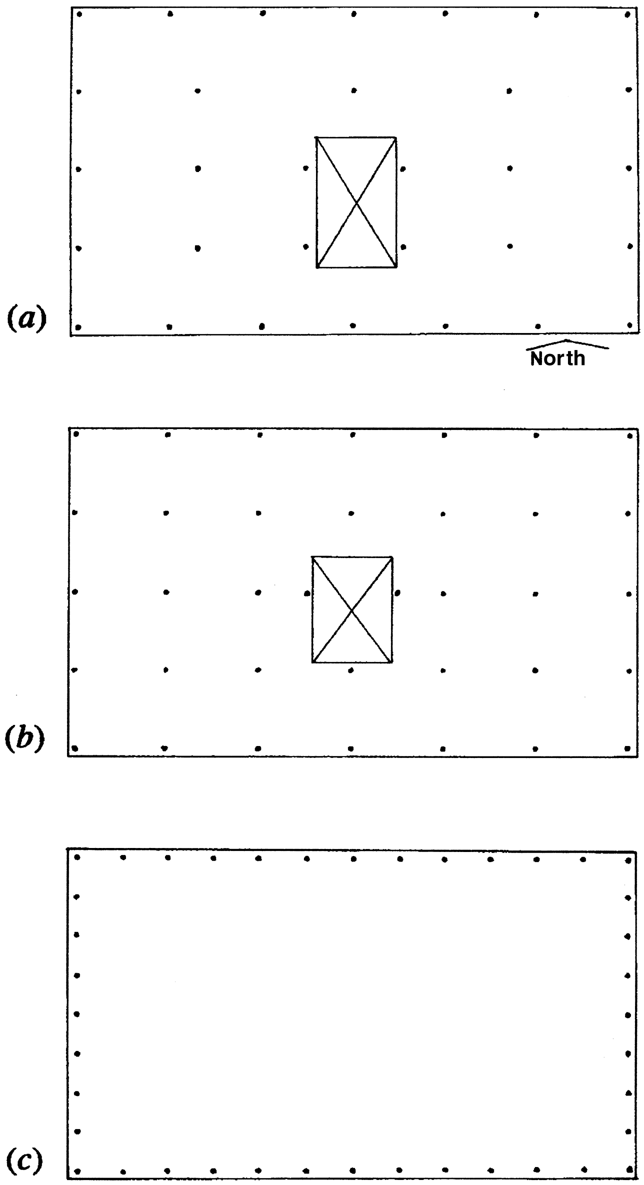


Figure 9.13 Options for layout of rigid frame bracing.

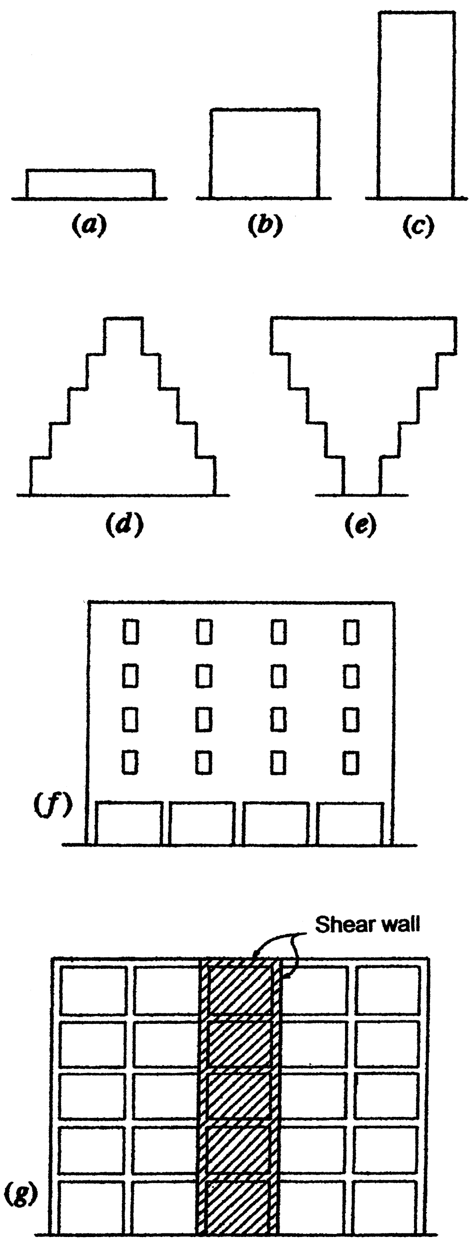


Figure 9.14 Vertical massing considerations for lateral bracing.

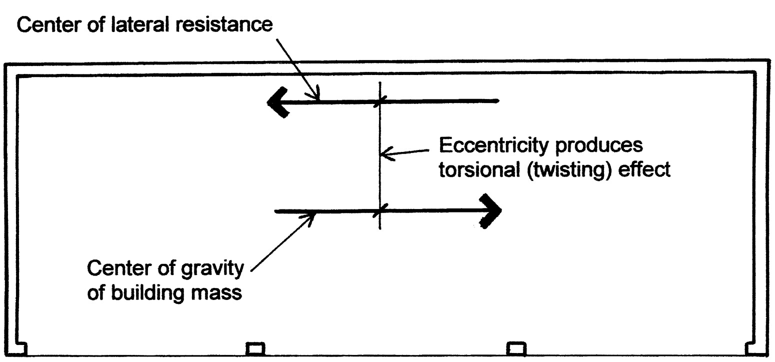


Figure 9.15 Torsional twisting of the three-sided building.

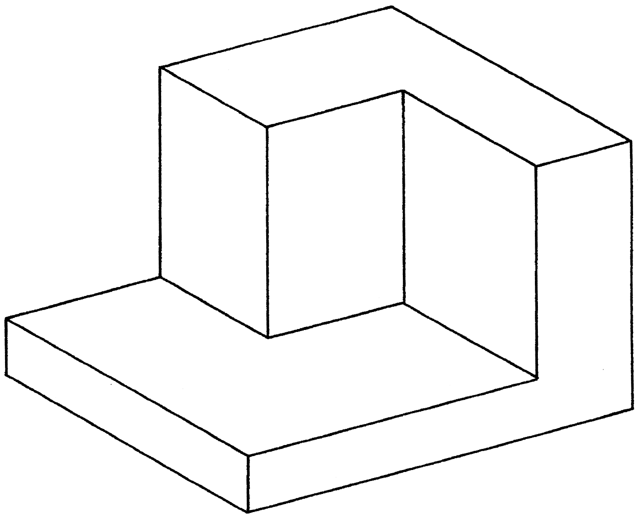


Figure 9.16 Example of a multi-massed building.

parts of this building will have different responses. If the building structure is developed as a single system, the building movements will be very complex, with extreme twisting effects and considerable stress at the points of connection between the discrete parts of the mass. Depending on the magnitude of the lateral forces, it may be possible to provide for these stress conditions, but other options are also possible.

If the two parts of the tower of the building in Figure 9.16 are actually separated, as shown in Figure 9.17*a* or *c*, the independent movements of the separated parts will be different due to their different stiffness. It may be possible to permit these independent movements by providing structural connections that are designed to tolerate the movements and the magnitude of the actual deformations. Thus the twisting effects on the building and the stress at the joints between the parts of the mass may be avoided.

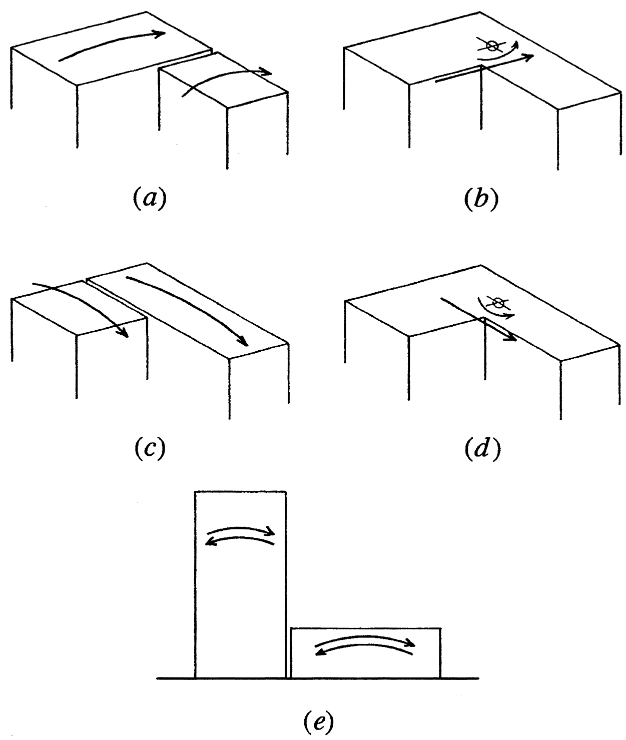


Figure 9.17 Effect of separation in multi-massed buildings.

Separation joints are quite difficult to achieve, as they usually occur at points where no jointing of the building is otherwise needed. Particularly difficult are exterior joints that must achieve continuity of water penetration seals and general weather-resistant functions. Joints for separation under lateral loads may need to retain some transfer functions for gravity loads. Still, joints for these functions have been achieved and shown to work for real earthquake experiences.

There is also the potential for difference in response movements of the tower and the lower portion of the building, as shown in Figure 9.17*e*. Actual structural separation may be created at this point to eliminate stress and simplify the design of the building parts.

However, it may not be feasible to achieve the separation of the structure and to provide for the connections of the rest of the building construction. Some continuity of the nonstructural parts of the building must be developed while achieving weather and water seals, extensions of piping, and other aspects of the general building elements.

The other option, of course, is to not provide structural separation and to design for the stresses between the building parts. A lot of study must be made to determine the best course of action in the design process.

Figure 9.18*a* shows an L-shaped building in which the architectural separation of the masses is accentuated. The linking element, although contiguous with the two parts, is unlikely to be capable of holding them together, especially during an earthquake. If it cannot, there are two forms of differential movement that must be provided for, as shown in Figures 9.18*b* and *c*. In addition to providing for these movements, it is also necessary to consider the bracing of the linkage element. If this element is not capable of being independently braced, it must be attached to one or the other of the larger parts for support, making for a quite complex study of the actions at the connection of the masses. A common example of this situation is the multistory apartment building with corner stairwells (see Figure 9.19).

Figure 9.20 shows three additional problems with linked buildings. When the separate parts move in opposite directions at the same time, the separation dimension must assure that they do not bump each other or be pulled apart (see Figure 9.20*a*). When they move in the same direction, a shearing action occurs at the joint, reaching a critical magnitude at the top of the building (see Figure 9.20*b*). A third action is the horizontal shearing effect at the linkage point, which is the most common type of problem that must be dealt with as it can occur in one-story buildings. The first two behaviors in Figure 9.20 are generally critical only for tall buildings.

Individual joined masses are sometimes so different in size or stiffness that the indicated solution is to simply attach

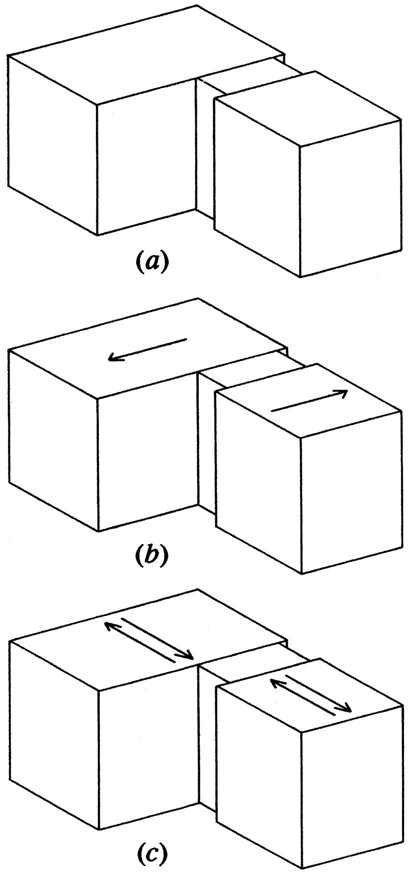


Figure 9.18 Consideration of linkage of building masses.

the smaller part to the larger and let it tag along. Such is the case for the buildings shown in Figures 9.21*a* and *b* in which the small lower portion and the narrow stair tower would be treated as attachments.

In some instances the tag-along relationship may be a conditional one, as shown in Figure 9.21*c*, where the smaller element extends a considerable distance from the larger mass. In this situation the movement of the smaller part to and away from the larger part may be adequately resisted by the attachment. However, some independent bracing of the extended end of the smaller part would be the best solution for movements parallel to the attachment.

The tag-along technique is often used for stairs, chimneys, entry canopies, carports, and other elements that extend from the building general mass. It is also possible, of course, to consider the total structural separation of the parts in some cases, with a linkage attachment that allows for movement between the parts.

Another classic problem of joined elements is that of coupled shear walls. These are shear walls that occur in sets in a single plane and are connected by the continuous construction of the wall. Figure 9.22 illustrates such a situation in a multistory building. The elements that serve to link such walls—in this example the spandrel panels beneath the windows—are wracked by the vertical shearing effect illustrated in Figure 9.22*b*. As the building rocks back and forth in an earthquake, this effect is rapidly reversed, developing the diagonal cracking shown in Figures 9.22*b* and *c*. This results in the X-shaped crack patterns shown in Figure 9.22*d*, which may be observed on the walls of many masonry, concrete, and stucco-surfaced buildings in regions of frequent seismic activity.

Forces applied to buildings must flow with some direct continuity through the elements of the structure, be transferred effectively from element to element, and eventually be resolved into the ground. Where there are interruptions in the normal flow of the forces, problems will occur. For example, in a multistory building the resolution of gravity forces requires a smooth vertical path; thus columns and bearing walls should be stacked on top of each other. If a column is removed in a lower story, a major problem is created, requiring the use of a heavy transfer girder or other device to deal with the discontinuity.

Figure 9.19 Earthquake damage on a low-rise apartment building with a typical donut plan (center courtyard). The linking corner stairwells get chewed up.

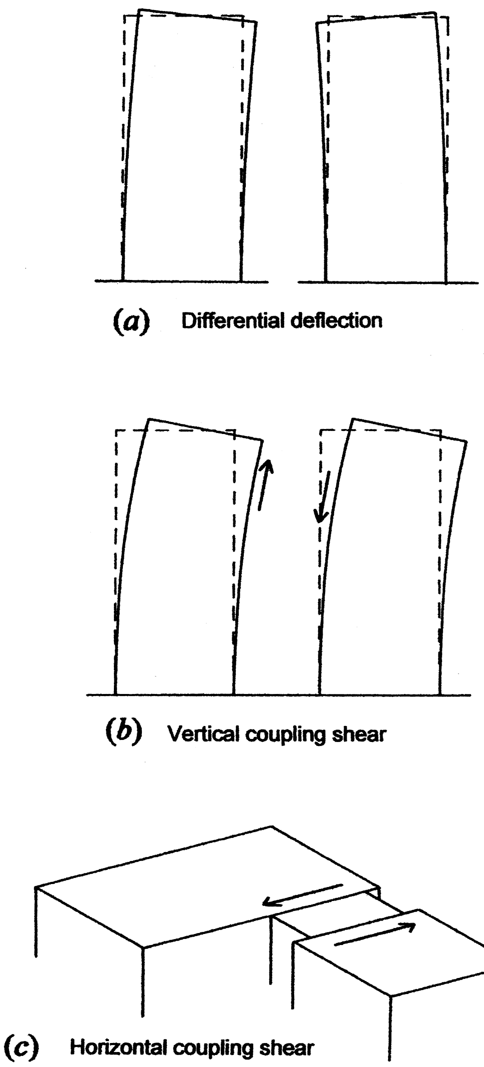


Figure 9.20 Lateral force effects in coupled masses.

A common type of discontinuity is that of openings in vertical or horizontal diaphragms. These can be a problem as a result of the location, size, or even shape. Figure 9.23*a* shows the plan of a horizontal diaphragm with an opening. The diaphragm is braced by four shear walls, and if it is considered to be uninterrupted, it will distribute its load to the walls in the manner of a continuous beam. If the relative size of the opening is as shown in Figure 9.23*a*, this assumption is a reasonable one. What must be done to assure the integrity of the diaphragm is to reinforce the edges and corners of the opening and be sure that the net diaphragm width at the opening is adequate for the shear force at that location.

If the opening in a horizontal diaphragm is as large as that shown in Figure 9.23*b*, it is usually not possible to maintain the continuity of the whole diaphragm. In the example the best solution would be to consider the diaphragm as consisting of four individual parts, each resisting some portion of the total lateral force. These parts are described as *subdiaphragms*. For openings of sizes between the two shown in Figure 9.23, judgment must be exercised as to the best course for design.

Another discontinuity that must sometimes be dealt with is that of the interrupted multistory shear wall. Figure 9.24*a* shows such a situation, with a wall that is not continuous down to its foundation. In this example it may be possible to use the structure at the second-floor level to redistribute the horizontal shear force to other shear walls in the same plane. However, the vertical forces in the shear wall ends, induced by the overturning effect, cannot be redirected, thus requiring that the columns at the ends of the walls continue down to the foundations.

Another case of an interrupted shear wall is shown in Figure 9.24*b*, where two multistory walls are shown as interrupted at the first story. One solution for this situation is to use the structure at the second floor to redistribute the lateral force to other parallel walls. Again, however, the overturn on the upper wall must be accommodated by the end columns being carried directly down to the foundations. Depending on the building planning, it may also be possible to use first-story walls in the same plane as the upper shear walls, as shown for the structure in Figure 9.24*a*.

Figure 9.25 shows an X-braced frame structure with a situation similar to that of the shear wall in Figure 9.24*a*. The individual panels of X-bracing are sufficiently similar in

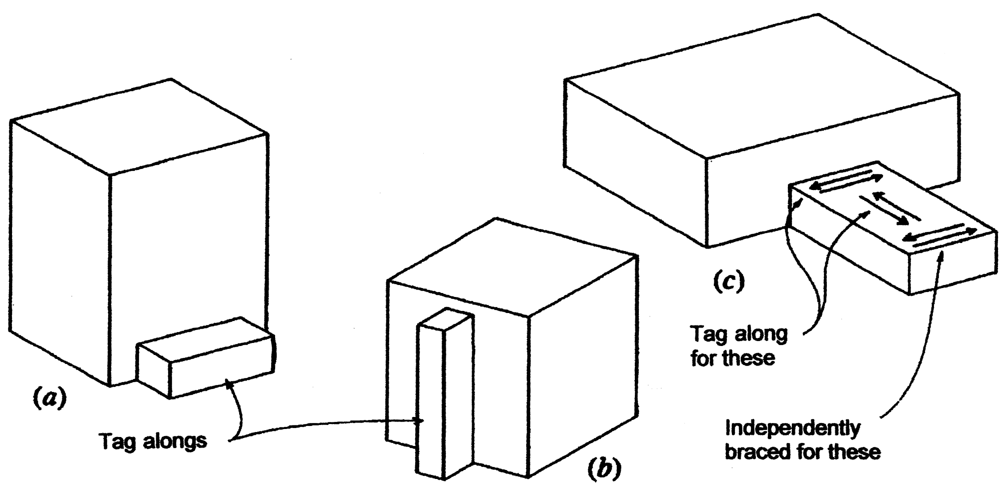


Figure 9.21 Tag-along relationships.

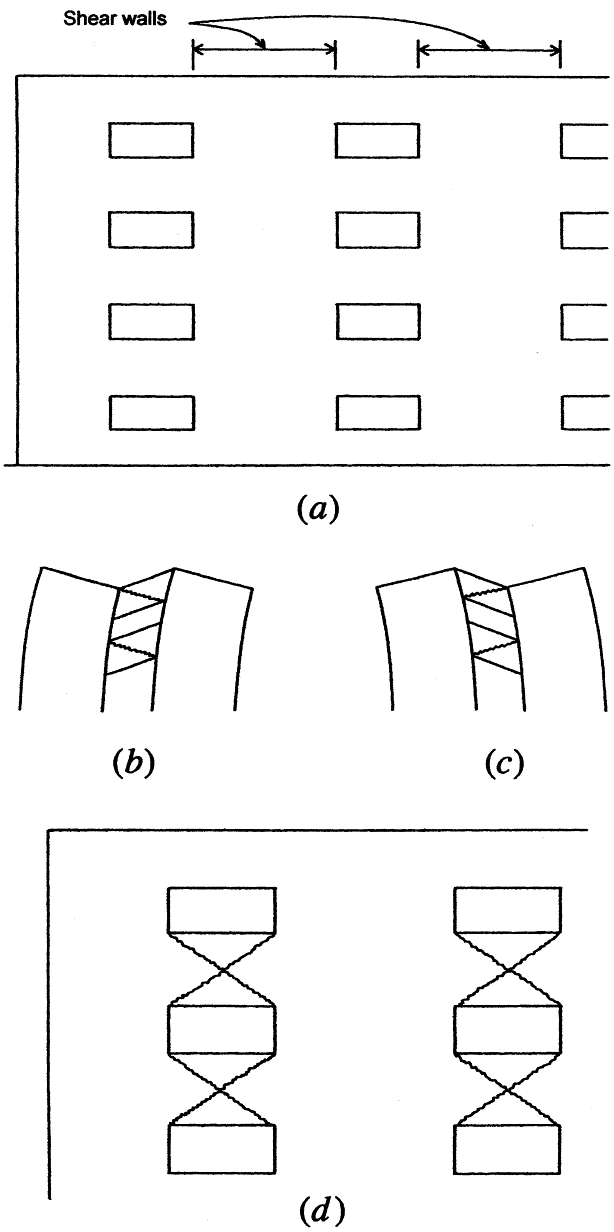


Figure 9.22 Construction failures with coupled shear walls.

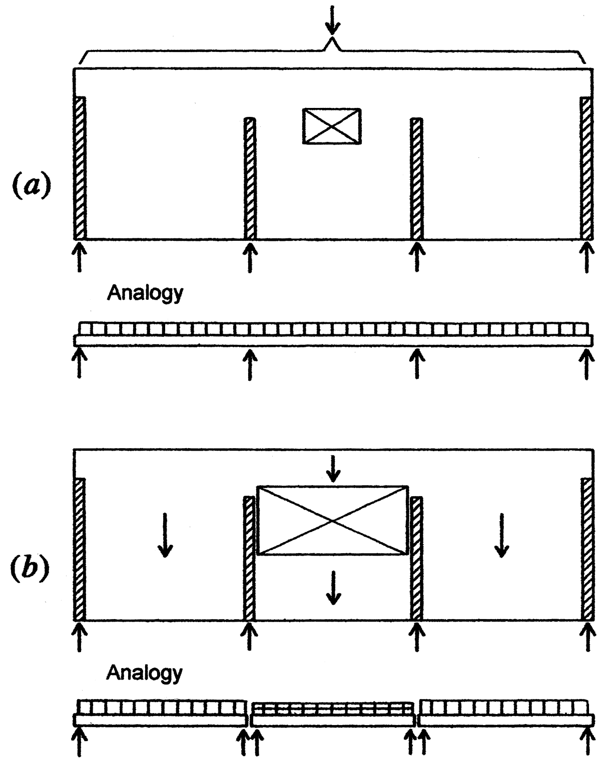


Figure 9.23 Effect of openings in horizontal diaphragms.

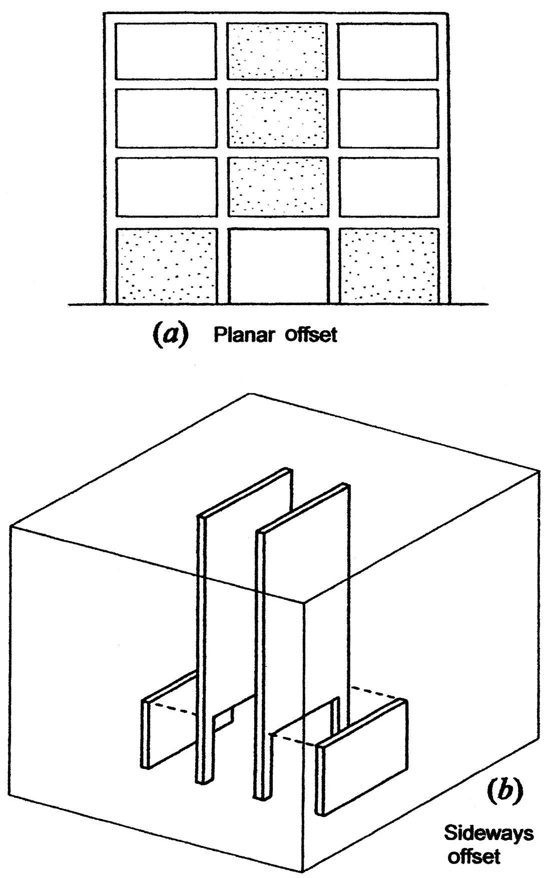


Figure 9.24 Effects of offsets in multi-massed buildings.

function to the individual panels of the shear wall to make the situation have the same general options and requirements for a solution.

Discontinuities are common in multistory and multi-massed buildings. They add to common problems of dissymmetry to create many difficult situations for analysis and design and require careful study for the proper assumptions of behavior and the special needs of the construction. In areas of high risk for earthquakes the codes often require a dynamic analysis for these conditions.

Special Problems

Vulnerable Elements

There are many commonly used elements of buildings that are especially vulnerable to damage due to lateral-force effects

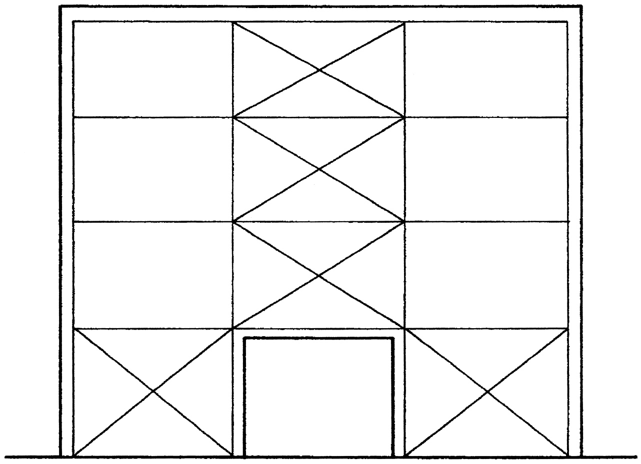


Figure 9.25 Offset trussed bracing.

(see Figure 9.26). Just about anything that sits on top of or hangs on the outside of a building is highly vulnerable to being blown away by high winds. These objects—such as signs, canopies, and balconies—must be securely anchored to hang onto the object and in a way that does not harm the building. Detached objects flying through the air are also potential sources for damage to other buildings, so codes are now quite concerned about this occurrence.

Movements during earthquakes can also damage many nonstructural objects; some typical situations are the following:

Suspended Ceilings. These can swing and bump into other elements of the construction. Their supports may also be subjected to a major downward jolting effect from vertical movements.

Cantilevered Elements. Balconies, canopies, parapets, cornices, and chimneys should be designed for significant seismic force in the direction perpendicular to that of the cantilever. Again, damage to both the cantilevered element and the supporting building must be considered.

Miscellaneous Suspended Objects. Light fixtures, signs, HVAC equipment, loudspeakers, catwalks, and other objects that are supported by hanging must be studied for the potential of pendulum-like movements. They may be fixed at the point of suspension or (preferably) braced with ties or struts.

Piping and Wiring. Movement during seismic actions can cause rupture of piping or conduits for wiring that are installed in a conventional manner. In addition to the usual allowances for thermal expansion, provisions should be made for flexing and extension of the rigid pipes and conduit. In addition, provision must be made for automatic cutoff of services when severe motions occur—especially critical for gas and electric service, a dangerous combination.

Stiff But Weak Elements. Any parts of the building construction that are stiff but not strong are usually subject to damage from movements of the building. This is true for lateral-force effects but is also a consideration for gravity live loads, foundation settlement, thermal expansion, and any other shape change sources. Vulnerable items include window glazing, plastered surfaces, wall and floor tile of ceramic or other cast material, and exterior veneers of brick, tile, or stone. In most cases use of control joints is advised to permit a degree of motion without fractures.

The Soft or Weak Story

Any discontinuity that constitutes an abrupt change in a structure is usually a source of some exceptional distress due to lateral-load effects. This is true for a static load situation but is often of critical concern for dynamic loading. Any abrupt increase or decrease in stiffness will result in some form of magnification of deformation and stress in a

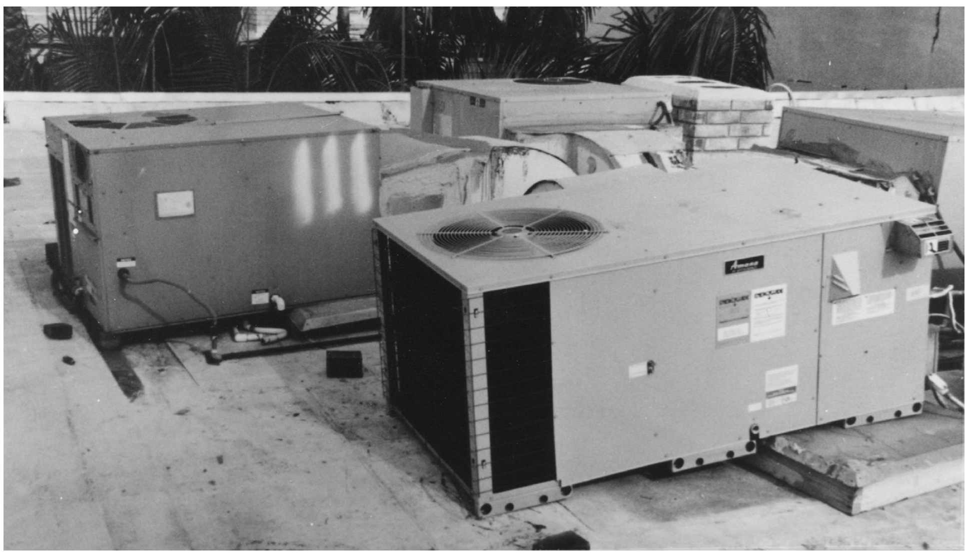


Figure 9.26 These heavy rooftop HVAC units slipped sideways during an earthquake. Lateral movements are often most extensive at the building top, which is, unfortunately, a preferred location for a lot of very heavy equipment. Supports that achieve a degree of lateral movement separation between the building and the supported object are increasingly being used for this situation.

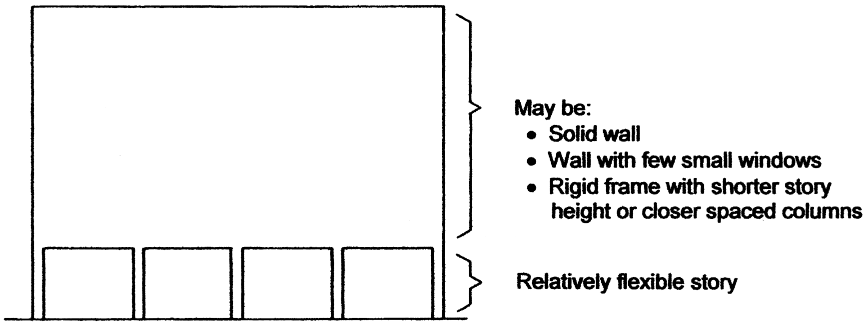


Figure 9.27 The soft-story condition.

structure subjected to energy loading. Openings, notches, necking-down points, and other form variations produce these abrupt changes in either the horizontal or vertical structure. One critical situation of this type is the so-called *soft story*, as shown in Figure 9.27.

The soft story can—and indeed sometimes does—occur at an upper level. However, it is more common at the ground floor level, where it is typically sandwiched between a very stiff foundation below and some rigid structure above. The tall, open ground floor is a popular architectural feature—not always merely a design style but frequently desired for functional reasons.

If the tall, relatively open ground floor is necessary, Figure 9.28 presents some possibilities for having this feature with a reduction of the soft-story effect. The methods shown consist of the following:

Bracing some of the open bays (Figure 9.28*a*). This consists of providing a number of stiff bracing elements around the building perimeter. These may be shear walls, trusses, or very stout rigid frames.

Keeping the building periphery open while providing a rigidly braced interior (core) structure (Figure 9.28*b*).

Increasing the number and/or stiffness of the ground floor columns for an all-rigid-frame structure (Figure 9.28*c*).

Using tapered or arched forms for the ground floor columns to increase their stiffness (Figure 9.28*d*).

Developing a separated, rigid ground floor as an upward extension of a heavy foundation structure (Figure 9.28*e*).

The soft story may potentially be used as a method for providing critical vibration damping or energy absorption. It may thus conceivably have a positive effect. However, the major stress concentrations and deformations must be carefully determined by a dynamic analysis.

Another case of vulnerable discontinuity is that of the *weak story*, which occurs when a single story is disproportionately weaker than stories above or below it. The conditions that produce this are similar to those for the soft story, except that the issue here is strength in terms of energy capacity, rather than stiffness.

The building in Figure 9.29 had a tall first story that could have presented a soft-story condition. However, the structural designer strengthened and stiffened it with very sturdy reinforced concrete members. When a severe earthquake occurred near the building, the first story survived, but the second story suffered from a weak-story condition and pancaked with a complete collapse of the second-floor columns. This caused the upper floors to simply drop vertically and tear free from the end service towers. Oddly enough, there was little apparent damage to the upper curtain wall, except for the missing second floor.

9.2 WIND EFFECTS ON BUILDINGS

Wind is moving air. The air has a particular mass (density or weight) and moves in a particular direction at a particular velocity (speed). It thus has kinetic energy of the form expressed as

$$E = \frac{1}{2}mv^2$$

When the moving fluid air encounters a stationary object, there are several effects that combine to exert a force on the object. The nature of this force, the many variables that affect it, and the translation of the effects into criteria for structural design are dealt with in the following discussion.

Wind Conditions

Of primary concern in wind evaluation is the maximum velocity achieved by the wind. This refers to sustained velocity and not to gust effects. A gust is a pocket of higher velocity wind within the general moving fluid air mass. The resulting effect of a gust is that of a very brief increase, or surge, in the wind velocity, usually of only a few seconds duration and most often consisting of not more than a 15% increase in the sustained wind velocity. Because of its higher velocity and its slamming effect, the gust actually represents the most critical effect of wind in most cases. For design work wind pressures are quoted in relation to sustained velocities, but the quantification of design pressures assumes a corresponding gust effect.

Winds are measured regularly at a large number of locations. The standard measuring point is at 10 m (approximately 33 ft) above the surrounding terrain, which provides a fixed reference with regard to the drag effects of the ground surface and the increase of velocity at higher

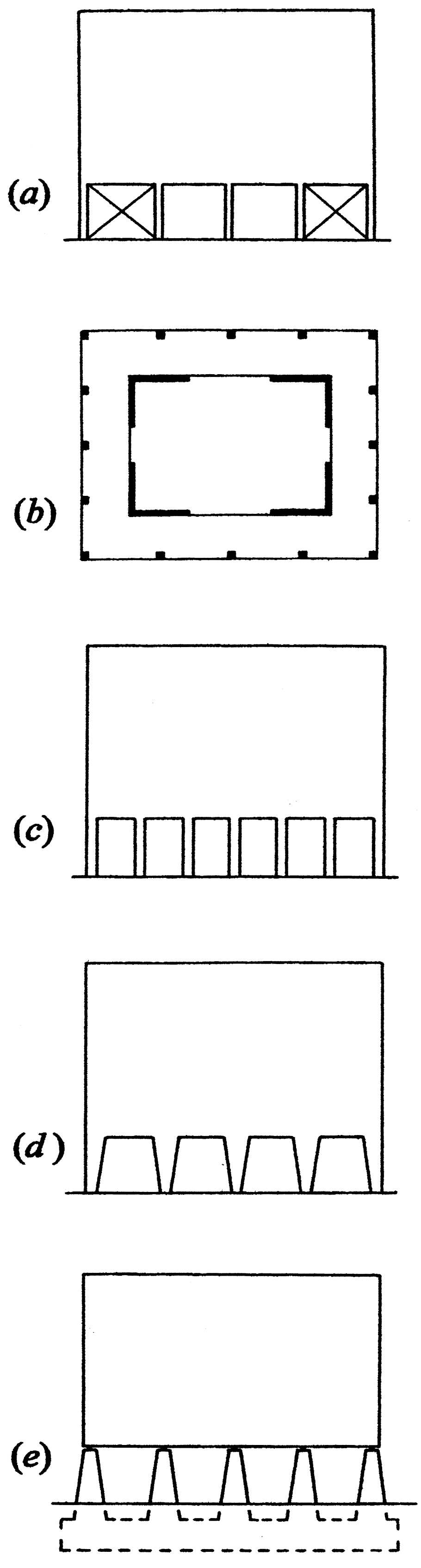


Figure 9.28 Some remedies for the soft story.

locations. The graph in Figure 9.30 shows the correlation between wind velocity, measured in miles per hour, and the pressure generated on a stationary object. The various levels of velocity are indicated with regard to a general evaluation of the effect.

Although wind conditions are usually generalized for a given geographic area, they vary considerably for specific sites because of surrounding terrain, of landscaping, or of nearby structures. Each building design must consider the effects of these conditions.

Wind Effects

The effect of wind on stationary objects in its path includes the following actions (see Figure 9.31):

Direct Positive Pressure. Surfaces facing the wind and perpendicular to its path receive a direct impact effect from the moving mass of air, which generally produces the major portion of force on the object, unless the object is highly streamlined in form.

Aerodynamic Drag. Because the wind does not stop upon encountering a stationary object but flows around it, there is a drag effect on surfaces that are parallel to the direction of the wind. These surfaces may also have inward or outward pressures exerted on them, but it is the drag that adds to the general force on the object in the direction of the wind path.

Negative Pressure. On the leeward side of the object (opposite from the wind direction) there is usually a suction effect, consisting of pressure outward on the surface of the object. By comparison to the direction of pressure on the windward side, this is called negative pressure. This pressure may also be exerted on sides parallel to the wind direction.

These three effects combine to produce a net force on the object in the direction of the wind that tends to move the object along with the wind. In addition to these there are other possible effects on the object that can occur due to the turbulence of the air or to the nature of the object. Some of them are as follows (see Figure 9.31):

Rocking and Flapping Effects. During windstorms, the wind velocity and its direction are seldom constant. Gusts and swirling winds are ordinary, so that an object in the wind path tends to be buffeted, rocked, flapped, and so on. Objects with loose parts, or with connections having some slack, or with highly flexible surfaces (such as fabric surfaces that are not taut) are most susceptible to these effects.

Harmonic Effects. Anyone who plays a wind instrument appreciates that wind can produce vibration, whistling, flutter, and so on. This is a matter of some match between the velocity of the wind and the natural period of vibration of the object or of its parts.

Figure 9.29 Spectacular failure of a weak story.

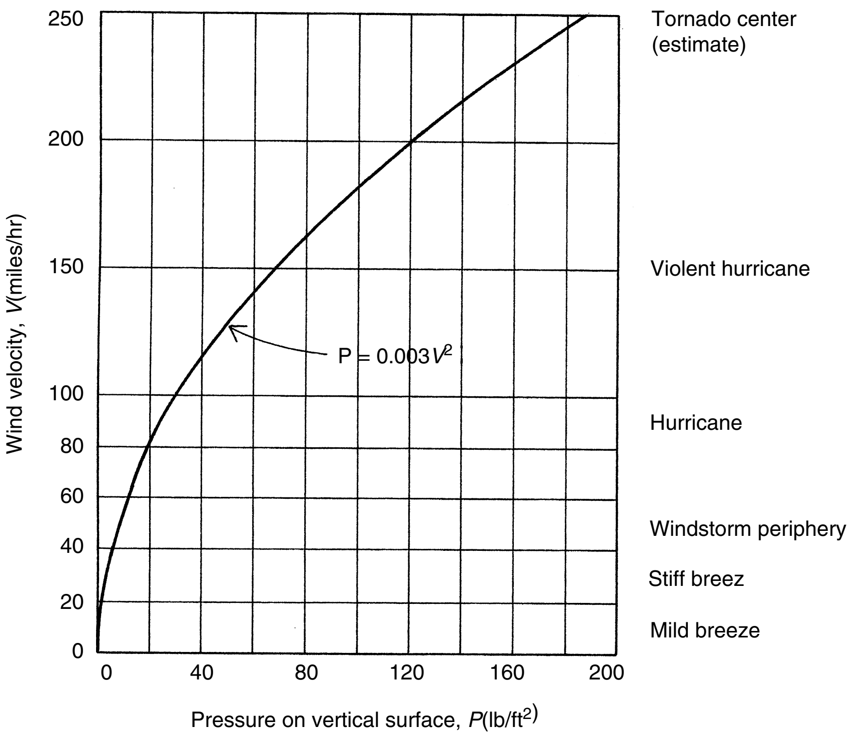


Figure 9.30 Relation of wind velocity and pressure.

Clean-Off Effect. Objects with protruding or attached parts tend to become smoothed off by the wind. Parts of the building such as balconies, parapets, and canopies will get blown away, as will signs, light fixtures, chimneys, and so on.

The critical condition of individual parts or surfaces of an object may be caused by any one or some combination of the above effects. Damage can be partial or local or be total with regard to the object.

If an object is resting on the ground, it may be collapsed or it may be slid, rolled over, or lifted from its position on the ground. Various aspects of the wind, of the object in the path of the wind, and of the surrounding environment will affect the critical wind effects. With regard to the wind itself some considerations are:

Magnitude of sustained velocities
Duration of high-level velocities
Presence of gusts, swirling, and so on
Prevailing direction of the wind (if any)

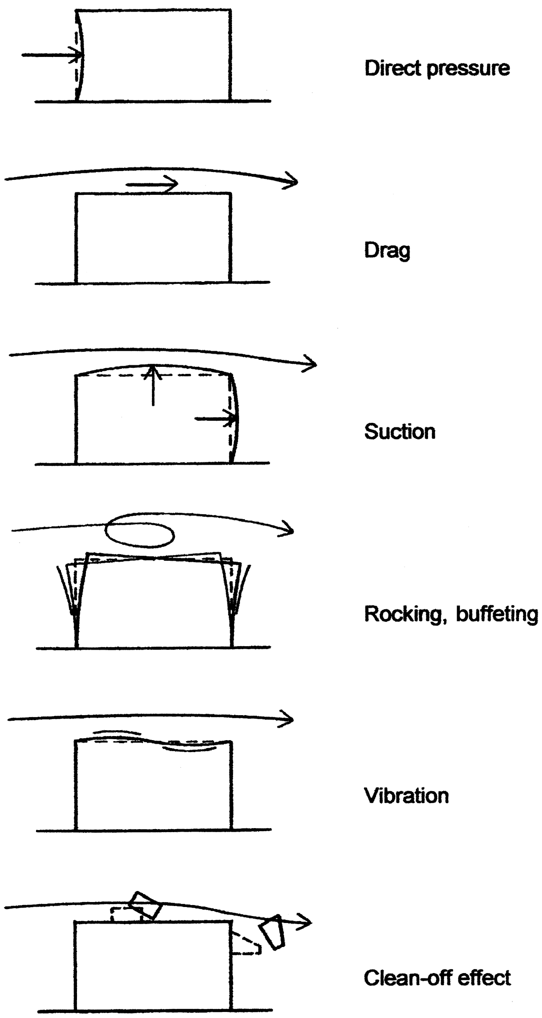


Figure 9.31 General effects of wind.

With regard to objects some considerations are:

Size of the object (relates to the relative effect of gusts, variations in pressures at some distance above ground, etc.)
Aerodynamic shape of the object
Period of vibration of the object or of its parts
Stiffness of surfaces, tightness of joints, and so on

With regard to the environment, possible effects may result from the sheltering or funneling caused by ground forms, landscaping, or adjacent structures. These effects may result in an increase or a reduction of the general wind effects or they may cause turbulence and a very unsteady wind condition.

The true behavior of an object during a wind storm can only be found by subjecting it to a real wind situation. Wind tunnel tests in a laboratory are useful because they can create tests on demand, as they have done to create a background for data and procedures used in design.

The major effects on buildings can be generalized to some degree because we know a bracketed range of characteristics that cover the most common conditions. Some of the generalizations made are as follows:

Most buildings are boxy and bulky in shape.
Most buildings have closed and relatively smooth surfaces.
Most buildings fit snugly to the ground.
Most buildings have relatively stiff structures.

These and other considerations allow a simplification of wind investigation by permitting a number of variables to be eliminated or lumped into a few modifying constants. If a building does not comply with the assumed conditions, codes may provide other forms of investigation, which will probably be more complex. For very unusual situations, such as elevated buildings, open structures, highly flexible structures, and any unusual aerodynamic shapes, it may be advisable to do a more thorough investigation, including the possible use of wind tunnel tests.

Design Considerations

Basic Design Wind Pressure

The primary effect of wind is visualized in the form of pressures normal to the building's exterior surface. The basis for this pressure begins with a conversion of the kinetic energy of the moving air mass into an equivalent static pressure using the basic formula from fluid flow mechanics,

$$p = Cv^2$$

in which C is a general constant accounting for the air mass, the units used, and a number of the assumptions previously described. With wind in miles per hour and the pressure in pounds per square inch, the average C value for the total effect on a simple box-shaped building is 0.003; this is the value used in plotting the graph in Figure 9.30. It should be noted that this pressure, called the *base pressure*, represents not the actual effect on a building surface but rather the total effect on the building from all the wind effects.

Inward Pressure on Exterior Walls

Surfaces directly facing the wind are usually required to be designed for the full base pressure, although this is somewhat conservative, because the windward force usually accounts for only about 60% of the total force on the building. However, gusts will usually have their major impact on the windward wall, so the design practice is not quite so unbalanced.

Overall Horizontal Force on the Building

As described before, this total force is an aggregate of the effects on all the building surfaces. There are presently two possible methods for determining this force. A simplified method uses the base pressure applied to the profile of the building as a single force but is limited to relatively simple cases. If a building does not qualify for the simplified method, a more complex method must be used, consisting of determination of the force on each surface of the building.

Horizontal Sliding of Building

In addition to the possible collapse of the lateral resistive system, there is the chance that the building may be simply slid off its foundations. For a tall building with a shallow foundation there may also be a problem regarding development of lateral soil pressure for resistance. The weight of the building generates a friction at the foundation-to-soil interface; otherwise resistance must come from passive horizontal soil pressure on the leeward side of the foundation structure.

Overturn Effect

As with horizontal sliding, the dead weight tends to resist the overturn, or toppling, effect. Overturn may be a problem for the building as a whole but is more often a problem for individual bracing elements within the structural system consisting of individual shear walls, trussed bents, or rigid-frame bents.

Wind on Building Parts

Elements projecting from the building are vulnerable for the previously discussed clean-off effect. Codes sometimes require these to be designed for pressures higher than the base pressure. Gusts are particularly hard on these usually smaller elements.

Harmonic Effects

Vibration, fluttering, whipping, and multimodal swaying can only be determined by a true dynamic analysis.

Effect of Openings

If the surface of a building is closed and reasonably smooth, the wind will slip around it in a fluid flow. Openings or building forms that tend to cup the wind can greatly affect the total wind force on the building. It is quite difficult to account for these effects in a static force analysis, except in a very empirical manner. Cupping of the wind can be a major effect when the entire side of a building is open, for example. Garages, hangars, band shells, and other buildings of similar form must be designed for an increased force that can only be estimated unless a wind tunnel test is performed.

Torsional Effect

If a building is not symmetrical in terms of its wind profile or if the lateral resistive system is not symmetrical within the building, the wind force may produce a twisting effect. This effect is the result of a misalignment of the center of the wind force and the center of stiffness of the lateral resistive system, which will produce an added force on some of the elements of the structure.

Building Code Requirements for Wind

Where wind is a regional problem, local codes are often developed in response to local conditions. Complete design for wind effects on buildings includes a large number of both architectural and structural concerns. The following is a discussion of some of the requirements from The American Society of Civil Engineers (ASCE, Ref. 1).

Basic Wind Speed

This is the maximum wind speed (or velocity) to be used for specific locations. It is based on recorded wind histories and adjusted for some statistical likelihood of occurrence. For the United States recommended minimum wind speeds are taken from maps provided in the ASCE standard. As a reference point, the speeds are those recorded at the standard measuring position of 10 m (approximately 33 ft) above the ground surface.

Wind Exposure

This refers to the conditions of the terrain surrounding the building site. The ASCE standard uses three categories, labeled B, C, and D. Qualifications for categories are based on the form and size of wind-shielding objects within specified distances around the building,

Simplified Design Wind Pressure (p_s)

This is the basic reference equivalent static pressure based on the critical wind speed and is determined as

$$p_s = \lambda I p_{s30}$$

where

λ = factor for building height and exposure
I = importance factor
p_{S30} = simplified design wind pressure for exposure B, at height of 30 ft, and for $I = 1.0$

The importance factor for ordinary circumstances of building occupancy is 1.0. For other buildings factors are given for facilities that involve hazard to a large number of people, for facilities considered to be essential during emergencies (such as windstorms), and for buildings with hazardous contents.

The design wind pressure may be positive (inward) or negative (outward, suction) on any given surface. Both the sign and the value for the pressure are given in the design standard. Individual building surfaces, or parts thereof, must be designed for these pressures.

Design Methods

Two methods are described in the ASCE standard for the application of wind pressures:

Method 1 (Simplified Procedure). This method is permitted to be used for relatively small, low-rise buildings of simple symmetrical shape. It is the method described here and used for the examples in Chapter 10.

Method 2 (Analytical Procedure). This method is much more complex and is prescribed to be used for buildings that do not fit the limitations described for method 1.

Uplift

Uplift may occur as a general effect, involving the entire roof or even the whole building. It may also occur as a local phenomenon such as that generated by the overturning moment on a single shear wall

Overturning Moment

Most codes require that the ratio of the dead-load resisting moment (called the restoring moment, stabilizing moment, etc.) to the overturning moment be 1.5 or greater. When this is not the case, uplift effects must be resisted by anchorage capable of developing the excess overturning moment. Overturning may be a critical problem for the whole building, as in the case of relatively tall and slender tower structures. For buildings braced by individual shear walls, trussed bents, and rigid-frame bents, overturning is investigated for the individual bracing units.

Drift

Drift refers to the horizontal deflection of the structure due to lateral loads. Code criteria for drift are usually limited to requirements for the drift of a single story (horizontal movement of one level with respect to the next above or below). As in other situations involving structural deformations, effects on the building construction must be considered; thus the detailing of curtain walls or interior partitions may affect limits on drift.

Special Problems

The general design criteria given in most codes are applicable to ordinary buildings. More thorough investigation is recommended (and sometimes required) for special circumstances such as the following:

Tall Buildings. These are critical with regard to their height dimension as well as the overall size and number of occupants inferred. Local wind speeds and unusual wind phenomena at upper elevations must be considered.

Flexible Structures. These may be affected in a variety of ways, including vibration or flutter as well as simple magnitude of movements.

Unusual Shapes. Open structures, structures with large overhangs or other projections, and any building with a complex shape should be carefully studied for the special wind effects that may occur. Wind tunnel testing may be advised or even required by some codes.

Examples of investigation for wind effects on buildings are presented in the design cases in Chapter 10.

General Design Considerations for Wind

The relative importance of design for wind as an influence on the general building design varies greatly among buildings. The location of the building site is a major consideration, with the basic design pressure varying considerably from the lowest risk wind area to the highest. Other important variations include the dead weight of the construction, the height of the building, the type of structural system (especially the lateral resistive system), the aerodynamic shape of the building, and the existence of large openings, recessed portions of the surface, and so on.

The following is a discussion of some general issues of design of buildings for wind effects. Any of these factors may be more or less critical in specific situations.

Influence of Dead Load

Dead weight of the building construction is generally an advantage in wind design because of its stabilizing effect in resisting uplift, overturn, sliding, and vibration or flutter. However, the stresses that result from various load combinations, all of which include dead load, may offset these gains when dead load is excessive. Foundation settlement in soft soils is usually due primarily to dead load, for example.

Critical Shape Considerations

Various aspects of the building form can cause increase or reduction in wind effects. Some potentially critical situations, as shown in Figure 9.32, are as follows:

Flat versus curved forms as they pertain to aerodynamic shape

Building height as related to various responses and to wind pressure change

Openings in the building surface that cup the wind

Projections of any kind that increase drag and require design for their own responses

Relative Stiffness of Structural Elements

In most buildings the lateral bracing consists of two elements: the horizontal distributing system and the vertical system. An important aspect of the behavior of elements of these systems relates to their relative stiffness. Various situations that can occur are discussed in Section 9.4 and are illustrated in the building design case examples in Chapter 10.

Stiffness of Nonstructural Elements

When the vertical elements of the lateral resistive system are relatively flexible, as with rigid frames and wood shear walls that are short in plan length, considerable lateral force may be transferred to nonstructural elements of the building construction. Wall finishes of masonry veneer, plaster, and even drywall can produce rigid planes whose stiffness exceeds that of the structure to which they are attached. If this is the case, the finish material may take the load initially, with the structure going to work only after the finish fails. This result

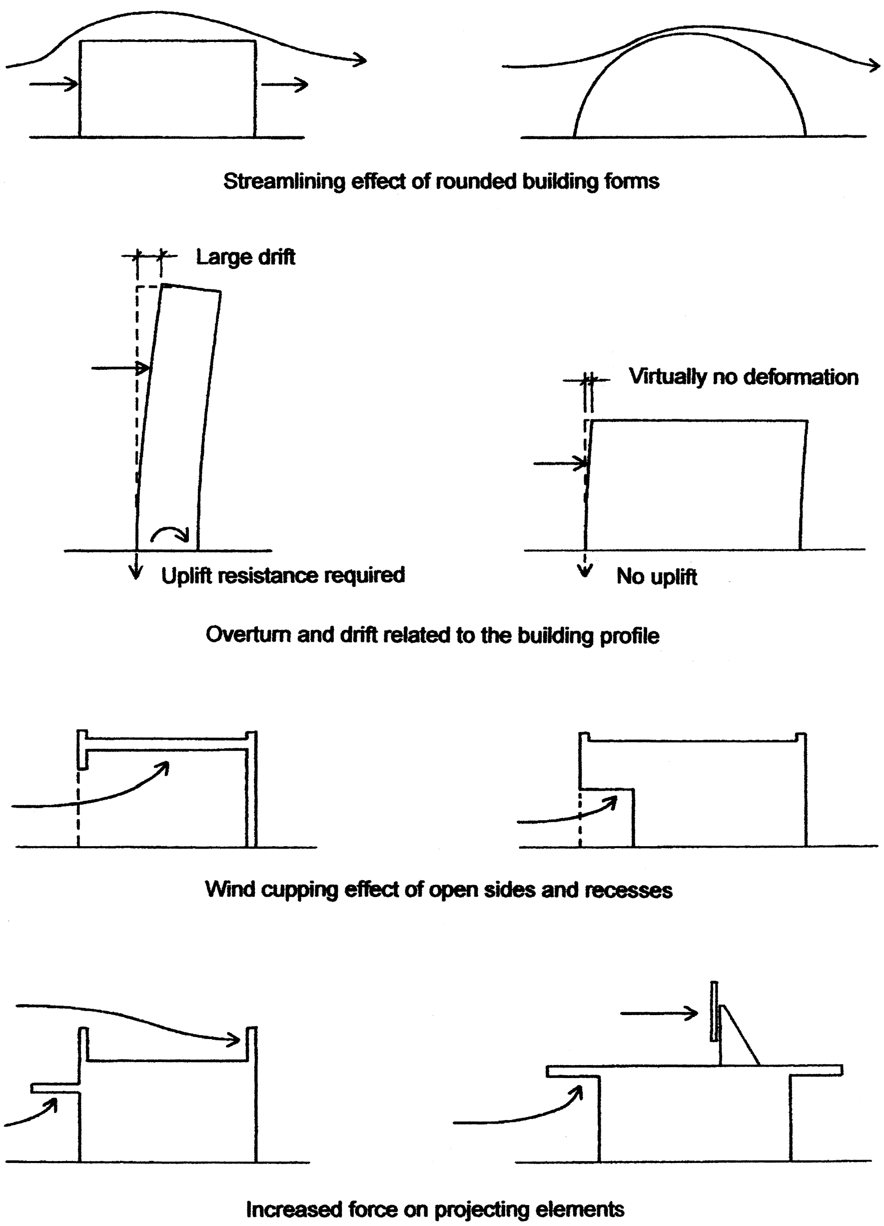


Figure 9.32 Wind effects related to building form.

is not entirely a matter of relative stiffness but also relates to the method of attachment of the finish to the structure. Development of the construction details should be done with careful consideration of this problem.

Allowance for Movement of the Structure

All structures deform when loaded. The actual dimension of movement may be insignificant, as in the case of a sitecast concrete shear wall, or it may be considerable, as in the case of a slender rigid frame. The effect of these movements on other elements of the building construction must be considered. The case of transfer of lateral load to nonstructural finishes, as just discussed, is one example of this problem. Another common example is that of windows and doors. Glazing must be installed so as to allow for some movement of the glazing with respect to the frame. Window and door frames must be installed with some allowance for movement of the frame with respect to the structure. Otherwise, load may be transferred to the windows and doors may be jammed.

9.3 EARTHQUAKE EFFECTS ON BUILDINGS

Following a major earthquake, it is usually possible to trace its history through the recorded seismic shocks for the region over an extended period of time. Foreshocks and aftershocks can be identified, extending over a considerable period of time, some of which may themselves be significant events.

Earthquakes are usually rather short in duration, often lasting only a few seconds and seldom more than a minute or so. During the earthquake, there are usually one or more major peaks of magnitude of motion. These peaks represent

the *intensity,* or magnitude, of the quake. This is a dynamic energy event, and intensity relates more specifically to the acceleration, rather than simple motion of the ground.

Modern recording equipment and practices provide us with representations of the ground movements at various locations, thus allowing us to simulate the effects of major earthquakes. Figure 9.33 is a symbolism of the graphic form of a recording made during an earthquake. In this example the graph is plotted in terms of acceleration of the ground, expressed in a percentage of the acceleration of gravity, as it varies over time. Using this recording, it is possible to do a playback of the event and apply the motion to a laboratory model of a structure.

Laboratory playbacks are used in research and sometimes in design of lateral resistive structures. Most building design work, however, is done with data, criteria, and procedures that have evolved through a combination of practical experience, theoretical studies, and some empirical relationships derived from research and testing. The results of the current collective knowledge are put forth in the form of recommended design procedures that are incorporated in the model building codes.

Although it may seem like a gruesome way to achieve it, we advance our level of competency in design every time there is a major earthquake that results in some major structural damage to buildings. Engineering societies and other groups routinely send investigating teams to the sites of major earthquakes to report on the effects on buildings in the area. Of particular interest are the effects on recently built buildings, because these are in effect full-scale tests of the validity of our most recent design techniques. Response of older buildings is usually predictable, based on observations from many previous earthquakes; it is what we are doing now in design that is our major concern. Each new edition of the model building codes usually reflects some results of this cumulative growth of knowledge culled from the latest disasters.

Effects of Earthquakes

Ground movements caused by earthquakes can have several types of damaging effects. Some of the major effects are as follows:

Direct Movement of Structures. This is the movement of structures caused by their attachment to the ground. The two major effects of this motion are a general destabilizing effect caused by the rapid back-and-forth and up-and-down motion of the structure and a series of forces generated by the momentum and inertia of the structure.

Ground Surface Faults. Surface faults may consist of cracks, vertical shifts, general settlement of an area, landslides, and so on. These may be major or minor, with minor ones causing small damage and major ones causing collapse of the structure.

Tidal Waves. Ground shocks under water can set up large waves on the surface of bodies of water that may cause major damage to shoreline areas.

Flooding, Fires, Gas Explosions, and So On. Ground faults or movements may cause damage to dams, reservoirs, river banks, buried pipelines, and so on. These can cause independent disasters.

Although all of these possible effects are of concern, we deal in this book only with the first effect—the direct motion of structures. Concern for this effect motivates us to provide for some degree of dynamic stability (general resistance to shaking) and some quantified resistance to energy loading of the structure.

The force effect caused by motion is generally directly proportional to the dead weight of the structure—or, more precisely, to the total dead weight borne by the structure. This weight also partly determines the character of dynamic response of the structure. The other major influences on the structure's response are its fundamental period of vibration and its efficiency in energy absorption. The vibration period is

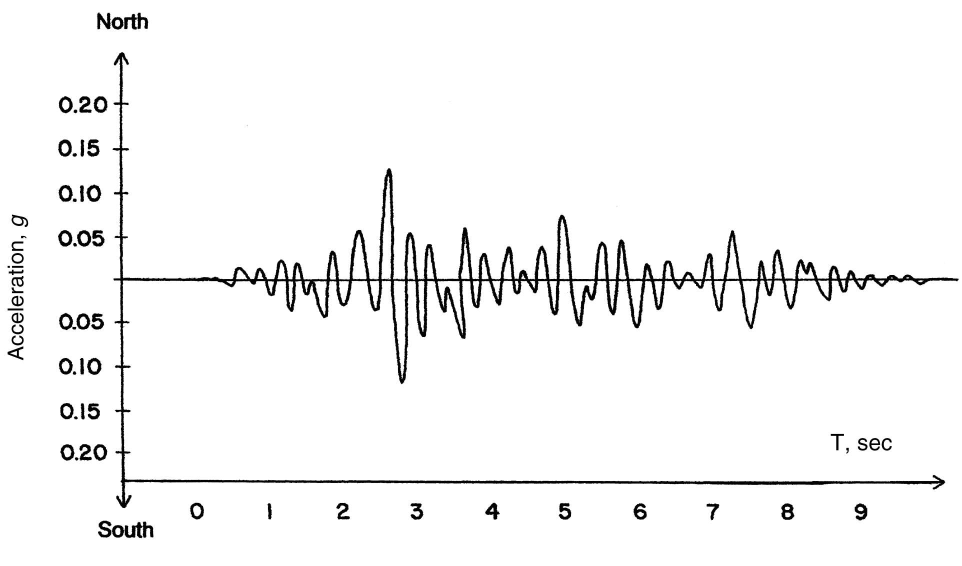


Figure 9.33 Characteristic form of ground acceleration graph for an earthquake.

basically determined by the mass, the stiffness, and the size of the structure. Energy efficiency is determined by the elasticity of the structure and by various factors such as stiffness of supports, the number of independently moving parts, and the rigidity of connections.

A relationship of major concern is that which occurs between the period of the structure and that of the earthquake. Figure 9.34 shows a set of curves, called *spectrum curves*, that represent this relationship as derived from a large number of earthquake "playbacks" on structures with different periods. The upper curve represents the major effect on a structure with no damping. Damping of the motion of the structure results in a lowering of the magnitude of the effects, but a general adherence to the basic form of the response remains.

The general interpretation of the spectrum effect is that the earthquake has its major direct force effect on buildings with shorter fundamental periods. These tend to be buildings with stiff lateral bracing systems, such as shear walls and X-braced frames, and with buildings that are small in size and/or squat in profile.

Damping may occur due to the materials or construction details of the structure. A notable source of this type of damping is that occurring due to yielding and deformation of connections. However, the major usual source of damping is that due to the nonstructural parts of the building construction and their interference with the smooth vibration of the structure.

For very large, flexible structures, such as tall towers and high-rise buildings, the fundamental period may be so long that the upper part of the structure does not respond to the ground movements until is has been through a few back-and-forth cycles. This can produce a whiplash effect, with different levels of the structure moving in opposite directions at the same time, as shown in Figure 9.35. Analysis for this behavior requires the use of dynamic methods that are beyond the scope of this book.

The three general cases of structural response are illustrated in Figure 9.36. Referring to the spectrum curves in Figure 9.34, for buildings with periods below that representing the upper cutoff of the curves (approximately 0.3 sec), the response is that of a rigid structure with virtually no flexing. For this case, the earthquake force is reduced only by damping and the structural response is basically one of shear.

For buildings with a period slightly higher than 0.3 sec but lower than 1.0 sec, there will be a steady decrease in the magnitude of the earthquake force. Structural response here will be a combination of shear and flexure. This effect is due to the measurable motion of the building, which uses up some of the energy of the earthquake force.

As the building period increases above 1.0 sec, the behavior approaches that for the slender tower in Figure 9.35, with the actual movement and flexing of the building strongly affecting its response to the earthquake force.

In addition to the movement of the building as a whole, there are the independent movements of distinct parts of the building. These will each have their own periods of vibration, and the total motion occurring in the structure can thus be quite complex.

General Effects on Buildings

The principal concern in structural design for earthquake forces is for the behavior of the lateral resistive structural system of the building. Failure of any part of this system, or of connections between the parts, can result in major damage to the building, including the possibility of total collapse.

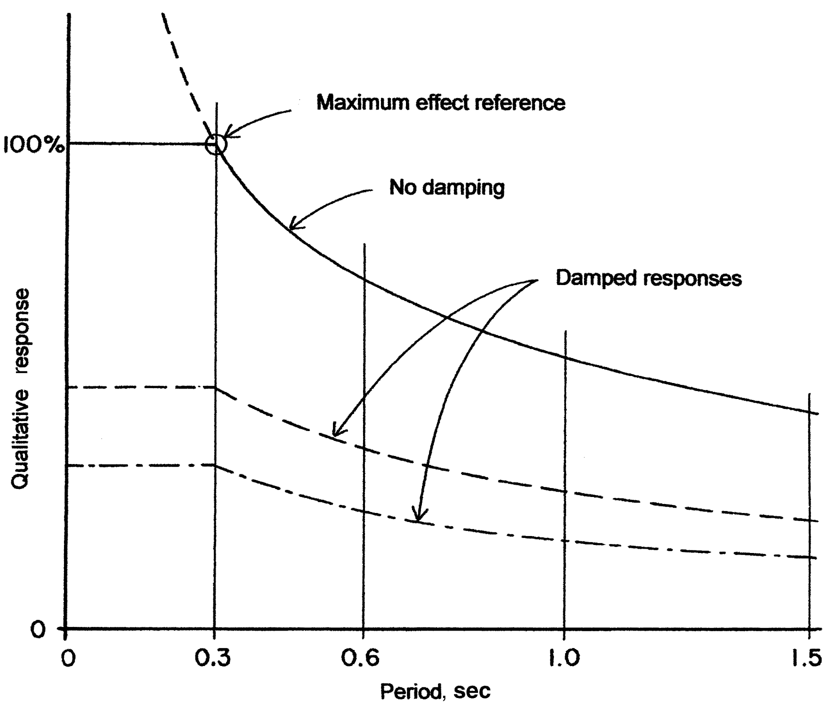


Figure 9.34 Spectrum response curves.

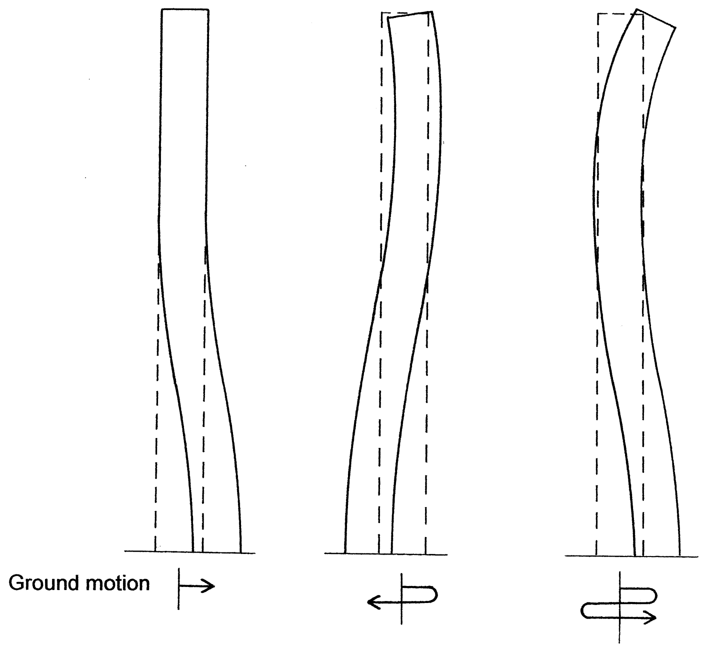


Figure 9.35 Earthquake motion of a tall building.

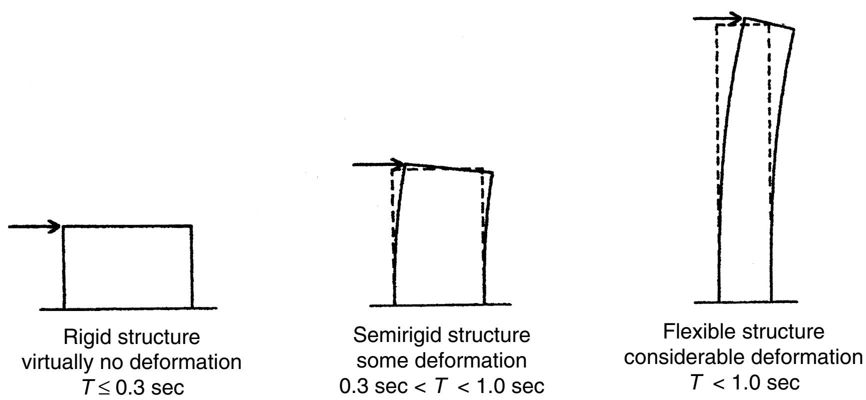


Figure 9.36 Seismic response of buildings with different fundamental periods.

Even if there is no significant damage to the bracing system, its deformations may result in considerable damage to other parts of the building. Safety is the critical concern, but for the building to remain usable after the earthquake, nonstructural damage should also be considered.

It is well to remember, however, that the earthquake shakes the whole building. The survival of the structure is critical, but it is a limited accomplishment if suspended ceilings fall, windows shatter, pipes burst, and elevators are derailed.

A major design consideration is that of tying the building together so that it is literally not shaken apart. With regard to the structure, this means that the various separate elements must be positively secured to one another. The detailing of the construction connections is a major part of the structural design for earthquake resistance.

In some cases it is desirable to allow for some degree of independent motion of parts of the building. This is especially critical in situations where a secure attachment between the structure and various nonstructural elements, such as window glazing, can result in undesired transfer of force to the nonstructural parts. In these cases use must be made of connecting materials and details that allow for the securing of the parts in place while still permitting relative independence of motion.

Design for lateral loads due to earthquakes is in many ways similar to that for the horizontal forces due to wind. The actions of resistance by the bracing structure are mostly of the same character. Just as for wind, however, there are many special considerations for seismic actions. The discussion in the next section deals with the use of code provisions for analysis and design for earthquakes.

Building Code Requirements for Earthquake Effects

Model building codes, such as the *International Building Code* (IBC, Ref. 2), generally present the most up-to-date, complete guidelines for design for earthquakes. Model codes are usually revised into new editions every few years, and earthquake design criteria and procedures are changed with every new edition.

The edition of the IBC used for this book contains a vast amount of material that describes the requirements for

designing for earthquakes. A full explanation of this material is well beyond the scope of this book. The following discussion describes the procedures in general terms and summarizes the various critical issues involved.

A critical determination for seismic design is what is called the *base shear*, which is essentially the total lateral shear force assumed to be delivered by the building to its below-grade lateral support—typically meaning the building foundations. In addition to the delivery at the base, this force is distributed in some fashion throughout the lateral resistive structural system.

As an equivalent static force effect, the base shear is expressed as some percent of the building weight. In early design codes this took the simple form of

$$V = 0.1W$$

or simply 10% of the building weight. This simple formula was embellished over the years to incorporate variables that reflect such issues as the degree of potential risk due to the location of the site, the dynamic response of the building, potential building–site interaction, and the relative importance of the building for postevent disasters. Thus the formula for base shear now takes a general form that may be described as

$$V = QW$$

in which the modifying factor Q represents a summary of a large number of variables. The principal issues dealt with by these variables are as follows:

Degree of Risk. This relates to the region in which the site exists, with a level of risk assigned to the region. Maps are used by the codes to identify these regions and their degree of risk.

Proximity to Seismic Faults. Extensive geological studies have identified the locations of many major seismic faults, typically indicated by lines on a map. The distance of a site from one of these faults establishes another layer of concern for risk.

Importance. Buildings are classified and assigned a degree of importance. This refers to considerations such as the importance of the building in terms of aid after the earthquake, the number of people typically occupying the building, and the presence of hazardous contents in the building.

Nature of the Building's Seismic Response. This includes considerations for a number of issues, such as the shape of the building, the degree of irregularity of the structure, the type of lateral bracing system, the size of the building (in particular its height), and the fundamental period of the building response to lateral movement.

Redundancy in the Bracing Structure. Many structures fail in earthquakes because of the failure of a single component of the bracing system. The codes now penalize for this condition and promote the design of a bracing system with redundancy consisting of multiple elements or multiple modes of failure.

Regularity of the Bracing Structure. Codes now define conditions for qualification of a structure as *regular* and, by inference, one that is *irregular*. This condition may relate to the building plan or to the form of the vertical elements of the bracing structure. An irregular condition is not necessarily forbidden; it merely means that a higher lateral force will be required and a dynamic analysis for the behavior of the structure may also be required.

Site–Structure Relations. Site conditions may produce a variety of problems, some of which are not solely related to seismic activities and some that are. One that the code deals with is that of so-called *structure–site interaction.* This refers to dynamic conditions that involve the response of the site and that of the building and their simultaneous occurrence.

Irregularity of the Structure

The conditions producing irregularity are specified in code tables. Although the code reference is to structures, it is ordinarily assumed that the general building form also has a major influence on behavior when it relates to the form of the structure.

Figure 9.37 illustrates a number of form irregularities. Descriptions in Figure 9.37 include several types mentioned in earlier discussions in the chapter, such as the three-sided building, the soft story, the diaphragm with openings, the interrupted vertical structure, and the multimassed building.

Concern for the conditions shown in Figure 9.37 is presented not as an argument against diversity in architectural form but merely to make designers conscious of implications for seismic response. Simplicity, symmetry, and regularity are all desirable conditions for structural purposes, but there are many other possible concerns for building design in general.

One possible solution with complex building forms is to divorce the form of the structure from that of the building to some degree. Figure 9.38 shows two cases where this is partly achieved. In the plan view, the exterior wall is seen to meander, albeit in a symmetrical manner. If this is accepted as a *structural* definition of the perimeter, the result is a complex, multimassed, irregular structure. As defined by the columns, however, the structure is quite orderly and—if developed as the bracing system—neatly eliminates all structural irregularities in the plan. The seismic lateral load generated by the meandering walls is simply grabbed by the horizontal structure and distributed to the highly regular vertical bracing structure.

Figure 9.38*b* also shows a vertical section through a multistory building with a series of setbacks—not very good form for structural regularity, especially when asymmetrical and abruptly occurring as shown here. However, the bracing is achieved with a large trussed element within the building. This bracing system is quite regular and a very simple solution, assuming it can be integrated in the interior space planning.

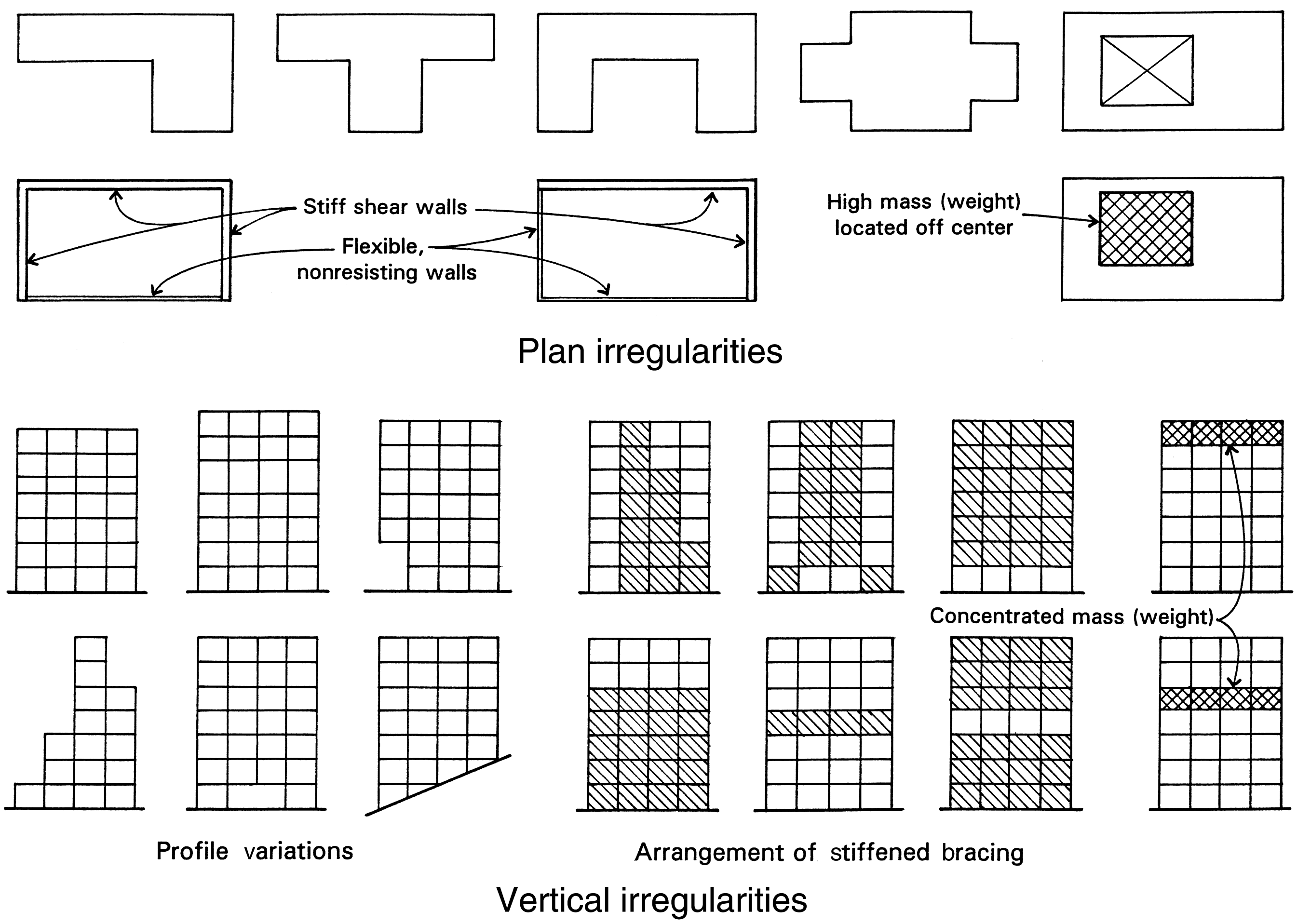


Figure 9.37 Examples of structural forms with irregular seismic response.

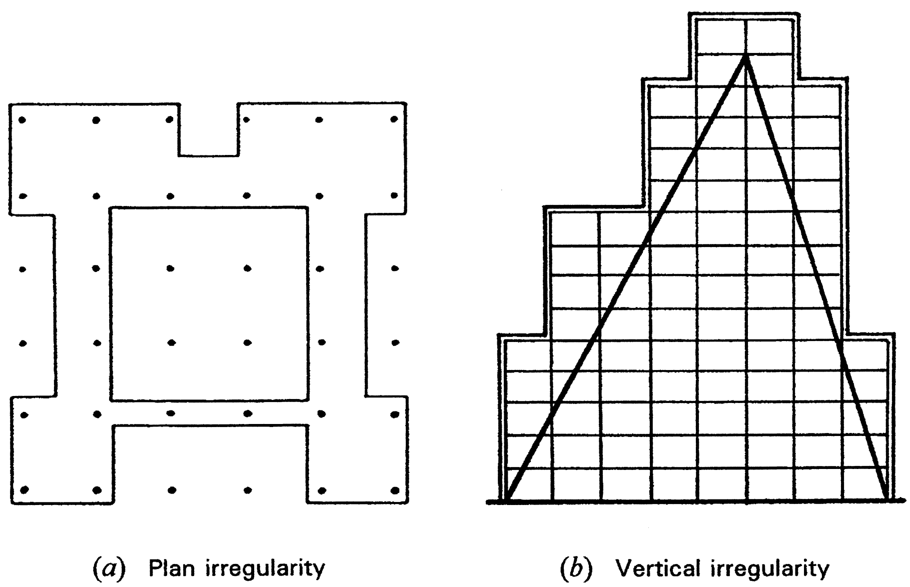


Figure 9.38 Irregular building forms with regular bracing structures.

For plan irregularities, one solution involves the use of seismic separation joints. Examples of this technique are shown in Figure 9.17, where the multimassed building situation is generally described.

Distribution of Seismic Base Shear

The total horizontal seismic force, computed as the base shear (V), must be distributed vertically and horizontally to the elements of the lateral resistive system. This process begins with consideration of the disposition of the building mass (weight), which essentially develops the actual earthquake force. However, for the purpose of simulating dynamic response, the distribution of forces for the investigation of the structure's behavior may be modified.

Codes require a redistribution of the lateral forces at the various levels of multistory buildings. These forces are assumed to be applied at the levels of the horizontal diaphragms, although the redistribution is intended to modify the form of the loading to the vertical bracing system. A principal effect of this modification is to move some of the lateral load to upper levels of the building, more realistically simulating the nature of the response of the vertical cantilever to the dynamic loads.

The total shear applied at any level of the building is generally assumed to be distributed to the vertical elements of the system in proportion to their stiffnesses (resistance to lateral deflection). If the lateral bracing elements are placed symmetrically and their collective center (the center of stiffness) corresponds to the center of gravity of the building mass, this simple assumption may be adequate. Such pure symmetry is not the usual condition, however, and there are two possible conditions that must be considered for modification of the simple assumption of direct distribution.

The first condition concerns the coincidence of the center of resistance and the center of gravity. If these two are not aligned, there will be a torsional effect in the horizontal diaphragm. The torsion will produce shears that must be added to those resulting from the direct resistance. This is so common that the codes now require a minimum amount of torsion, called *accidental eccentricity*, as a safety measure, even when perfect alignment apparently occurs.

The second possible modification of the horizontal distribution concerns the relative stiffness of the horizontal diaphragm. The two major considerations that affect this condition are the aspect ratio of the diaphragm (length-to-width ratio between vertical bracing elements) and the basic construction of the diaphragm. Wood and formed steel decks are flexible, while concrete decks are stiff. The actions of diaphragms in this respect are discussed in Section 9.4.

Priorities of the Codes

Codes in general are developed for protection of the public, life safety being the major concern. Unless life safety is an issue, codes are not concerned with the preservation of the building appearance, loss of critical building services, general destruction of nonstructural elements, continuing functional use of the building, or the protection of the investment of the building owners. Designers or building owners with concerns beyond life safety should consider the code criteria to be minimal and go on to evaluate other effects on the building.

Mitigation of Seismic Effects

Most design work for earthquake resistance has been aimed at strengthening structures or controlling their behavior in some way. A different approach is that of trying to reduce the impact of the earthquake itself by mitigation (reduction) of the seismic effects applied to the building. Various ways of achieving this include:

- Architectural design to reduce the building's vulnerability and unwanted responses
- Use of seismic separation joints
- Use of base isolation
- Use of motion- and shock-absorbing devices within the lateral resistive structure
- Modification of ground materials to improve the response of the site and the building–site interaction

The general objective of these efforts is to modify the form and/or magnitude of the energy load on the building and its bracing structure, that is, to reduce the demand for seismic resistance. A second objective, which may be quite important for some buildings, is the reduction of actual dimensions of movement of the building.

Use of Separation Joints

The means of achieving seismic separation are discussed in Section 9.4. Reasons for separation include a desire to isolate individual components of a multimassed building or to eliminate the structural connection between parts of a continuous building or a building group.

When successfully achieved, seismic separation permits the separate parts to function individually for seismic response. This may eliminate some undesirable action—such as severe torsion—that would otherwise occur if the connected parts functioned as a unit. In other cases, the purpose may be to disconnect a part that would individually experience little movement from one that is likely to experience considerable movement.

A difficult design decision pertaining to any separation joint (seismic, temperature expansion, shrinkage control, differential foundation settlement, etc.) regards the actual dimension of the separation, usually represented by the tolerable dimension of movement within the joint. This involves judging how much each part can be allowed to move, which can only be determined by a deformation investigation of the two separated structures.

Usually, deformation of even the simplest of structures can only be approximated, and the more complex the structure or the loading, the more approximate the computed dimension of movement. For a building under dynamic loading from

an earthquake, modified by ground conditions and by consideration for the effects of nonstructural construction, computation of an actual dimension of movement is hypothetical. This needs to be understood when estimating how useful any separation joint might be.

In fact, separation may be only partly achieved by design. This is how the base isolator and the in-line shock absorber function, as discussed in the next section. These devices also achieve a form of separation while still maintaining some force transfer capabilities. In a way, they are measured separation devices. The traditional separation device, on the other hand, is generally visualized as achieving full separation.

Base Isolation

The way to achieve what amounts to a form of shock absorption between the ground and a building is by the use of *base isolators*. Since seismic force is induced in a building by movement of the supporting ground, the isolation consists of absorbing some of this motion in the building-to-ground connection.

It must be noted that this represents not a total elimination of the effects of the earthquake but merely a reduction of the horizontal movement and the dynamic energy load that is delivered to the lateral bracing system. Serious efforts to produce a good seismic response of the building should still be undertaken. A comprehensive bracing system is still needed and all efforts to reduce nonstructural damage should still be considered.

This is not just a matter of seismic resistance by the building, however. Of possibly greater significance in some situations is reducing the effects of movement on the building's occupants or contents. Consider how valuable the mitigation effort can be in reducing the earthquake motion trauma for bed-ridden patients in a hospital, for valuable and sensitive equipment or machinery, for priceless objects in a museum, or for the fragile and irreplaceable construction elements of highly valued historic buildings.

Typically, base isolation is achieved by placing isolation devices between the bottoms of building supports (columns and bearing walls) and their foundations (Figure 9.39). The base refers to that of the building, specifically, the base as defined in the seismic design codes—the location where the major earthquake force is delivered to the building's lateral bracing system. This is, in many ways, the right spot for the shock-absorbing system—the exact location of the delivery of the punch from the earthquake.

A problem with most base isolation methods is that they deal primarily only with horizontal movements. All earthquakes also have some vertical motion, although it is typically smaller than the horizontal movements. Vertical effects are also often less critical as the building is ordinarily designed for the vertical-force effects of gravity.

Shock Absorbers and Motion Modification

One form of mitigation is that involving the use of devices that provide a shock-absorbing action within the bracing system. These may be used primarily for their damping effects on dynamic vibrations of the structure. Thus, instead of swinging back-and-forth repeatedly, the structure quickly comes to a stop, eliminating some of the effects of rapid stress reversal. However, another benefit is that of actual absorption of some of the dynamic energy of the earthquake.

Piston-type shock absorbers have been used in both new construction and as parts of the systems for protecting existing vulnerable buildings. Three methods used are shown in Figure 9.40:

- A truss bracing system, as shown in (*a*), with dampers replacing the truss diagonals. This method has been used for historic buildings with minor intrusion in interior spaces.
- A chevron form bracing system with dampers replacing the chevron connection as shown in (*b*).
- A special application involving the use of a base control system with a combination of ordinary base isolators and piston dampers. Since each has different dynamic behaviors, they work together in a manner described as *out of phase*, with each exhibiting its own behavior to provide a double energy-absorbing function. The result is a net reduction of seismic force and motion beyond what either element is capable of alone.

Figure 9.41 shows the form of installation of an in-line shock-absorbing damper device as a diagonal member in a steel frame. The structure illustrated was developed for a laboratory test. Simple raw force absorption is a major feature of this device, but its actual effect on movement is possibly

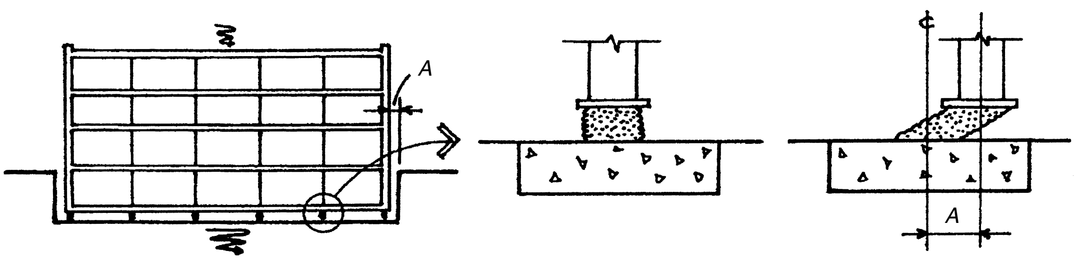


Figure 9.39 Base isolation is usually achieved with two basic elements: a buffering support device (isolator) and a horizontal restraint system. Restraint may be developed with a surrounding structure, such as a basement wall. Isolators have a maximum displacement capacity (dimension *A* in the figure), which is matched to the spaced edge distance.

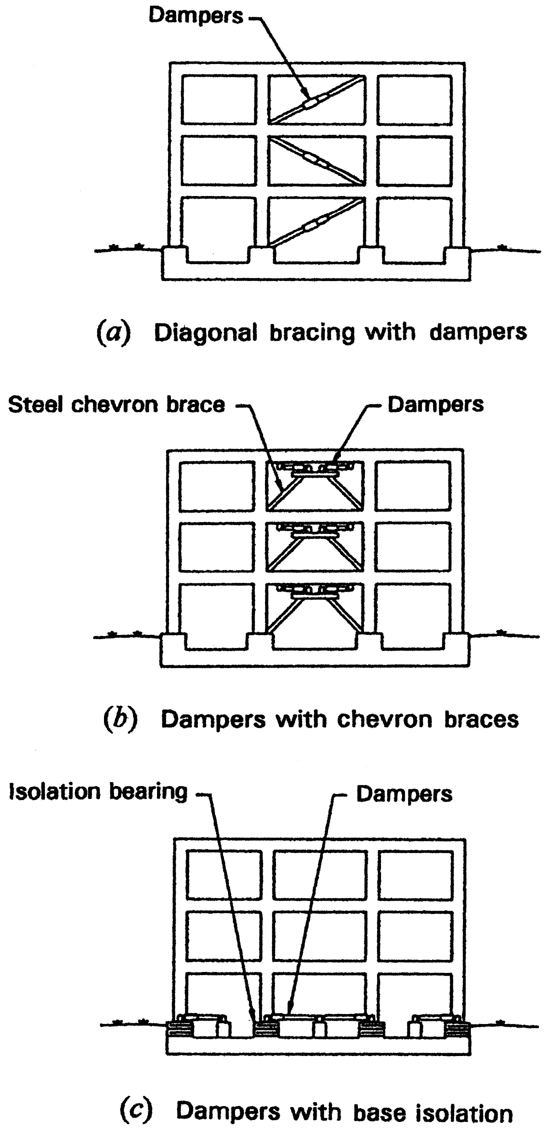


Figure 9.40 Use of in-line shock absorbers for reduction of energy loading and damping of seismic movements. Illustration courtesy of Taylor Devices, Inc.

of greater significance. The graphs in Figure 9.42 show the nature of this effect. In Figure 9.42*a*, the characteristic response of an unmodified structure is shown, with the structure swaying significantly through several back-and-forth cycles (up and down in the graph). Natural damping causes this swaying to diminish with time, but often not without several discernable cycles of vibration.

The curve in Figure 9.42*b* shows the nature of the modification achieved by base isolation. Two effects should be noted: the reduction of the amplitude (actual magnitude of movement) and the more rapid damping (faster stopping of discernable movement). As shown in Figure 9.42*c*, the in-line shock absorber does somewhat the same thing as the base isolator. However, the shock absorber is designed to stop the movement faster—essentially before the completion of one full cycle and ideally pretty much after only a half cycle. Finally, in Figure 9.42*d*, the coupled base isolator and in-line shock absorber can be seen to affect a significant stop to the swaying and a major reduction of the amplitude. Use of any of these options may be appropriate for different situations in terms of design goals and the building user's needs.

In ordinary situations, some energy damping and shock absorbing are performed by the failing of various elements of the building construction. Every fractured pane of window glazing, buckled partition wall, and dropped ceiling absorbs some energy and relieves the dynamic effect on the bracing structure. In the structure itself, yielding of reinforcing bars is assumed to occur in conjunction with major cracking of brittle concrete or masonry, thus giving those otherwise brittle structures a yield character. The yielding of steel gives a nature of toughness to a structure that may otherwise be subject to sudden failure by brittle fracture. The combination of the cracking of the mass of concrete or masonry and the development of inelastic strain in the steel absorbs a lot of energy from an earthquake.

For the concrete or masonry structure that has been designed to develop yield in the steel reinforcement, the effective upper limit of energy loading is likely to occur with some major cracking and extensive nonrecoverable deformations. This state of deformation is also true for a steel structure that is allowed to develop yield-level stresses and plastic hinging. Yielding of the structure may prevent full collapse, but the structure is likely to be unusable and not feasibly repaired after the earthquake. The purpose of shock absorbers is to reduce the need for reliance on yielding of steel in the bracing structure.

In general, design for mitigation of seismic forces is developed with the aim of softening the blow and keeping the structure usable. Some energy-absorbing elements might be lost, but the structure is essentially unharmed or, at worst, slightly damaged and easily repaired. And, the building occupants, the nonstructural construction, and the building contents have all experienced a softer ride than the earthquake promised. In the case of essential emergency facilities, such as hospitals, fire stations, communication centers, and power plants, a critical goal is to have the facility continue in operation immediately following the earthquake.

Architectural Design for Mitigation

Any effect that functions to reduce the impact of an earthquake in terms of the resistance required by the bracing system for a building can be classified as a form of mitigation. In this regard, the general architectural design offers a great potential for mitigation efforts.

Decisions involving the building form, dimensions, use of materials, and particular details have great influence on how the building responds to an earthquake. Add considerations for site selection and development, and many of the factors that affect the building–site interaction are involved, and the response required by the bracing structure is strongly influenced. Certain architectural features may be strongly

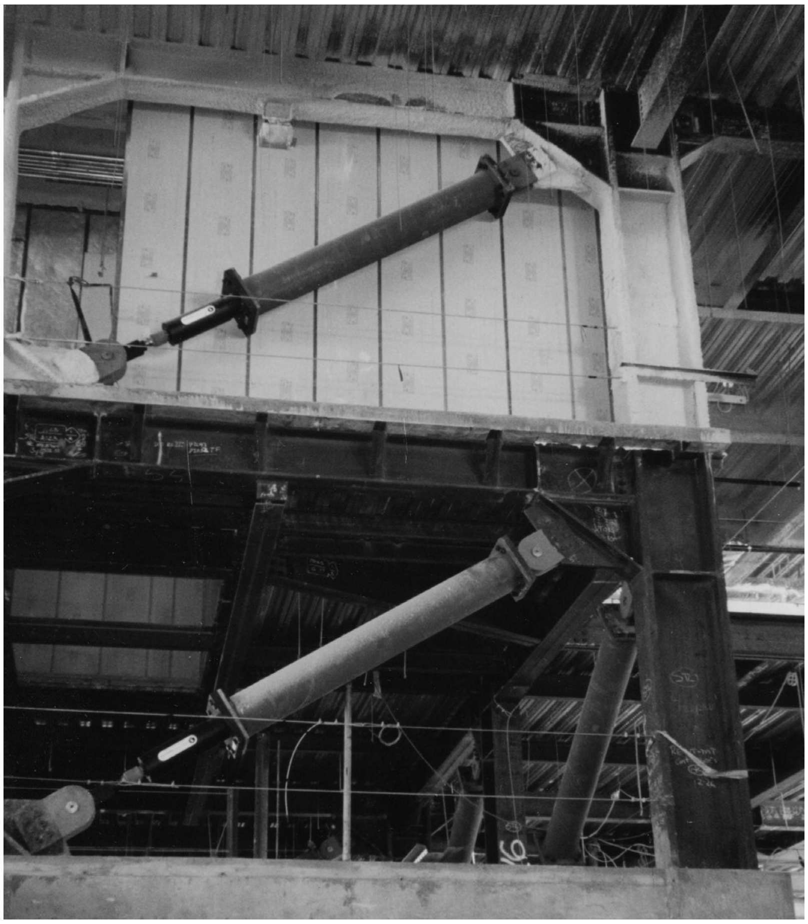


Figure 9.41 Testing of an in-line shock-absorbing device as a diagonal in a steel frame. Photo provided by Taylor Devices, Inc.

desired, but their effects should be considered and better options carefully studied.

Ground Modification

Many engineers and architects are not aware of what can be done these days to modify existing ground conditions that affect the structural performance of building sites. Use of methods such as dynamic compaction, jet grouting, soil nailing, and slurry cut-off walls can modify or stabilize site conditions that were once considered to be insurmountably bad. The state of the art of geotechnical engineering has grown immensely and building and site designers should become aware of the currently available technologies for site modification.

For example, if a serious soil liquefaction problem can be significantly reduced by a certain technique, it may reduce requirements for the design of the lateral bracing system. More importantly, it may effectively reduce the possibility for exaggerated movements during an earthquake—a positive gain in terms of trauma to building occupants or sensitive contents. This is the kind of gain that can be obtained with base isolation, for example, but the expensive base isolators may not be required if the site is sufficiently modified. Other soil problems, as described in Chapter 8, may also be reduced in magnitude or totally eliminated by soil modification methods.

9.4 ELEMENTS OF LATERAL RESISTIVE SYSTEMS

This section presents a discussion of the ordinary structural elements used to develop bracing systems for resistance to the effects of wind and earthquakes. Design examples of building structures utilizing many of these elements are included in the case studies in Chapter 10.

Horizontal Diaphragms

Most lateral resistive structural systems for buildings consist of combinations of horizontal elements and vertical elements. The horizontal elements are most often the roof and floor decks. When the deck is of sufficient strength and stiffness to be developed as a rigid plane, it is called a *horizontal diaphragm*.

General Behavior

A horizontal diaphragm typically functions by collecting the lateral forces at a particular level of the building and then distributing them to the vertical elements of the lateral bracing system. For wind forces the lateral loading of the horizontal diaphragm is usually through the attachment of the exterior walls to its edges. For seismic forces the loading is partly a result of the weight of the deck itself and partly a result of the weights of other parts of the building attached to it.

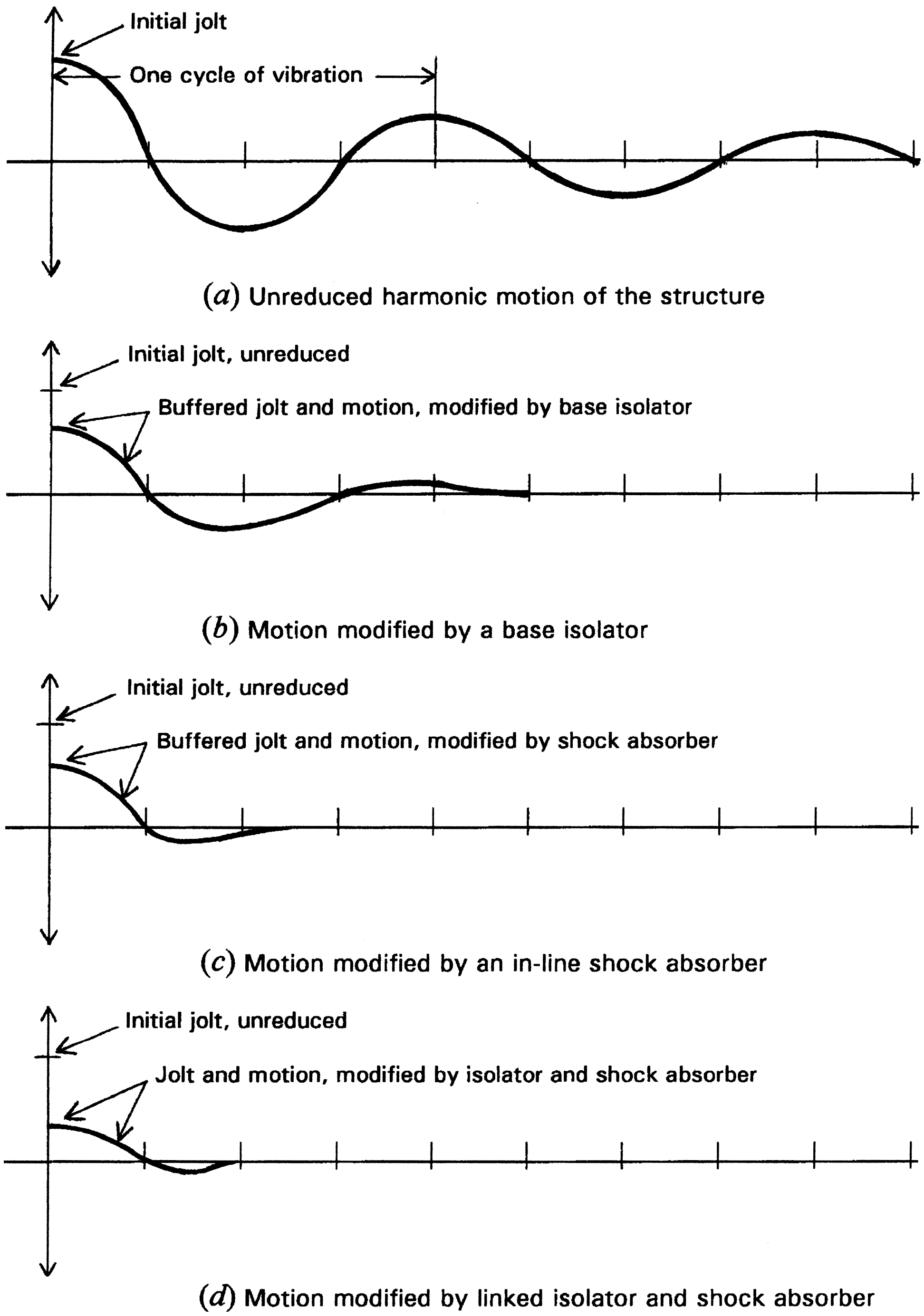
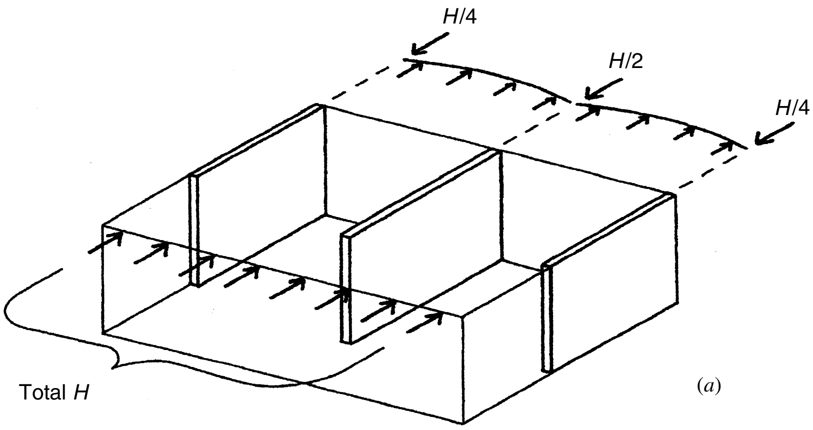


Figure 9.42 Harmonic vibration of a structure due to an initial displacing dynamic jolt.

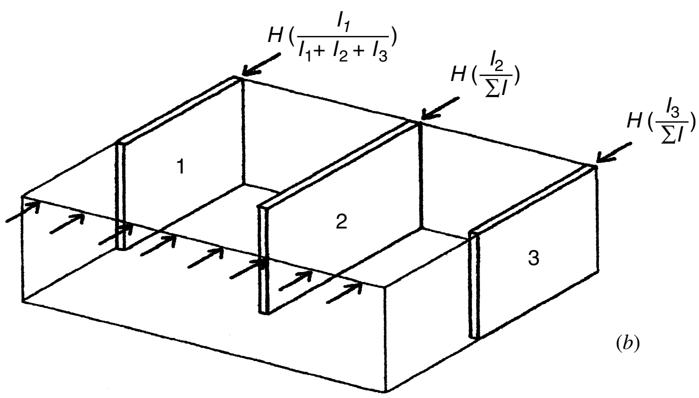
The particular structural behavior of the horizontal diaphragm and the manner in which loads are distributed to the vertical bracing elements depend on a number of considerations, as described in the following discussion.

Relative Stiffness of the Diaphragm. If the horizontal diaphragm is relatively flexible, it may deflect so much that its continuity is negligible and the distribution of loads to the relatively stiff vertical elements is essentially on a load periphery basis. If the deck is quite rigid, on the other hand, the distribution to vertical elements will be essentially in proportion to their relative stiffness with respect to each other. The possibilities for these two situations are illustrated in Figure 9.43.

Torsional Effects. If the centroid of the lateral forces in the horizontal diaphragm does not coincide with the centroid of stiffness of the vertical elements, there will be a twisting effect (called a *torsional effect*) on the structure as well as the direct force effect. Figure 9.44 shows a structure in which this effect occurs. Torsion is usually of significance only if the



Peripheral distribution with horizontal diaphragm



Prorportionate stiffness distribution with rigid horizontal diaphragm

Figure 9.43 Distribution of forces to vertical bracing elements.

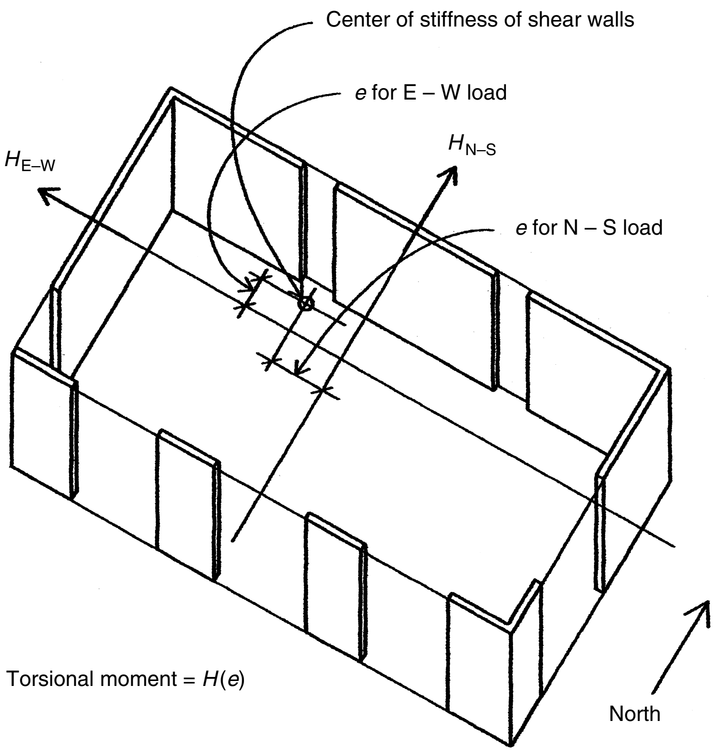


Figure 9.44 Torsional effect of lateral force.

horizontal diaphragm is relatively stiff. This stiffness is a matter of the materials and construction of the diaphragm and its depth-to-span ratio between vertical elements. In general, wood and steel decks are quite flexible, whereas concrete decks are quite stiff.

Relative Stiffness of the Vertical Elements. When vertical elements share load from a rigid horizontal diaphragm, as shown in the lower part of Figure 9.43, their relative stiffness must usually be determined in order to establish the manner of sharing. The determination is comparatively simple when the elements are similar in type and materials, such as all-plywood shear walls. When vertical elements are different, such as a mix of plywood and masonry walls, their actual deflections must be computed in order to establish the distribution, which often requires laborious computations. Deflection computations are not only complex but also usually quite considerably hypothetical, and simplified methods are often employed for these cases.

Use of Control Joints. The general approach in design for lateral loads is to tie the whole bracing structure together to assure its overall continuity of movement. Sometimes, however, because of the irregular form or large size of a building, it may be desirable to control its behavior under lateral loads by the use of control joints. These joints usually function to permit independent motion of separate parts of the building and/or its bracing structure. They may also be used to assure specific load transfers to selected elements while eliminating load transfers to other elements.

Design and Usage Considerations

In performing their basic tasks, horizontal diaphragms have a number of potential stress problems. A major consideration is that of the shear stress in the plane of the diaphragm caused by the spanning action of the diaphragm, as shown in Figure 9.45. This action results in shear stress in the material as well as a force that must be transferred across joints in the deck when the deck is composed of separate elements such as panels of plywood or units of formed sheet metal.

The sketch in Figure 9.46 shows a typical plywood framing detail at the joint between two panels. The stress in the deck at this point must be passed from one panel through the edge nails to the framing member and then back out through the nails to the adjacent panel.

As is the usual case with shear stress, both diagonal tension stress and diagonal compression stress are induced simultaneously with the shear stress. The diagonal tension is critical in concrete, while the diagonal compression is a potential source of buckling in decks composed of thin panels of plywood and other decking. In plywood decks the thickness of the plywood relative to the spacing of framing members must be considered. It is also a reason why plywood must be nailed to all the framing, not just the edge support members. In metal decks the gauge of the sheet metal and spacing of stiffening ribs must be considered. Tables of allowable loads for decks incorporate limits for these considerations.

Diaphragms with continuous deck surfaces are usually designed in a manner similar to that for webbed steel beams. The web (deck) is designed for the shear, and the flanges (edge-framing elements) are designed to take the moment, as shown in Figure 9.47. The edge members are called *chords* and they must be designed for the tension and compression forces at the edges. With diaphragm edges of some length, the latter function usually requires that the edge-framing members be spliced for continuity of the forces. In most cases there are ordinary elements of the framing system, such as spandrel beams or top plates of stud walls, that have the potential to function as chords for the diaphragm.

The diaphragm shear capacities for commonly used decks of various materials are available from the codes or from load tables prepared by deck material manufacturers or industry trade organizations. Loads for plywood and other wood product decks are given in Table 9.1, which is reproduced from a reference published by an industry organization.

A special situation is a horizontal system that consists partly or wholly of a braced (trussed) frame. These may be used when there are a large number of openings in the deck or when the shear capacity required is beyond the deck capability.

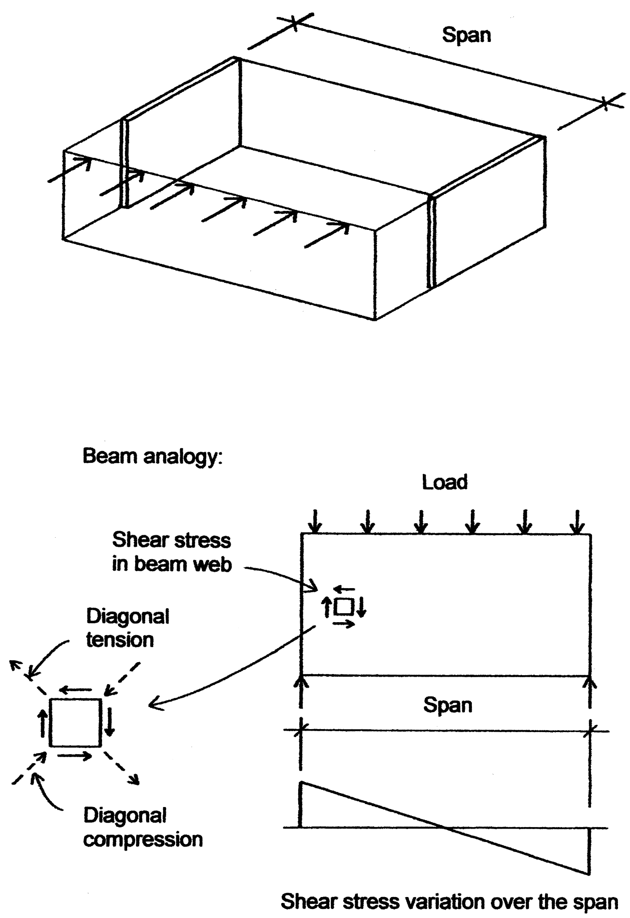


Figure 9.45 Beam functions of a horizontal diaphragm.

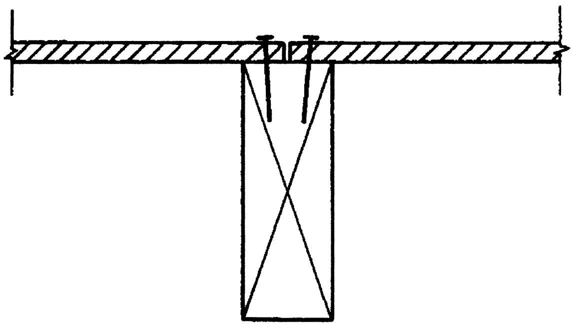


Figure 9.46 Panel edge nailing of a panel diaphragm.

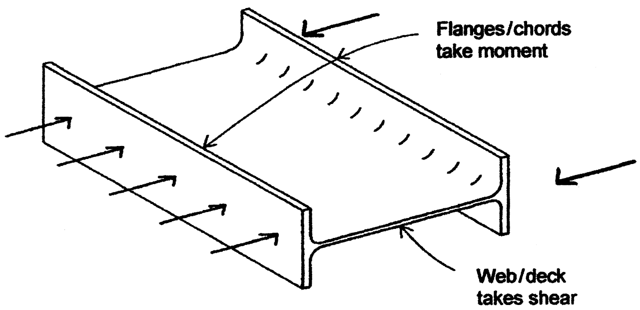


Figure 9.47 Beam analogy for a horizontal diaphragm.

Typical Construction

A very common horizontal diaphragm is the manufactured wood product panel deck for the simple reason that wood frame construction is so popular (see Figure 9.48). Floor decks are mostly plywood, although roof decks are also made with other types of panels. Attachment of the deck to framing was formerly mostly done with common wire nails, and deck shear capacities are still given for this attachment. Mechanically driven fasteners are now mostly used, although their capacities are rated in terms of equivalency to common nails. Wood panel decks are quite flexible and should be investigated for deflection when spans are large or span-to-depth ratios are high.

Decks of boards or timber, usually with tongue-and-groove joints, were once popular but are given low rating for shear capacity. Where the expensive timber decks are desired, it is common to attach a plywood deck to the top of the timber elements to serve as the actual horizontal diaphragm.

Steel decks offer possibilities for use as diaphragms for either roofs or floors. Acceptable shear capacities and recommended installation details should be obtained from industry organization publications. Stiffnesses are generally comparable to those for wood decks. For floor decks, a concrete fill is usually applied to the top of the deck, which greatly improves the diaphragm strength and stiffness. Actually, concrete fill is now often used on wood floor decks as well, for the added acoustic separation and reduction of bouncing effect, resulting here also in strengthening and stiffening of the diaphragm.

Sitecast concrete decks provide the strongest and stiffest diaphragms. Precast concrete deck units can also be used as diaphragms; they are frequently covered with a sitecast topping, which further enhances their diaphragm capacity.

Many other types of roof deck construction may function adequately for diaphragm action, especially when shear forces are low and only minor shear stress resistance is required. Acceptability of any decking system is subject to approval by code-enforcing agencies.

Stiffness and Deflection

As spanning elements, the relative stiffness and actual dimensions of deformation of horizontal diaphragms depend on a number of factors, such as:

- Materials of the construction
- Continuity of the spanning diaphragm over a number of supports
- Span-to-depth ratio of the diaphragm
- Effect of various special conditions, such as chord length changes, yielding of connections, and influence of large openings

With respect to their span-to-depth ratios, most horizontal diaphragms approach the classification of deep beams. As shown in Figure 9.49, even the shallowest of diaphragms, such as the usual maximum 4 : 1 case allowed for a plywood deck, tends to present a fairly stiff flexural member. As span-to-depth ratio falls below about 3 : 1, the deformation character of the diaphragm approaches that of a very stiff beam, for which the deflection is primarily determined by shear deformation rather than by flexural deformation.

Vertical Diaphragms

Vertical diaphragms, or *shear walls*, as they are commonly called, are usually the walls of buildings, most frequently exterior walls. As building walls, in addition to their shear wall functions, they must fulfill various architectural functions and may also serve as bearing walls for gravity loads. Location of walls, materials used, and construction details must be developed with all functions in mind.

The most common shear wall constructions are those of sitecast concrete, masonry, and wood frames of studs with structural panel surfaces. Choice of the type of construction may be limited by the magnitude of the value of maximum shear stress caused by lateral loads but will also be influenced by fire code requirements and the satisfaction of various other wall functions.

General Behavior

Some of the structural functions usually required of vertical diaphragms are the following (see Figure 9.50):

Direct Shear Resistance. This usually consists of the transfer of a lateral force in the plane of the wall from an upper level of the wall to a lower level or to the bottom of

Table 9.1 Load Values for Horizontal Panel Diaphragms

DIAPHRAGMS: RECOMMENDED SHEAR (POUNDS PER FOOT) FOR HORIZONTAL APA PANEL DIAPHRAGMS WITH FRAMING OF DOUGLAS-FIR, LARCH OR SOUTHERN PINE[a] FOR WIND OR SEISMIC LOADING

Panel Grade	Common Nail Size	Minimum Nail Penetration in Framing (inches)	Minimum Nominal Panel Thickness (inch)	Minimum Nominal Width of Framing Member (inches)	Blocked Diaphragms: Nail Spacing (in.) at diaphragm boundaries (all cases), at continuous panel edges parallel to load (Cases 3 & 4), and at all panel edges (Cases 5 & 6)[b]: 6; Nail Spacing (in.) at other panel edges (Cases 1, 2, 3 & 4): 6	4 / 6	2-1/2[c] / 4	2[c] / 3	Unblocked Diaphragms: Nails Spaced 6" max. at Supported Edges[b]: Case 1 (No unblocked edges or continuous joints parallel to load)	All other configurations (Cases 2, 3, 4, 5 & 6)
APA STRUCTURAL I grades	6d	1-1/4	5/16	2	185	250	375	420	165	125
				3	210	280	420	475	185	140
	8d	1-3/8	3/8	2	270	360	530	600	240	180
				3	300	400	600	675	265	200
	10d[d]	1-1/2	15/32	2	320	425	640	730	285	215
				3	360	480	720	820	320	240
APA RATED SHEATHING, APA RATED STURD-I-FLOOR and other APA grades except Species Group 5	6d[e]	1-1/4	5/16	2	170	225	335	380	150	110
				3	190	250	380	430	170	125
			3/8	2	185	250	375	420	165	125
				3	210	280	420	475	185	140
	8d	1-3/8	3/8	2	240	320	480	545	215	160
				3	270	360	540	610	240	180
			7/16	2	255	340	505	575	230	170
				3	285	380	570	645	255	190
			15/32	2	270	360	530	600	240	180
				3	300	400	600	675	265	200
	10d[d]	1-1/2	15/32	2	290	385	575	655	255	190
				3	325	430	650	735	290	215
			19/32	2	320	425	640	730	285	215
				3	360	480	720	820	320	240

(a) For framing of other species: (1) Find specific gravity for species of lumber in the AFPA National Design Specification. (2) Find shear value from table above for nail size for actual grade. (3) Multiply value by the following adjustment factor: Specific Gravity Adjustment Factor = [1 – (0.5 – SG)], where SG = specific gravity of the framing. This adjustment shall not be greater than 1.

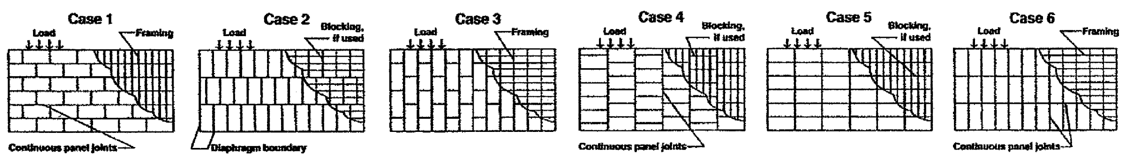
(b) Space nails maximum 12 in. o.c. along intermediate framing members (6 in. o.c. when supports are spaced 48 in. o.c.).

(c) Framing at adjoining panel edges shall be 3-in. nominal or wider, and nails shall be staggered where nails are spaced 2 inches o.c. or 2-1/2 inches o.c.

(d) Framing at adjoining panel edges shall be 3-in. nominal or wider, and nails shall be staggered where 10d nails having penetration into framing of more than 1-5/8 inches are spaced 3 inches o.c.

(e) 8d is recommended minimum for roofs due to negative pressures of high winds.

Notes: Design for diaphragm stresses depends on direction of continuous panel joints with reference to load, not on direction of long dimension of sheet. Continuous framing may be in either direction for blocked diaphragms.



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the wall. This produces the typical situation of shear stress and the accompanying diagonal tension and compression stresses.

Cantilever Moment (Overturn) Resistance. In resisting upper level lateral forces, walls act like vertical cantilever beams, developing opposed tension and compression forces at the wall edges and transferring an overturning moment to the wall base.

Horizontal Sliding Resistance. The direct transfer of the lateral load at its base produces a tendency for the wall to slip horizontally off of its supports.

The shear stress function is usually considered independently of other structural functions of the wall. The maximum shear stress generated by the lateral load is compared to a rated capacity of the wall construction.

Figure 9.48 Conventional wood-framed roof structure with panel decking. A hierarchy of framing is typically required, graduating upward from the smallest and closest spaced pieces that directly support the deck. For the major span required here, two additional framing elements are used, producing a rafter–purlin–girder framing system.

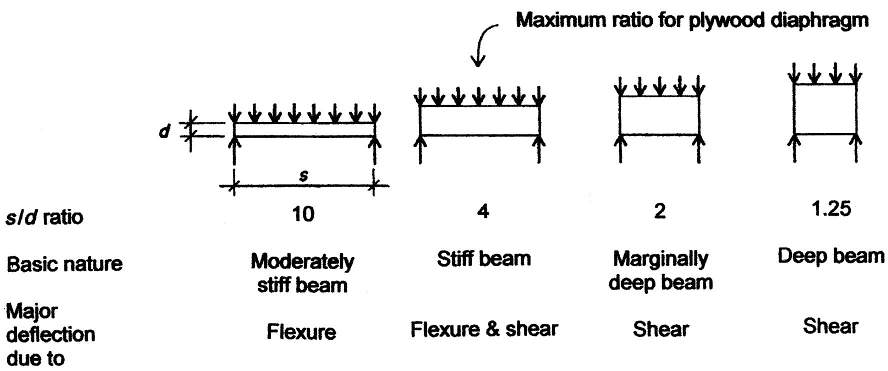


Figure 9.49 Behavior of horizontal diaphragms related to span-to-depth ratios.

Although the possibility exists for the buckling of walls as a result of the diagonal compression effect, this is usually not critical because other limitations exist to constrain wall slenderness. Diagonal tension is most critical for wall finishes of plaster, tile, and masonry veneer.

As in the case of horizontal diaphragms, the moment effect on the wall is considered to be resisted by the two vertical edges of the wall acting as beam flanges or truss chords. These edges must be investigated for this concentrated force effect. In many cases the edges are constructed as columns and must be designed for a combination of lateral and gravity loading.

The overturn effect of the lateral loads must be resisted by the gravity dead load with a safety factor of 1.5. The form for this analysis is as shown in Figure 9.51. If the tiedown force is actually required, it is developed by anchorage of the edge-framing elements of the wall. For seismic effects, only 85% of the dead load should be used to resist uplift effects when using the allowable stress method. Specific examples of this analysis are shown in Chapter 10.

Resistance to horizontal sliding at the base of a shear wall should be developed by connection to the supports. Anchor bolts for wood-framed walls provide this function. For masonry and concrete walls sufficient anchorage is usually provided by the dowels for vertical reinforcement, aided by the horizontal sliding friction for the heavy walls.

Design and Usage Considerations

An important judgment that must often be made in design work for lateral loads is that of the manner of distribution of a total lateral force between a number of shear walls that share the load. In some cases the existence of symmetry or of a flexible diaphragm may simplify this consideration. In other cases, however, the relative stiffness of the walls must be determined for this computation.

If considered in terms of static force and elastic stress–strain conditions, the relative stiffness of a wall is inversely proportional to its deflection under a unit load. Figure 9.52 shows the form of deflection of a shear wall for two assumed conditions. In Figure 9.52*a*, the wall is considered to be fixed at its top and bottom, thus flexing in a double curve with an inflection point at midheight. This is the case for most concrete and masonry walls that are rigidly connected to framing or upper walls at their tops and to heavy supports at the bottom.

In Figure 9.52*b* the wall is considered to be fixed at its bottom only, thus functioning as a vertical cantilever. This is the case for most independent, freestanding, one-story walls. A third possibility is shown in Figure 9.52*c*, in which a relatively short wall pier is assumed to be fixed at its top only by continuous wall construction, which produces the same deflection as in (*b*), as shown with an assumption of a flexible

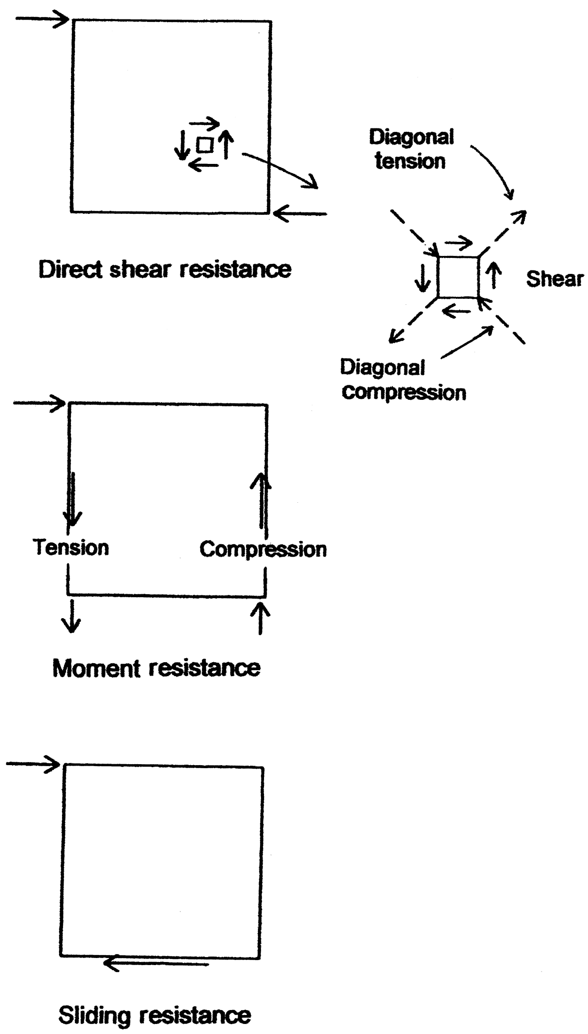
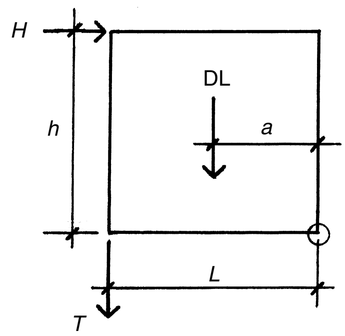


Figure 9.50 Functions of a shear wall.



To determine T:

For wind: $DL(a) + T(L) = 1.5[H(h)]$

For seismic: $0.85[DL(a)] + T(L) = H(h)$

Figure 9.51 Overturn analysis for a shear wall.

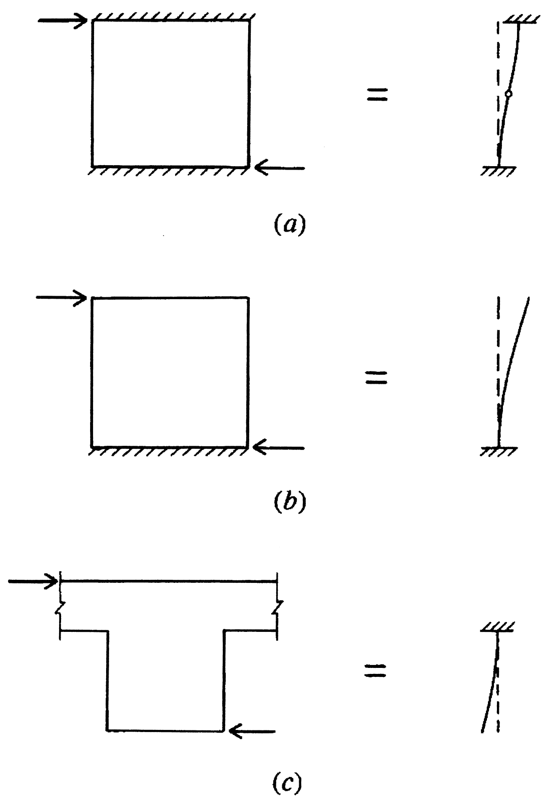


Figure 9.52 Shear wall support conditions affecting deflection.

support at the bottom. If the bottom is fixed, the deflection will be of the same form as in (*a*).

In some instances the deflection of the wall may result largely from shear distortion, rather than from flexural distortion (see discussion for the horizontal diaphragm and the deep beam effect). Furthermore, stiffness in resistance to dynamic loads is not quite the same as stiffness in resistance to static loads. The following recommendations are made for single-story shear wall:

For wood-framed walls with height-to-length ratios of 2 or less, assume the stiffness to be proportional to the plan length of the wall.

For wood-framed walls with height-to-length ratios over 2 and for concrete and masonry walls use pier rigidity values, available from various references for masonry design.

Avoid situations in which walls of significantly great differences in stiffness share loads.

Avoid mixing walls of different construction.

Two situations of mixed shear wall construction are shown in Figure 9.53. The upper figure shows a mix of wood-framed and masonry walls in a single row. In this case, the conservative design assumption would be that the full load is taken by the much stiffer masonry walls.

In the lower figure in Figure 9.53 a mix of walls share the load from a horizontal diaphragm. In this case, the critical concern is for the relative stiffness of the horizontal diaphragm: If it is very stiff, the wise decision is to use only

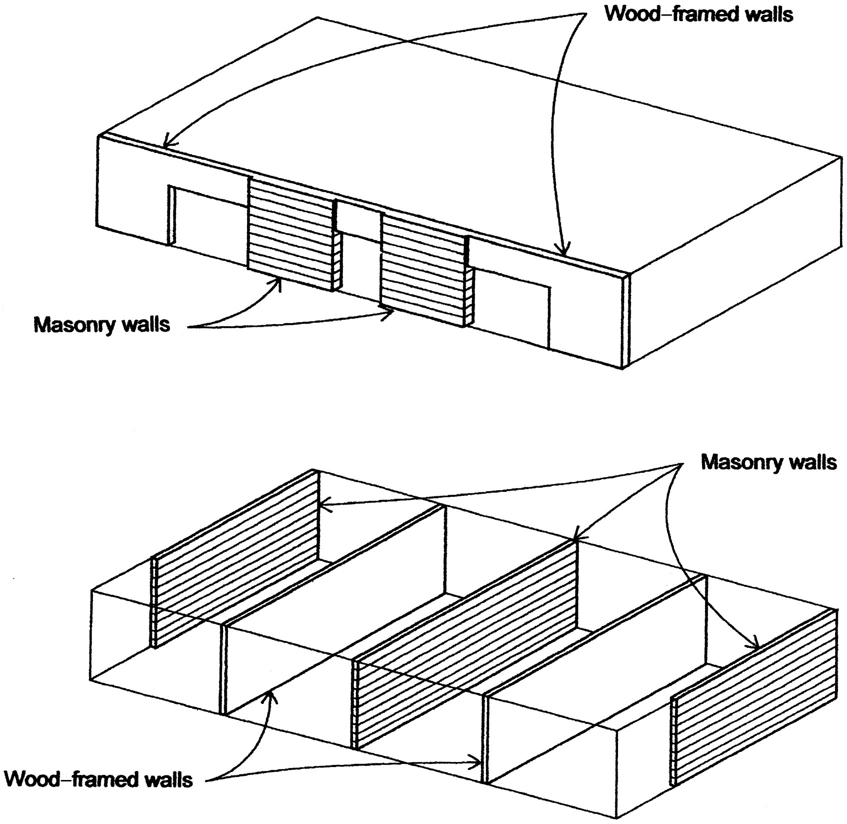


Figure 9.53 Interacting shear walls of mixed construction.

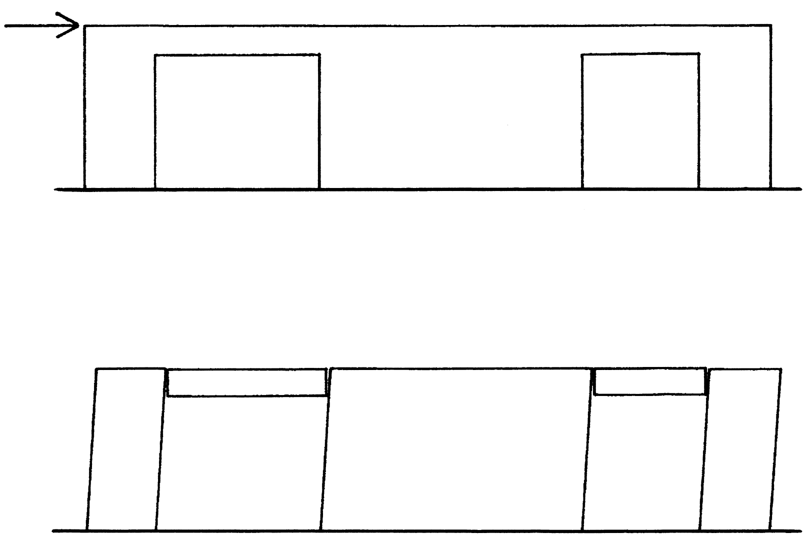


Figure 9.54 Effect of wall deflection on headers over wall openings.

the masonry walls; if it is flexible, the load may be shared on a peripheral basis by all the walls.

Wall deflections often directly affect other construction in the wall plane. Figure 9.54 shows a typical situation where cracking is likely between the walls and the connected headers. This is best resolved by using expansion control joints at the ends of the headers.

Typical Construction

The only wood-framed shear wall extensively used in the past was the plywood-covered wall. Now, the action of various other construction has been recognized, and many are rated for shear capacity (see Figure 9.55).

Plywood is still the strongest material used for this purpose, but many other products are now recognized as potential shear wall coverings. One product now widely used is OSB (oriented strand board), a pressed fiberboard material made with large wood chips. The main popular feature of OSB is its relatively low cost, although it also now has the appeal of being produced from fast growth trees which are not usable for standard lumber or plywood plies.

Figure 9.55 Garage sidewall with stucco fastened directly to studs without sheathing. Lateral bracing with let-in braces.

For all walls there are various considerations (good carpentry, fire resistance, available products, cost, etc.) that work to establish a certain minimum construction. In many situations this minimum is really adequate for low levels of shear loading, and most necessary add-ons consist of details for achieving load transfers into and out of the walls. Increasing wall strength beyond the minimum construction usually means increasing the dimensions or quality of elements in the system or adding reinforcement of some kind. It well behooves the designer to find out the standards for basic minimum construction, what its lateral load limits are, and what can most easily be done to improve it when necessary.

Long-accepted practices for construction may actually not be adequate for some common situations. A prime example is masonry construction—developed over centuries without steel reinforcement—and found to be really inadequate for major earthquakes.

Load Capacity

Load capacities for wood-framed walls with panel covering are provided in most building codes. These are derived from tables supplied by wood industry organizations who have largely supported the funding for the research to establish the capacity values. Table 9.2 is an example of such tables. Use of data from this table is illustrated in examples in Chapter 10. The table is quite complex as it incorporates a large number of variables.

Many buildings are built with exterior walls of masonry materials. The majority of these are frame structures with masonry materials applied as finishes. Structural masonry is also used in a few cases.

For wind resistance, various forms of structural masonry may be used. Masonry construction is tightly controlled by specifications, and the minimum code-acceptable forms all have some shear wall potential.

For seismic actions the only masonry construction ordinarily acceptable is that classified as reinforced masonry, as described in Chapter 7. The most common form of this consists of construction produced with hollow units of precast concrete, called concrete blocks—or now, more officially, CMUs, for *concrete masonry units*. Design of this construction is documented in various references and generally follows methods developed for reinforced concrete.

Most concrete design is based on the current edition of the ACI code (Ref. 16), although building codes are often slow to keep up with the latest editions in some cases. The general design of reinforced concrete structures is described in Chapter 6 and some examples are shown in Chapter 10.

Concrete offers the strongest form of construction for shear walls. These may be used in buildings with mixed forms of construction, but they occur mostly in buildings with a general concrete structure. Concrete frames have a great potential for lateral resistance as rigid frames. However, when solid walls occur with the frames, the great stiffness of the walls often makes them the principal resisting elements for lateral loads.

For tall buildings, a common building form is one that uses a heavy core with solid walls around stairs, elevators, restrooms, and duct shafts. This core may be braced significantly to provide the lateral-force-resisting structure for the whole building. The strongest and stiffest form of construction for such a core is one using solid concrete walls.

Stiffness and Deflection

In order to prevent major damage to other parts of the building construction, it is desirable for the lateral resistive structure to have a small amount of deflection. For shear walls there are a number of factors that affect deflection—starting with the basic materials of the wall construction.

The most flexible walls are generally those consisting of a wood frame with attached surfacing. Deflection of these walls is affected by the following:

Change in length of the chords (wall end framing) due to the overturning moment on the wall

Table 9.2 Load Values for Wood Panel Shear Walls

SHEAR WALLS: RECOMMENDED SHEAR (POUNDS PER FOOT) FOR APA PANEL SHEAR WALLS WITH FRAMING OF DOUGLAS-FIR, LARCH, OR SOUTHERN PINE[a] FOR WIND OR SEISMIC LOADING[b]

			Panels Applied Direct to Framing					Panels Applied Over 1/2" or 5/8" Gypsum Sheathing				
Panel Grade	Minimum Nominal Panel Thickness (in.)	Minimum Nail Penetration in Framing (in.)	Nail Size (common or galvanized box)	Nail Spacing at Panel Edges (in.) 6	4	3	2[e]	Nail Size (common or galvanized box)	Nail Spacing at Panel Edges (in.) 6	4	3	2[e]
APA STRUCTURAL I grades	5/16	1-1/4	6d	200	300	390	510	8d	200	300	390	510
	3/8	1-3/8	8d	230[d]	360[d]	460[d]	610[d]	10d[f]	280	430	550	730
	7/16			255[d]	395[d]	505[d]	670[d]					
	15/32			280	430	550	730					
	15/32	1-1/2	10d[f]	340	510	665	870	—	—	—	—	—
APA RATED SHEATHING; APA RATED SIDING[g] and other APA grades except species Group 5	5/16 or 1/4[c]	1-1/4	6d	180	270	350	450	8d	180	270	350	450
	3/8			200	300	390	510		200	300	390	510
	3/8	1-3/8	8d	220[d]	320[d]	410[d]	530[d]	10d[f]	260	380	490	640
	7/16			240[d]	350[d]	450[d]	585[d]					
	15/32			260	380	490	640					
	15/32	1-1/2	10d[f]	310	460	600	770	—	—	—	—	—
	19/32			340	510	665	870	—	—	—	—	—
			Nail Size (galvanized casing)					Nail Size (galvanized casing)				
APA RATED SIDING 303[g] and other APA grades except species Group 5	5/16[c]	1-1/4	6d	140	210	275	360	8d	140	210	275	360
	3/8	1-3/8	8d	160	240	310	410	10d[f]	160	240	310	410

(a) For framing of other species: (1) Find specific gravity for species of lumber in the AFPA National Design Specification. (2) For common or galvanized box nails, find shear value from table above for nail size for actual grade. (3) Multiply value by the following adjustment factor: Specific Gravity Adjustment Factor = [1 − (0.5 − SG)], where SG = specific gravity of the framing. This adjustment shall not be greater than 1.

(b) All panel edges backed with 2-inch nominal or wider framing. Install panels either horizontally or vertically. Space nails maximum 6 inches o.c. along intermediate framing members for 3/8-inch and 7/16-inch panels installed on studs spaced 24 inches o.c. For other conditions and panel thicknesses, space nails maximum 12 inches o.c. on intermediate supports.

(c) 3/8-inch or APA RATED SIDING 16 oc is minimum recommended when applied direct to framing as exterior siding.

(d) Shears may be increased to values shown for 15/32-inch sheathing with same nailing provided (1) studs are spaced a maximum of 16 inches o.c., or (2) if panels are applied with long dimension across studs.

(e) Framing at adjoining panel edges shall be 3-inch nominal or wider, and nails shall be staggered where nails are spaced 2 inches o.c.

(f) Framing at adjoining panel edges shall be 3-inch nominal or wider, and nails shall be staggered where 10d nails having penetration into framing of more than 1-5/8 inches are spaced 3 inches o.c.

(g) Values apply to all-veneer plywood APA RATED SIDING panels only. Other APA RATED SIDING panels may also qualify on a proprietary basis. APA RATED SIDING 16 oc plywood may be 11/32 inch, 3/8 inch or thicker. Thickness at point of nailing on panel edges governs shear values.

Typical Layout for Shear Walls

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Shear strain in the plywood panels
Deformation in the nailed joints
Yield of anchorage connections
Movement of supports due to moment or shear

As shown in Figure 9.56*a*, shear walls tend to be quite stiff, mostly in the class of deep beams as discussed for the horizontal diaphragm. In this case flexure in the wall affects primarily only the end chords and the wall surface is, in the main, deformed only by the shear action.

The two general cases for flexure are the cantilever and the doubly fixed pier, as described in Figure 9.52. For flexural deformation, the doubly fixed pier is assumed to take the shape of an S curve, as shown in Figure 9.56*b*, which effectively cuts the cantilever distance for flexural deflection by half. This is of considerable note where flexure is critical (for piers short in

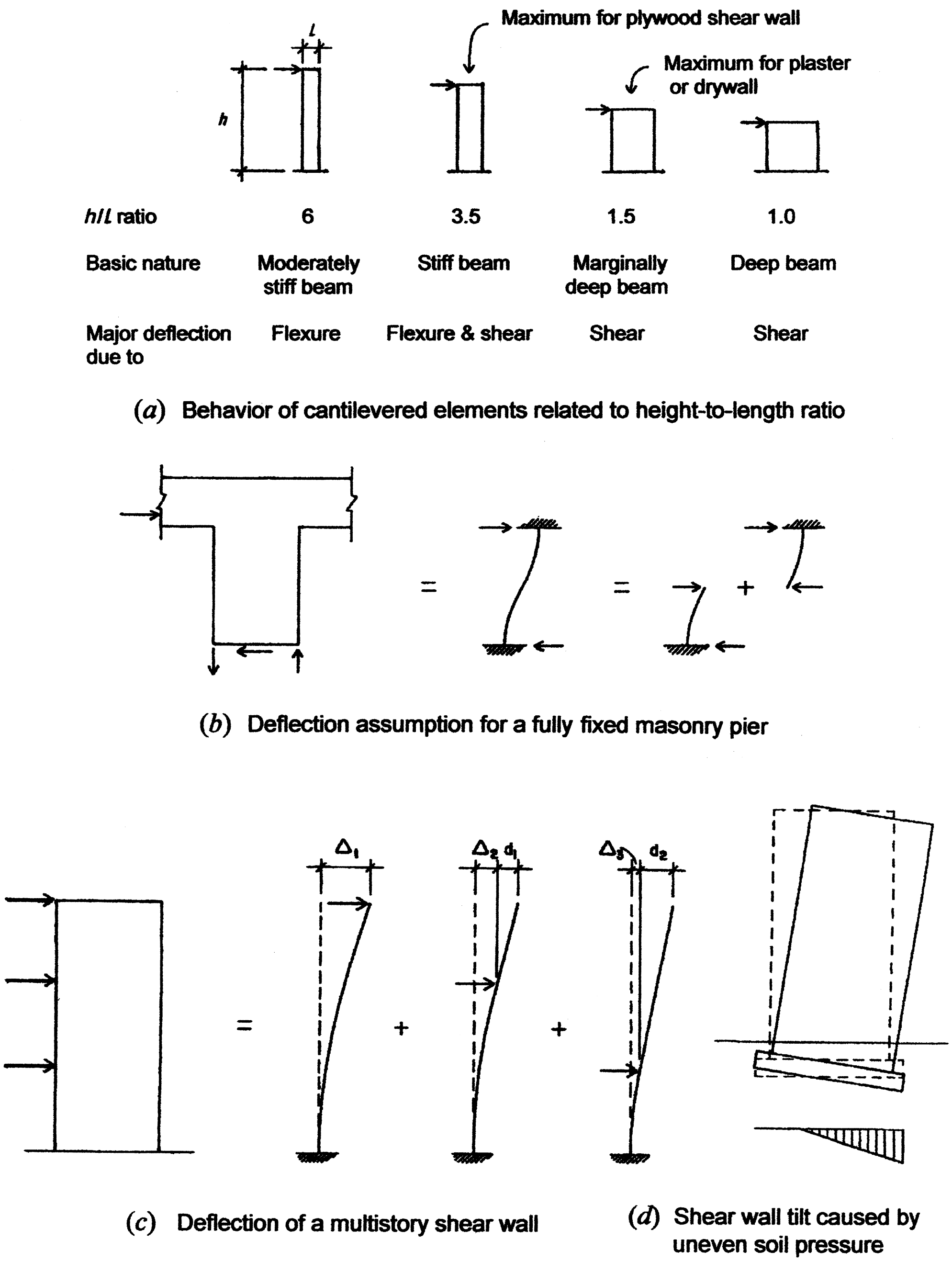


Figure 9.56 Aspects of deflection of shear walls.

length), but not of much concern for the long wall in which shear is the main source for lateral deflection.

For the multistory shear wall, as shown in Figure 9.56*c*, the deflection at the top may be found as the sum of the individual deflections of the three loads (Δ_1, Δ_2, and Δ_3) plus the projected deflections d_1 and d_2. In some situations this free cantilever deflection may be considerably reduced by the restraining effect of the horizontal structures at the upper levels of the wall.

Rotation at the base of the wall due to soil deformation can also contribute to deflection of shear walls (see Figure 9.56*d*). This is especially true for tall walls on isolated foundations placed on relatively compressible soils, such as loose sands and soft clay—a situation to be avoided if at all possible. Unless a very dense and stable soil is available for bearing, a deep foundation system may be advisable.

Braced Frames

Although there are actually other ways to brace a frame against lateral loads, the term *braced frame* is used to refer to frames that utilize trussing. Trusses are used mainly for vertical bracing elements in combination with the usual horizontal diaphragms. It is possible, however, to use a trussed frame for a horizontal system or to combine vertical and horizontal trussing in a truly three-dimensional trussed framework. The latter is more common for open tower structures, such as

those for large electrical transmission lines and radio and television transmitters.

Use of Trussing for Bracing

Post-and-beam systems, consisting of separate vertical and horizontal members, may be inherently stable for gravity loading, but they must be braced in some manner for lateral loads. The three basic ways of achieving this are through shear panels, moment-resistive joints between the members, or trussing. The trussing, or triangulation, of a frame with rectangular arrangements of the members is typically achieved by inserting diagonal members between joints in the frame. The diagonals usually do not participate in resistance of gravity loads.

If single diagonals are used, they must serve a dual function: acting in tension for the lateral loads in one direction and in compression when the load direction is reversed (see Figure 9.57). Because long tension members are more efficient than long compression members, frames are often braced with a crisscrossed set of diagonals (called *X-bracing*) to eliminate the need for the compression members. The symmetrical form of X-bracing makes it an architectural form that designers sometimes choose for the building exterior (see Figure 9.2) or the interior (see Figure 9.58).

In any event, trussing causes the lateral loads to induce only axial forces in the members of the frame, as compared to the behavior of rigid frames. It also generally results in a frame which is stiffer for both static and dynamic loading, having less deflection than most rigid frames.

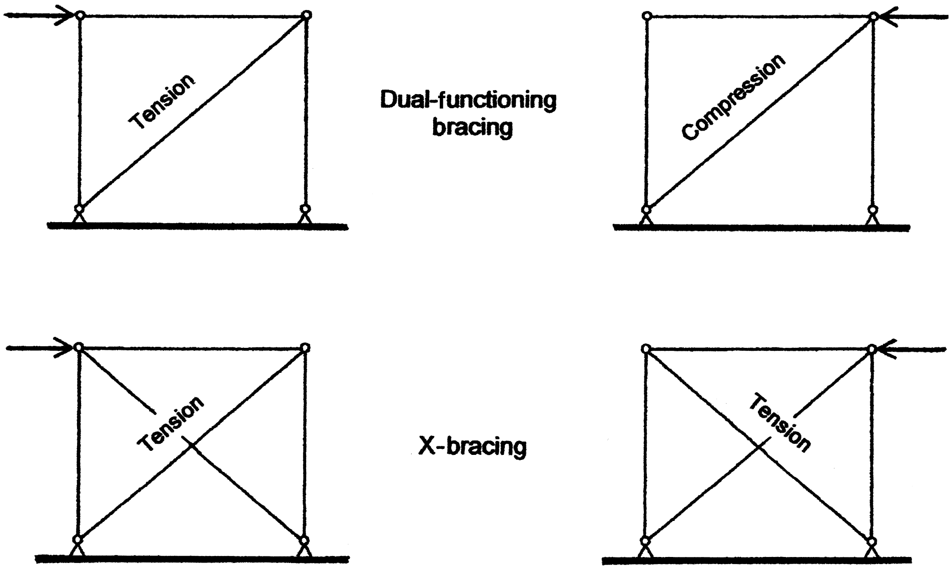


Figure 9.57 Trussed bracing.

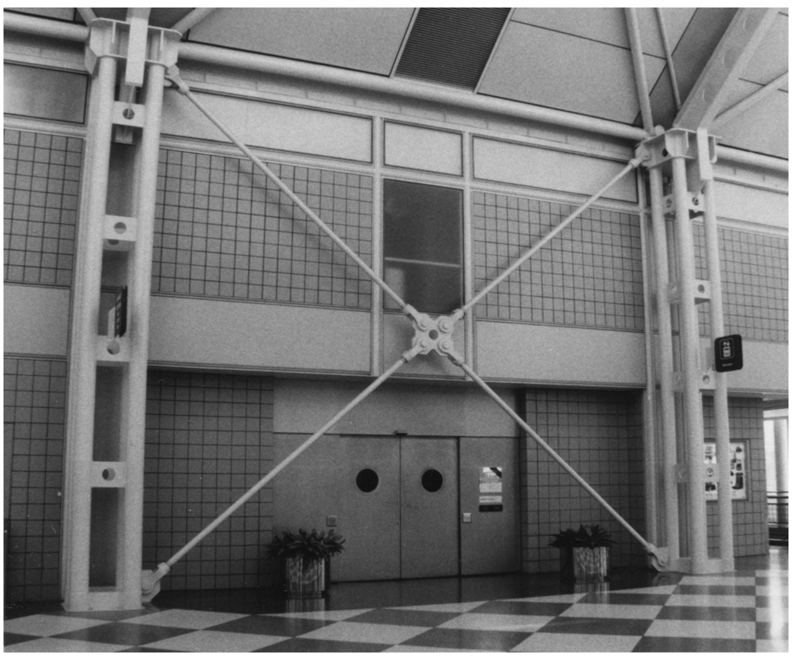


Figure 9.58 X-bracing may be dramatically displayed on both the exterior and interior of buildings. It is tricky to avoid blocking of traffic but is achieved in this example in the United Airlines Terminal in Chicago.

Single-story, single-bayed buildings may be braced as shown in Figure 9.57. Single-story, multibayed buildings may be braced by bracing less than all of the bays in a single plane of bracing, as shown in Figure 9.59*a*. The continuity of the horizontal framing is used in the latter situation to permit the rest of the bays to tag along.

Similarly, a single-bayed, multistoried towerlike frame as shown in Figure 9.59*b* may have its frame fully braced, whereas the more common form of the frame for a multistoried building, as shown in Figure 9.59*c*, is usually only partly braced. In this case, the partial bracing allows for more free use of the space on the building interior.

Just about any type of floor construction used for multistoried buildings usually has sufficient capacity for diaphragm action in the lateral bracing system. Roofs, however, often utilize light construction, sometimes have a nonflat form (gable, arch, etc.), or have a lot of large openings. In the latter case, the roof deck structure may not be functional as a horizontal diaphragm. For such roofs, or for floors with large openings, it is possible to use a horizontal truss for the bracing system, as shown in Figure 9.60.

For single-span structures, trussing may be utilized in a variety of ways for the combined gravity and lateral-load-resistive system. Figure 9.61*a* shows a typical gable roof with the rafters tied at their bottom ends by a horizontal member.

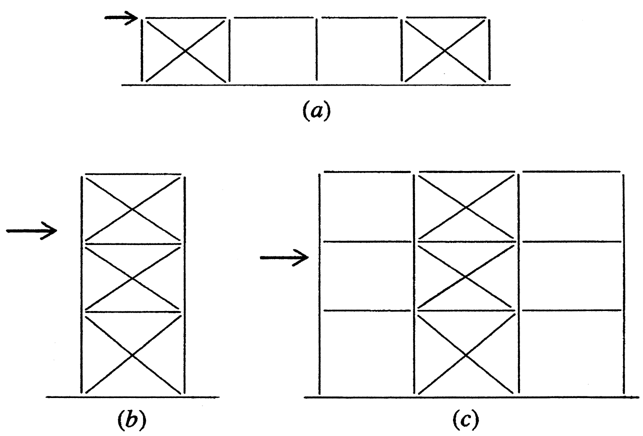


Figure 9.59 Bracing of frames with X-bracing.

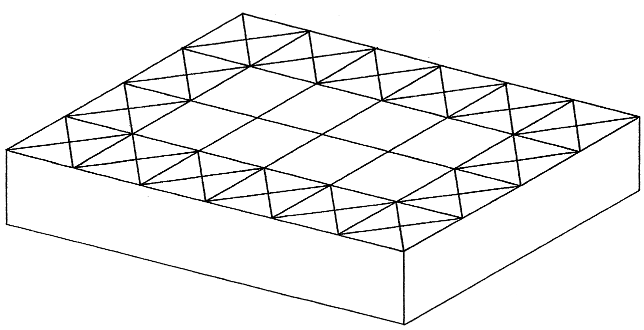


Figure 9.60 Partial trussing of a roof structure for use as horizontal bracing.

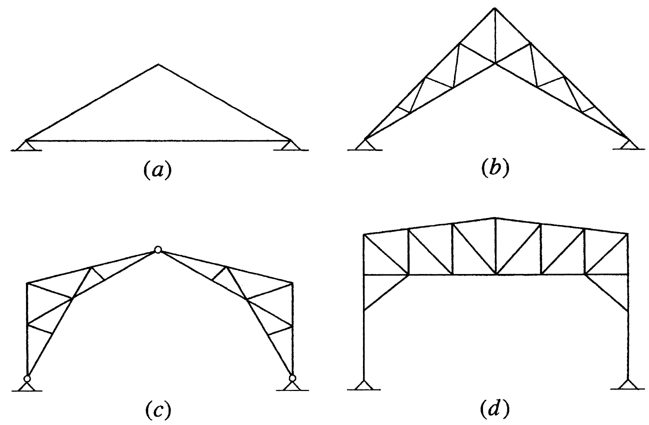


Figure 9.61 Forms of single-span trussed frames.

The tie, in this case, serves the dual functions of resisting the outward thrust due to gravity loads and working as one of the members of the single-triangle, trussed structure that is rigidly resistive to lateral loads. Thus the wind force on the sloping roof surface, or the horizontal earthquake force caused by the roof construction, is resisted by the triangular form of the rafter–tie combination.

The horizontal tie shown in Figure 9.61*a* may not be architecturally desirable in all cases. Some other possibilities for the single span—all producing more openness beneath the structure—are shown in Figures 9.61*b* through *d*. Figure 9.61*b* shows the so-called *scissors truss*, which can be used to produce a ceiling surface that reflects the gable form of the roof. Figure 9.61*c* shows a trussed bent that is a variation on the three-hinged arch. The structure shown in Figure 9.61*d* consists of a single-span truss that is supported by columns at its ends. If the columns are pin jointed at the bottom of the truss, the structure lacks basic resistance to lateral loads and must be separately braced. If the column is continuous to the top of the truss, it can be used in rigid-frame action for resistance to lateral loads. Finally, if the knee braces shown in the figure are used, stability can be achieved with the pin-topped columns.

Eccentrically Braced Frames

The knee brace is one form of what is called *eccentric bracing* because the diagonal members connect at one or more ends to a point within the length of the frame members. The range of trussed bracing is shown in Figure 9.62.

Concentric bracing (Figure 9.62*a*) connects to joints of the rectilinear frame, producing a conventional form of trussing. As used for lateral bracing, this may be achieved with single diagonals, but it is also frequently used in the form of *X-bracing* to avoid the problem of buckling of the long compression members.

As shown in Figure 9.63*b*, *V-bracing*, and its inverted form, *chevron bracing*, connects to a beam/column joint at one end but within the beam span at the other end. This results in a failure mode that involves the developing of a

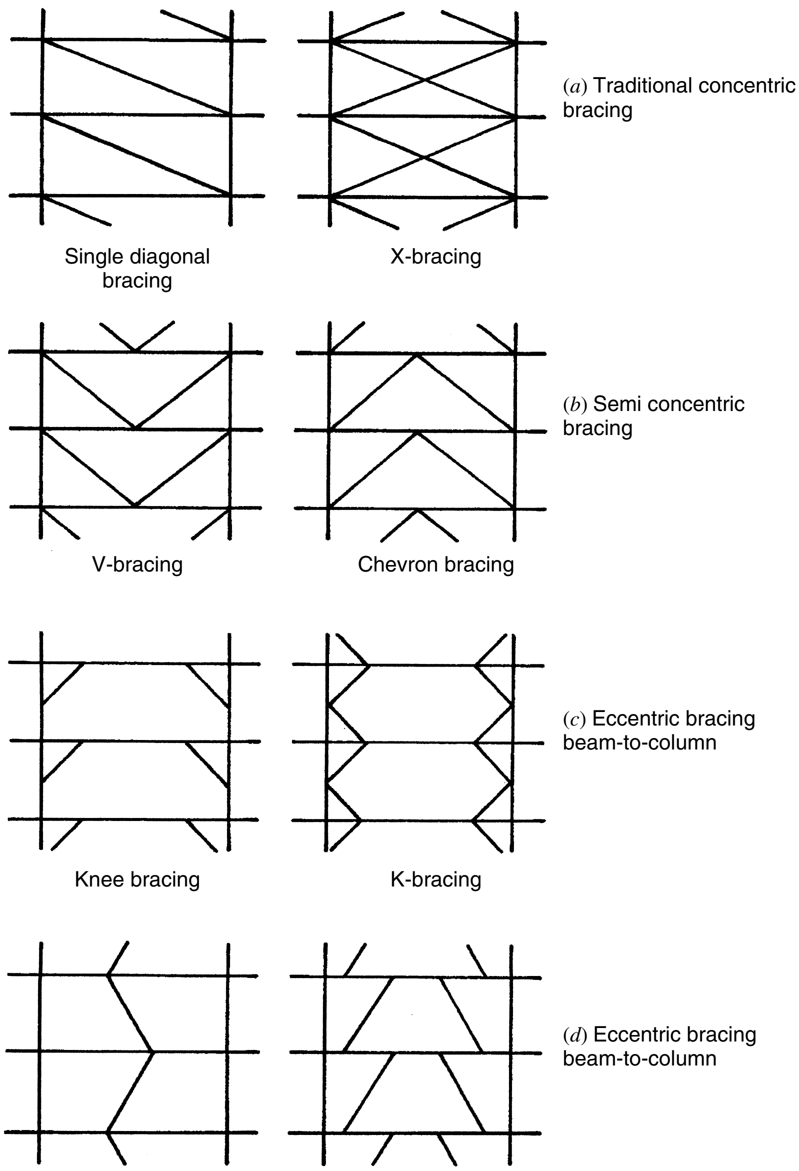


Figure 9.62 Options for development of the braced frame in multistory buildings.

plastic hinge in the beam. This bracing utilizes the energy-absorbing plastic behavior of the frame while still achieving the stiffness of the trussed bracing.

V-bracing is popular for building exteriors, where windows can be placed in the center of the bay. Chevron bracing, on the other hand, is popular for interiors, where a door or hallway can be accommodated. The open center of the bay is also preserved with *knee bracing* and *K-bracing*. Single-diagonal and X-bracing do not allow for this planning, which partly accounts for the development of the other forms.

Both knee bracing and K-bracing (Figure 9.63*c*) are traditional forms of fully eccentric bracing, with end connections to frame members rather than at joints. Both have been used for bracing frames for wind forces for many years. They are still often used for wind bracing, but they are not favored for seismic bracing due to the bending that is induced in the columns.

An unusual form of eccentric bracing is shown in Figure 9.62*d*, where tilted members are connected to beams at different levels. While quite a ways from conventional trussing, it nevertheless provides a trusslike action for the frame. When used with a rigid frame, it provides the stiffening of a truss system plus a redundant energy-absorbing capacity.

Use of the various forms of trussed bracing depends on the dimensions of beam spans and column heights and the size of frame members. In high-rise buildings, for example, columns in lower stories are very large and stiff, and the effects of bracing in producing column bending are of less concern. For tall framed buildings and for open framed towers, it is not uncommon to change forms of bracing several times in the total frame height, as sizes and proportions of members change.

On the other hand, in low-rise buildings columns tend to be small, and even if beam spans are long, the beams may be deep and stiff. Thus, V-bracing and chevron bracing

(*a*) (*b*)

Figure 9.63 Use of V-bracing for a perimeter bracing system in a low-rise building.

are favored for low-rise buildings. Figure 9.63 shows the use of V-bracing for a two-story building with relatively small columns and deep beams. As shown in the lower photo, the top connection of the V-brace is made to the beam end, rather than to the column.

Planning of Bracing

Some of the issues to be considered in using braced frames are the following:

Diagonal members must be placed so as not to interfere with the action of the gravity-resistive structure or with nonstructural building functions. If the bracing members are designed as axial-force members, they must be located and attached to avoid loadings other than those required for their bracing functions.

As mentioned previously, the reversibility of the lateral loads must be considered. As shown in Figure 9.58, such consideration requires that diagonal members be dual functioning (as for single diagonals) or redundant (as for X-bracing) with one set of diagonals working for the load from one direction and the other set working for the reversal loading.

Although the diagonal bracing elements usually function only for lateral loading, the vertical and horizontal elements must be considered for all the various combinations of gravity and lateral load. Thus the total frame must be analyzed for all the possible loading conditions, and each member must be designed for the particular load combination that represents its peak-response demand.

Long, slender bracing members, especially those in X-braced systems, may have considerable sag due to their own dead weight, and some form of support may be considered. An exception is with the use of highly tightened steel rods.

The trussed structure should be "tight." Connections should be made in a manner to assure that they will be initially free of slack and will not loosen under load reversals or repeated loadings. This means avoiding connections such as those using nails, loose pins, and unfinished bolts.

The deformation of the truss must be considered, as it may relate to its function as a distributing element, as in the case of a horizontal structure, or to the establishing of its relative stiffness, as in the case of a series of vertical elements that share loads. It may also relate to some effects on nonstructural elements that are attached to the structure.

In most cases it is not necessary to brace every individual bay of a rectangular frame system. In fact, this is often not possible for architectural reasons. As shown in Figure 9.59, walls consisting of several bays can be braced by trussing only a few, or even a single bay, with the rest of the structure tagging along.

The braced frame can be mixed with other bracing systems in some cases. Figure 9.64 shows the use of a braced frame for the vertical resistive structure in one direction and a set of shear walls in the other direction. In this example the two systems act independently, except for the possibility of torsion on the building, and there is no need for a deflection analysis to determine lateral-load sharing.

Figure 9.65 shows a structure in which the end bays of the roof framing are X-braced. For loading in the direction shown, these braced bays take the highest shear in the horizontal structure, allowing the deck to be designed for a lower shear stress.

Although buildings and their structures are often planned and constructed in two-dimensional components (horizontal

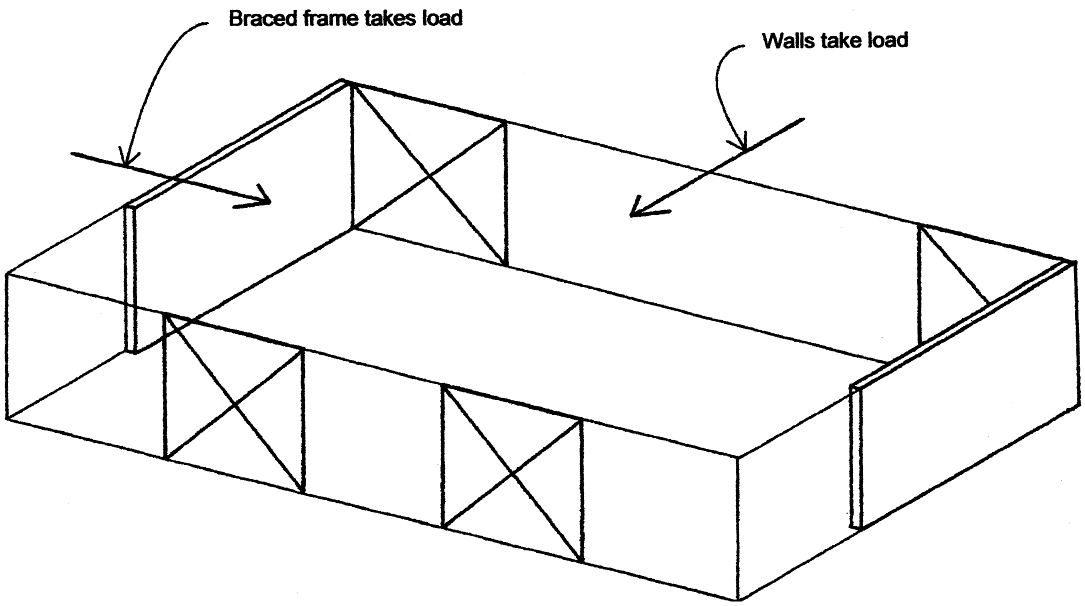


Figure 9.64 Mixed vertical elements for lateral resistance.

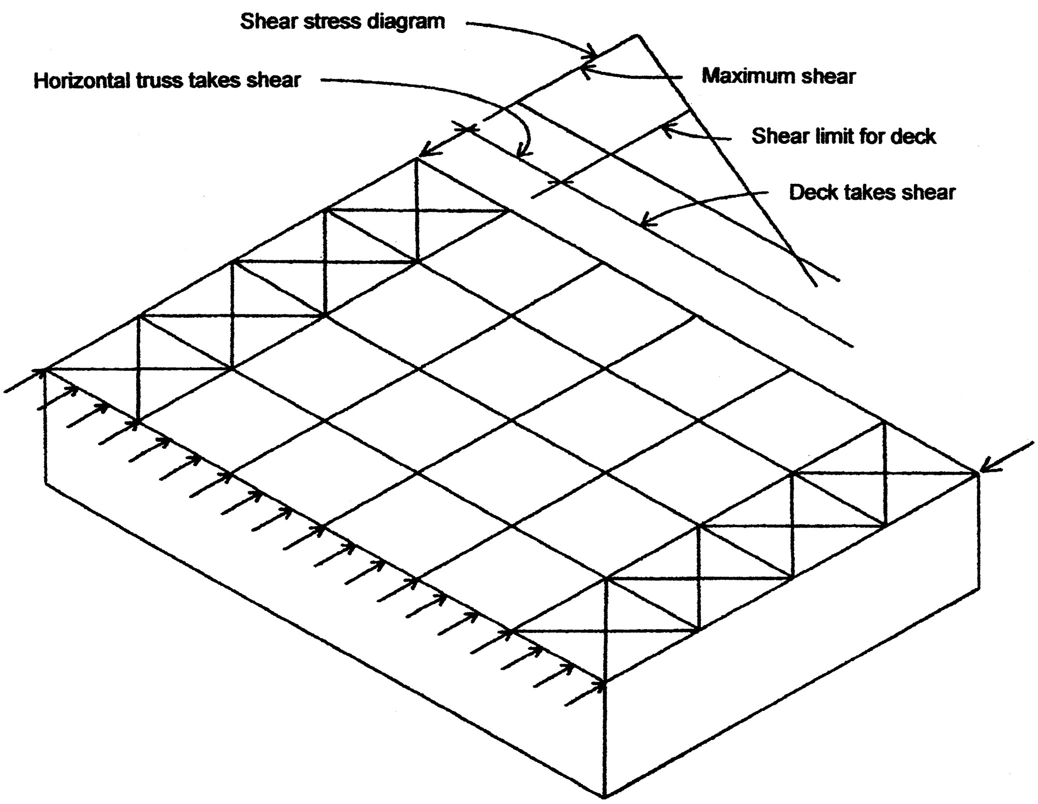


Figure 9.65 Horizontal bracing designed as a mixed system.

floor, vertical planar wall or framing bent), it must be noted that a building is truly three dimensional. The design for lateral bracing is thus a three-dimensional problem and the individual elements of the system must be clearly seen to interact in three dimensions. While the single triangle may be stable in its own plane, the three-dimensional structure composed of multiple planar triangles may not be truly stable.

Typical Construction

Development of the construction details for trussed bents is in many ways similar to the design of spanning trusses. The materials used (generally wood or steel), the form of individual truss members, the type of jointing (nails, bolts, welds, etc.), and the magnitudes of the forces are all major considerations. Since many of the members of the complete trussed bent serve dual roles for gravity and lateral loads, member selection is seldom based on lateral-force effects alone. Quite often trussed bracing is produced by simply adding diagonals (or X-bracing) to a system already conceived for gravity loads and for the general development of the desired architectural forms and spaces.

Figure 9.66 shows some details for wood framing with added diagonal members. Wood members are most often rectangular in cross section and metal connecting devices of various forms are used in the assembly of the framework. Figure 9.66*a* shows a typical beam-and-column assembly with diagonals consisting of pairs of wood members bolted to the frames. When X-bracing is used, and the diagonals need take only tension forces, slender steel rods can be used; a possible detail for this is shown in Figure 9.66*b*. For the wood diagonal an alternative to the bolted double members is one that uses gusset plates with single-member diagonals. If construction details make the protruding members in Figure 9.66*a*, or even the gusset plates in Figure 9.66*b*, undesirable, a bolted connection like that shown in Figure 9.66*d* may be used. Whatever the form, size, or arrangement of the wood members, there are most likely a variety of manufactured steel connecting devices that can be used to create these joints.

A contributing factor in the deformation of trussed bents under any loading may be movements that occur within the joints. Bolted connections are especially vulnerable when used in shear resistance since the drilled holes for bolts must be slightly oversized to facilitate assembly. Other contributing factors are the shrinkage of the wood and bending of bolts. Tightness of bolted joints can be increased by the use of shear developers, such as split-ring connectors.

Gusset plates may consist of plywood or thin sheet metal for small structures. For larger bents gussets will usually be thick steel plates with bolts for connectors. Plywood gussets ordinarily use nails or screws. Thin sheet metal gussets may use nails or screws but are also formed with protruding metal points that are embedded in the wood by pressure.

Figure 9.67 shows some details for the incorporation of diagonal bracing in steel frames. As with wood structures, bolt loosening is a potential problem. For shear-type connections, highly tensioned bolts are preferred. A completely welded connection will produce the stiffest joint, but on-site bolting is much preferred over welding. The typical joint consists of some parts that are welded to members in the shop (off-site) and some attachment with bolts in the field (on-site).

Various steel elements may be used for diagonal members, depending mostly on the magnitude of loads and the form of the frame members. A popular diagonal consists of a pair of angles which are typically bolted to a plate or the web of a portion of a structural tee shape. This is the detail shown in Figure 9.67*c*. Many other options are possible, including steel rods and steel pipes or tubular sections.

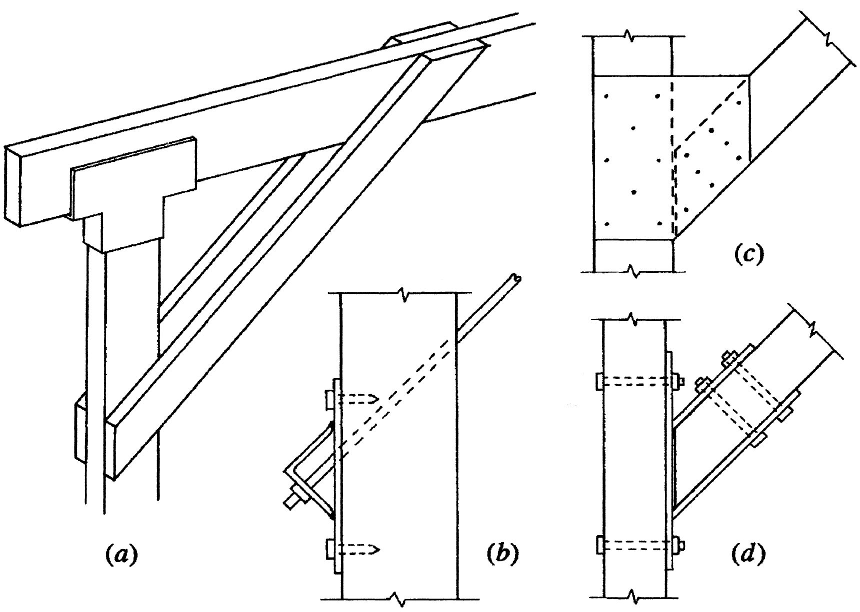


Figure 9.66 Framing of trussed wood bents.

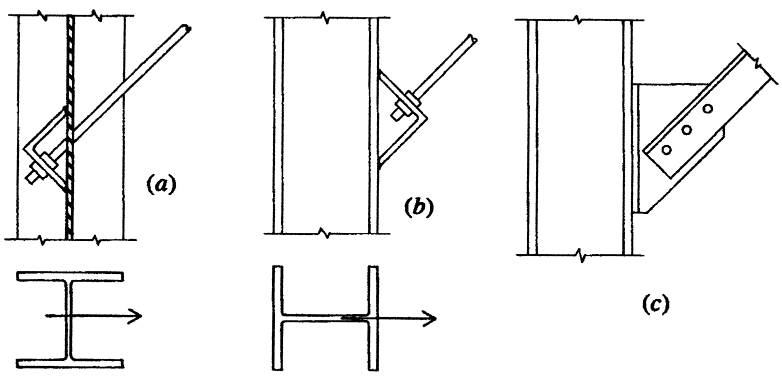


Figure 9.67 Framing of trussed steel bents.

Stiffness and Deflection

As has been stated previously, the braced frame is typically a relatively stiff structure. This is based on the assumption that the major contribution to the deflection of the structure is the shortening and lengthening of the members of the frame as they experience the tension and compression forces due to the truss action. However, the two other potentially significant contributions to the lateral movement of the braced frame are:

Deformation in the Frame Connections. This involves primarily the behavior of the connecting devices—nails, bolts, and so on—as has been discussed. However, it may also have to do with deformations of the connected members at a joint.

Movement of the Supports. This includes the possibility of deformation of the soil-supported foundations and the yielding of anchorage connections. For foundations on soil, there will be an uneven development of pressure due to lateral load, which may be critical for highly deformable soils.

It is good design practice in general to study the connection details for braced frames with an eye toward reduction of deformation within the connections. This may also involve considerations for the materials and forms of the frame members.

The maximum lateral deflection, or drift, of a single-story X-braced frame is usually caused primarily by the tension elongation of the diagonals. This condition is based on the assumption that the slender compression diagonal buckles at a relatively low force and the horizontal and vertical members are not highly stressed by the truss action. As shown in Figure 9.68, the elongation of the diagonal moves the rectangular-framed bay into a parallelogram form. The approximate value of the deflection, *d* in Figure 9.69, can be derived as follows.

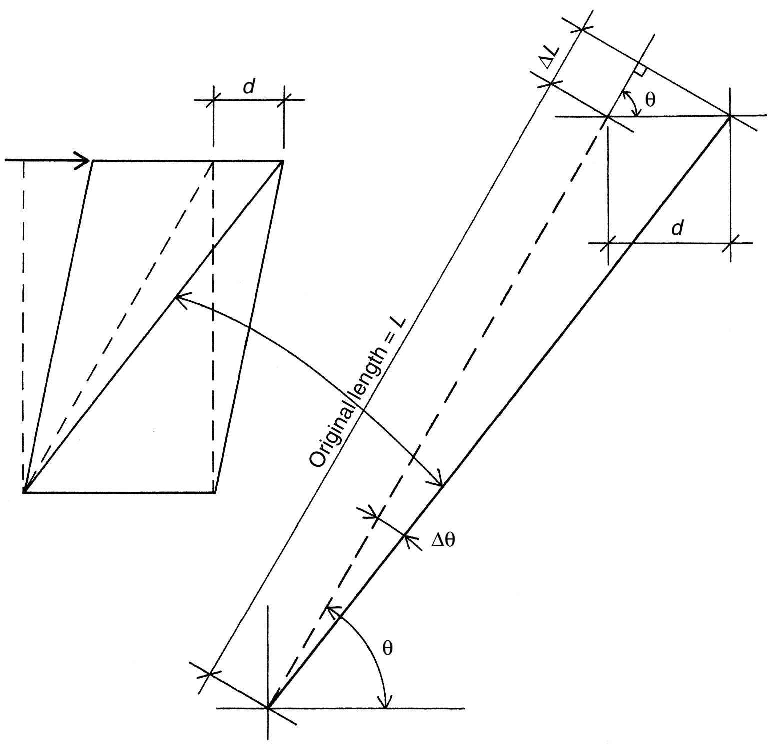


Figure 9.68 Deflection of a single-bay braced frame.

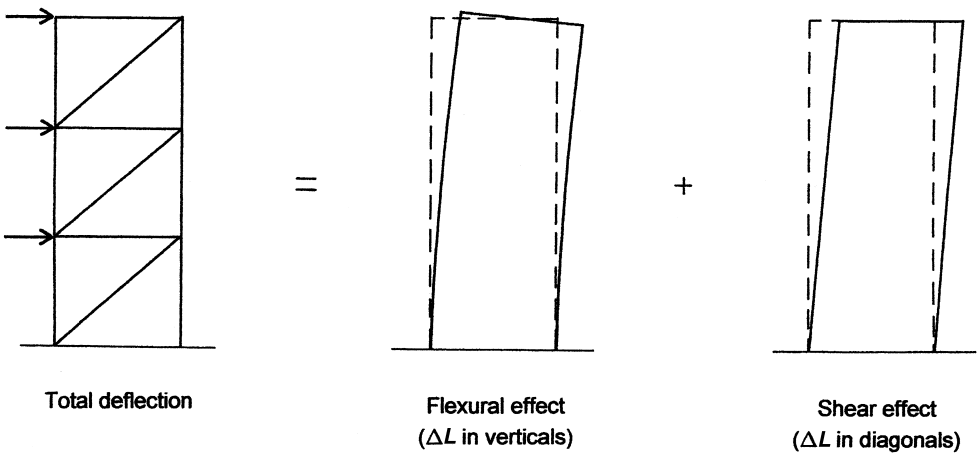


Figure 9.69 Deflection of a multistory trussed bent.

Assuming the change in the angle of the diagonal to be small, the change of length in the diagonal may be used to approximate one side of the triangle of which d is the hypotenuse. Thus

$$d = \frac{\Delta L}{\cos\theta} = \frac{TL/AE}{\cos\theta} = \frac{TL}{AE\ \cos\theta}$$

where

T = tension in diagonal
A = cross-sectional area of diagonal
E = elastic modulus of diagonal
θ = angle of diagonal from horizontal

The deflection of multistory trussed bents has two components, both of which may be significant. As shown in Figure 9.69, the first effect is caused by the change in length of the vertical members of the frame due to the overturning moment. The second effect is caused by elongation of the diagonals, as discussed for the single-story frame. These deflections occur in each story of the frame and can be computed individually and summed up for the whole frame. Although this effect is also present in the single-story frame, it becomes more pronounced as the frame gets taller with respect to its width.

Moment-Resistive Frames

Frames that have rigid connections between the members are commonly called *rigid frames.* Code references now use the term *moment-resistive frame,* but the common term of rigid frame is still used for general description and will be used in this discussion.

General Behavior

In most cases rigid frames are actually the most flexible of the basic types of lateral resistive systems, unless the frame members themselves are exceptionally stiff. This deformation character makes the rigid frame a structure that absorbs energy loading through deformation (like a coil spring) as well as through its sheer brute strength. The net effect is that the structure actually works less hard in force resistance, especially for dynamic loads.

Most rigid frames consist of steel or concrete. Steel frames have either welded or bolted connections between members to develop moment transfers. Concrete frames achieve moment connections through the monolithic form of the cast concrete and the continuity of the steel reinforcement. Because concrete is brittle, the concrete rigid frame depends on the plastic yield of the reinforcement to develop a yield behavior of the rigid frame.

A complete presentation of the design of rigid frames is beyond the scope of this book. Such design can only be done with computer-aided methods that are capable of dealing with the high degree of indeterminacy of the basic structural form and presently used strength methods. For seismic loadings it is also usually required to use dynamic energy loading analysis. Simplifying assumptions sometimes permit the use of reasonable approximate analyses; examples of which are shown in Chapter 10.

For lateral seismic loads, the design codes have recently promoted the inclusion of redundancy in the bracing system. This may be achieved with multiple elements, so that failure of a single element does not mean immediate collapse of the whole structure. In this regard, the rigid frame has an advantage in that it develops many internal stress conditions instead of a single one. It also lends itself to being part of a *dual bracing system,* that is, one in which two separate systems are coupled to share lateral forces, with their different responses serving to provide a mutual damping effect for dynamic loads.

Deformation analysis is a critical part of the design of rigid frames. Excessive deformations have the potential for causing movements that can be sensed by occupants or of damage to nonstructural parts of the construction. The need to limit deformations often results in the size of vertical elements of the frame being determined principally by stiffness requirements rather than by stress limits.

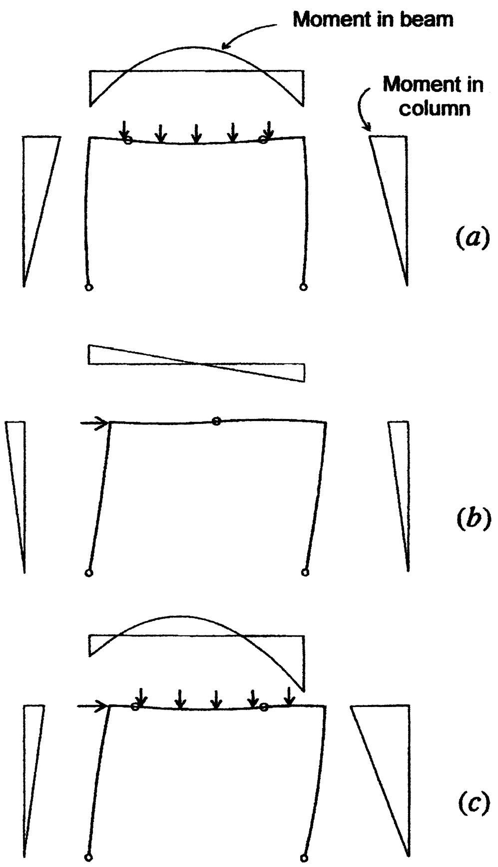


Figure 9.70 Loading and response for a single-span rigid frame with pin-based columns: (*a*) under gravity alone; (*b*) under lateral load alone; (*c*) under combined gravity and lateral loads.

Loading Conditions and Responses

Unlike shear walls or trussing, rigid frames are not able to be used for lateral bracing alone. Actions of lateral forces must always be combined with those due to gravity. This often means that several loading combinations must be investigated, with different parts of the bracing system being critical for different load combinations.

Figure 9.70*a* shows the form of deformation and the distribution of internal bending moments in a single-span rigid frame, as induced by gravity loads. If the frame is not required to resist lateral loads (due to other bracing), the members and connections of the frame may be designed for this condition alone. Observable conditions of note include the bending moment in the columns, the rotation of the column base, the sign of moment at the beam-to-column joint, the sign of moment and nature of corresponding stresses at various points in the beam, and the location of inflection in the beam. Selection of member shapes and sizes and the details for the frame joints will be based on all of these situations and the quantified effects they produce.

If the frame under gravity load (Figure 9.70*a*) is symmetrical, there should be no lateral deflection, and the only deformations of concern are the outward bulging of the columns and the vertical deflection of the beam at midspan.

Under action of lateral loading, the form of deformation and distribution of internal bending moment will be as shown in Figure 9.70*b*. If the gravity and lateral loadings are combined, the net effect will be as shown in Figure 9.70*c*. Observing the effects of the combined loading, we note the following:

Horizontal deflection at the top of the frame (called drift) must now be considered, in addition to other deformations mentioned previously for gravity load.

The maximum value for the moment at the beam-to-column connection is increased on one side and decreased (or even possibly reversed) on the other side. The entire form of the connection may need to change for this situation.

The direction of the lateral load is reversible, so that if the frame is not symmetrical, different parts of the frame may respond critically to different load combinations. In fact, because load factors vary for different load combinations, gravity load alone may be critical in some cases.

While single-span rigid frames are often used for buildings, the multispan or multistory frame is the more usual case. Figure 9.71 shows the response of a two-story, two-bay wide frame to lateral loads (*a*) and to a gravity load on a single beam (*b*). The response to lateral loads is essentially similar to that of the single-span frame in Figure 9.70. For gravity loads the multiunit frame must be analyzed for a more complex set of potential load combinations, because the live load portion of the gravity loads must be considered to be random and thus may or may not occur in any given beam span.

Lateral loads produced by winds will generally produce the response shown in Figure 9.71*a*. Because of its size and relative flexibility, however, a multistory rigid frame may respond so slowly to seismic movements that upper levels of the frame experience a whiplash-like effect; thus separate levels may be moving in opposite directions at a single moment (see Figure 9.35). Figure 9.72 illustrates a type of response that may occur if the two-story frame experiences this action. Only a true dynamic analysis can ascertain whether this action occurs and is of critical concern for a particular structure.

Of course, any lack of symmetry in a rigid frame may cause much more complex responses in terms of deformations and internal force effects. This is an argument not for pure symmetry in all cases but for the more complex investigations for effects and more work in design that is required

Approximate Analysis for Gravity Loads

Most rigid frames are statically indeterminate and require the use of some method beyond simple statics for their analysis. Simple frames of few members may be analyzed by some *hand* method using handbook coefficients, moment distribution, and so on. If the frame is complex, consisting of several bays

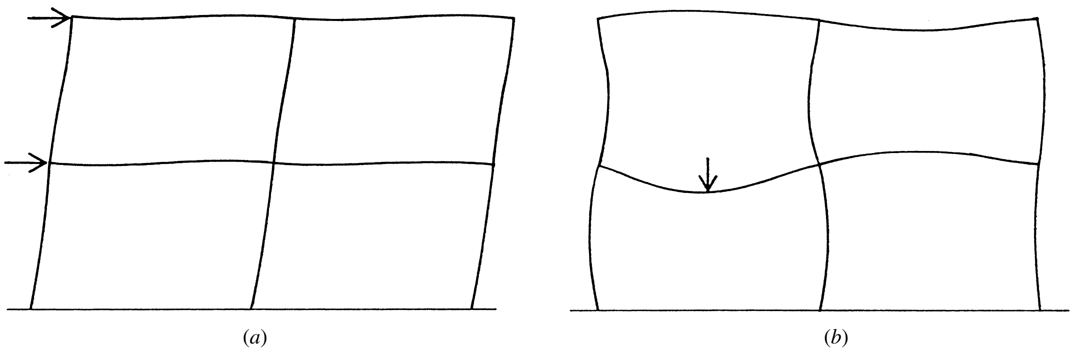


Figure 9.71 Load responses of a multistory rigid frame.

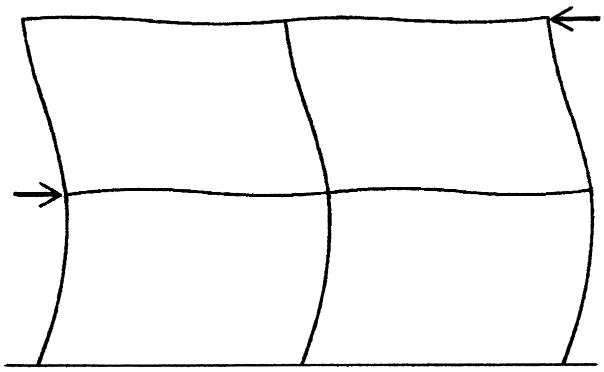


Figure 9.72 Slow response of a multistory rigid frame to seismic movements.

and levels or having a lack of symmetry, the analysis will be quite laborious unless performed with a computer-aided method.

For preliminary design it is often useful to have the results of some approximate analysis which can be quickly performed. Internal forces, member sizes, and deflections so determined may be used for a first design try to establish the feasibility of the design.

For the simple, single-span bent shown in Figure 9.73, the analysis for gravity loads is quite simple, since a single loading condition exists as shown and the only necessary assumption is that for the relative sizes of the members. For the frame with pin-based columns an analogy is made to a continuous three-span beam on rotation-free supports. If the column bases are fixed, the ends of the analogous beam are considered as fixed.

For multibayed, multistoried frames, an approximate analysis may be performed using the beam analysis factors as described for continuous beams in Chapter 6. This is more applicable to concrete frames, of course, but can also be used for quick approximations for welded steel frames. An example of this analysis is shown for the three-story case study building in Chapter 10.

Even when using approximate analyses, it is advisable to find values separately for dead loads and live loads. The results can then be combined as required for various critical combinations of dead load, live load, wind load, and seismic load.

Approximate Analysis for Lateral Loads

For frames that are complex—due to irregularities, lack of symmetry, tapered members, and so on—analysis is hardly feasible without the computer. This is also true for analyses that attempt to deal with the true dynamic behavior of the structure under seismic load. For quick approximation for preliminary design, however, use of various approximation methods is likely to continue.

For the simple bent shown in Figure 9.74, the effects of the single lateral force may be quite simply visualized in terms of the load-deflected shape, the reaction forces, and the variation of moment in the members.

If the columns are pin based and of equal stiffness, it is reasonable to assume that the horizontal reactions at the base are equal, thus permitting an investigation of the frame by static analysis. If the column bases are fixed, the frame is able to be analyzed only by indeterminate methods, although an approximate analysis can be done by assuming the location of the inflection point in the columns. In truth, the real condition is likely to be somewhere between these two cases. A preliminary design may use both analyses and design for the worst of both, with adjustments made when the true conditions of the construction at the base and supports are established.

For multibayed frames, such as that shown in Figure 9.75*a*, an approximate analysis may be done in a manner similar to that for the single-bay frame. If the columns are all of equal stiffness, the total load is simply divided by the number of columns. Assumptions about the column base condition would be the same as for the single-bay frame. If the columns are not all of the same stiffness, an approximate distribution can be made on the basis of relative stiffness.

Figure 9.75*c* illustrates the basis for an approximation of the horizontal shear forces in the columns of a multistory building. As for the single-story frame, the individual column shears are distributed on the basis of assumed column stiffnesses. For the upper level columns the inflection point is assumed to occur at midheight, unless column splice points are used to control its location.

General Design Considerations

From a structural point of view, there are many advantages of the rigid frame as a lateral bracing system for seismic effects. Current codes tend to favor it over shear walls or

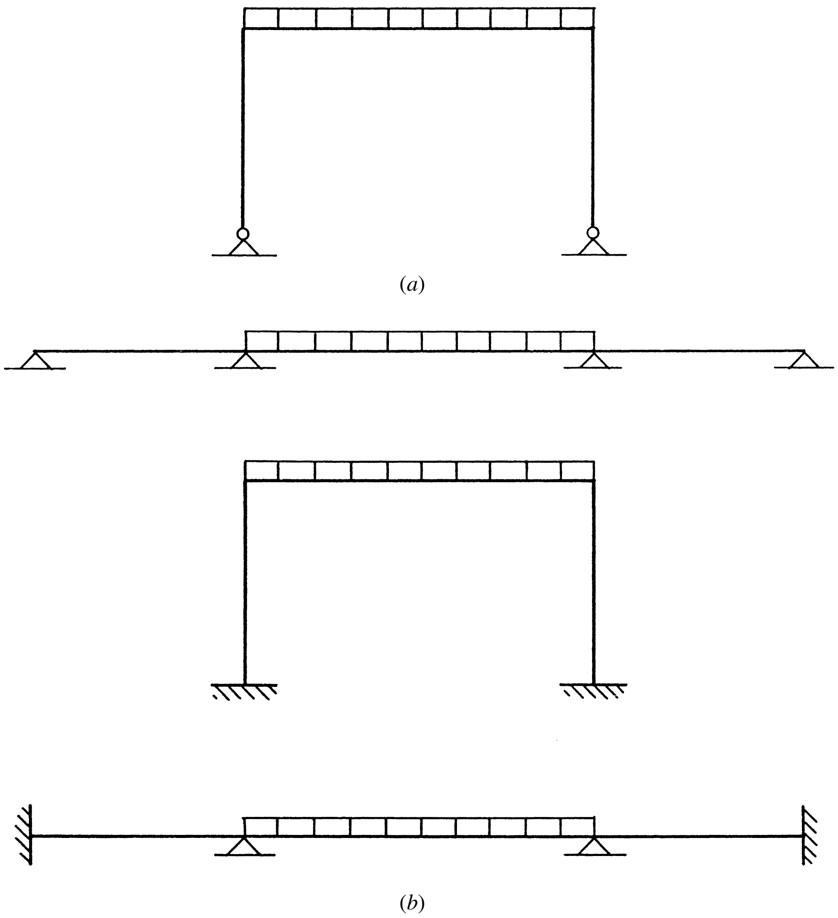


Figure 9.73 Continuous beam analogy for a single-span rigid frame.

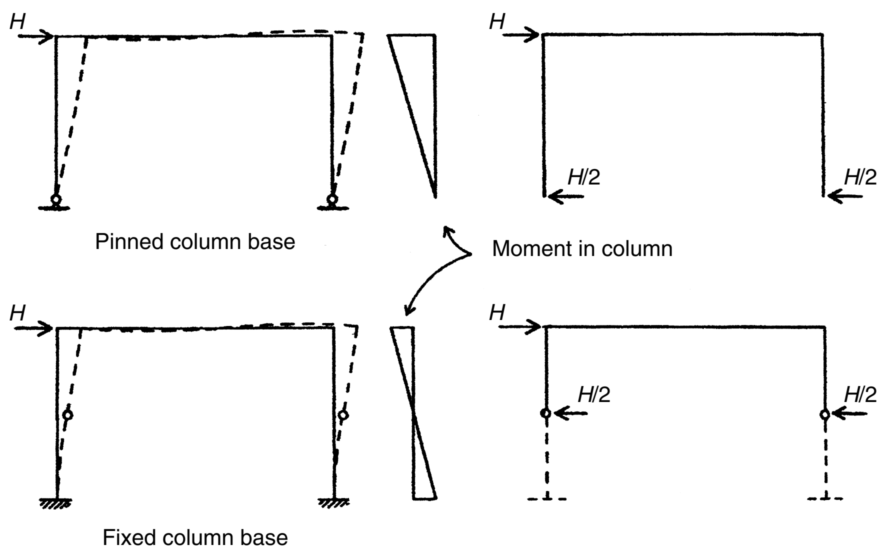


Figure 9.74 Effects of column base conditions on a single-span bent under lateral load.

braced frames by requiring less design load due to its reduced stiffness. For tall frames the combination of slow reaction time and various damping effects means that forces tend to dissipate rapidly in the remote portions of the frame.

Architecturally, the rigid frame offers the least potential for interference with planning of open spaces within the building interior and in exterior walls. It is thus highly favored by architects whose design style includes the use of ordered rectangular grids and open spaces of cubical form. Geometries other than rectangular ones are possible also; what remains as the primary advantage is the absence of need for diagonal bracing or solid structural walls.

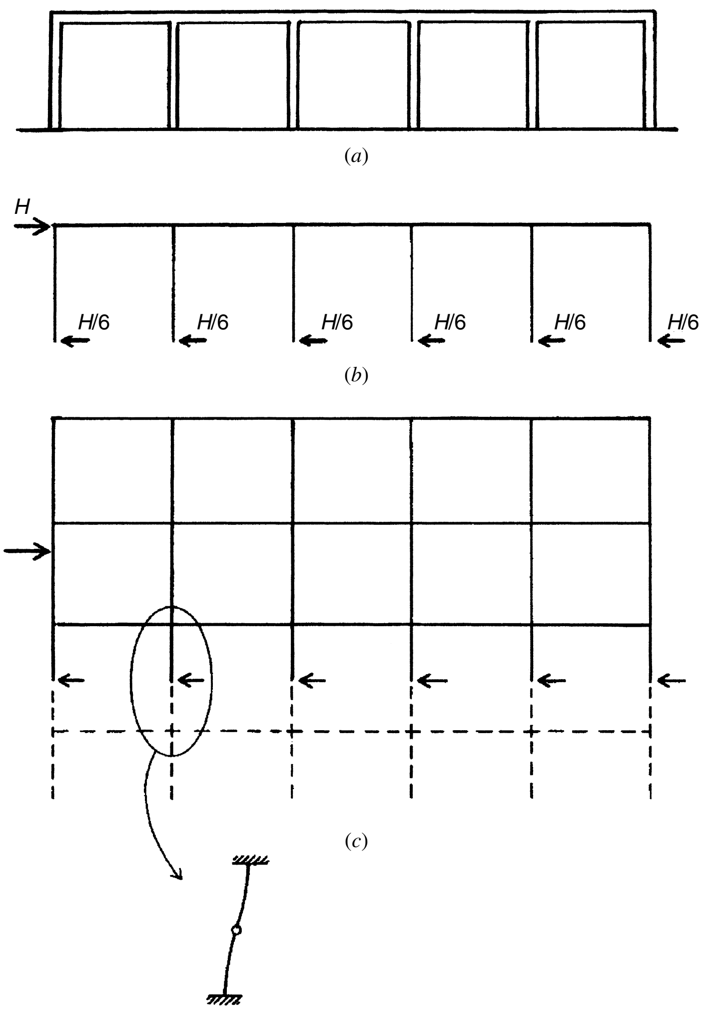


Figure 9.75 Distribution of lateral shear in multiunit frames.

Some of the disadvantages of rigid frames are the following:

Lateral deflection, or drift, of the structure is likely to be a problem. As a result, it is often necessary to stiffen the frame—mostly by increasing the number and/or size of the columns.

Connections must be stronger, especially in steel frames. Addition of heavily welded steel joints, added reinforcing for bar anchorage, shear, and torsion in concrete frames and for cumbersome moment-resistive joints in wood frames can add appreciably to construction cost and time.

Dynamic analysis for seismic loading is usually required for frames with any degree of complexity.

When rigid frames are to be used for the lateral bracing system for a building, there must be a high degree of cooperation between the planner of the building form and the structural designer. The building planning must accommodate some regularity and alignment of columns that constitute the vertical members of the rigid-frame planar bents. Large floor and roof openings must be placed so as not to interrupt beams that are parts of the bents.

There must, of course, be coordination between the designs for gravity and lateral loads. If a rigid frame is used for bracing, its horizontal members will undoubtedly be parts of the general framing for gravity loads. However, in most cases, much of the horizontal structure will be used only for gravity loads. In the system that utilizes only the bents that occur at exterior walls (called a *perimeter bracing system*), none of the interior beams or columns will be involved in the bent actions; thus their planning is free of concerns for the bent development.

Isolated rigid frames may be used within a building to provide lateral bracing for a much lighter general construction system. A common use of this type is one in which steel bents are used in wood frame buildings, usually to accommodate open spaces in place of solid shear walls. A building that is mostly produced with precast concrete elements may have selected bents of sitecast concrete with its characteristic rigid-frame actions.

Selection of members and construction details for rigid frames depends a great deal on the materials used. The following discussion treats separately the problems of bents of steel and reinforced concrete.

Steel Frames

Steel frames with moment-resistive connections were used for early skyscrapers. Fasteners consisted of rivets until the advent of welding and high-strength bolts. Figure 9.76 shows a typical rigid steel frame for a low-rise office building. Shown here is the use of welding for the moment connections (flange to flange) and bolting for shear connections of beam webs to column faces. This was a widely used system for connection until recent earthquakes revealed a high vulnerability. New and better versions of this connection are still being developed, so the simple form shown here is becoming less used.

Another form of rigid frame is the trussed bent, as shown in Figure 9.61*d*. In that illustration the rigid connection is achieved with a knee brace, but it can also be developed by simply extending the column to the top of the truss.

For tall buildings a currently popular system is one that uses a perimeter bracing arrangement with stiff, closely spaced columns and heavy spandrel beams. The stiff members, which experience little flexural curving, produce a bent that is close in character to the pierced solid wall, and drift in this case is very limited. Figure 9.77 shows the erected frame for such a structure. Note that the bents are discontinuous at the corners, thus avoiding the high concentration of forces on the corner columns which develop due to torsion on the tower structure.

Reinforced Concrete Frames

Sitecast concrete frames with monolithic columns and beams have a natural rigid-frame action. It is also possible, of course, to brace such a structure with concrete shear walls placed at the building plan core. Even when the frame alone is used for bracing, selected bents may be designed for bracing by deliberately stiffening the columns and/or beams in the planes of the selected bents.

For seismic resistance both columns and beams must be specially reinforced at their ends for the shears and torsions at the member ends. Beams in bracing bents ordinarily use continuous top and bottom reinforcing with continuous spaced lateral loop ties that serve the triple functions of resisting shear, torsion, and buckling of compressive reinforcement. As with steel frames, recent experiences with earthquakes have resulted in more stringent requirements for the detailing of reinforcement at the joints of concrete rigid frames.

Structures with precast concrete elements are often difficult to develop as rigid frames, unless the precast units themselves are cast as individual bents instead of the usual single, linear elements. Moment-resisting joints for these structures are difficult to achieve, the more common system being the use of sitecast shear walls for bracing.

Figure 9.78 shows two concrete structures with sitecast exterior frames. These may be developed with perimeter bents or may have the major lateral bracing achieved with interior frames or shear walls. The perimeter frame is most advantageous for resistance of torsion on the building and may be used when the building form results in such action for seismic loads. For buildings subjected mainly to wind or with very symmetrical plans, the interior bracing may be preferred, allowing for a lighter exterior frame.

Interaction of Frames and Diaphragms

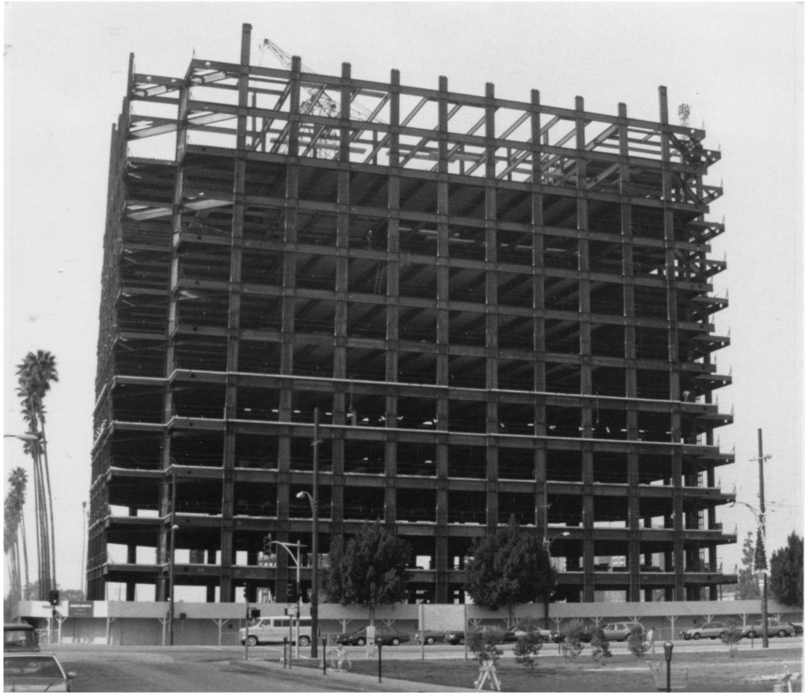
Most buildings consist of combinations of horizontal decks, walls, and some framing of wood, steel, or concrete. The planning and design of the lateral resistive structure requires judgments and decisions regarding the roles of these elements and their interaction during the application of lateral loads.

Coexisting, Independent Elements

Most buildings have some solid walls, that is, walls with continuous surfaces mostly free of openings. In one way or

(*a*) (*b*)

Figure 9.76 Steel structure with welded moment connections between beams and columns and welded splices in columns for rigid-frame action.



(*a*)

(*b*)

Figure 9.77 Office building with very rigid frame. Top photo shows proportions of stout members of frame. Bottom photo shows detachment of the frame bents at the building corner.

(*a*)

(*b*)

Figure 9.78 Reinforced concrete rigid frames at the building exterior. Top photo shows perimeter of the beam/column frame in the plane of the building exterior wall; the frame may or may not be the bracing system. Bottom photo shows the exterior beam/column bents outside the building enclosing walls. Here also the frame may or may not be the primary bracing system.

another, these walls are likely to become involved in some response to lateral loads.

Most solid walls tend to be relatively stiff in terms of resistance to lateral deformation in the plane of the wall. This has two aspects that must be considered: If the wall is nonstructural, it must be protected from damage during the application of the lateral loads; if it is a structural shear wall, its relationship to other elements of the bracing system must be considered—especially when some form of framing coexists with the walls.

A frame may be designed for gravity loads only, with shear walls providing lateral resistance. If so, some members of the frame will act as collectors, stiffeners, shear wall end members, or diaphragm chords. If walls are not intended for carrying gravity loads, care must be taken in development of construction details to assure that is indeed the case.

Load Sharing

In some cases both walls and frames may be used for lateral bracing at different locations or in different directions.

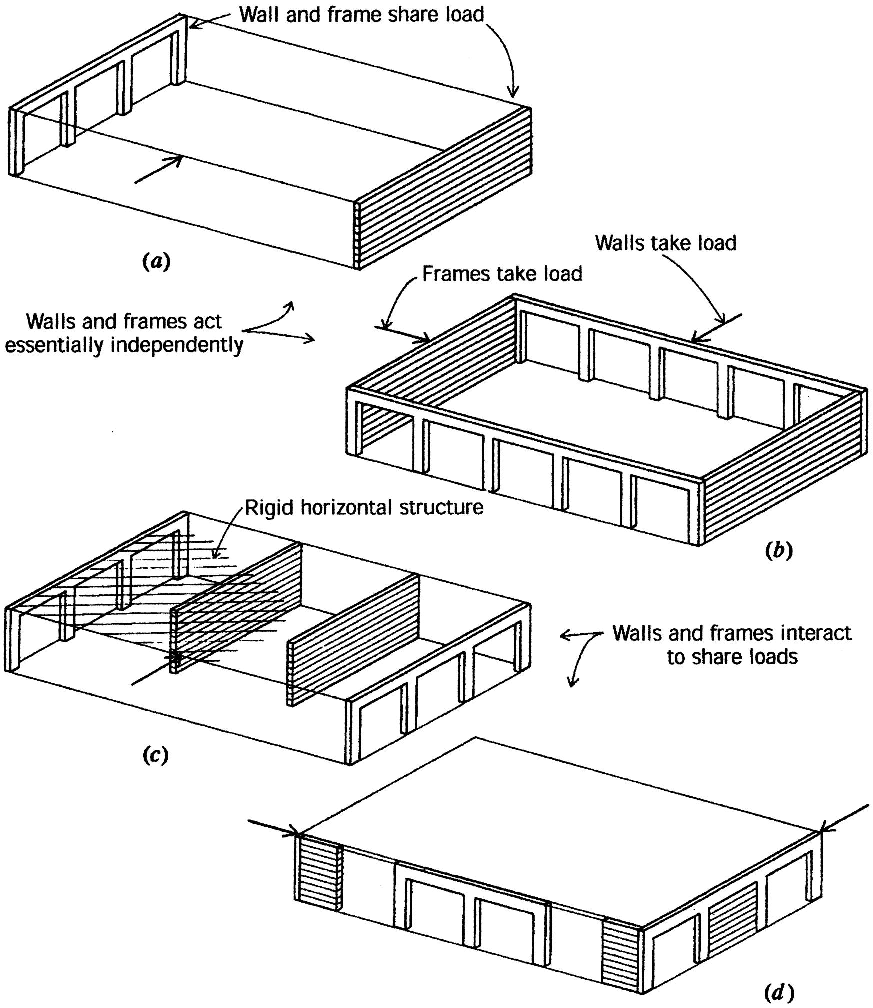


Figure 9.79 Mixed lateral bracing systems.

Figure 9.79 shows two such situations. In Figure 9.79*a* a shear wall is used at one end of the building and a frame at the other end for the wind in one direction. In Figure 9.79*b* walls are used for the lateral loads from one direction and frames for the loads from the other direction. In both of these cases the walls do not actually interact, that is, they act independently with regard to load sharing.

Figure 9.79*c* shows a situation in which walls and frames interact to share a direct load. If the horizontal structure is a rigid diaphragm, the load sharing will be done on the basis of the relative stiffness of the vertical elements. This relative stiffness must be established on the basis of the computed deflection resistance of the elements.

Figure 9.79*d* shows a situation in which walls and a frame interact to share a load in the same plane. This is a highly indeterminate situation, and most designers would assume that the stiffer walls take the entire load.

Collectors and Ties

Transfer of loads from horizontal to vertical elements in a lateral bracing system frequently involves the use of some structural elements that serve the functions of struts, drags, ties, collectors, and so on. These elements often serve two functions—as part of the gravity-resistive system and for functions in the bracing system.

Figure 9.80 shows a bracing structure consisting of a horizontal diaphragm and a number of exterior shear walls. For loading in the north–south direction the framing members labeled A serve as chords for the diaphragm. In most cases they are also parts of the roof edge or top of the wall framing. For the lateral load in the east–west direction they serve as *collectors*. This latter function permits us to consider the shear stress at the roof edge to be a constant along the entire length of the edge. The collector gathers this stress from the roof edge and distributes it to the isolated shear walls, thus functioning as a tension/compression member in the gaps between the walls. These framing members also serve to tie the separate walls together so that they all deflect the same in sharing the loads.

Collectors B and C in Figure 9.80 gather load from the diaphragm and distribute it to the three shear walls. When the load direction is northward, they act in compression; for this action they are referred to as *struts* in column action. For this function they are subject to the usual concerns for buckling, and the construction must be reviewed for development of resistance to the buckling of the long

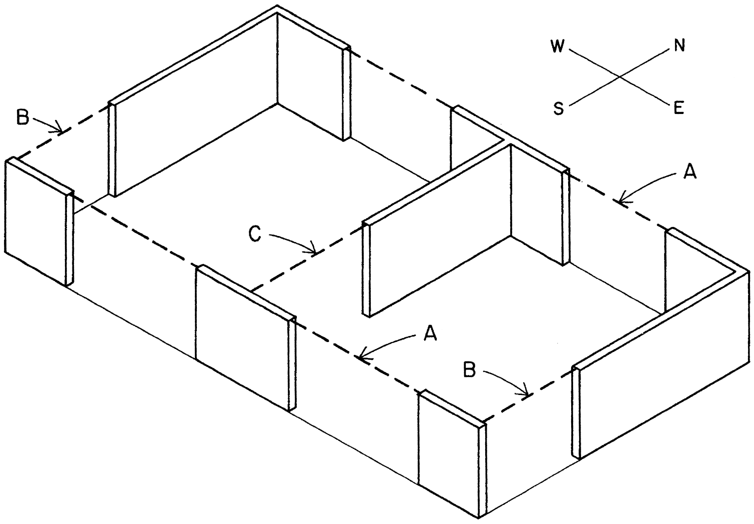


Figure 9.80 Collectors and ties in a box system.

compression member. When the load direction switches to southward, these members act in tension and are referred to as *drag struts*. This name actually refers to their dual use—as compression struts and tension drags. For the drag function they must be attached to the shear wall with a tension-resistive connection and must be investigated for any critical net cross-sectional stress in tension. Members A serve the drag strut functions for load in the east and west directions.

Anchorage Elements

The attachment of elements in the lateral resistive system to one another, to collectors, or to supports usually involves the use of some type of anchorage device. There are a great variety of these devices, encompassing the range of situations with regard to load transfer conditions, magnitude of the forces, and various materials and forms of the connected elements.

Tiedowns

Resistance to vertical uplift is sometimes required for vertical elements of a bracing system. Where upward wind force (suction) occurs on a roof surface, the roof construction should be anchored, especially with very light roof materials or structures. For wood-framed shear walls anchorage is achieved with a device that anchors the end-framing member to the supporting structure. For steel frames anchorage is usually achieved with steel anchor bolts attached to foundations or to other framing. For concrete and masonry walls or frames this is less often a problem, due to the usual considerable weight of the construction; however, these structures are typically anchored by the usual extension of the steel reinforcement into supports, whether needed for uplift anchorage or not.

The illustrations in Figure 9.81 show some of the devices that are used for anchoring wood structures, for which

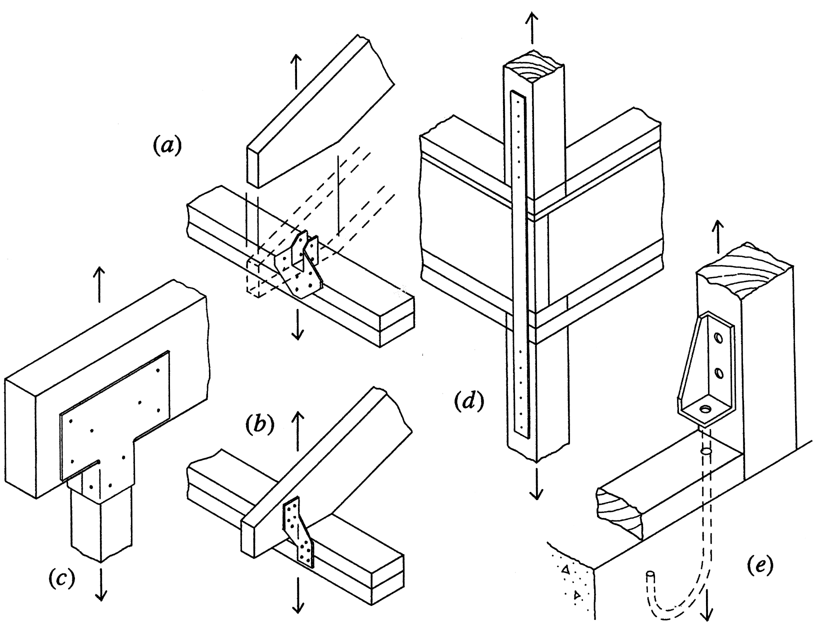


Figure 9.81 Anchorage devices for wood frame structures.

anchorage is quite often critical. These devices are ordinarily provided by manufacturers who have obtained certification of load capacity from code approval agencies.

The term *tiedown* or *hold-down* is used mostly for the devices shown in Figures 9.81*d* and *e*, which are commonly used details for anchorage of shear wall framing. The devices for roof anchorage shown in Figures 9.81*a* and *b* are made from thin sheet steel; similar devices of other forms are used for most of the assembly joints in light wood frame construction. The device shown in Figure 9.81*c* may be made with sheet steel or with thicker steel plates, depending on the size of the connected wood timber members.

Horizontal Anchors

In addition to the transfer of vertical gravity load and lateral shear load at the edges of horizontal diaphragms, there is usually a need for resistance to the horizontal pulling away of walls from the diaphragm edge. In many cases the connections that are provided for other functions also serve to resist this action. Design codes usually require that this type of anchorage be a "positive" one, not relying on such things as the withdrawal of nails or lateral force on toe nails. Figure 9.82 shows some of the means for achieving this type of action.

Shear Anchors

The lateral shear force from the edge of a horizontal diaphragm must be transferred from the diaphragm into a collector or some other intermediate element or directly into a vertical bracing element. Except for sitecast concrete structures, this process usually involves some means of attachment. For wood structures the transfer is usually achieved through the lateral loading of nails, bolts, or lag screws for which the codes or industry specifications provide tabulated load capacities. It may also involve the use of intermediate devices where direct connection of wood members is not achieved.

For steel deck diaphragms the transfer is usually achieved by welding the deck to supporting steel framing. If the vertical bracing system is a steel frame, these members are usually also parts of the general steel frame structure for the building. If the vertical structure is concrete or masonry, edge loads from wood and metal decks are usually transferred first into some framing which is then bolted in place through anchor bolts set in the concrete or masonry. As in other situations, the full effects of combined gravity and lateral loadings must be investigated for these elements and their attachments.

Another shear transfer situation occurs at the base of a shear wall, where the wall must be attached to its supports. For wood frame walls supported on concrete or masonry, this is usually achieved by bolting of the wood frame sill to anchor bolts cast into the concrete or masonry.

For concrete and masonry shear walls the base shear is often adequately resisted by friction, due to the weight of the walls and the usual rough surfaces of contact. However, the joint is typically considerably enhanced by the steel

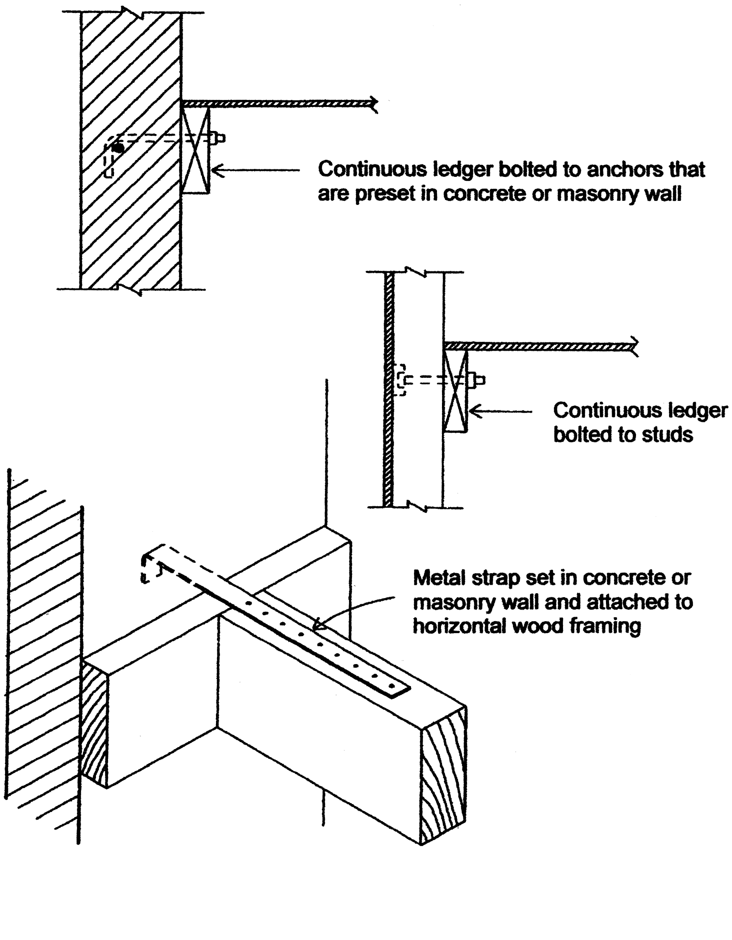


Figure 9.82 Anchoring of horizontal wood diaphragms.

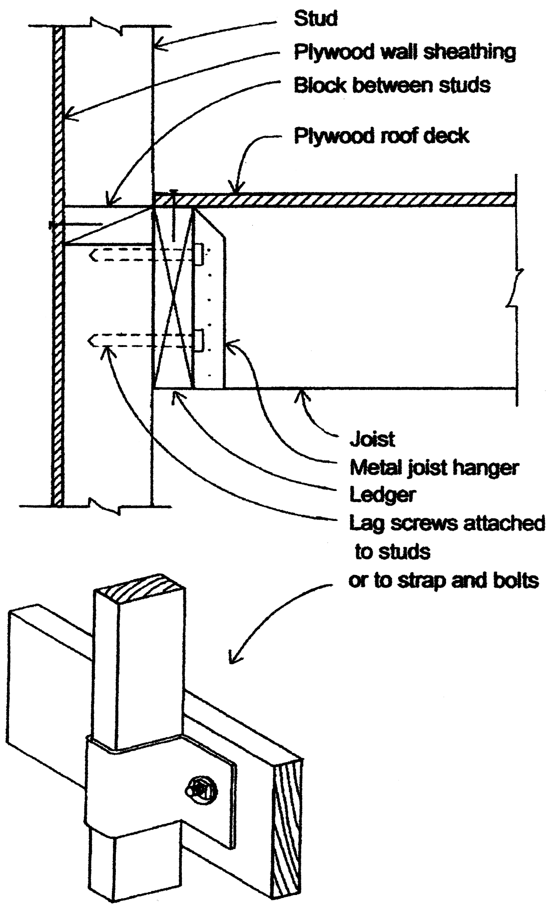


Figure 9.83 Transfer of loads between horizontal and vertical elements of the lateral bracing system.

reinforcement extending from the foundations into the wall bottom.

Transfer of Forces

The complete transfer of forces from the horizontal to the vertical bracing elements can be quite complex. Figure 9.83 shows a joint between a horizontal plywood diaphragm and a vertical plywood shear wall. For reasons other than lateral-load resistance, it is desired here that the wall studs continue past the level of the horizontal deck. This necessitates the use of a continuous edge-framing member, called a *ledger*, which serves as the support of the deck for gravity loads as well as a chord and collector for the diaphragm stresses. This ledger is shown to be attached to the faces of the studs with lag screws at each stud. The functioning of this joint involves the following:

The vertical gravity load is transferred from the ledger to the studs by shear in the lag screws. Nailing of the plywood to the ledger is structurally not required for gravity loads but is typically used simply to hold the plywood panels in place. Nailing is critical, however, for transfer of lateral load from the deck.

The lateral shear stress in the roof diaphragm is transferred to the ledger through lateral force on the edge nails in the direction parallel to the wall. This stress is in turn transferred to the studs from the ledger by lateral shear on the lag screws. The horizontal blocking is fit tightly between the studs to provide for the transfer of the lateral load to the wall plywood, which is nailed to the blocking.

Outward loading on the wall, by seismic force or wind suction, is resisted by the lag screws in tension. This is generally not considered to be a really good positive connection, although the load magnitude should be considered in making this evaluation. A more positive connection is achieved by bolting the studs to the ledger in a form such as that shown in the bottom illustration in Figure 9.83.

The functions described for the wood structure must also be investigated for the design of the horizontal-to-vertical connection in all situations, with various combinations of wood, steel, concrete, and masonry elements.

CHAPTER

10

Building Structures Design Examples

Work in Chapter 10 consists of discussions of the design of structural systems for several example buildings. The purpose of this work is to illustrate the process of dealing with the design of the whole building structure, whereas work in earlier chapters is focused on limited topics. The work here is linked to other chapters by using elements of the buildings here for some of the example exercise problems in other chapters. Buildings of similar size, shape, and purpose often have several alternatives for their basic construction, with each choice generally satisfying the goals for the building. To illustrate that situation, several different schemes are presented for some of the buildings used here for examples.

10.1 GENERAL CONCERNS FOR STRUCTURAL DESIGN

This section treats a number of issues that relate to the general work of designing building structures. Many of these issues are also discussed in other parts of the book.

Design Process and Methods

In general, the design work for a building consists of the conceptualization and decision process by which the final form and fabric of the finished building is descriptively determined. The output of the *design work* is the recorded description of the desired object. The work of generating the ideas and recording them is called *designing*. The displayed collection of recorded ideas, usually in some combination of graphic and written documents, is called the *design*. The person who generates the ideas is the *designer*.

Design work may be viewed as the collection of decisions that determine the finished image of the designed object. Designers exert some judgment in making some of these decisions, although other sources strongly influence some decisions. How this all works for a specific design case depends on many factors. Some buildings are built using mostly predesigned, prefabricated, off-the-shelf parts, which reduces the design work to selecting and arranging of cataloged items. Other buildings may use newly developed materials or old materials put together in new ways, requiring considerable imagination and innovation in the design work. A particular building may be simple in its form and use, while others are complex and multifunctioning and present problems that are difficult to analyze. What exactly is involved in design work and how it is accomplished varies from case to case, even when the design work is done by the same persons.

Figure 10.1 presents an image of the design process viewed as a succession of activities. While a particular design is finished when the designer completes the design work and communicates the description to the persons who will create the actual object, the continuation of the project through final occupancy and use of the building has effects on final evaluation of the design and on ongoing design activities of the working designer.

Design of buildings is often thought of as being primarily the function of the architect. While it is true that architects often serve as prime designers for buildings, there is typically a long list of other participants in the design process. The fact that innovation and creative design occur to the extent that they do is more impressive when the collective restraints of all of these influential parties is considered. Clearly, effective designers learn to deal with these realities as well as with the creative process of design.

It is foolish to think of design work as flowing easily from one decision to the next, progressing smoothly toward a final statement. A final, conclusive statement must be achieved,

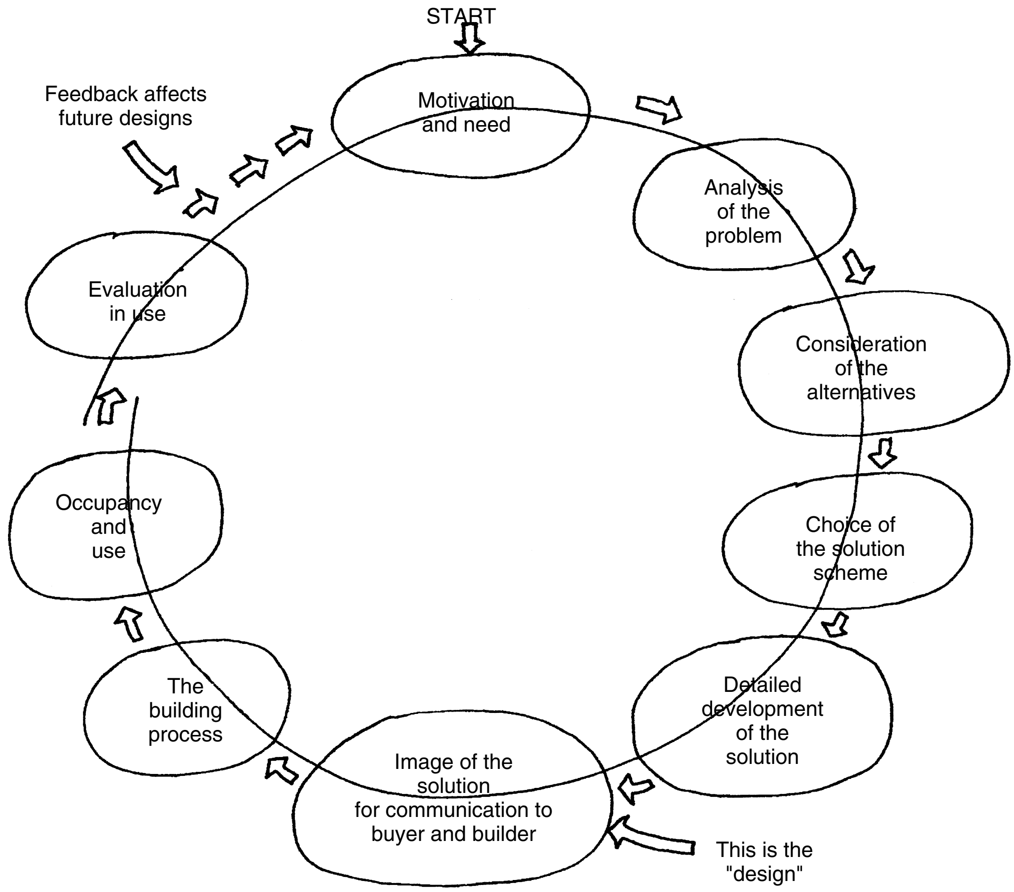


Figure 10.1 Building design process.

but getting there usually involves some false starts, some steps backward, and a lot of dilemmas.

Many designers never really finish a design; they just quit. That is, there is almost always another idea to explore, another alternative to consider, or another approach to try. Given open-ended time and infinite resources, the design of an individual object may never be completed to the total satisfaction of the designer. In real situations, however, time is usually short, resources are limited, and designers are forced to quit working at some specified time (the ominous deadline). The next idea or other alternatives will have to wait for the next design project.

Design work needs to have a schedule of some kind. Decisions are unavoidably sequential and interactive. Critical decisions must be frozen in stages, and the costs in time and resources involved in going back to reconsider decisions must be acknowledged.

Responsibility for various design decisions must be clearly established. This is especially true for two kinds of decisions: those that critically affect the time schedule and those that have major effects on the final solution. Going back needs to be in the hands of the prime controlling designer and is a major management task with regard to the work of the whole design team.

It is difficult to generalize about a specific level of mathematics or a set of ideas and design skills that need some mastery in order to allow for professional work in the investigation and problem-solving activities related to building design work. Even if a person's role, area of involvement, and level of activity can be tightly defined, it is hard to say just what is and what is not required in mathematical training. A great deal of the engineering work for buildings is nonmathematical, and what is mostly consists of arithmetic with some occasional simple trigonometry and algebra. With the advent of the computer and the growing stock of user-friendly programs, even much of that work is eliminated.

Design Standards and Aids

There are many sources for information, guidance, and assistance to support work in structural design for buildings. Some of these sources, which were used in preparation of this book, are listed in the References at the back of the book. A great deal more is available.

Industry Standards

There are many organizations that serve the general interests of various factions of the building industry. Groups are formed on the basis of common interest in a single material, in a type of product, in a special method of building, and so on. In many cases the organizations perform promotional activities for the particular industry group, but they also serve to police the group for some adherence to mutually accepted

standards. These standards are in many cases published and widely distributed; they often become basic references for design practices and for appropriate portions of building codes.

Table 10.1 indicates several areas of basic concern in building structures and the organizations that provide essential information relating to those areas. The publications of several of these organizations are listed in the References. In some cases the publications provide general data about products, but in most situations it is advisable to obtain such information directly from manufacturers or suppliers of the products, as variations are possible and the actual products available in a region should be used.

There are also organizations, such as the American Society for Testing and Materials (ASTM) and the American National Standards Institute (ANSI), that deal with the industry as a whole, providing references used by many groups.

For any actual design work, it is wise to determine what specific standards are used in references by the building code with jurisdiction for any proposed work. Standards change regularly, but local codes are sometimes slow to adopt the changes. This is a general problem with any reference material but is a chronic one with regard to building codes.

Building Codes

Building codes are the legal ordinances enacted by some governmental entity (city, county, state) for regulation of the construction of buildings in its jurisdiction. The code is the basis for granting or refusing to grant a permit for construction. These ordinances are frequently revised on the basis of recommendations by experts or for various political reasons. Local builders, trade unions, real estate development interests, and others often exert pressure to influence the regulations in favor of their interests.

Model building codes are prepared as recommendations by various organizations. The standards of the AISC, ACI, AFPA, and others are the usual references for applicable portions of the codes. While the model codes have no legal jurisdiction, many government entities adopt the model codes for the majority of material in their own codes.

Model codes are revised frequently in the attempt to keep up to date with current research and experience. However, local codes often lag behind in adopting new codes and standards. To get a building permit, some respect must be paid to the local code, although designers may also use the latest model codes and standards for their design work.

Professional Organizations

There are a number of professional organizations that produce journals and various other publications with recommendations for aspects of building design. For structures, a leading contributor is the American Society of Civil Engineers (ASCE). In conjunction with the ANSI, the ASCE publishes the leading reference for design loads for structures (see Ref. 1).

Textbooks and Handbooks

There are textbooks of considerable variety available for every subject in the area of structural design. Many of these are of a form designed to be used in courses in engineering schools, covering topics from an introductory level to highly advanced problems. These books are intended basically for students and practitioners in the field of structural engineering.

For persons with limited background in mathematics and science, engineering texts are difficult to use. The best of these texts need some companion resource (teacher, tutor, etc.) in order to be effectively understood. In the main, self-taught engineers are about as rare these days as self-taught lawyers and doctors.

In spite of these difficulties, it is possible to learn a great deal about the design of building structures without pursuing a degree in structural engineering. This book can provide such a starting point.

Mathematical computations are actually a small fraction of the work to be done in developing the complete design of

Table 10.1 Industry Organizations

Acronym	Name of Organization	Areas of Concern
AISC	American Institute of Steel Construction	Steel construction, rolled products, steel connectors
AISI	American Iron and Steel Institute	Steel construction, steel products, cold-formed (light-gauge) products
SJI	Steel Joist Institute	Prefabricated light steel trusses (open-web joists)
SDI	Steel Deck Institute	Formed sheet steel products
AWS	American Welding Society	Welding—materials and processes
ACI	American Concrete Institute	Cement, concrete construction
PCA	Portland Cement Association	Cement, concrete construction
CRSI	Concrete Reinforcing Steel Institute	Standards and design for reinforced concrete
AFPA	American Forest and Paper Association	Structural wood materials and products, design standards
AITC	American Institute of Timber Construction	Products and design for timber and glued-laminated construction
APA	The Engineered Wood Association	Miscellaneous fabricated wood products, fiber, plywood, various structural panels
MIA	Masonry Institute of America	Masonry products and construction

a building structure. In addition, the vast majority of design problems are simple and repetitive. Thus it is possible to learn much of what is required to design ordinary structures and to perform a great deal of the total work in design without recourse to any complex mathematical computations or highly sophisticated engineering investigative procedures.

Most handbooks, such as the manuals of the AISC and CRSI, are compilations of data and various aids for shortcut design work. They also will usually have some amount of text in a form designed to explain the use of the materials in the book. They are not self-standing as beginning texts but have great practical value as learning sources for professional design work. They are also usually quite up to date with industry practices.

Much of what goes into any current building construction is in the form of standardized products. The design and detailing of these products are preestablished, so that using them requires little in the way of computations. This situation is generally increasing, so that much of structural design work consists of selections and specifications from manufacturer's catalogs. Many of these catalogs actually consist of handbooks of predesigned structures.

Structural Planning

Planning a structure requires the ability to perform two major tasks. The first task is the logical arranging of the structure, regarding form, dimensions, and proportions. The second task is the ordering of the elements for basic stability and interaction. These issues must be faced, whether the building is simple or complex, small or large, and of ordinary construction or totally unique. Spanning beams must be supported and have depths adequate for the spans; thrusts of arches must be resolved; columns above should be centered over columns below; and so on.

Consider the building plan shown in Figure 10.2*a*. The general building form is defined by the combination of the solid portions of walls, the windows and doors, and the dashed lines outlining edges and openings of the roof above.

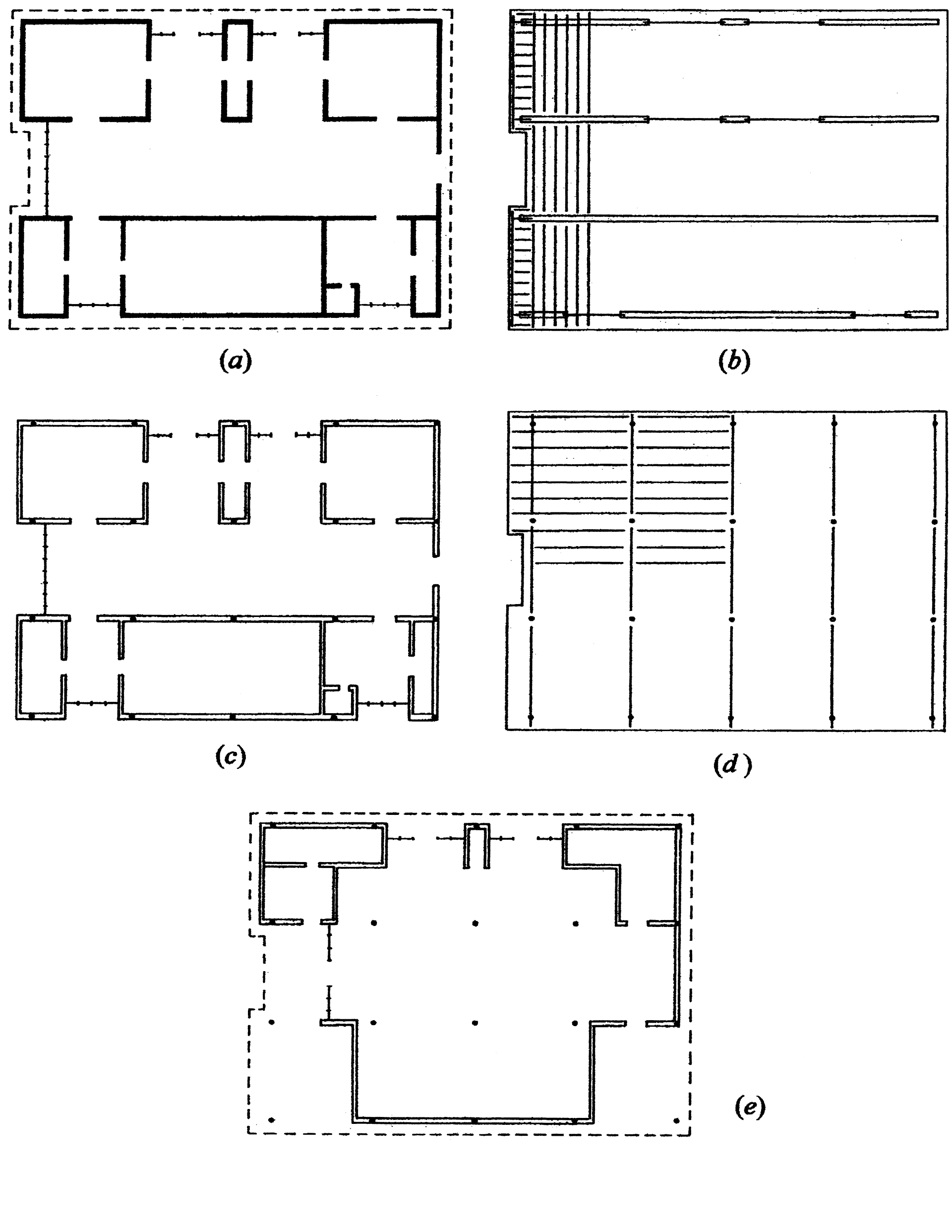


Figure 10.2 Developing the structural plan.

If the existence of a single ceiling height and a flat roof are assumed, the overall building form is easily visualized.

A possible solution for the roof structure is shown in Figure 10.2*b*, consisting of the use of some walls for bearing walls to support a roof-framing system. Beams are placed in the gaps between portions of the walls and are most likely supported by posts within the walls. A possible problem with this solution is the requirement to commit some walls as permanent, possibly inhibiting future rearrangement of spaces. However, if this is not a problem, this solution is probably the simplest and least costly choice.

Figure 10.2*c* shows the plan for another possibility for the development of a support system for the roof structure using columns only, the columns being located so as to be incorporated into the walls. The same framing system used with the bearing walls could be used, but another possibility is shown in Figure 10.2*d*. The walls in this case could be developed free of structural functions. However, lateral as well as gravity loads must be considered, and rigid-frame or trussed bracing must be used if the walls are completely nonstructural.

Figure 10.2*e* shows a possibility for rearrangement of some of the interior walls to create larger open space. This is more easily achieved with the column support system, especially if lateral bracing is accomplished by the exterior walls.

Hopefully, the architectural planning and the structural planning are done interactively, not one after the other. The more the architect knows about structural problems and the structural designer knows about architectural problems, the more likely it is possible that effective communication and an interactive design development may occur.

Most buildings are not as simple in form as this single-story one. Multiple levels and more complex roof forms will present different problems to be solved in arranging the structure.

Choice of Structure

Although each individual building is a unique situation if all variables are considered, the majority of building design problems are highly repetitive. Problems usually have many alternative solutions, each with its own pluses and minuses in terms of various points of comparison. Development of the final design involves the comparative evaluation of definable alternatives and the eventual selection of one.

When problems are truly new in terms of a new building use, a scale jump, or a new performance demand situation, there is a real need for innovation. Usually, however, when new solutions to old problems are presented, their merits must be compared to established previous solutions in order to justify them. In its broadest context the selection process includes the consideration of all possible alternatives—those well known, those new but unproven, and those only imagined.

Selection may be done in one single stroke, similar to buying a new car off the dealer's showroom floor. For predesigned package building systems this may be the actual case. Usually, however, selection consists of a series of related decisions, starting with broad ones of basic system type, form and materials, and progressing to increasingly detailed ones of shape of parts, connections, finishes, and so on.

Usually the broader the decision, the more difficult it is to make. Ideally, most of the detailed decisions should be anticipated when making the initial broad ones. However, the operational difficulty of this is immense. Quite often it is necessary to explore some alternatives in considerable detail before well-informed broad decisions can be made. The more innovative the solution or unique the problem, the more this is required.

Simply knowing all the reasonable alternatives is in itself a considerable task. Information about most building materials and products is not disseminated in a way that makes uniform, objective evaluations for comparison easy to perform. There are very few well-organized information sources that present the various alternatives in a comprehensive, impartial way. Consequently, knowing about alternatives is usually a fragmentary and imperfect undertaking, highly dependent on the personal experience and particular resources of the individual designer.

Assuming that a designer is able to know of a reasonable range of possible alternatives, the task of choosing between them must still be faced. Ideally, this calls for some organized system of evaluation of characteristics, including considerations of cost, time, fire behavior, energy use, installation problems, and so on. For all of the reasons discussed previously, this information is difficult to obtain in a uniform shape that is easily handled for evaluation analyses.

A major aspect of this problem is simply that we have a highly dynamic society. We continually create new situations and problems for designers, produce new materials and products, shift our priorities (e.g., from dollar cost to energy use to pollution), and generally keep two steps ahead of anyone trying to organize and conduct design work. Any effort to deal with this problem requires the recognition of its essential dynamic nature.

As design work progresses from initial broad decisions to increasingly detailed ones, there is an ever-present possibility that previous decisions may need to be reconsidered. The farther back this reaches in the design progression, the more it may disturb the smooth flow of the work. This places a great pressure on the earliest, broadest decisions. For this reason, most designers consider the preliminary design to be the most sensitive activity, requiring the most sound and experience-based judgments.

Economics

Dealing with estimates of dollar cost is a very difficult, but necessary, part of structural design. For the structure itself, the bottom-line cost is the delivered cost of the finished structure, usually measured in units of dollars per square foot of the building plan or in total cost as a percentage of the total building cost. For individual components, such as a single wall, units may be used in other forms. Individual

cost factors or components, such as cost of materials, labor, transportation, installation, testing, and inspection, must be aggregated to produce a single unit cost for the entire structure.

Designing for control of the cost of the structure is only one aspect of the design problem, however. The more meaningful cost is that for the entire building construction. It is possible that certain cost-saving efforts applied to the structure may result in increases in cost for other parts of the construction. The truly cost-saving structure is often the one that produces significant savings of nonstructural costs, in some cases at the expense of less structural efficiency and possibly some increase in the cost of the structure itself.

Real cost figures can only be determined by those who deliver the completed construction. Estimates of cost are most reliable in the form of actual offers or bids for the construction work. The farther the cost estimator is from the actual requirement to deliver the goods, the more speculative the estimate.

Designers, unless they are in the actual employ of the builder, must base any cost estimates on educated guesswork deriving from some comparison with similar work recently done in the same region. This kind of guessing must then be adjusted for the most recent developments in terms of local markets, competitiveness of builders and suppliers, and the general state of the economy.

Cost estimating requires a lot of training and experience and an ongoing source of reliable, timely information. Various sources are available in the form of publications and computer data banks.

The following are some general rules for efforts that can be made in structural design work in order to have an overall, general cost-saving attitude.

Reduction of Material Volume. This is usually a means of reducing cost. However, unit prices for different grades or levels of quality must be noted. Higher grades of wood or steel may be proportionally more expensive than the higher stress values they represent; more volume of cheaper material may be less expensive.

Use of Standard, Commonly Stocked Products. Special sizes or shapes may have premium prices. This applies to lumber sizes, rolled steel shapes, and other products; within a wide range of choices, some few may be most common and cheapest in unit cost.

Reduction in the Complexity of Systems. Simplicity in purchasing, handling, managing of inventory, and so on, will be reflected in lower bids as builders anticipate simpler tasks. Use of the fewest number of different grades of material, size of fasteners, and other such variables is as important as the fewest number of different parts. This is especially true for assemblage on the building site; large inventories may not be a problem in a factory but usually are on the building site.

Use of Construction Familiar to Builders. Cost reduction is usually achieved when materials, products, and construction methods are highly familiar to local builders and construction workers. If alternatives exist, choice of the "usual" one is the best course.

Reduce Labor Cost. Labor cost is often greater than material cost. Labor for building forms, installing reinforcement, pouring concrete, and finishing concrete is the major cost for sitecast concrete construction. Savings in these areas are usually much more significant than savings of material volume. Also, labor at the building site is usually more expensive than labor in a factory setting.

Coordinate Construction Schedules. For buildings of an investment nature, time is money. Speed of construction may be a major advantage. However, getting the structure up fast is not a true advantage unless the other aspects of the construction can take advantage of the time gained. Steel frames often go up quickly, only to stand around and rust while the rest of the work catches up.

Computer-Aided Design

This book is not essentially about how to practice the professions of architecture and structural engineering. In this day and age, every design engineer is heavily involved in using computers for much of the design work. Software for the engineer is available from many sources—some free and some quite expensive. For the parts of structural design work that involve considerable research from large data sources, computer use is essential. Computers can also aid the work of design itself, although not equally for all types of work.

For much of the structural design work—especially that which may not require use of very complex mathematical analyses—designers may work well with a simple pocket calculator and pencil and paper. This applies to much of the preliminary design work, where very approximate answers may be sufficient to support broad decisions. All of the work in this book consisting of computations for the example problems was done in this manner.

As computer hardware and software continue their hectic pace of growth, many routine tasks may become easy to perform with computer aid. What is really exciting, however, is the possibility for new levels of design activity not really feasible without the computer and mostly still in the imaginations of most designers. Stay tuned for more news.

Structural Design Loads and Methods

This section treats the issues of determination of loads on the structure and selection of the basic structural design method to be used.

Load Sources

Structural tasks are defined primarily in terms of the loading conditions imposed on the structure. There are many potential sources of load for building structures. Designers must consider all the potential sources and the logical combinations with which they may occur. Building codes

currently stipulate both the load sources and the form of combinations to be used for design. The following loads are listed in ASCE *Minimum Design Loads for Buildings and Other Structures* (Ref. 1), hereinafter referred to as ASCE 2005:

D = dead load
E = earthquake-induced force
L = live load, except roof load
L_r = roof live load
S = snow load
W = load due to wind pressure

Additional special loads are listed but these are the commonly occurring loads. The following is a description of some of these loads.

Dead Loads

Dead load consists of the weight of the materials of which the building is constructed, such as walls, partitions, columns, framing, floors, roofs, and ceilings. In the design of a beam or column, the dead load used must include an allowance for the weight of the structural member itself.

Table 10.2, which lists the weights of many construction materials, may be used in the computation of dead loads. Dead loads are due to gravity and they result in downward vertical forces.

Dead load is generally a permanent load, once the building construction is completed, unless remodeling or rearrangement of the construction occurs. Because of this permanent, long-time, character, the dead load requires certain considerations in design, such as the following:

It is always included in design loading combinations, except for investigations of singular effects, such as deflections due to only live load.

Its long-time character has some special effects causing permanent sag and requiring reduction of design stresses in wood structures, development of long-term, continuing settlements in some soils, and producing creep effects in concrete structures.

It contributes some unique responses, such as the stabilizing effects that resist uplift and overturn due to wind forces.

Although weights of materials can be reasonably accurately determined, the complexity of most building construction makes the computation of dead loads possible only on an approximate basis.

Building Code Requirements

Structural design of buildings is most directly controlled by building codes, which are the general basis for the granting of building permits—the legal permission required for construction. Building codes (and the permit-granting process) are administered by a local unit of government: city,

Table 10.2 Weight of Building Construction

	psf[a]	kPa[a]
Roofs		
3-ply ready roofing (roll, composition)	1	0.05
3-ply felt and gravel	5.5	0.26
5-ply felt and gravel	6.5	0.31
Shingles: Wood	2	0.10
Asphalt	2–3	0.10–0.15
Clay tile	9–12	0.43–0.58
Concrete tile	6–10	0.29–0.48
Slate, 3 in.	10	0.48
Insulation: Fiber glass batts	0.5	0.025
Foam plastic, rigid panels	1.5	0.075
Foamed concrete, mineral aggregate	2.5/in.	0.0047/mm
Wood rafters: 2 × 6 at 24 in.	1.0	0.05
2 × 8 at 24 in.	1.4	0.07
2 × 10 at 24 in.	1.7	0.08
2 × 12 at 24 in.	2.1	0.10
Steel deck, painted: 22 gauge	1.6	0.08
20 gauge	2.0	0.10
Skylights: Steel frame with glass	6–10	0.29–0.48
Aluminum frame with plastic	3–6	0.15–0.29
Plywood or softwood board sheathing	3.0/in.	0.0057/mm
Ceilings		
Suspended steel channels	1	0.05
Lath: Steel mesh	0.5	0.025
Gypsum board, 1/2 in.	2	0.10
Fiber tile	1	0.05
Drywall, gyspum board, 1/2 in.	2.5	0.12
Plaster: Gypsum	5	0.24
Cement	8.5	0.41
Suspended lighting and HVAC, average	3	0.15
Floors		
Hardwood, 1/2 in.	2.5	0.12
Vinyl tile	1.5	0.07
Ceramic tile: 3/4 in.	10	0.48
Thin-set	5	0.24
Fiberboard underlay, 0.625 in.	3	0.15
Carpet and pad, average	3	0.15
Timber deck	2.5/in.	0.0047/mm
Steel deck, stone concrete fill, average	35–40	1.68–1.92
Concrete slab deck, stone aggregate	12.5/in.	0.024/mm
Lightweight concrete fill	8.0/in.	0.015/mm
Wood joists: 2 × 8 at 16 in.	2.1	0.10
2 × 10 at 16 in.	2.6	0.13
2 × 12 at 16 in.	3.2	0.16
Walls		
2 × 4 studs at 16 in., average	2	0.10
Steel studs at 16 in., average	4	0.20
Lath. plaster—see *Ceilings*		
Drywall, gypsum board, 1/2 in.	2.5	0.10
Stucco, on paper and wire backup	10	0.48

(*Continued*)

Table 10.2 (*Continued*)

	psf[a]	kPa[a]
Windows, average, frame + glazing:		
Small pane, wood or metal frame	5	0.24
Large pane, wood or metal frame	8	0.38
Increase for double glazing	2–3	0.10–0.15
Curtain wall, manufactured units	10–15	0.48–0.72
Brick veneer, 4 in., mortar joints	40	1.92
1/2 in., mastic-adhered	10	0.48
Concrete block:		
Lightweight, unreinforced, 4 in.	20	0.96
6 in.	25	1.20
8 in.	30	1.44
Heavy, reinforced, grouted, 6 in.	45	2.15
8 in.	60	2.87
12 in.	85	4.07

[a]Average weight per square foot of surface, except as noted. Values given as /in. (per in.) or /mm (per mm) are to be multiplied by actual thickness of material.

county, or state. Most building codes, however, are based on some model code.

Model codes are more similar than different and are in turn largely derived from the same basic data and standard reference sources, including many industry standards. In the several model codes and many city, county, and state codes, however, there are some items that reflect particular regional concerns. With respect to control of structures, all codes have materials (all essentially the same) that relate to the following issues:

Minimum Required Live Loads. All building codes have tables that provide required values to be used for live loads. Table 10.3 contains some loads as specified in ASCE 2005 (Ref. 1).

Wind Loads. These are highly regional in character with respect to concern for local windstorm conditions. Model codes provide data with variability on the basis of geographic zones.

Seismic (Earthquake) Effects. These are also regional with predominant concerns in the western states. These data, including recommended investigations, are subject to quite frequent modification, as the area of study responds to ongoing research and experience.

Load Duration. Loads or design stresses are often modified on the basis of the time span of the load, varying from the life of the structure for dead load to a few seconds for a wind gust or a single major seismic shock. Safety factors are frequently adjusted on this basis. Some applications are illustrated in the work in the design examples.

Load Combinations. These were formerly mostly left to the discretion of designers but are now quite commonly stipulated in codes, mostly because of the increasing use of ultimate strength design and the use of factored loads.

Design Data for Types of Structures. These deal with basic materials (wood, steel, concrete, masonry, etc.), specific structures (rigid frames, towers, balconies, pole structures, etc.), and special problems (foundations, retaining walls, stairs, etc.). Industry-wide standards and common practices are generally recognized, but local codes may reflect particular local experience or attitudes. Minimal structural safety is the general basis, and some specified limits may result in questionably adequate performances (bouncy floors, cracked plaster, etc.).

Fire Resistance. For the structure, there are two basic concerns, both of which produce limits for the construction. The first concern is for structural collapse or significant structural loss. The second concern is for containment of the fire to control its spread. These concerns produce limits on the choice of materials (e.g., combustible or noncombustible) and some details of the construction (cover on reinforcement in concrete, fire insulation for steel beams, etc.).

The work in the design examples in Chapter 10 is based largely on criteria from ASCE 2005 (Ref. 1).

Live Loads

Live loads technically include all the nonpermanent loadings that can occur in addition to the dead loads. However, the

Table 10.3 Minimum Floor Live Loads

Building Occupancy or Use	Uniformly Distributed Load (psf)	Concentrated Load (lb)
Apartments and Hotels		
Private rooms and corridors serving them	40	
Public rooms and corridors serving them	100	
Dwellings, One and Two Family		
Uninhabitable attics without storage	10	
Uninhabitable attics with storage	20	
Habitable attics and sleeping rooms	30	
All other areas except stairs and balconies	40	
Office Buildings		
Offices	50	2000
Lobbies and first-floor corridors	100	2000
Corridors above first floor	80	2000
Stores		
Retail		
First floor	100	1000
Upper floors	75	1000
Wholesale, all floors	125	1000

Source: ASCE 2005 (Ref. 1), used with permission of the publisher, American Society of Civil Engineers.

term as commonly used usually refers only to the vertical gravity loadings on roof and floor surfaces. These loads occur in combination with the dead loads but are generally random in character and must be dealt with as potential contributors to various loading combinations.

Roof Loads

In addition to the dead loads they support, roofs are designed for a uniformly distributed live load. The minimum specified live load accounts for general loadings that occur during construction and maintenance of the roof. For special conditions, such as heavy snowfalls, additional loadings are specified.

The minimum roof live load in pounds per square foot is specified in ASCE 2005 (Ref. 1) in the form of an equation, as follows:

$$L_r = 20R_1R_2 \quad \text{in which} \quad 12 \leq L_r \leq 20$$

In the equation R_1 is a reduction factor based on the tributary area supported by the structural member being designed (designated as A_t and quantified in square feet) and is determined as follows:

$$\begin{aligned} R_1 &= 1 \quad \text{for } A_t \leq 200\,\text{ft}^2 \\ &= 1.2 - 0.001\,A_t \quad \text{for } 200\,\text{ft}^2 < A_t < 600\,\text{ft}^2 \\ &= 0.6 \quad \text{for } A_t \geq 600\,\text{ft}^2 \end{aligned}$$

Reduction factor R_2 accounts for the slope of a pitched roof and is determined as follows:

$$\begin{aligned} R_2 &= 1 \quad \text{for } F \leq 4 \\ &= 1.2 - 0.05F \quad \text{for } 4 < F < 12 \\ &= 0.6 \quad \text{for } F \geq 12 \end{aligned}$$

The quantity F in the equations for R_2 is the number of inches of rise per foot for a pitched roof (for example: $F = 12$ indicates a rise of 12 in 12 in; that is, an angle of 45°).

The design standard also provides data for roof surfaces that are arched or domed and for special loadings for snow or water accumulation. Roof surfaces must also be designed for wind pressures on the roof surface, both inward and outward. A special situation that must be considered is that of a roof with a low dead load and a significant wind load that exceeds the dead load.

Although the term *flat roof* is often used, there is generally no such thing; all roofs must be designed for some water drainage. The minimum required pitch is usually $\frac{1}{4}$ in./ft, or a slope of approximately 1 : 50. With roof surfaces that are close to flat, a potential problem is that of *ponding*, a phenomenon in which the weight of the water on the surface causes deflection of the supporting structure, which in turn allows for more water accumulation (in a pond), causing more deflection, and so on, resulting in an accelerated collapse condition.

Floor Live Loads

The live load on a floor represents the probable effects created by the occupancy. It includes the weights of human occupants, furniture, equipment, stored materials, and so on. All building codes provide minimum live loads to be used in the design of buildings for various occupancies. Since there is a lack of uniformity among different codes in specifying live loads, the local code should always be used. Table 10.3 contains a sample of values for floor live loads as given in ASCE 2005 (Ref. 1) and commonly specified by building codes.

Although expressed as uniform loads, code-required values are usually established large enough to account for ordinary concentrations that occur. For offices, parking garages, and some other occupancies, codes often require the consideration of a specified concentrated load as well as the distributed loading. This required concentrated load is listed in Table 10.3 for the appropriate occupancies.

Where buildings are to contain heavy machinery, stored materials, or other contents of unusual weight, these must be provided for individually in the design of the structure.

When structural framing members support large areas, most codes allow some reduction in the total live load to be used for design. These reductions, in the case of roof loads, are incorporated in the formulas for roof loads given previously.

The following is the method given in ASCE 2005 (Ref. 1) for determining the reduction permitted for beams, trusses, or columns that support large floor areas.

The design live load on a member may be reduced in accordance with the formula

$$L = L_0\left(0.25 + \frac{15}{\sqrt{K_{\text{LL}}A_T}}\right)$$

where

L = reduced design live load per square foot of area supported by the member

L_0 = unreduced live load supported by the member

K_{LL} = live-load element factor (see Table 10.4)

A_T = tributary area supported by the member

Table 10.4 Live-Load Element Factor

Element	K_{LL}
Interior columns	4
Exterior columns without cantilever slabs	4
Edge columns with cantilever slabs	3
Corner columns with cantilever slabs	2
Edge beams without cantilever slabs	2
Interior beams	2
All other members not identified above	1

Source: ASCE 2005 (Ref. 1), used with permission of the publisher, American Society of Civil Engineers.

Here, L shall not be less than $0.50L_0$ for members supporting one floor and L shall not be less than $0.40L_0$ for members supporting two or more floors.

In office buildings and certain other building types, partitions may not be permanently fixed in location but may be erected or moved from one position to another in accordance with the requirements of the occupants. In order to provide for this flexibility, it is customary to require an allowance of 15 to 20 psf, which is usually added to other dead loads.

Lateral Loads (Wind and Earthquake)

As used in building design, the term *lateral load* is usually applied to the effects of wind and earthquakes, as they induce horizontal forces on stationary structures. Design methods and criteria for wind and earthquake effects is extensively discussed in Chapter 9. Development of design for lateral loads is also discussed for several of the example buildings in this chapter.

Load Combinations

The various types of load sources, as described in the preceding section, must be individually considered for quantification. However, for design work the possible combination of loads must also be considered. Using the appropriate combinations, the design load for individual structural elements must be determined. The first step in finding the design load is to establish the critical combinations of load for the individual element. Using ASCE 2005 (Ref. 1) as a reference, the following combinations are to be considered. As this process is different for the two basic methods of design, they are presented separately.

Allowable Stress Method

For this method the individual loads are used directly for the following possible combinations:

Dead load only
Dead load + live load
Dead load + roof load
Dead load + 0.75(live load) + 0.75(roof load)
Dead load + wind load or 0.7(earthquake load)
Dead load + 0.75(live load) + 0.75(roof load) + 0.75(wind load) or 0.7(earthquake load)
0.6(Dead load) + wind load
0.6(Dead load) + 0.7(earthquake load)

The combination that produces the critical design situation for individual structural elements depends on the load magnitudes and the loading condition for the elements. Demonstrations of the use of these combinations are given in the building design cases in this chapter.

Strength Design Method

Some adjustment of the percentage of loads (called factoring) is done with the allowable stress method. However, factoring is done with all the loads for the strength method. The need here is to produce a load higher than the true anticipated load (called the *service load*)—the difference representing a margin of safety. The structural elements will be designed at their failure limits with the design load and they really should not fail with the actual expected loads.

For the strength method the following combinations are considered:

1.4(Dead load)
1.2(Dead load) + 1.6(live load) + 0.5(roof load)
1.2(Dead load) + 1.6(roof load) + live load or 0.8(wind load)
1.2(Dead load) + 1.6(wind load) + (live load) + 0.5(roof load)
1.2(Dead load) + 1.0(earthquake load) + live load + 0.2(snow load)
0.9(Dead load) + 1.0(earthquake load) or 1.6(wind load)

Use of these load combinations is demonstrated in the examples in the building design cases in the remaining sections of this chapter.

Determination of Design Loads

The following example demonstrates the use of some of the code-mandated load combinations.

Figure 10.3 shows the plan layout for the framed structure of a multistory building. The vertical structure consists of columns and the horizontal floor structure of a deck and beam system. The repeating plan unit of 24 × 32 ft is called a column bay. Assuming lateral bracing of the building to be achieved by other structural elements, the columns and beams shown here will be designed for dead load and live load only.

The load to be carried by each element of the structure is defined by the unit loads for dead load and live load and the *load periphery* for the individual elements. The load periphery for an element is established by the layout and dimensions of the framing. Referring to the labeled elements in Figure 4.1, the load peripheries are as follows:

Beam A: $8 \times 24 = 192\ \text{ft}^2$
Beam B: $4 \times 24 = 96\ \text{ft}^2$
Beam C: $24 \times 24 = 576\ \text{ft}^2$ (Note that beam C carries only three of the four beams per bay of the system, the fourth being carried directly by the columns.)
Column 1: $24 \times 32 = 768\ \text{ft}^2$
Column 2: $12 \times 32 = 384\ \text{ft}^2$
Column 3: $16 \times 24 = 384\ \text{ft}^2$
Column 4: $12 \times 16 = 192\ \text{ft}^2$

For each of these elements the unit dead load and unit live load from the floor are multiplied by the floor areas computed for the individual elements. Any possible live-load reduction is made for the individual elements based on their load periphery area.

Additional dead load for the elements consists of the dead weight of the elements themselves. For the columns

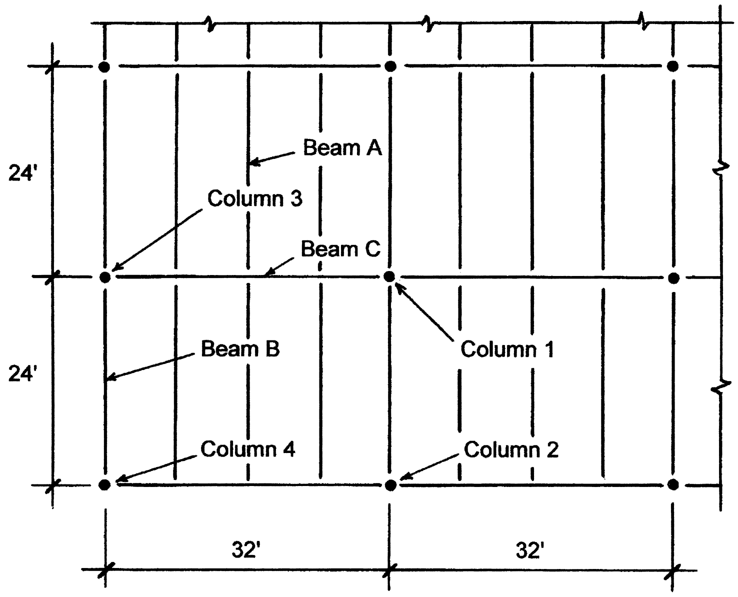


Figure 10.3 Reference for determination of design loads.

and beams at the building edge, another additional dead load consists of the portion of the exterior wall construction supported by the elements. Thus column 2 carries an area of the exterior wall defined by the multiple of the story height times 32 ft. Column 3 carries 24 ft of wall and column 4 carries 28 ft of wall (12 + 16).

The column loads are determined by the indicated supported floor, to which is added the weight of the columns. For an individual story column this would be added to loads supported above this level—from the roof and any upper levels of floor.

The loads as described are used in the defined combinations. If any of these elements are involved in the development of the lateral bracing structure, the appropriate wind or earthquake loads are also added. Floor live loads may be reduced. Reductions are based on the tributary area supported and the number of levels supported by members.

Computations using this process are described in the example cases in the rest of this chapter.

Design Methods

Use of allowable stress as a design condition relates to the classic method of structural design known as the *working stress method* and now called the *allowable stress design* (ASD) method. The loads used for this method are generally those described as *service loads*; that is, they are related to the service (use) of the structure. Deformation limits are also related to service loads.

Even from the earliest times of use of stress methods, it was known that for most materials and structures the true ultimate capacity was not predictable by use of elastic stress methods. Compensating for this with the working stress method was mostly accomplished by considerations for the establishing of the limiting design stresses. For more accurate predictions of true failure limits, however, it was necessary to abandon elastic methods and to use true ultimate strength behaviors. This led eventually to the so-called *strength method* for design, presently described as the LRFD method, or *load and resistance factor design* method.

The procedures of the stress method are still applicable in many cases, especially for design for deformation limitations. However, the LRFD methods are now very closely related to more accurate use of test data and risk analysis and purport to be more realistic with regard to true structural safety.

Although the LRFD methods are presently highly favored by academics and the writers of codes and standards, their use in design work is still trickling down to the design offices. However, with new generations of designers highly trained with computers and with strong encouragement from industry groups (ASCE, ACI, etc.), the ASD methods will soon fade out of sight. In this book, we present both methods for our readers' learning process.

The ASD Method

The allowable stress method generally consists of the following:

The service (working) load conditions are visualized and quantified as intelligently as possible. Adjustments may be made here by the determination of various statistically likely load combinations (dead load plus live load plus wind load, etc.), by consideration of load duration, and so on.

Stress, stability, and deformation limits are set by standards for the various responses of the structure to the loads: in tension, bending, shear, buckling, deflection, uplift, overturning, and so on.

The structure is then evaluated (investigated) for its adequacy or is proposed (designed) for an adequate response.

An advantage obtained in working with the stress method is that the real usage condition (or at least an intelligent

guess about it) is kept continuously in mind. The principal disadvantage comes from its detached nature regarding real failure conditions, since most structures develop much different forms of stress and strain as they approach their failure limits.

The Strength Design Method (LRFD)

In essence, the allowable stress design method consists of designing a structure to *work* at some established appropriate percentage of its total capacity. The strength method consists of designing a structure to *fail*, but at a load condition well beyond what it should have to experience in use. A major reason for favoring of strength methods is that the failure of a structure is relatively easily demonstrated by physical testing. What is truly appropriate as a working condition, however, is pretty much a theoretical speculation. The strength method is now largely preferred in professional design work. It was first largely developed for design of concrete structures but has now generally taken over all areas of structural design.

Nevertheless, it is considered necessary to study the classic theories of elastic behavior as a basis for visualization of the general ways that structures work. Ultimate responses are usually some form of variant from the classic responses (because of inelastic materials, secondary effects, multimode responses, etc.). In other words, the usual study procedure is to first consider a classic, elastic response and then to observe (or speculate about) what happens as failure limits are approached.

For the strength method, the process is as follows:

The service loads are quantified as in the first step for the stress method and then are multiplied by an adjustment factor (essentially a safety factor) to produce the *factored load.*

The form of response of the structure is visualized and its ultimate (maximum, failure) resistance is quantified in appropriate terms (resistance to compression, to buckling, to bending, etc.). This quantified resistance is also subject to an adjustment factor called the *resistance factor.* Use of resistance factors is discussed for concrete beams in Chapter 6.

The usable resistance of the structure is then compared to the ultimate resistance required (an investigation procedure) or a structure with an appropriate resistance is proposed (a design procedure).

10.2 BUILDING ONE

General Considerations

Building One is a split-level, single-family house, a common sight in small-town and suburban United States (see Figure 10.4). The house itself is primarily achieved with light wood frame construction, the most popular form for this type of building. As assured as the use of wood may be for the above-ground construction, however, it is even more assured that the ground-contacting portions of the construction will be achieved with concrete or masonry construction. While some details of the supported wood structure are shown here, the principal attention is given to the supporting construction for the building and some of the site construction, shown here as using sitecast concrete.

Support and Site Structures

Figure 10.5 shows some possible forms for the common elements of the supporting structure. The letters on the details refer to locations indicated on the building section in Figure 10.4.

Detail C in Figure 10.5 shows the support of a wood stud wall on top of a concrete wall. A common connecting device for this situation is the steel anchor bolt which is cast into the top of the concrete wall. This must be located with some precision in order to be used to bolt down the sill for the stud wall. The anchor bolts serve to hold the wood frame in position during construction. However, they may also be required to transfer lateral and uplift loads to the foundations due to wind or earthquake forces.

The concrete wall is shown supported on a simple strip footing in detail D of Figure 10.5. For the loads from this small, light building, this footing is likely to be only a small amount wider than the wall and likely to be achieved with no reinforcement. However, there is a special problem for this particular wall that may require a different form of footing; this is discussed later.

Detail D also shows the edge of the concrete floor slab for the garage. This is most likely not actually connected to the wall but simply floats on a prepared base on top of the ground. For passenger car wheel loads, the minimum 3.5-in.-thick slab is most likely adequate here. With the size of this garage, it is probably possible to pour this slab in a single unit, with separating joints occurring only at the surrounding walls.

Details F and G show some conditions for the concrete slab floor at the living room level. Detail F shows the use of a shallow foundation at the building edge, a common form used in mild climates where protection from frost action is not a concern. In cold climates, the footing must be placed considerably below the exterior surface of the ground, which is typically achieved by introducing a wall between the slab edge above and the footing below.

Detail G in Figure 10.5 shows an interior partition wall supported directly on the floor slab. In this case the wall sill is anchored by devices installed after the slab is cast. This is a practical solution and acceptable in most cases for interior walls that are not required to function as shear walls. Precisely locating required anchor bolts and somehow holding them in midair while concrete is poured around them are not that easy, so this detail is truly a simplification of the building process.

Figure 10.6 shows a site plan which indicates a sloping lot. While changes of the ground profile can be made within the site, a common situation is the need to meet existing ground contours at the lot edge. If lowering, raising, or simply leveling

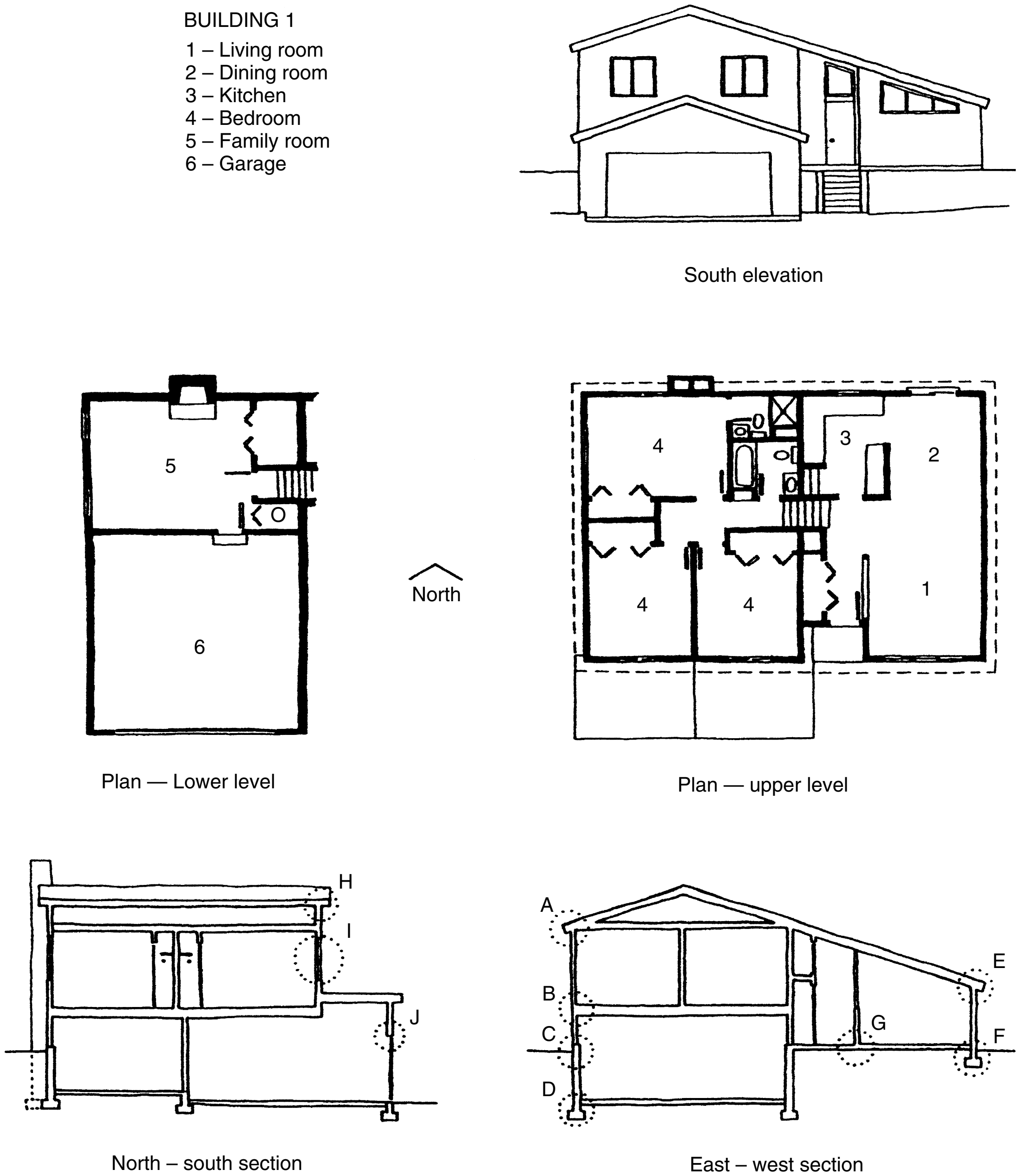


Figure 10.4 General form of Building One.

out of the site is required, some special construction may be necessary at the lot edges to protect the adjoining properties.

Another concern for the site is the relation between the original ground surface and the constructed surfaces. The term *original surface*, as used here, refers to an undisturbed condition before any constructed filling to raise the surface level, whether the fill is part of the current work or from some previous activity. Cutting down of the site for construction may expose materials that need consideration. Or, on the other hand, raising finished concrete slab surfaces and bottoms of walls above the original grade may require some special consideration for the support of construction placed on top of the fill and some tall foundation walls.

Figure 10.7 shows structural plans for the two levels of the building that sit directly on the ground. Various details of this construction and some of the site construction elements are shown in Figures 10.7 and 10.8. The following discussion relates to items shown in Figures 10.7 and 10.8.

Lower Level Floor

The building section in Figure 10.4 and the plan of the lower level in Figure 10.7 show that the floor changes level between the garage and family room (detail 6). The easiest way to achieve this is simply to cast a strip grade beam/footing and cast the two floor slabs separately. This footing/wall also supports the dividing wall in this case.

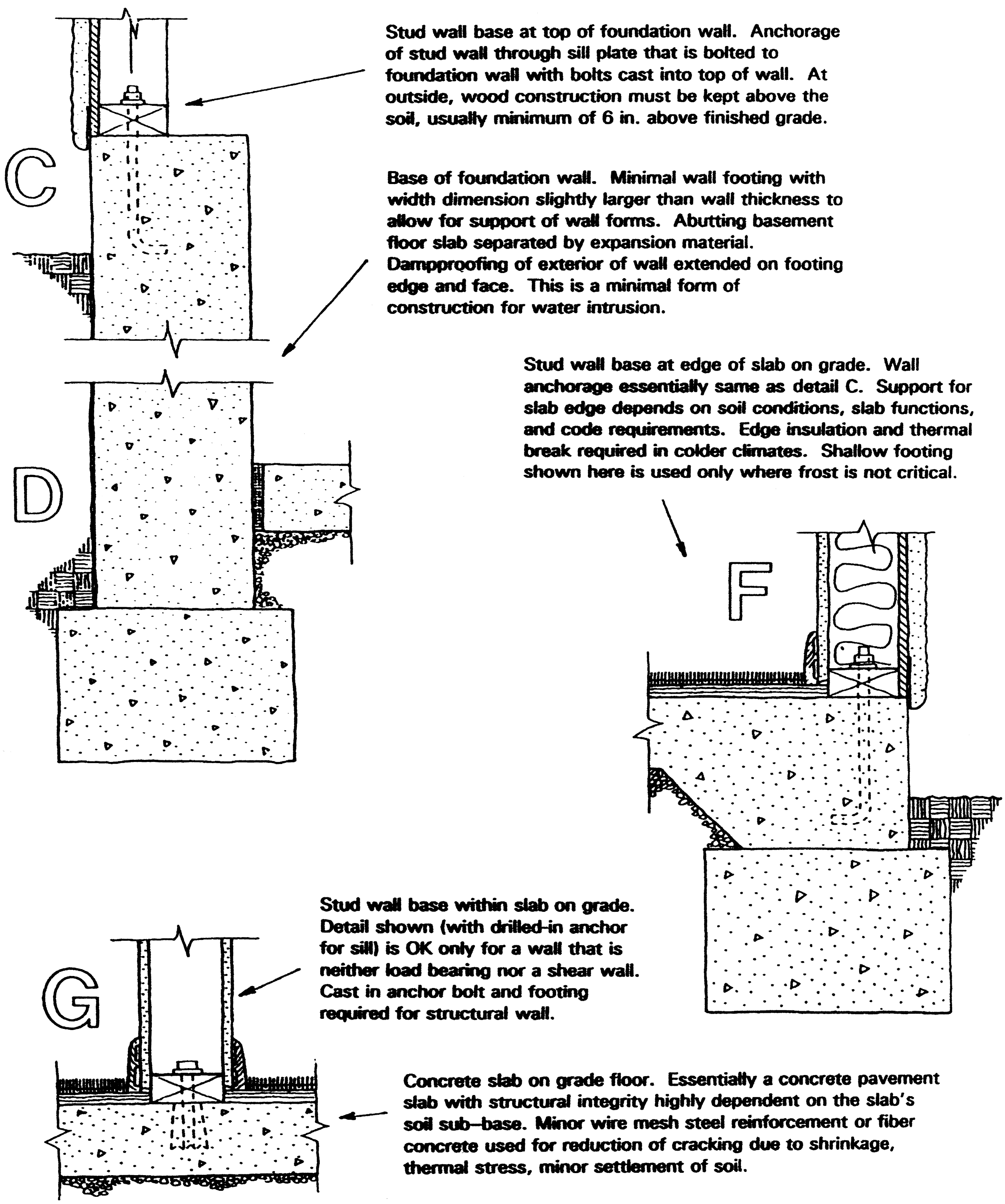


Figure 10.5 Construction details for Building One.

The garage floor continues to become the driveway at the entrance. The driveway slab and garage floor slab are essentially similar, but there should be a separation joint here and some support for the slab edges (detail 5). The grade beam shown here is merely an extension of the wall-supporting construction at the edges of the door opening.

Laterally Unsupported Wall

The site plan shows that the finish ground surface slopes along the west side of the building. At the rear of the house the level approaches that of the middle (living room) level, while at the front it approaches the level of the garage floor. While this foundation wall serves to partly form a basement

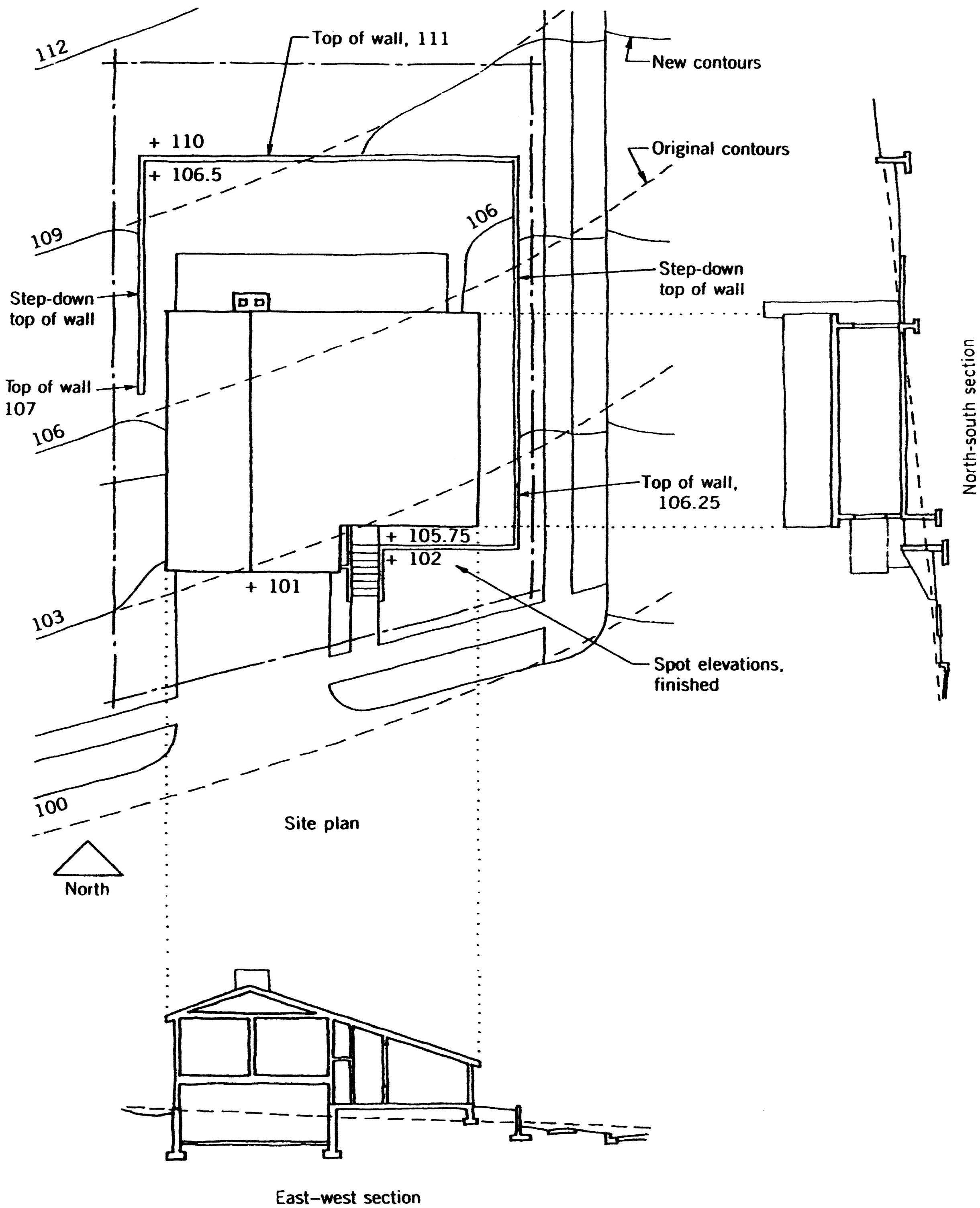


Figure 10.6 Site plan and sections for Building One.

space (or half basement), it does not have the usual lateral support of a full basement wall.

The bottom of the wall is adequately braced by the edge of the concrete floor slab, but its top merely supports the wood stud wall. If balloon framing (with continuous studs) is used, it may be possible to use the wall studs for lateral support. However, it is probably better to build this wall as a cantilever retaining wall, with the footing as shown in details 1 and 2 in Figure 10.7.

At the rear of the house, however, this wall can span the short horizontal distance from the side walls to the fireplace, so a simple wall footing is adequate. The wall between the

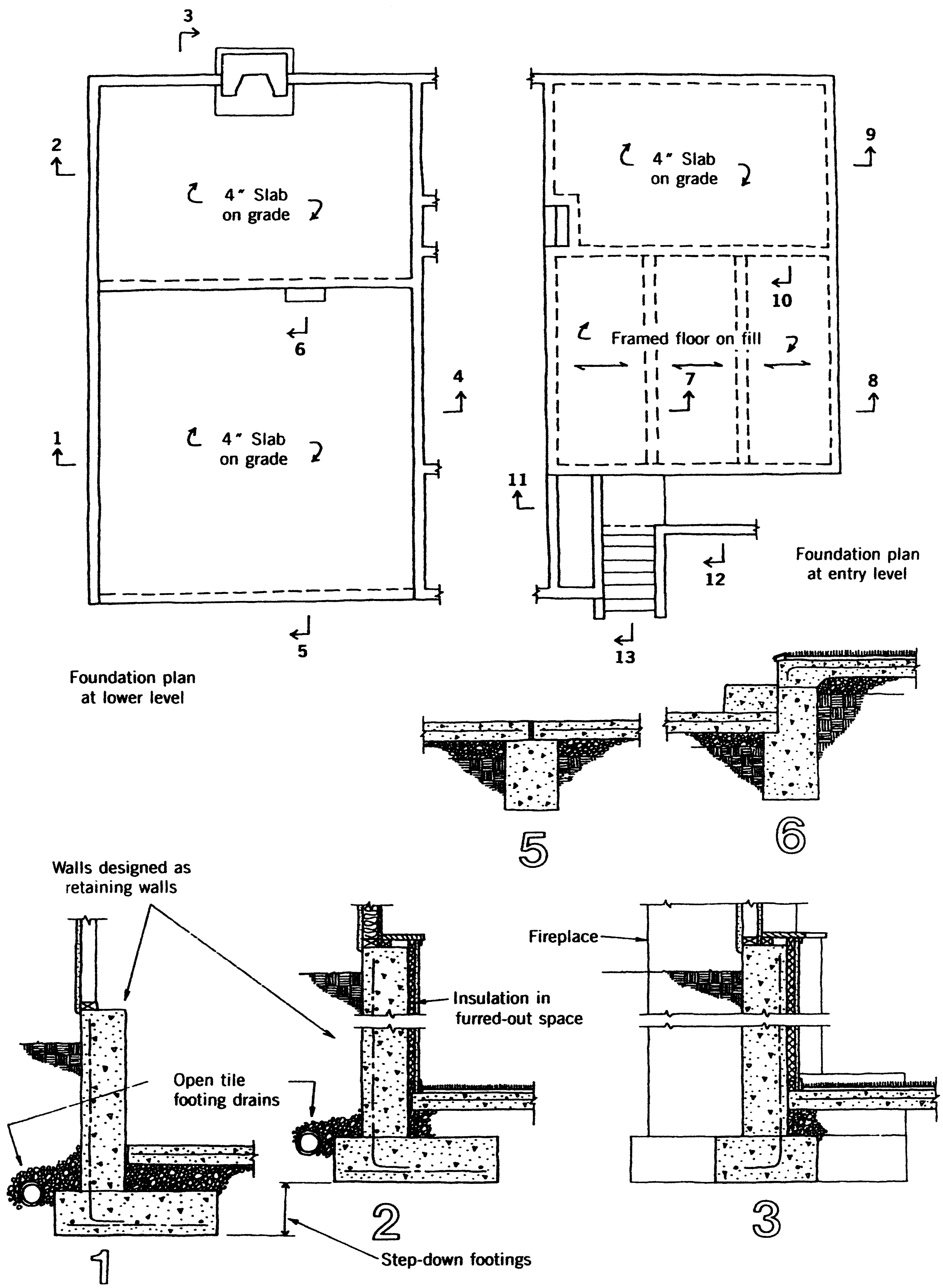


Figure 10.7 Foundation and site details for Building One.

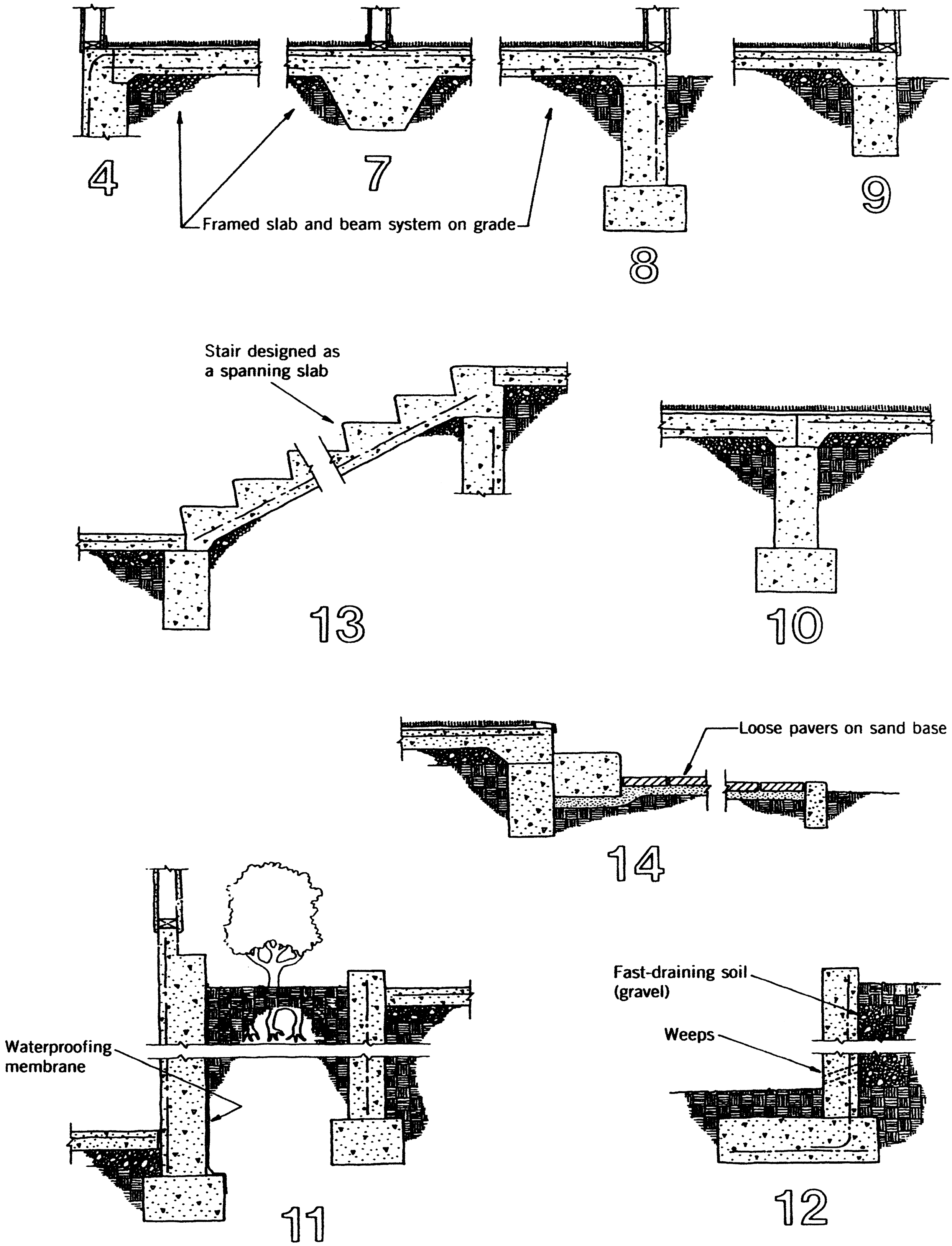


Figure 10.8 Foundation and site details for Building One.

two levels at the center of the house also has a difference of level on two sides but should be adequately braced by other construction (detail 4 in Figure 10.8).

Framed Floor on Grade

The floor plan for the middle level in Figure 10.7 and details 4, 7, and 8 in Figure 10.8 show a special type of sitecast concrete construction. This consists of a spanning structure cast directly on soil materials, commonly called a *framed floor on grade*. The usual reason for using such a structure is the lack of confidence in the supporting material regarding future settlement. This is typically due to the condition that exists here, in which the finished level is some distance above the original undisturbed ground surface. Regardless of the quality of work for compaction of this fill, some settlement must be anticipated. While this slight movement must be accepted for exterior construction (drives, walks, patios, etc.), it is a different problem for building floors—especially ones that support walls or other interior construction.

Depending on circumstances, this structure may be achieved in a number of ways, as follows:

One-Way Slab. If supporting walls or grade beams are relatively close together, a simple one-way slab may be used. This consists of simply adding some reinforcing bars to the usual slab on grade construction. However, if the span is more than 8 ft or so, the slab must be thickened and may become unfeasible. In this case the shortest span is about 18 ft, which would require a very thick slab, even though the live load is low.

Two-Way Slab. If supporting edge structures define rectangular areas of approximately square shape, it may be possible to use a two-way spanning slab. This would allow the use of a slightly thinner slab in comparison to the one-way spanning slab. This is an option for our case, as the bay size is close to square (18 × 22 ft, approximately), but something thicker than the 3.5-in.-thick slab would be required.

Precast Spanning Units. If a considerable area of floor is involved, a simple solution may be to use precast floor units. For this small project, however, it is not necessary and probably not practical.

Post tensioned, Prestressed Slab. This is sometimes a solution for larger projects but is much more complicated and expensive than other alternatives here.

One-Way Spanning Slab-and-Beam System. This amounts basically to simply casting the usual slab on grade with the addition of some extra reinforcement and the creation of beams by trenching slightly below the general surface developed for the bottom of the slab. This trenching can also be used (as it is more frequently) to create a strip footing for a wall, as shown in detail 7 in Figure 10.8.

The slab-and-beam system on grade may be designed in the same manner as the usual spanning system for floors or roofs. Some details for the system are shown in Figure 10.9. The three-span slab is reinforced with three sets of bars, designated bars A, B, and C in the figure. These are placed and used as follows:

Bars A are the lowermost bars and are supported to keep them above the subgrade materials. These bars provide for positive bending moments (tension in the bottom of the slab) and in this case would be provided as continuous bars, approximately 21 ft long. If this is an impractical length, they could be spliced at one of the beams.

Bars B are placed on top of the lower set, at right angles, and provide for shrinkage reinforcement.

Bars C are the top layer, providing for negative bending moment (tension in the top of the slab).

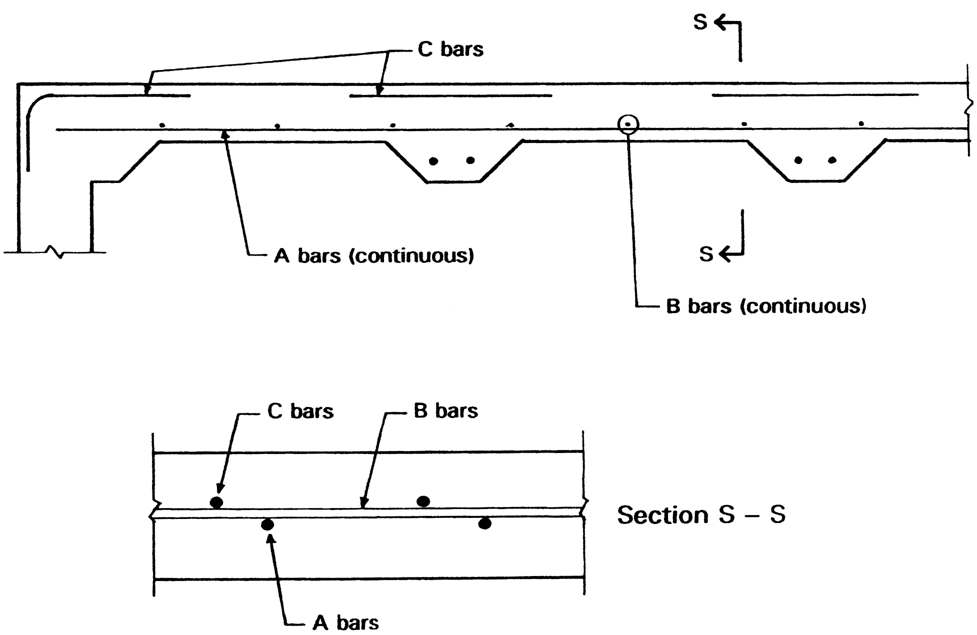


Figure 10.9 Reinforcement for the spanning slab-and-beam system.

As mentioned previously, most of the wall construction shown in the sections could be achieved with masonry (most likely CMUs) and frequently are. This applies to construction for both the building foundation and site structures.

10.3 BUILDING TWO

This building consists of a simple, single-story, box-shaped building. Lateral bracing is developed in response to wind load. Several alternatives are considered for the building structure.

General Considerations

Figure 10.10 shows the general form, the construction of the basic building shell, and the form of the wind-bracing shear walls for Building One. The drawings show a building profile with a generally flat roof (with minimal slope for drainage) and a short parapet at the roof edge. This structure is generally described as a *light wood frame* and is the first alternative to

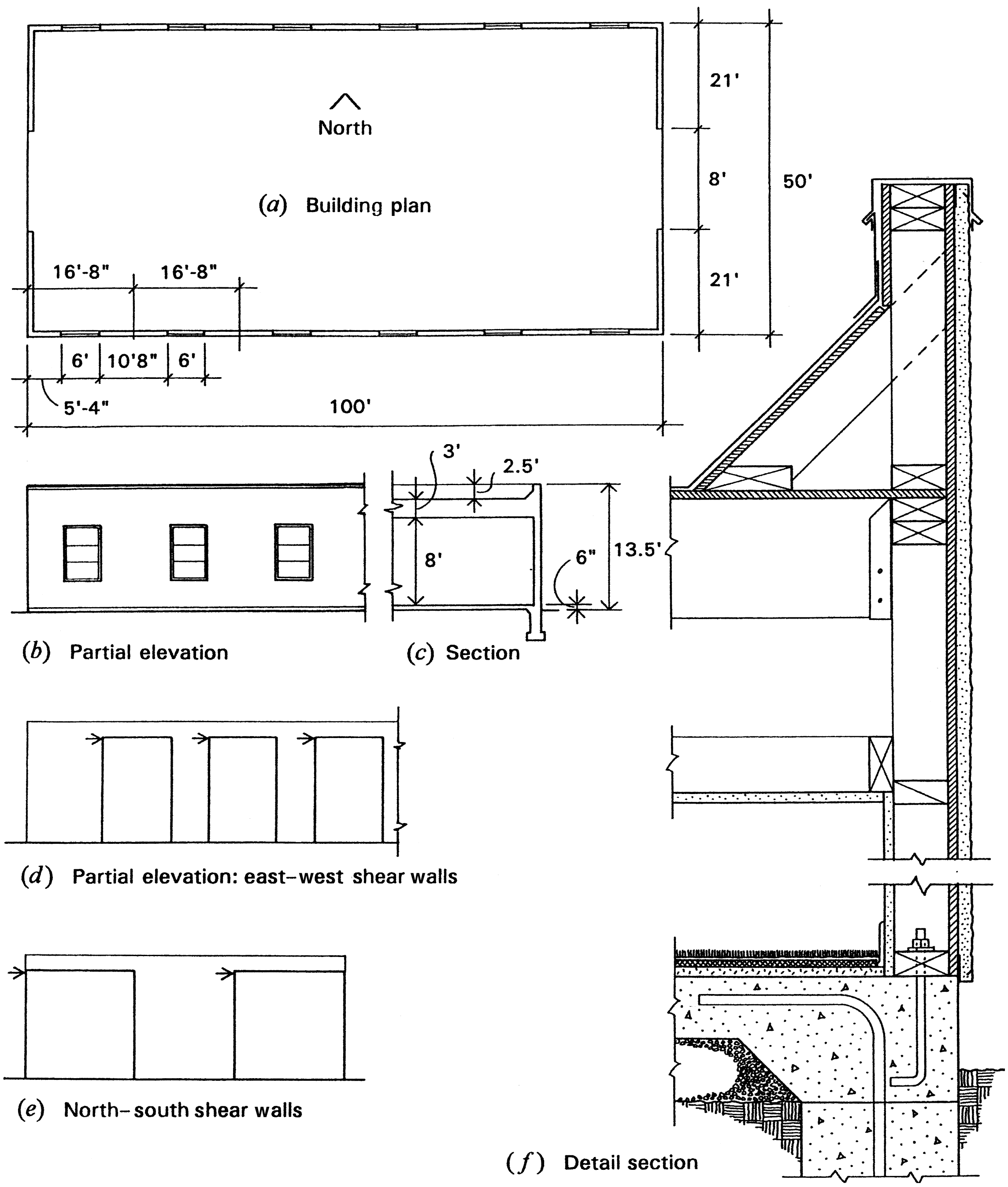


Figure 10.10 Building Two, general form of the light wood alternative.

be considered for Building One. The following data are used for design:

Roof live load = 20 psf (reducible)
Wind load, determined from the ASCE standard (Ref. 1)
Wood-framing lumber of Douglas fir-larch

Although we will consider the light wood frame building as a singular alternative for this building size and form, there are actually a considerable number of alternative choices for the construction. Some choices for the construction may be made on the basis of regional location. The relative severity of local wind or earthquake conditions may also affect choices. We show here a design for a mild climate with minor wind loads.

Design of the Wood Structure for Gravity Loads

With the construction as shown in Figure 10.10*f*, the roof dead load (in pounds per square feet) is determined as follows:

Three-ply felt and gravel roofing	5.5
Glass fiber insulation batts	0.5
Plywood roof deck 2 in. thick	1.5
Wood rafters and blocking (estimate)	2.0
Ceiling framing	1.0
Drywall ceiling 2 in. thick	2.5
Ducts, lights, etc.	3.0
Total roof dead load for design	16.0

Assuming a partitioning of the interior as shown in Figure 10.11*a*, various possibilities exist for the development of the spanning roof and ceiling framing systems and their supports. Interior walls may be used for supports, but a more desirable situation is obtained by using interior columns that allow for rearrangement of interior spaces. The roof framing system shown in Figure 10.11*b* is developed with two rows of interior columns placed at the location of the corridor walls. If the partitioning shown in Figure 10.11*a* is used, these columns may be totally out of view if they are incorporated in the wall construction. Figure 10.1*c* shows a second possibility for the roof framing using the same column layout as in Figure 10.11*b*. For this example the scheme shown in Figure 10.11*b* is arbitrarily selected for illustration of the design of the elements of the structure.

Installation of membrane-type roofing ordinarily requires at least a $\frac{1}{2}$-in.-thick roof deck. Such a deck is capable of up to 32-in. spans in a direction parallel to the face ply grain (the long direction of ordinary 4 × 8-ft panels). If rafters are not over 24 in. on center—as they are likely to be for the schemes shown in Figures 10.11*b* and *c*—the panels may be placed so that the plywood span is across the face grain.

An advantage in the latter arrangement is the reduction in the amount of blocking between rafters required at panel edges not falling on a rafter.

A common choice of grade for rafters is No. 2, for which Table 4.1 yields an allowable bending stress of 1035 psi

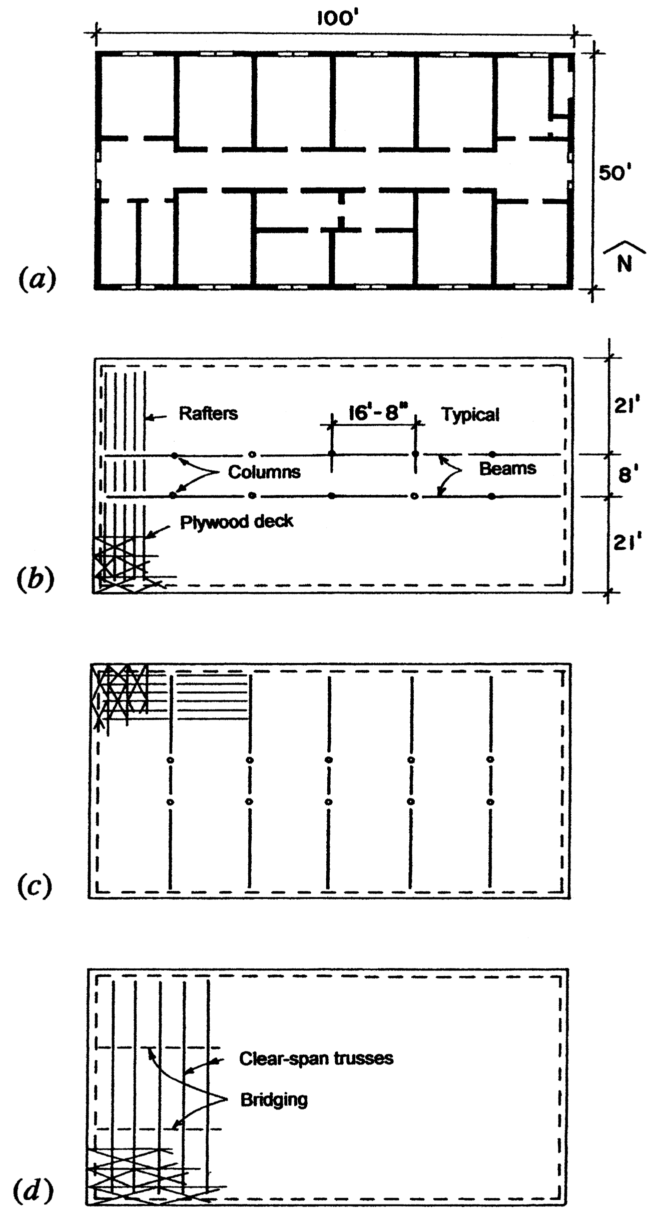


Figure 10.11 Developed plan for interior partitioning and alternatives for the roof framing.

(repetitive use) and a modulus of elasticity of 1,600,000 psi. Since the data for this case fall approximately within the criteria for Table 4.8, possible choices from the table are for either 2 by 10s at 12-in. centers or 2 by 12s at 16-in. centers.

A ceiling may be developed by direct attachment to the underside of the rafters. However, the construction as shown here indicates a ceiling at some distance below the rafters, allowing for various service elements to be incorporated above the ceiling. Such a ceiling might be framed independently for short spans (such as at a corridor) but is more often developed as a *suspended ceiling*, with hanger elements from the overhead structure.

The wood beams as shown in Figure 10.11*b* are continuous through two spans, with a total length of 33 ft, 4 in. and beam spans of 16 ft, 8 in. For the two-span beam

the maximum bending moment is the same as for a simple span, the principal advantage being a reduction in deflection. The total load area for one span is

$$A = \left(\frac{21+8}{2}\right) \times 16.67 = 242\,\text{ft}^2$$

This permits the use of a live load of 16 psf. Thus the unit of the uniformly distributed load on the beam is found as

$$w = (16\,\text{psf LL} + 16\,\text{psf DL}) \times \frac{21+8}{2} = 464\,\text{lb/ft}$$

Adding a bit for the beam weight, a design for 480 lb/ft is reasonable, for which the maximum bending moment is

$$M = \frac{wL^2}{8} = \frac{480 \times (16.67)^2}{8} = 16{,}673\,\text{ft-lb}$$

A common minimum grade for beams is No. 1. The allowable bending stress depends on the beam size and the load duration. Assuming a 15% increase for load duration, Table 4.1 yields the following:

For a 4-by member: $F_b = 1.15(1000) = 1150$ psi
For a 5 by or larger: $F_b = 1.15(1350) = 1552$ psi
Then, for a 4 by:

$$\text{Required } S = \frac{M}{F_b} = \frac{16{,}673 \times 12}{1150} = 174\,\text{in.}^3$$

From Table A.8, the largest 4-in.-thick member is a 4 by 16 with $S = 135.7$ in.^{3}, which is not adequate. (Note: Deeper 4-by members are available but are quite literally unstable and thus not recommended.) For a thicker member the required S may be determined as

$$S = \frac{1150}{1552} \times 174 = 129\,\text{in.}^3$$

for which possibilities include a 6 by 14 with $S = 167$ in.^{3} or an 8 by 12 with $S = 165$ in.^{3}

Although the 6 by 14 has the least cross-sectional area and ostensibly the lower cost, various considerations of the development of construction details may affect the beam selection. This beam could also be formed as a built-up member from a number of 2-by members. Where deflection or long-term sag are critical, a wise choice might be to use a glued-laminated section or even a steel rolled shape. For heavily loaded beams shear is often critical, which may also affect the choice.

A minimum slope of the roof surface for drainage is usually 2%, or 1/4 in./ft. If drainage is achieved as shown in Figure 10.12*a*, this requires a total slope of 6.25 in. from the center to the roof edge. There are various ways of achieving this sloped surface, including simply tilting of the rafters.

Figure 10.12*b* shows some possibilities for the details of the construction at the center of the building. As shown here,

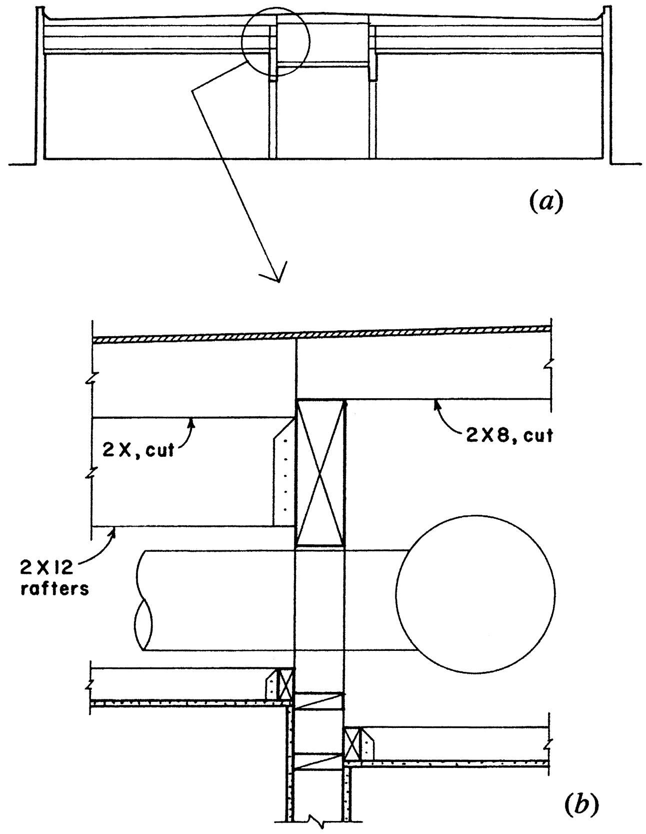


Figure 10.12 Construction details.

the rafters are kept flat and the roof profile is achieved by attaching cut 2-by members to the tops of the rafters and using a short profiled rafter at the corridor. Ceiling joists for the corridor are supported by the corridor walls. Other ceiling joists are supported at their ends by the walls and at intermediate points by suspension from the rafters.

The typical column at the corridor supports a load approximately equal to the entire spanning load for one beam, or

$$P = 480 \times 16.67 = 8000\,\text{lb}$$

This is a light load, but the column height requires something larger than the 4-by size. (See Table 4.10.) If a 6 by 6 is not objectionable, it is adequate in the lower stress grades. However, it is common to use a steel pipe or tubular shape, either of which can probably be accommodated within a stud partition wall.

Design for Lateral Loads

Design of the building structure for wind includes consideration of the following:

Inward and outward pressures on exterior building surfaces, causing bending of the wall studs and an addition to the gravity loads on the roof

Total lateral (horizontal) force on the building, requiring bracing by the roof diaphragm and the shear walls

Uplift on the roof, requiring anchorage of the roof to its supports, especially for light roof construction

Total effects of uplift and lateral forces, possibly resulting in overturn (toppling) of the whole building or of individual vertical bracing elements

Uplift on the roof depends on the roof shape and its height above ground. For this low, flat-roofed building, the ASCE standard (Ref. 1) requires an uplift pressure of 10.7 psf. In this case, the uplift pressure does not exceed the roof dead weight of 16 psf, so anchorage of the roof construction is not required. However, common use of metal framing devices for light wood frame construction provides an anchorage with considerable resistance.

Overturning of the building is not likely for a building with this squat profile (50 ft wide by only 13.5 ft high). Even if the overturning moment caused by wind exceeds the restoring moment due to the building weight, the sill anchor bolts will undoubtedly hold the building down in this case. Overturn of the whole building is more critical for towerlike building forms or for extremely light construction. Of separate concern is the overturn of individual elements of the bracing system—in this case the individual shear walls, which will be investigated later.

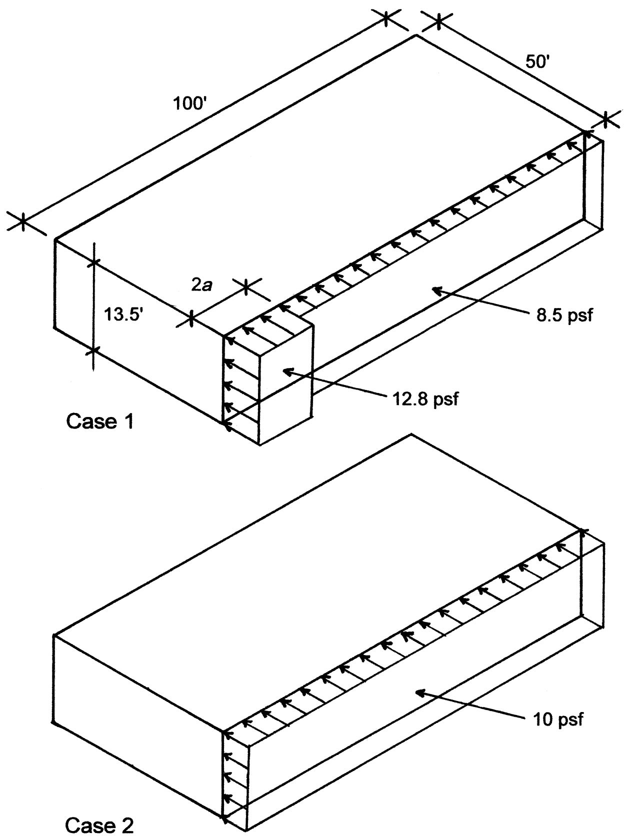


Figure 10.13 Wind pressure on the south wall.

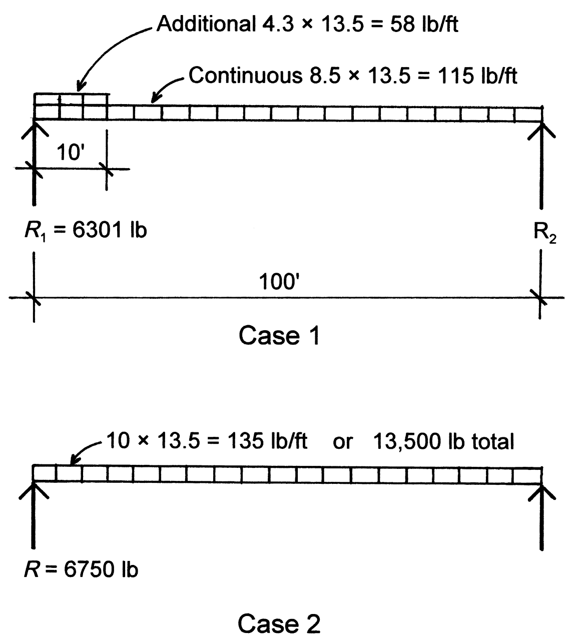


Figure 10.14 Loads on the analogous beam.

Wind Force on the Bracing System

The building's bracing system must be investigated for wind effects in the two directions of the building's axes: east–west and north–south.

The horizontal wind pressure on the north and south walls of the building is shown in Figure 10.13. This pressure is generated by a combination of positive (direct, inward) pressure on the windward side and negative (suction, outward) pressure on the lee side of the building. The pressure shown as case 1 in Figure 10.13 is obtained from data in the ASCE standard (Ref. 1). (See discussion of wind loads in Chapter 9.) The ASCE standard provides for two zones of pressure: a general one and a small special increased area of pressure at one end. The values shown here are derived by considering a critical wind velocity of 90 mph and an exposure condition B, as described in the standard.

The range for the increased pressure in case 1 is defined by the dimension *a* and the height of the windward wall. The value of *a* is established as 10% of the least plan dimension of the building or 40% of the wall height, whichever is smaller, but not less than 3 ft. For this example, *a* is determined as 10% of 50 ft, or 5 ft. The distance for the pressure of 12.8 psf in case 1 is thus $2(a) = 10$ ft.

The design standard also requires that the bracing system be designed for a minimum pressure of 10 psf on the entire surface of the wall. This sets up two cases (1 and 2 in Figure 10.13) that must be considered. Because the concern for the design is the generation of the maximum effect on the roof diaphragm and the end shear walls, the critical conditions may be determined by considering the development of the reaction forces and maximum shear for an analogous beam subjected to the two loadings. This analysis is shown in Figure 10.14, from which it is apparent that the critical concern for the end shear walls (the reactions) and the maximum effect in the roof diaphragm is derived from the case 2 loading.

The actions of the horizontal wind force resisting system in this regard are illustrated in Figure 10.15. The initial wind force comes from the wind pressure on the building's vertical sides. The wall studs here span vertically to resist this uniformly distributed loading, as shown in Figure 10.15*a*. Assuming the wall function to be as shown in Figure 10.15*a*, the north–south wind force delivered to the roof edge is determined as

$$\text{Total } W = (10\,\text{psf})(100 \times 13.5) = 13{,}500\,\text{lb}$$

$$\text{Roof edge } W = 13{,}500 \times \frac{6.75}{11} = 8284\,\text{lb}$$

In resisting this load, the roof functions as shown in Figure 10.14. Thus the end reaction and the maximum diaphragm shear are found as

$$R = V = \frac{8284}{2} = 4142\,\text{lb}$$

which produces a maximum unit shear in the 50-ft-wide diaphragm of

$$v = \frac{\text{shear force}}{\text{roof width}} = \frac{4142}{50} = 82.8\,\text{lb/ft}$$

From Table 9.1 a variety of selections is possible. Variables include the class of the panel, the panel thickness, the width of supporting rafters, the nail size and spacing, the use of blocking, and the layout pattern of the panels. Assuming a plywood deck with a minimum thickness for the flat roof of $\frac{1}{2}$ in. (given as $\frac{15}{32}$ in the table), a possible choice is as follows:

APA rated sheathing, $\frac{15}{32}$ in. thick, 2-by rafters, 8d nails at 6 in. at all panel edges, unblocked diaphragm, allowable shear 180 lb/ft

This is pretty much minimal construction. Had closer nail spacing been required, it would be possible to reduce the spacing for some parts of the roof at a greater distance from the ends. This variation is possible because of the shape of the shear diagram for uniformly distributed loading, as shown in Figure 10.16*b*.

The moment diagram shown in Figure 10.16*c* indicates a maximum value of 104 kip-ft at the center of the span. This moment is used to determine the maximum force in the diaphragm chords at the roof edges. This force must be developed in both compression and tension as the wind direction changes.

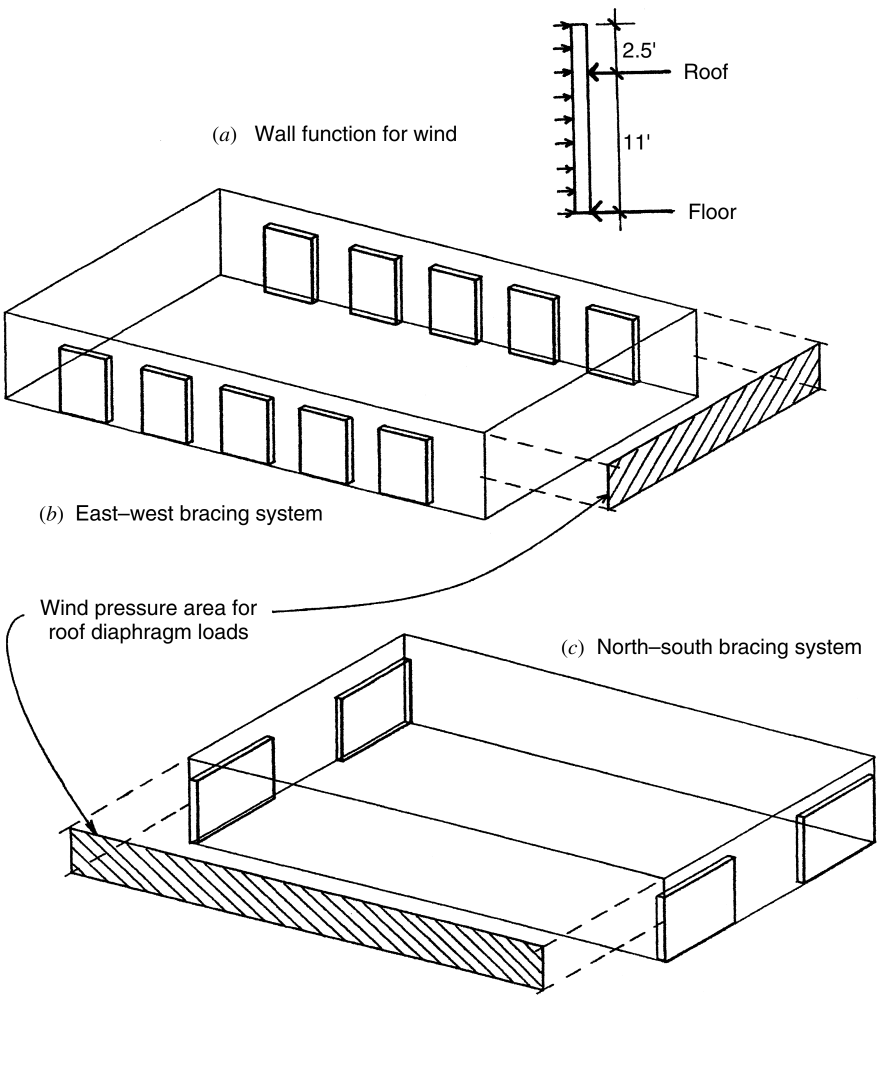


Figure 10.15 Wall functions and wind pressure development.

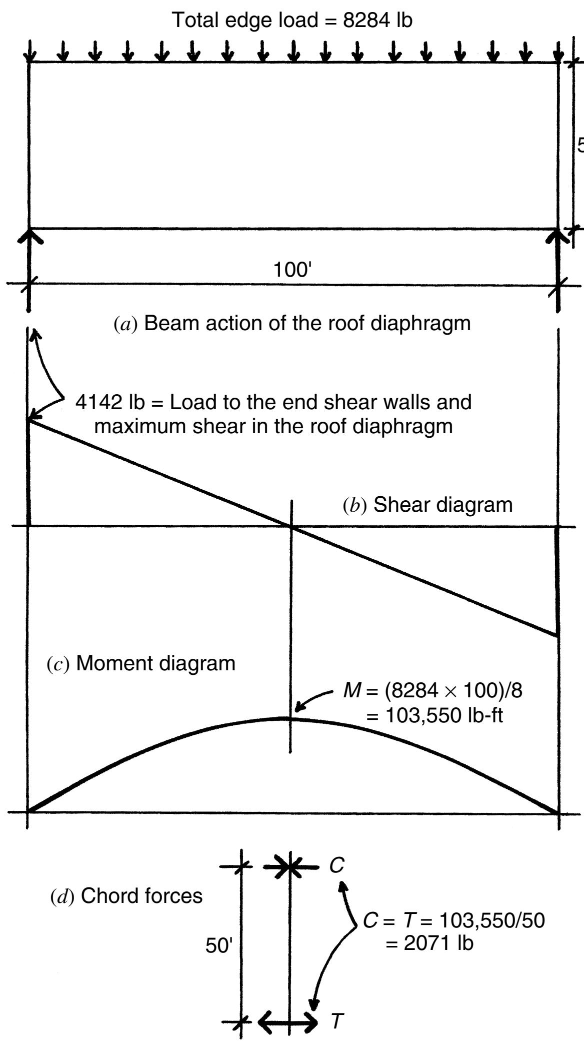


Figure 10.16 Spanning functions of the roof diaphragm.

With the construction as shown in Figure 10.10*f*, the top plate of the stud wall is the most likely element to be utilized for this function. In this case, the chord force of 2071 lb, as shown in Figure 10.16, is quite small, and the doubled 2-by member should be capable of resisting the force. However, the building length requires the use of several pieces to create this continuous plate, so the splices for the double member should be investigated.

The end reactions for the diaphragm, as shown in Figure 10.16*a*, must be developed by the two 21-ft-long end shear walls, as shown in Figure 10.10*f*. Thus the total shear force is resisted by a total of 42 ft of shear wall and the unit shear in the walls is

$$v = \frac{4142}{42} = 98.6\,\text{lb/ft}$$

As with the roof deck diaphragm, there are various considerations for the selection of the wall construction. Various materials can be used for both the exterior and interior surfaces of the stud wall. The common form of construction shown in Figure 10.10*f* indicates gypsum drywall on the inside and a combination of plywood and stucco (cement plaster) on the outside of the wall. However, with a combination of wall surfacing materials, it is common practice to consider only the strongest of the materials to be the shear-resisting element. In this case that means the plywood sheathing on the exterior surface of the wall. From Table 9.2, a possible choice is

> APA rated sheathing, 3/8 in. thick, with 6-in. spacing at all panel edges, shear capacity 200 lb/ft

Again, this is minimal construction. For higher loads, a greater resistance can be obtained by using better panel material, thicker panels, larger nails and/or closer nail spacing, and—sometimes—thicker studs. Unfortunately, the nail spacing cannot be graduated here as with the roof deck, since the shear is constant in magnitude throughout the wall height.

Although the wood sheathing would most likely indeed be considered alone as the shear-resisting surfacing, the other materials will participate as the wall is deformed by the load. In fact, the stucco—being securely attached to the wall sheathing and being the stiffest of the materials—will tend to take the shear force at first, only passing the load to the sheathing when the stucco cracks. This presents a problem which is discussed in Chapter 9 regarding damage to nonstructural elements of the construction.

Figure 10.17*a* shows the loading condition for the investigation of the overturning effect on the end shear wall. Overturn is resisted by the so-called restoring moment caused by the dead load on the wall—in this case, a combination of the wall weight and the portion of the roof dead load supported by the wall. Safety is considered adequate if the restoring moment is at least 1.5 times the overturning moment. For the example, the two moments

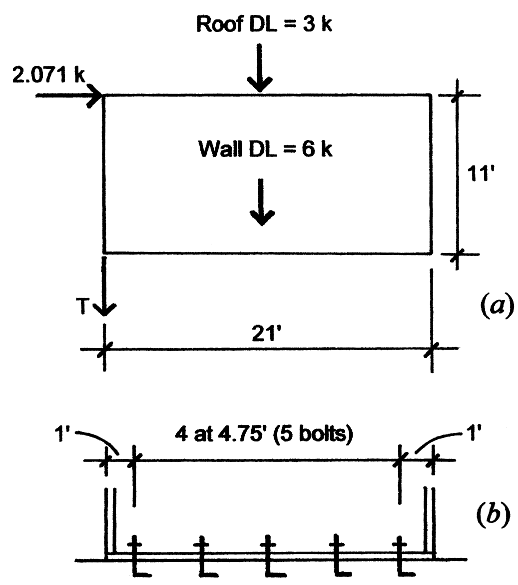


Figure 10.17 Functions of the end shear wall.

are determined as

$$\text{Overturning } M = (2.071)(11)(1.5) = 34.2\text{ kip-ft}$$

$$\text{Restoring } M = (3+6)\left(\frac{21}{2}\right) = 94.5\text{ kip-ft}$$

which clearly indicates that overturning is not a problem and no tiedown force is required at the wall ends.

Details of the construction and other structural functions of the wall may provide additional resistance to overturn. For example, at the building corners, the two abutting walls will most likely be reasonably attached to each other and each will help to hold the other down. Nevertheless, many designers prefer to provide tiedown anchorage at the ends of all shear walls.

Finally, the walls will be bolted to the foundation with code-required sill bolts, which will provide some resistance to uplift and overturn effects. For various reasons, however, these are not relied on for uplift or overturn, although they can be used to resist lateral sliding of the wall. The usual minimum bolting is with 1/2-in. bolts spaced a maximum of 6 ft on center and with a bolt at each end of the wall within 12 in. of the wall end. This results in a bolting for this wall as shown in Figure 10.17*b*. The five bolts shown should be capable of developing the shear force for this wall.

The overturning moment is also exerted on the supporting foundation. This situation must be investigated for effects on the foundation structure and/or the soil pressures developed by the combination of gravity and moment effects.

A general area of concern has to do with the transfer of wind forces from element to element throughout the structural system. A critical joint to be considered here is that between the roof diaphragm and the shear walls. This transfer is actually between the roof deck panels and the wall panels. How each of these is attached to elements of the framing must be investigated, plus how the framing is attached to complete the transfers.

Alternative Steel and Masonry Structure

Alternative construction for Building Two is shown in Figure 10.18. In this case, the walls are made of concrete masonry units (CMUs) and the roof structure consists of a formed sheet steel deck supported by open-web steel joists (light, prefabricated steel trusses). The following data are assumed for design:

Roof dead load 15 psf, not including weight of the structure
Roof live load 20 psf (reducible)

Construction consists of:

K-series open-web joists
Reinforced CMU exterior walls
A 1.5-in. formed sheet steel deck
Deck surfaced with lightweight insulating concrete fill
Multiple-ply, hot-mopped felt and gravel roofing
Suspended ceiling with gypsum drywall

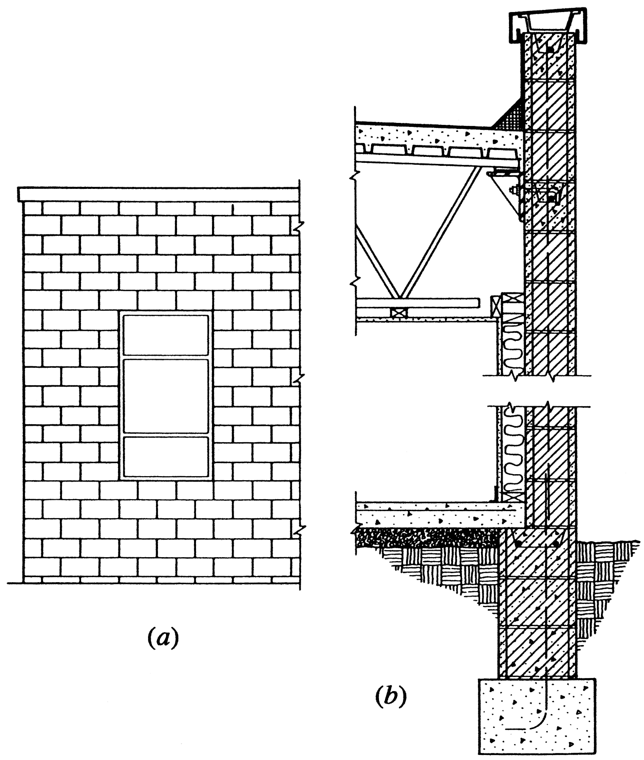
The section in Figure 10.18*b* indicates that the wall continues above the top of the roof and the trusses are supported at the wall face. The span of the trusses is thus approximately 48 ft.

As the construction section shows, the roof deck is placed directly on top of the trusses and the ceiling is attached to the bottom of the trusses. Minimum roof drainage of 2% may be achieved by tilting of parallel-chorded trusses or by using trusses with flat bottoms and tilted (sloped) tops. If the parallel-chorded trusses are used, the ceiling may be suspended. The following work assumes a constant depth of the trusses for design purposes.

Design of the Roof Structure

Spacing of the open-web joists must be coordinated with the selection of the roof deck and ceiling construction. A spacing of 4 ft is assumed here. From Table 5.10, with deck units assumed to achieve three spans or more, the lightest deck in the table can be used. The choice for the deck configuration depends on the type of materials placed on top of the deck and the method of attachment of the deck to the supports. For diaphragm action, the preferred attachment is by welding, so the widest stem width for the deck is commonly used.

Adding the weight of the deck to the other roof dead load produces a total dead load of 17 psf for the superimposed load



(*a*) (*b*)

Figure 10.18 Building Two: alternative steel and masonry structure.

on the joists. As illustrated in Section 5.2, the design for a K-series joist is as follows:

Joist live load = 4(20) = 80 lb/ft (or plf)
Superimposed dead load = 4(17) = 68 lb/ft (not including joist weight).
Total factored load = 1.2(68) + 1.6(80) = 82 + 128 = 210 plf + the joist weight

For the 48-ft span, possible choices from Table 5.10 are the following:

24K9 at 12.0 plf, total factored load = 1.2(12 + 68) + 128 = 224 plf, less than the table value of 313 plf
26K5 at 10.6 plf, total factored load = 1.2(10.6 + 68) + 128 = 222 plf, less than table value of 233 plf

Live-load capacity for L/360 deflection exceeds the requirement for both of these choices.

Although the 26K5 is the lightest choice, there may be reasons for using a deeper joist. For example, if the ceiling is directly attached to the bottoms of the joists, a deeper joist will provide more space for passage of building service elements. Deflection will also be reduced if a deeper joist is used. Pushing the live-load deflection to the limit means a deflection of $(1/360)(480 \times 12) = 1.9$ in. Even though this may not be critical for the roof surface, it can present problems with the sag of the ceiling. Choice of the 30K7 joist at 12.9 plf results in considerably less deflection at a small premium in additional weight.

Note that Table 5.10 is abridged from a larger table in the reference, and there are therefore many more choices for joists sizes. The example here is meant only to indicate the process for use of such references.

Specifications for open-web joists give requirements for end-support details and lateral bracing (see Ref. 10). If the 30K7 is used for the 48-ft span, for example, four rows of bridging are required.

Although the masonry walls are not designed for this example, note that the support indicated for the joists in Figure 10.18*b* results in an eccentric load on the wall. This induces bending in the wall, which may be objectionable. An alternative detail for the roof-to-wall joint is shown in Figure 10.19, in which the joists sit directly on the wall with the joist top chord extending to form a short cantilever. This is a common detail, and the reference supplies data and suggested details for this construction.

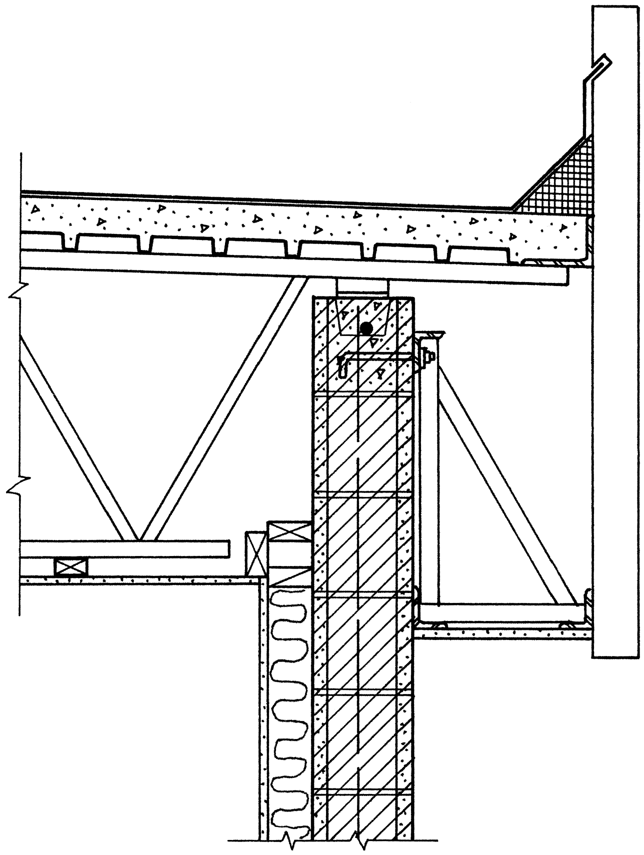


Figure 10.19 Building Two: variation of the roof-to-wall joint.

Alternative Roof Structure with Interior Columns

If a clear spanning roof structure is not required for this building, it may be possible to use some interior columns and a framing system for the roof with quite modest spans. Figure 10.20*a* shows a framing plan for a system that uses columns at 16 ft 8 in. on center in each direction. Although short span joists may be used with this system, it would also be possible to use a longer span deck, as indicated on the plan. This span exceeds the capability of the 1.5-in. deep deck, but decks with deeper ribs are available.

A second possible framing arrangement is shown in Figure 10.20*b*, in which the deck spans in the other direction and only two rows of columns are used. This arrangement allows for wider column spacing. Although this arrangement increases the beam spans, a major cost savings is represented by the elimination of 60% of the interior columns and their footings.

Beams in continuous rows can sometimes be made to simulate a continuous beam action without the need for moment-resistive connections. Use of beam splice joints off the columns, as shown in Figure 10.20*c*, allows for relatively simple connections and some advantages of the continuous beam. A principal gain thus achieved is a reduction of deflections.

For the beam in Figure 10.20*b*, assuming a slightly heavier deck, an approximate dead load of 20 psf will result in a beam load of

$$w = 16.67[1.2(20 + 1.6(16)]$$
$$= 827\text{ plf} + \text{the beam}, \quad \text{say } 900\text{ plf}$$

Note that the beam support periphery of 555 ft^2 qualifies the beam for a live-load reduction, from 20 to 16 psf.

For a simple beam, the maximum factored bending moment on the 33.3-ft span is

$$M = \frac{wL^2}{8} = \frac{(0.900)(33.3)^2}{8} = 125\text{ kip-ft}$$

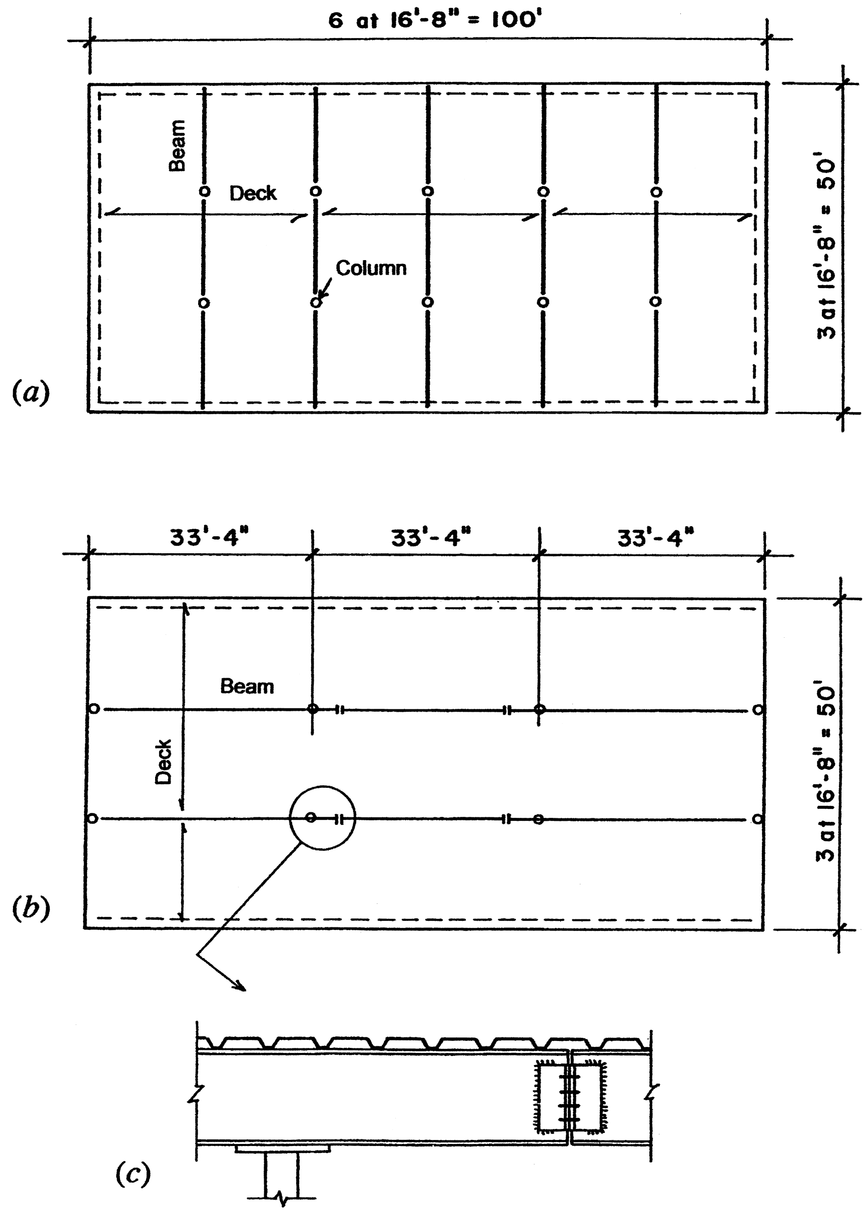


Figure 10.20 Building Two: options for roof framing with interior columns.

The required moment resistance of the beam is therefore

$$M_n = \frac{125}{0.9} = 139\,\text{kip-ft}$$

From Table 5.1, the lightest W-shape beam permitted is a W 14 × 34. From Figure 5.5, we observe that the total load deflection will be approximately L/240, which is usually not critical for roof structures. The live-load deflection will be less than half this, which is quite a modest value. It is assumed that the continuous connection of the deck to the beam top flange is adequate to consider the beam to have continuous lateral support, permitting the use of a resisting moment of M_p.

If the three-span beam is constructed with three simple-spanning beam segments, the detail at the top of the column will be as shown in Figure 10.21. Although this may be

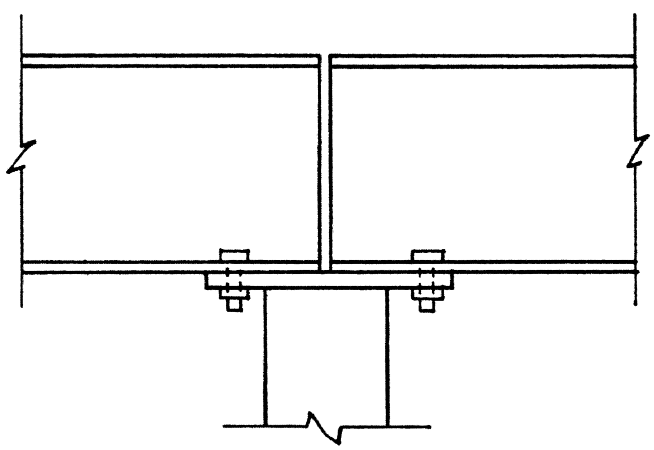


Figure 10.21 Framing at top of column with simple beam action.

possible, the detail shown in Figure 10.20*c*, with the beam-to-beam connection made off the column, is a better framing detail. With this construction, the beam will simulate the behavior of a continuous beam, with the maximum bending moment reduced by approximately 20%. The beam size may thus be slightly smaller, and deflection will also be reduced.

The total factored load on one simple-span beam is approximately the load on the column; thus

$$P = 0.900(33.3) = 30\,\text{kips}$$

Assuming a column height of 10 ft, the following choices may be found for the column:

From Table 5.9, a 3-in. pipe (nominal size, standard weight)

From Table 5.11, an HSS 3-in. square tube, with $\frac{3}{16}$-in.-thick wall

Alternative Truss Roof

If a gabled (double-sloped) roof form is desirable for Building Two, a possible roof structure is shown in Figure 10.22. The building profile shown in Figure 10.22*a* is developed with a series of trusses spaced at plan intervals, as shown for the beam and column rows in Figure 10.20*a*. The truss form is shown in Figure 10.22*b*. The complete results of an algebraic

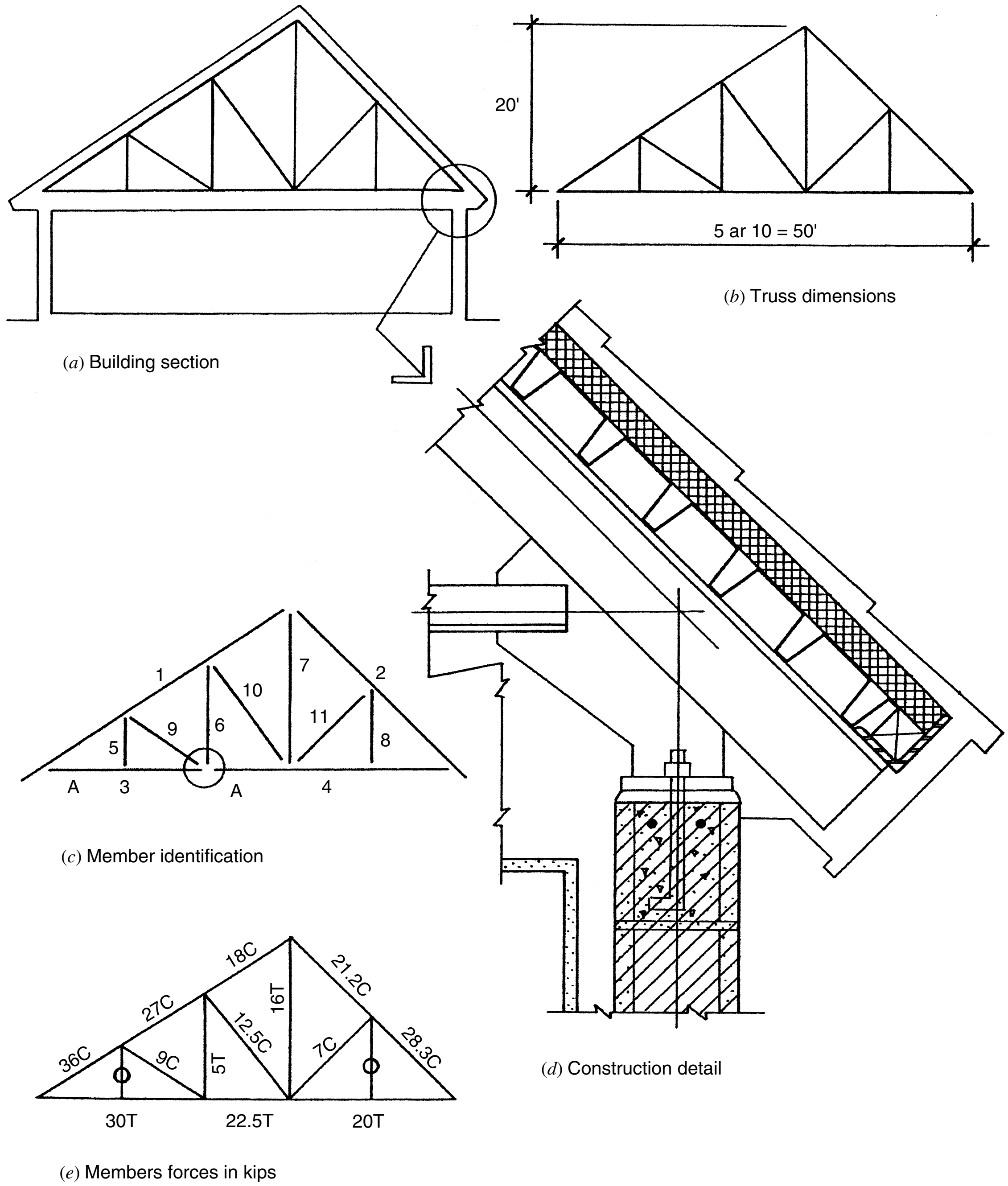


Figure 10.22 Building Two: alternative truss roof structure.

analysis for a unit loading on this truss are displayed in Figure 2.10. The true unit load for this case is approximately 10 times the unit load used in the analysis, so the true internal forces are as shown in Figure 10.22*e*.

The detail in Figure 10.22*d* shows the use of double-angle members and gusset plates. The truss top chord is extended to form the cantilevered edge of the roof. As with the open-web joists, support is provided by direct bearing on the masonry wall.

In trusses of this size, it is quite common to extend the chords without joints for as long as possible. Available lengths of angles depend on the size and the usual lengths in stock by local fabricators. Figure 10.22*c* shows a possible layout of the truss members that creates a two-piece top chord and a two-piece bottom chord. If any of these member lengths are difficult to obtain, more chord joints will be necessary.

The roof construction illustrated in Figure 10.22*d* shows the use of a long-span steel deck that bears directly on top of the top chord of the trusses. This option simplifies the framing by eliminating the need for intermediate framing between the trusses. For the truss spacing of 16 ft, 8 in. as shown in Figure 10.20*a*, the deck will be quite light, and this is a feasible system. However, the direct bearing of the deck adds a spanning function to the top chord, and the chords must be considerably heavier to work for this added task.

Various forms may be used for the members and joints of this truss. The loading and span are both quite modest here, so the truss members will be small and joints will have small axial forces from the members. Truss members will be small and joints will have small loads. The smaller joints will make it less likely that bolted jointing will be used. Members are likely to be welded in the fabricating shop to gusset plates. An alternative form also used is one with structural tees for the chords, which eliminates the gussets as the interior members will be directly welded to the tee stems (see Figure 5.30).

It is unlikely that the roof decks can be used for horizontal diaphragms, due to the slope of the roof surfaces. Horizontal bracing will most likely be achieved with a horizontal plane of bracing at the level of the truss bottom chords. This bracing will also be used as part of the lateral bracing for the trusses.

We will not carry the design of this truss any farther; the process can be seen in the design of the trussed roof structure for Building Six.

Other Alternatives for Building Two

Various other structures may be developed for this simple building. Possibilities include a rigid frame bent, a lamella frame, an arch rib system, and precast concrete spanning units. For any of these, the form and exposed structure may be a major part of the architectural design as well as relating to some particular usage of the space inside the building.

The gabled roof form created with the trusses can also be achieved with a rigid-frame bent. This structure could be developed with welded steel, glued-laminated wood, or precast concrete. An advantage with the rigid frame would be the elimination of the interior truss members if the use of that space is significant for the building use.

Foundations for Building Two

Foundations for Building Two would be quite minimal. For the exterior bearing walls, the construction provided will depend on concerns for frost and the location below the ground surface of suitable bearing material. Options for the wood structure are shown in Figure 10.23.

Where frost is not a problem and suitable bearing can be achieved at a short distance below ground, a common solution is to use the construction shown in Figure 10.23*a*, called a *grade beam.* This element is essentially a combined footing and short foundation wall in one. It is typically reinforced with steel bars in the top and bottom to give it some capacity as a continuous beam, capable of spanning over isolated weak spots in the supporting soil.

Where frost *is* a problem, local codes will specify some minimum distance from finished grade to the bottom of the

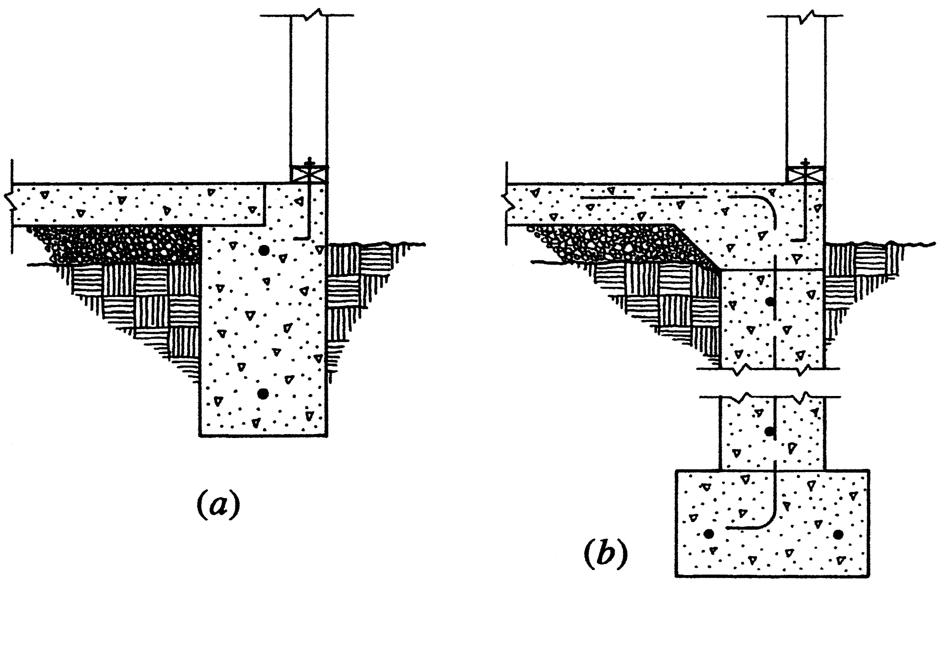
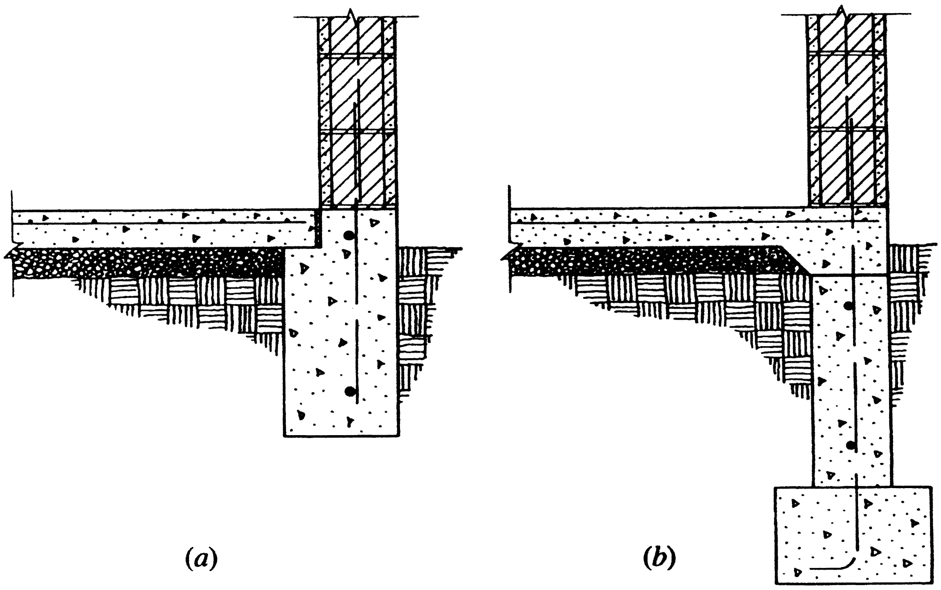


Figure 10.23 Building Two: options for the exterior wall foundation for the wood structure.



(*a*) (*b*)

Figure 10.24 Building Two: options for the exterior wall foundation for the masonry wall structure.

foundation. To reach this distance, it may be more practical to build a separate footing and foundation wall, as shown in Figure 10.23*b*. This short, continuous wall may also be designed for some minimal beamlike action, similar to that for the grade beam.

For either type of foundation, the light loading of the roof and a wood stud wall will require a very minimum width of the footing or grade beam—a measurement usually specified by building codes. If the bearing-type footing is not an option, this simple footing must be replaced by some form of deep foundation (driven piles or dug caissons), with a major increase in cost of the foundation system.

Figure 10.24 shows foundation details for the masonry wall structure similar to those for the wood structure. Extra weight of the masonry wall may require some more width for the footing or grade beam, but the general form of the construction will be quite similar. An alternative for the foundation wall for either structure is to use grout-filled concrete block walls instead of the cast concrete wall.

Footings for any interior columns for Building Two would also be minimal because of the relatively light loads from the roof structure and the low value for the roof live load.

10.4 BUILDING THREE

Figure 10.25 shows a building that consists essentially of stacking the plan for Building Two to produce a two-story building. The profile section of the building shows that the structure for the second floor is developed the same as the roof structure and walls for Building Two. Here, for both the roof and the second floor, the framing option chosen is that shown in Figure 10.11*b*.

Even though the framing layout is similar, the principal difference between the roof and floor structures has to do with

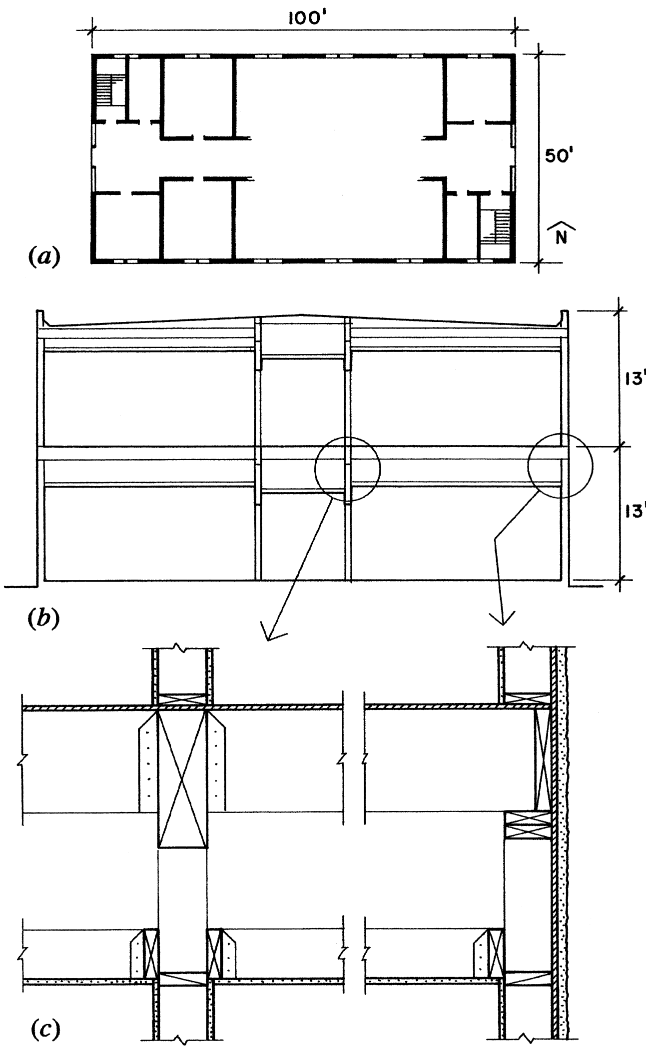


Figure 10.25 Building Three: general form and construction details.

the loadings. Both the dead load and live load are greater for the floor. In addition, the deflection of long-spanning floor members is a concern for both the depth dimension and the bounciness of the structure.

The two-story building sustains a greater total wind load, although the shear walls for the second story will be basically the same as for Building Two. The major effect in this building is the force generated in the first-story shear walls. In addition, there is a second horizontal diaphragm consisting of the second-floor deck.

Some details for the second-floor framing are shown in Figure 10.25*c*. Roof framing details are similar to those shown for Building Two in Figure 10.12*b*. As with Building Two, an option here is to use a clear-spanning roof structure, which would eliminate the need for the corridor columns in the second story.

Design for Gravity Loads

For design of the second-floor structure, the following construction is assumed. The weight of the ceiling is omitted, assuming it to be supported by the first-story walls:

Carpet and pad	3.0 psf
Fiberboard underlay	3.0 psf
Concrete fill, 1.5 in	12.0 psf
Plywood deck, 3/4 in.	2.5 psf
Ducts, lights, wiring, etc.	3.5 psf
Total, without joists and beams	24.0 psf

The minimum live load for office areas is 50 psf. However, the code requires the inclusion of an extra load to account for possible additional partitioning, usually 25 psf. Thus, the full design live load is 75 psf. At the corridor, the live load is 100 psf. Many designers would prefer to design the whole floor for a live load of 100 psf, thereby allowing for other arrangements or occupancies in the future. Because the added partition load is not required for this live load, it is only an increase of about 20% in the total load. With this consideration, the total design load for the joists is thus 124 psf. Of course, live-load reductions may be used for design of beams and columns.

With joists at 16-in. centers, the design superimposed uniformly distributed loads for a single joist are as follows:

$$\text{DL} = \frac{16}{12}(24) = 32\ \text{lb/ft} + \text{the joist}, \quad \text{say } 40\ \text{lb/ft}$$

$$\text{LL} = \frac{16}{12}(100) = 133\ \text{lb/ft}$$

and the total load is thus 173 lb/ft. For the 21-ft-span joists the maximum bending moment is

$$M = \frac{wL^2}{8} = \frac{173(21)^2}{8} = 9537\ \text{ft-lb}$$

For Douglas fir-larch joists of select structural grade and 2 in. nominal width, allowable bending stress from Table 4.1 is 1725 psi for repetitive member use. Thus the required section modulus is

$$S = \frac{M}{F_b} = \frac{9537(12)}{1725} = 66.3\ \text{in.}^3$$

Table A.8 shows no 2 by with this value; a possible choice is a 3 by 14 with $S = 73.151$ in.^{3} This a really expensive joist, and furthermore its deflection will be considerable, indicating a likely problem with bounciness. A better choice for this span and load is probably for one of the proprietary fabricated joists, deeper than 14 in.

The beams support both the 21-ft-span joists and the short 8-ft corridor joists. The total load periphery carried by one beam is thus 240 ft^2, for which a reduction of 7% is allowed for the live load (see discussion in Section 10.1). Using the same loading for both the corridor and offices, the beam load is determined as

DL = (30)(14.5)	= 435 lb/ft
+ beam weight (estimate)	= 50
+ corridor wall	= 150
Total DL	= 635 lb/ft
LL = (0.93)(100)(14.5)	= 1349 lb/ft
Total load on beam	= 1984, say 2000 lb/ft

For the uniformly loaded simple beam with a span of 16.67 ft,

Total load $= W = (2)(16.67) = 33.4$ kips
End reaction = maximum beam shear $= W/2 = 16.7$ kips
Maximum bending moment is

$$M = \frac{WL}{8} = \frac{33.4(16.67)}{8} = 69.6\ \text{kip-ft}$$

For a Douglas fir-larch, dense No. 1 grade beam, Table 4.1 yields values of $F_b = 1550$ psi, $F_v = 170$ psi, and $E = 1{,}700{,}000$ psi. To satisfy the flexural requirement, the required section modulus is

$$S = \frac{M}{F_b} = \frac{69.6(12)}{1.550} = 539\ \text{in.}^3$$

From Table A.8, choices are a 10 by 20 or a 12 by 18.

If the 20-in.-deep section is used, its effective bending resistance must be reduced (see discussion in Section 5.2). Thus the actual moment capacity of the 10 by 20 is reduced by the size factor from Table 4.3 and is determined as

$$\begin{aligned} M &= C_F \times F_b \times S \\ &= (0.947)(1.550)(602.1)(1/12) \\ &= 73.6\ \text{kip-ft} \end{aligned}$$

Because this still exceeds the requirement, the selection is adequate. A similar investigation will show the other option to also be adequate.

If the actual beam depth is 19.5 in., the critical shear force may be reduced to that at a distance of the beam depth from the support. Thus, an amount of load equal to the beam depth times the unit load ca be subtracted from the maximum shear. The critical shear force is thus

$$\begin{aligned} V &= \text{(actual end shear force)} \\ &\quad - \text{(beam depth in feet times unit load)} \\ &= 16.67 - 2.0(19.5/12) \\ &= 16.67 - 3.25 = 13.42\,\text{kips} \end{aligned}$$

For the 10 by 20 the maximum shear stress is thus

$$f_v = 1.5\frac{V}{A} = 1.5\frac{13{,}420}{185.25} = 108.7\,\text{psi}$$

This is less than the limiting stress of 170 psi as given in Table 4.1, so the beam is acceptable for shear resistance. However, this is a really big piece of lumber, and questionably feasible, unless this building is in the heart of a major timber region. It is probably logical to modify the structure to reduce the beam span or to choose a steel beam or a glued-laminated section in place of the solid-sawn timber.

Although deflection is often critical for long spans with light loads, it is seldom critical for the short-span, heavily loaded beam. The reader may verify this by computing the deflection of this beam, but the work is not shown here.

For the interior column at the first story, the design load is equal to the load on the second-floor beam plus the load from the second-story column. Because the roof load is approximately one-third of that for the floor, the design load is about 50 kips for the 10-ft-high column. Table 4.10 yields possibilities for an 8-by-10 or 10-by-10 section. For various reasons it may be more practical to use a steel member here—a round pipe or square tube—which may actually be accommodated within the corridor stud wall.

Columns must also be provided at the ends of the beams in the east and west walls. Separate column members may be provided at these locations, but it is common to simply build up a column from a number of bundled studs.

Design for Lateral Loads

Lateral resistance for the second story of Building Three is essentially the same as for Building Two. Design considerations here will be limited to the diaphragm action of the second-floor deck and the two-story end shear walls.

The wind-loading condition for the building is shown in Figure 10.26*a*. For the same design conditions assumed for wind on Building Two, the pressure used for horizontal force on the bracing system is 10 psf for the entire height of the exterior wall. At the second-floor level, the wind load delivered to the edge of the diaphragm is 120 lb/ft, resulting in the spanning action of the diaphragm as shown in Figure 10.26*b*. Referring to the building plan in Figure 10.25, observe that the opening for the stairs creates a void in the floor deck at the ends of the diaphragm.

Assuming the net width of the diaphragm to be reduced to 35 ft at the stairs, the unit shear stress for maximum shear is

$$v = \frac{6000}{35} = 171\,\text{lb/ft}$$

From Table 9.1, this requires only a minimum nailing for a $\frac{19}{32}$-in.-thick plywood deck, which is a minimum thickness used for floor decks. As discussed for the roof diaphragm for Building Two, the chord at the edge of the floor diaphragm must be developed by framing members to sustain a computed tension/compression force of 3 kips. Ordinary framing may be capable of this action if attention is paid to splicing for full-length continuity of the 100-ft-long edge member.

Construction details for the roof, floor, and exterior walls must be carefully studied to ensure that the necessary transfers of force are achieved. These transfers include the following:

The shear force from the roof deck to the second-story shear wall sheathing
The shear force from the second-story shear wall to the first-story shear wall below it
The overturning from the second-floor shear wall to the supporting wall below
The shear force from the second-floor deck to the first-story shear walls
The shear force from the first-story shear walls to the building foundations
The overturning moment from the first-story shear walls to the building foundations

In the first-story end shear walls, the lateral load is 5000 lb, as shown in Figure 10.26*d*. For the 21-ft-wide wall the unit shear is thus

$$v = \frac{5000}{21} = 238\,\text{lb/ft}$$

From Table 9.2, this resistance can be achieved with $\frac{3}{8}$-in.-thick APA rated sheathing, although nail spacing closer than the minimum of 6 in. is required at panel edges.

At the first-floor level, the investigation for overturn of the end shear wall is as follows:

$$\text{Overturning } M = (1.5)\{(2)(24) + (3)(13)\} = 130.5\,\text{kip-ft}$$

$$\text{Restoring } M = (3 + 2 + 11)(21/2) = 168\,\text{kip-ft}$$

There is thus no requirement for tiedown anchorage.

In fact, there are other resisting forces on this wall. At the building corners, the end walls are attached to the north and south walls, which would need to be lifted by the overturn on the end walls. At the sides of the building entrances, there will be a post in the end of the wall to support the end of the second-floor beams. All in all, there is probably no

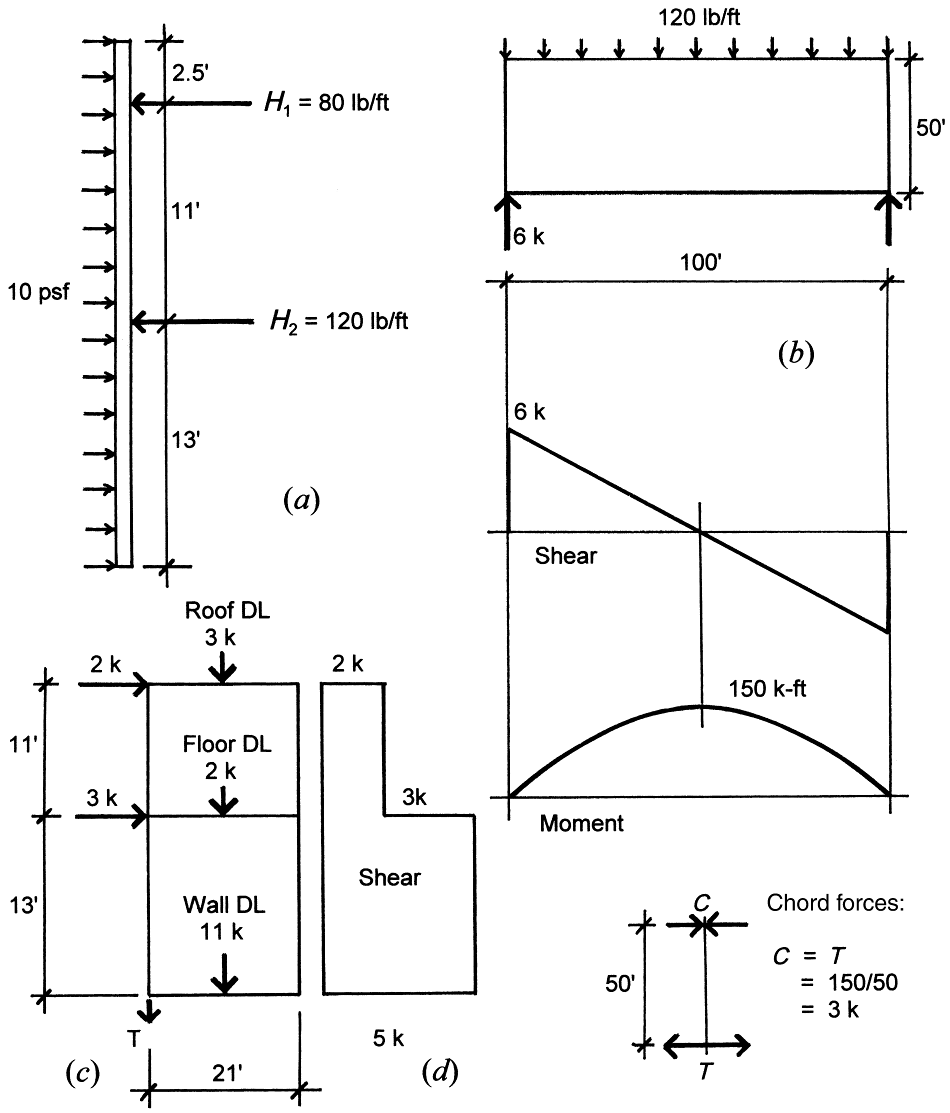


Figure 10.26 Development of lateral force due to wind.

basis for requiring an anchor at the ends of these shear walls. Nevertheless, the basic details for this masonry construction will typically provide for considerable anchorage at the wall ends and corners.

Alternative Steel and Masonry Structure

As for Building Two, an alternative construction for this building is one with masonry walls and interior steel framing, with the roof achieved as shown for Building Two. The second floor may be achieved as shown in Figure 10.27. Because of the heavier loads, the floor structure here consists of a steel framing system with rolled steel beams supported by steel columns on the interior and by masonry pilasters at the exterior walls.

The floor deck consists of formed sheet steel units with a structural-grade concrete fill. This deck spans between steel beams, which are in turn supported by larger beams that are supported by the columns. All of the elements in this system can be designed by procedures described in Chapter 5.

Figure 10.28 shows a framing plan for the second floor consisting of a variation of the roof framing plan for Building Two, as shown in Figure 10.18. Here, instead of the long-span roof deck, a shorter span floor deck is used with a series of beams at 6.25-ft spacing.

Although the taller masonry walls, carrying both roof and floor loads, have greater stress development than in Building Two, it is still possible that minimal code-required construction for the reinforced CMU masonry may be adequate for both gravity and lateral loads.

Another possibility for Building Three, depending on fire resistance requirements, is to use wood construction for the building roof and floor systems together with the masonry walls. Many buildings were built through the nineteenth and early twentieth centuries with masonry walls and interior structures of timber. A similar system in present-day construction uses a combination of wood and steel structural elements with exterior masonry walls, as illustrated for Building Six.

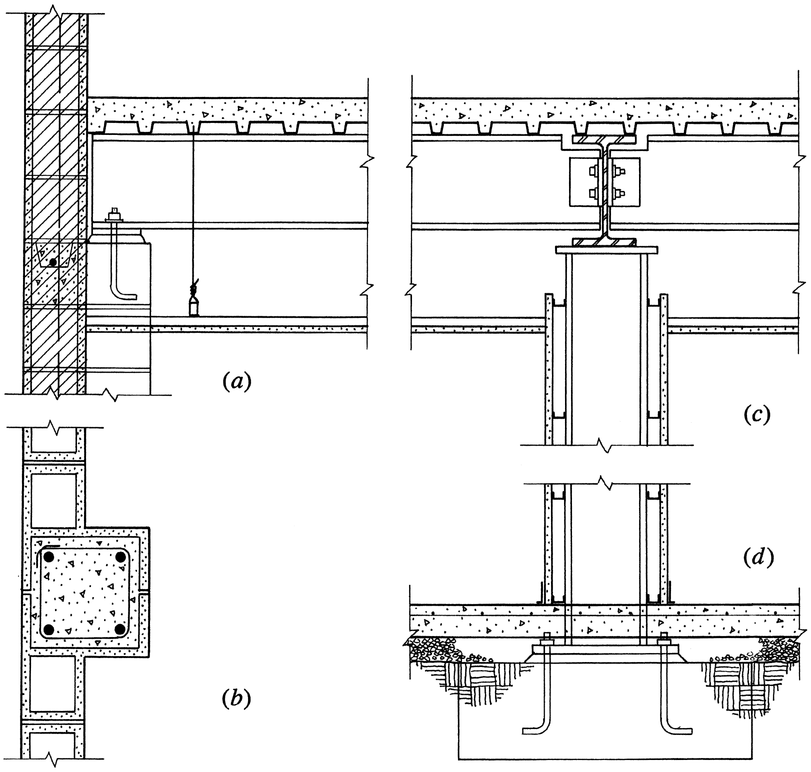


Figure 10.27 Building Three: alternative steel and masonry structure.

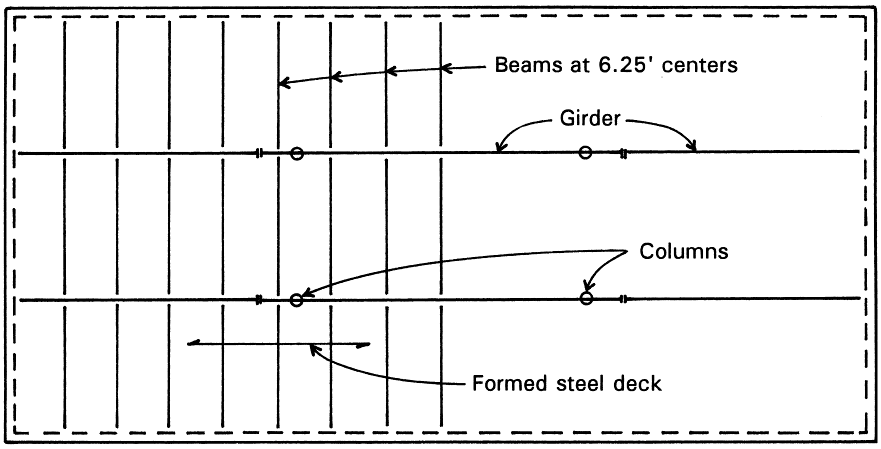


Figure 10.28 Building Three: second-floor steel framing plan.

10.5 BUILDING FOUR

Figure 10.29 shows a partial framing plan for the roof structure of a one-story industrial building. The system as shown, with 48-ft square bays, is repeated a number of times in each direction. The same live load and dead load given for Building Two will be used.

The plan as shown in Figure 10.29 indicates a series of girders supported by columns, a series of joists perpendicular to the girders, and a spanning roof deck supported by the joists. Selection of the joist spacing is affected by the following:

Load on a Single Joist. Based on the span and the load on a single joist, there is some feasible range for the spacing of individual manufactured joists. Upper limits for spacing relate to strength of the joists; lower limits relate to economy in terms of the number of joists used.

Selected Deck. Deck may be a deep-ribbed formed sheet steel product, wood structural panels, or a rafter-and-plywood panel with dimensions relating to practical use of the panels.

Point Loads on Girders. This relates to the lateral bracing for the girders and to logical panel units of the girders if they are trusses.

A very wide range of possible combinations exists for the choice of deck, joists, girders, columns, and exterior walls for this building. Two common solutions are discussed here.

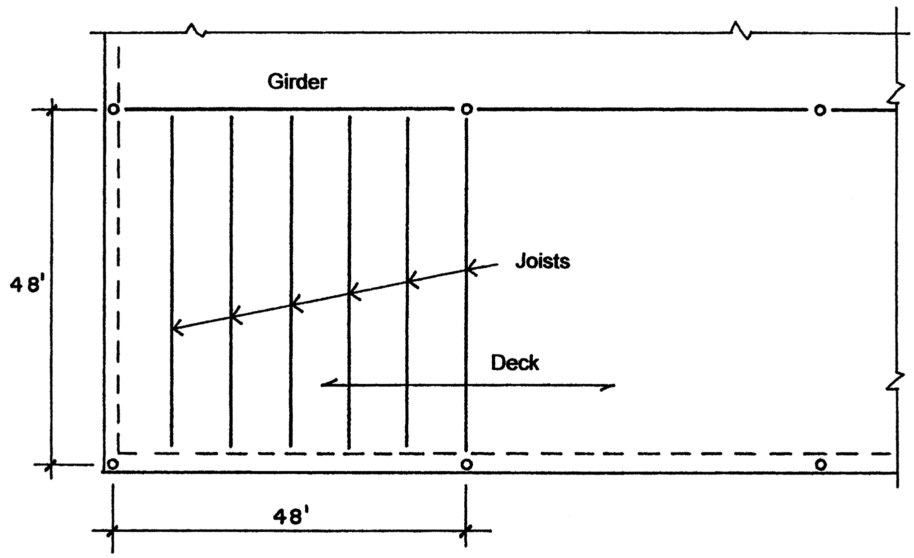


Figure 10.29 Partial roof framing plan for Building Four.

Alternative One: Manufactured Joists and W-Shape Girders
For this system, joists are placed at 4-ft centers and are supported by W-shape girders. For these conditions, choices for the joists are the same as for Building Two. The remaining discussion deals with the girders.

Because of the large area supported by a single girder, the roof live load drops to the minimum of 12 psf. Assuming a dead load with the joists of 20 psf, the average linear load on a girder is

$$w = [1.2(20)(48) + 1.6(12)]$$
$$= 2074\,\text{plf}, \quad \text{or} \quad 2.074\,\text{kips/ft}$$

In fact, the joist loads constitute concentrated loads at 4 ft on the girder, but the difference in the maximum bending moment on the girder is quite small. Adding some assumed loading due to the weight of the girder, a reasonable approximation is 2.2 kips/ft. For simple beam action, the maximum bending moment is

$$M = \frac{wL^2}{8} = \frac{(2.2)(48)^2}{8} = 633.6\,\text{kip-ft}$$

From Table 5.1, the lightest shape is a W 30 × 90.

Using the same scheme for the girder as with Building Two, with splice points off the columns, the girder maximum moment will drop by about 20%. Thus a new design moment is 0.80(633.6) = 507 kip-ft, and a possible selection is for a W 27 × 76.

For the column, the factored load is thus (2.2)(48) = 105.6 kips. Given a clear height of 20 ft and $K = 1$, the following are possible:

W 8 × 31 (see Table 5.10)
8-in. standard pipe (see Table 5.11)
6 × 6 × $\frac{5}{16}$-in. HSS tube (see Table 5.12)

Alternative Two: Manufactured Joists and Joist Girders
This system uses trusses instead of the W-shape girders. When used with manufactured steel joists, these are called *joist girders* and their use is discussed in Section 5.2. These trusses are ordinarily supplied by the same source from which the joists are obtained. With these predesigned elements, design is limited to determining layout, loading, and girder depth.

The spaced point loads of the joists determine a horizontal module for the joist girder. This dimension should approximately match the depth of the girder so that the truss layout does not approach the extremes shown in Figure 10.30. Also, for efficiency of the truss the depth in inches should approximate the span in feet; thus a depth of 4 ft seems a good choice here.

With a 4-ft joist spacing, the truss has 12 panels, and the unit load at the truss joints is equal to the total load on one joist. Using the load determined previously, the panel point load is 2.074(48) = 8.296, say 8.3 kips. As described in Section 5.2, the joist girder designation is thus 48G12N8.3K. This indicates a 48-in.-deep girder with 12 spaces for supported joists and a joist load of 8.3 kips.

Column options are the same as those for Alternative One, although the connection details between the girder and column are different; thus column shape may present a different favored choice here.

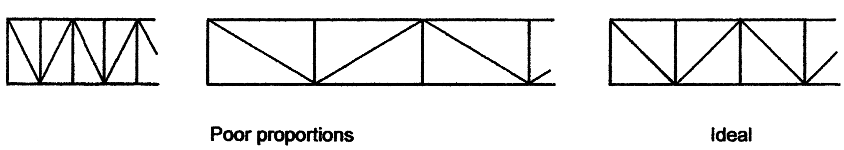


Figure 10.30 Layout of truss form for the truss girder.

10.6 BUILDING FIVE

The general use of trussing for horizontal bracing is discussed in Section 9.4. A common reason for using horizontal trussing is the absence of an adequate horizontal decking that can be used for diaphragm action. Building Five, as described here, has a roof of a form that precludes diaphragm development, requiring some other form of horizontal bracing structure at the roof level.

Figure 10.31 shows a large one-story building with a complex roof consisting of a series of sky windows. The roof structure consists of a series of clear-spanning trusses that define the planes for the windows. The remainder of the roof consists of sloped portions of decking that are supported by the top chord of one truss and the bottom chord of the next truss. This system is commonly called a *sawtooth roof*, and it has been widely used for large industrial facilities that are able to use the advantages of the potential daylighting and the natural ventilation provided by the expanse of windows.

Due both to its steep slope and its lack of continuity, the roof decking cannot perform diaphragm actions for the building as a whole. It can, however, contribute to the lateral bracing of the spanning trusses. The usual diaphragm action in the general plane of the roof is provided by the horizontal X-bracing in the structural bays adjacent to the outside walls. This diagonal framing combines with the bottom chords of the spanning trusses and the horizontal framing in the top of the walls to constitute a series of trusses at the building edges.

The bottom chords of the spanning trusses serve as distributing struts to carry the wind across the building, so that the horizontal trusses on both sides of the building work together for the wind from a single direction.

Because of the length of the building, the plan indicates the use of a center row of columns that are used to develop a three-span trussed bent, which serves to break the 240-ft-long roof into two units for the span of the horizontal trusses. Similar trussed bents are used at the building ends. Trussed bents could also be used along the long exterior walls, and the entire building lateral bracing system would be developed by trussing. However, the exterior walls might also be developed with wall construction capable of the required shear wall actions.

The structure illustrated would probably function for both wind and earthquake forces. A major concern for earthquakes

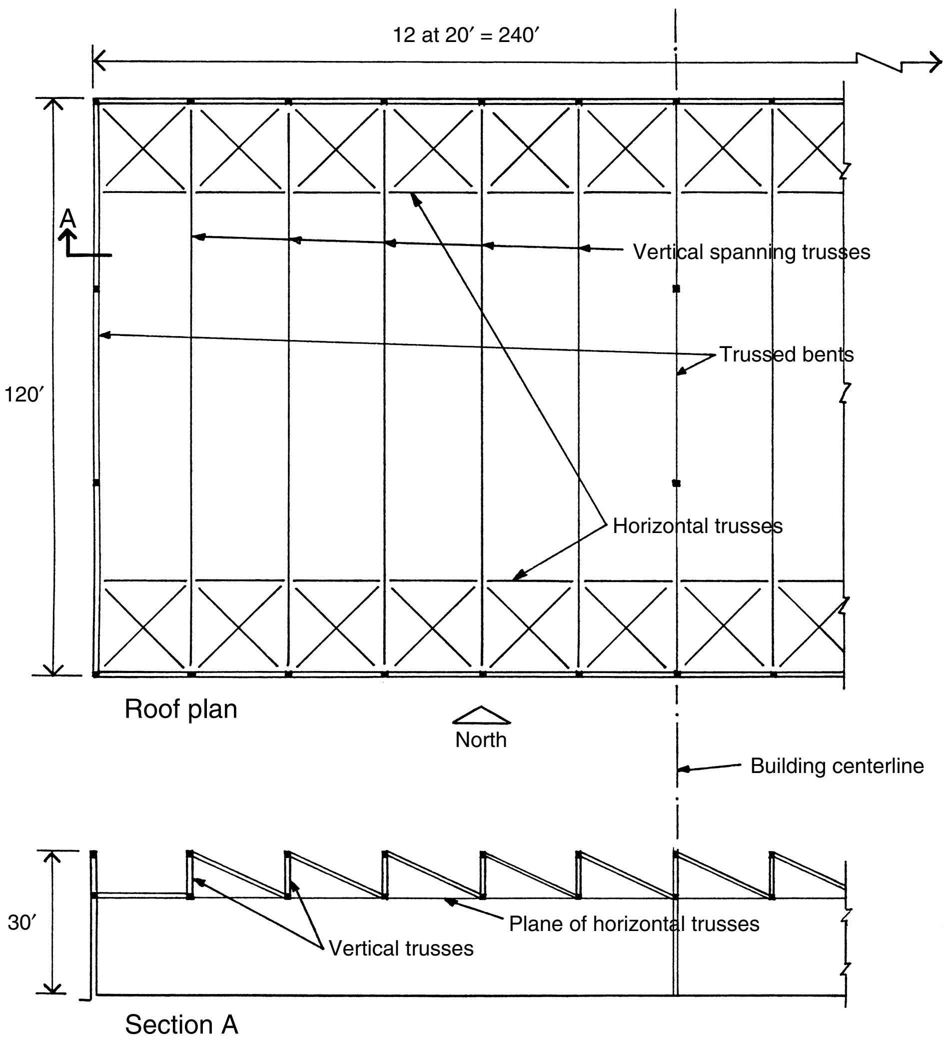


Figure 10.31 Building Five: form of the building structure.

is the shaking effect of the rapidly reversing earthquake vibrations, requiring very secure connections.

Design of the Horizontal Truss

We will illustrate here the development of the horizontal X-bracing along the long walls for resistance to wind load in the north–south direction. Figure 10.32 illustrates a unit edge loading developed by a wind pressure of 20 psf on the exterior wall. As shown in Figure 10.32*b*, the force on the 30-ft-high wall is resisted at the ground level and at 18 ft above ground by the trusses, which involves the beam functions shown in Figure 10.32*c*. The investigation of one of the trusses for this loading is shown in Figure 10.33.

The member forces shown in Figure 10.33*b* are relatively small, and the X members could be quite small with regard to the truss forces. However, several considerations must be made in selecting the X members:

The general form of the construction of the spanning trusses, especially the member shapes and joint details for the bottom chords.

The need to reduce lateral movements by lowering the tension stress in the X members. Slightly larger members might be used to reduce strain and the elongation of the long diagonals.

The problem of sag, since the X members are quite long (approximately 28 ft), and, if essentially designed for tension, typically they will have low bending resistance caused by their own weight on the horizontal span. This might be alleviated by providing some form of vertical support at the location of the crossing point for the X-bracing members. However, the sag may not actually be a problem for this building.

Although we have investigated the horizontal trusses for their basic task of bracing, they are also parts of the general structural system and their participation for other purposes—such as the lateral bracing of the spanning trusses or support of building service elements—may affect selection of members and development of construction details.

As shown in the building roof plan in Figure 10.31, the X-bracing is not used along the ends of the building. This leaves in question the situation for the lateral bracing of the trussed bent in this wall and the resolution of wind on the building end. It may thus be advisable to use the same X-bracing in this wall and thus to develop wind in

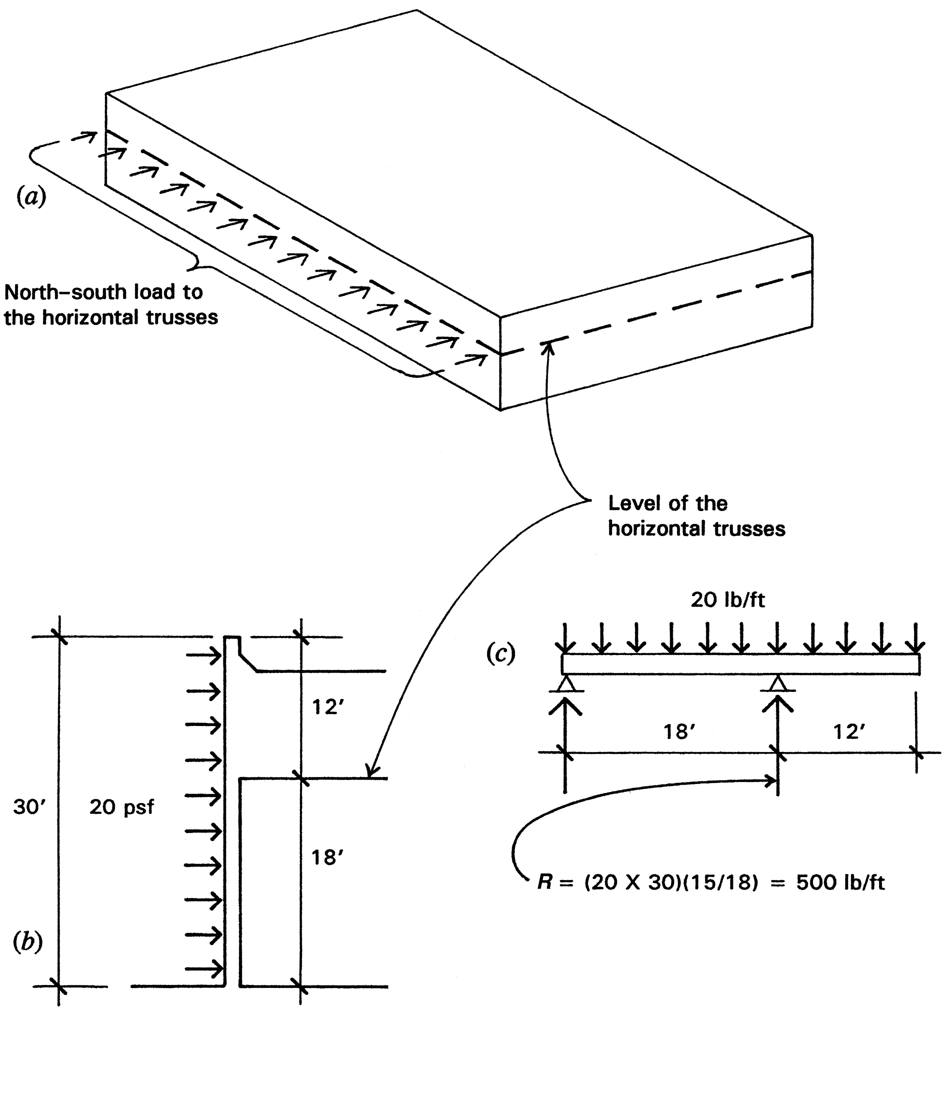


Figure 10.32 Wind load on the horizontal truss.

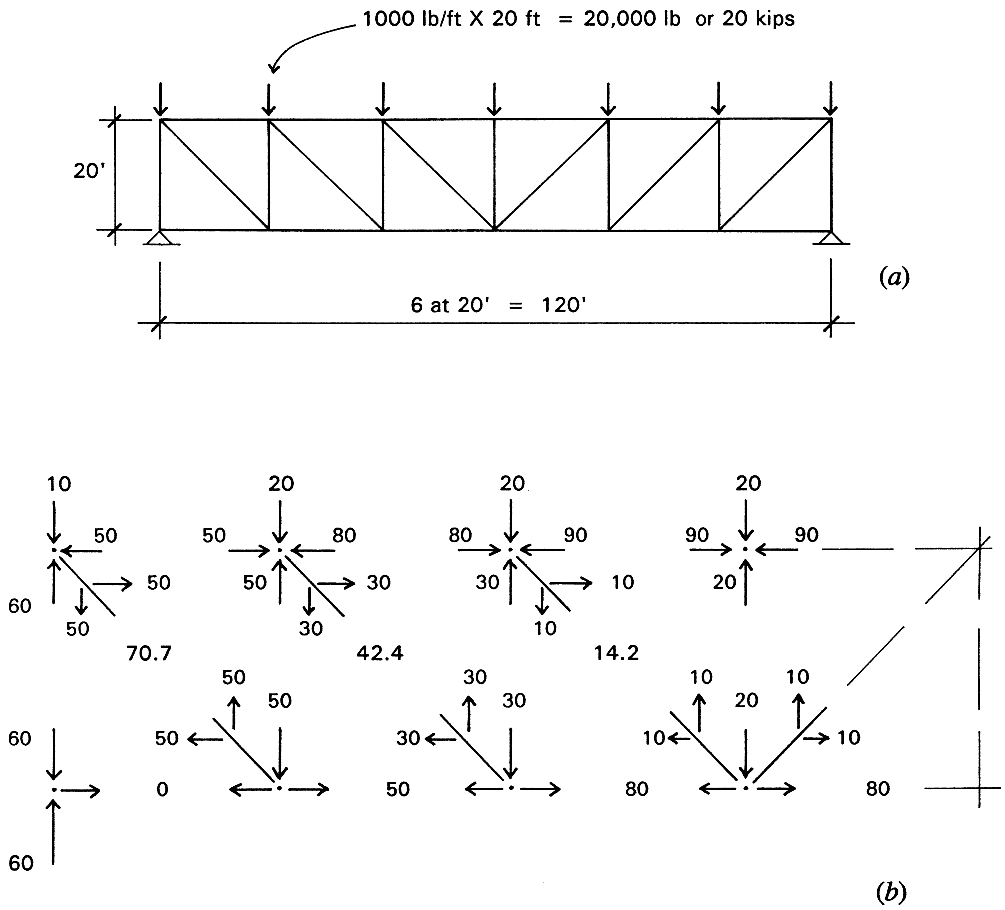


Figure 10.33 Investigation of the horizontal truss.

both directions with the X-bracing, rather than to involve the sawtooth roof construction in wind load resistance.

As the size of trusses increases, considerable deflection might be experienced under live loading. The two primary components of this deformation are the elongation of long tension members and the multiple deformations that can occur in truss joints. Members sustaining pure tension can be quite slender (with no buckling problems) and so may be highly stressed. Reduction of tension elongation may be achieved by simply using larger members.

Joints should be studied for reduction of deformation within the joints themselves. Welded joints with direct member attachment or with heavy gusset plates will generally produce the stiffest joints. For earthquakes, the stiffer structure will also reduce back-and-forth movement.

10.7 BUILDING SIX

This section presents solutions for a building that utilizes a form of construction known as *mill construction*, which evolved during the industrial revolution and was used extensively in the United States up to the early twentieth century. Its use extended from factory buildings to various commercial and public buildings. It is, in fact, a classic form of construction with many continuing applications.

Alternative One: Masonry and Timber

A general form for Building Six is shown in Figure 10.34, indicating the use of exterior masonry walls, a heavy timber roof structure, and interior timber framing for upper floors. This is a very historic form of mill construction, although still achievable with present technologies. It is not in general an economical form of construction and can probably only be justified for its appealing appearance.

A framing plan for the second and third floors is shown in Figure 10.35. The clear span for the office spaces is provided by girders at the location of the interior columns and the masonry wall piers. These girders cantilever to provide support for the balcony corridor. The structural floor deck consists of timber units that are exposed to view on the underside. Beams at 5-ft centers span between the girders and support the deck. Girders are supported by timber columns in the first and second stories. The following discussion deals with the basic components of this system.

Floor Deck

We assume the floors to be achieved with a standard unit appropriate to the span and the need for fire resistance. For finish, the minimum construction would include a fiber board underlay with finish materials applied to the top. The thick commercial deck units may be questionable for floor diaphragm action, so the attached fiberboard or a plywood

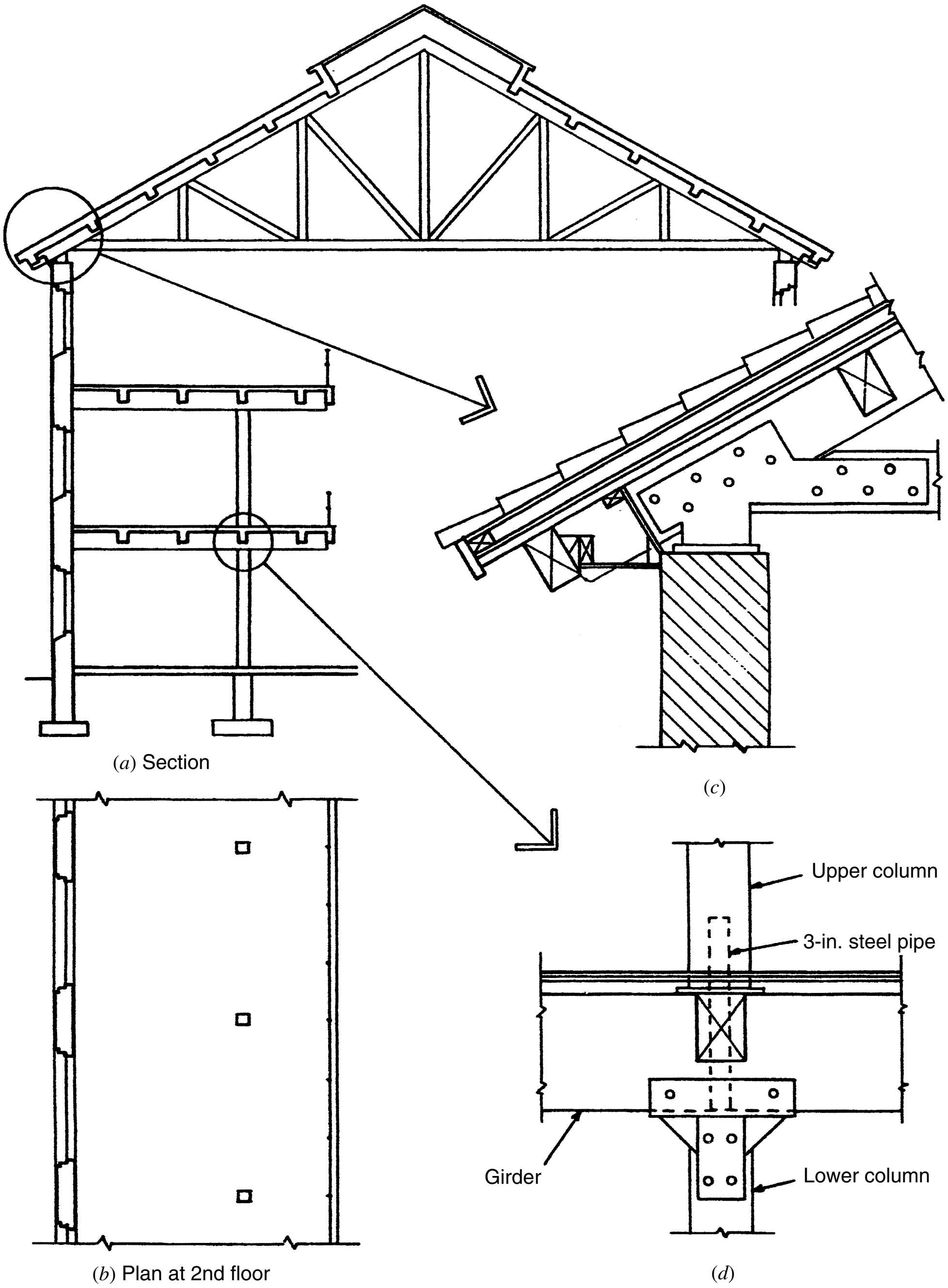


Figure 10.34 Building Six: general form of the wood and masonry alternative.

layer may be used for this purpose. We will assume a total dead load of 15 psf for this construction.

Girders and Beams

For the solid-sawn girders and beams we use Douglas fir-larch, No. 1 grade. From Table 4.1, $F_b = 1350$ psi, $F_v = 170$ psi. With a live load of 100 psf, the total load on one beam is (5)(15)(115) = 8625 lb. Adding about 300 lb for the total beam weight produces an approximate load of 9000 lb. For the maximum bending moment on this simple beam,

$$M = \frac{WL}{8} = \frac{(9000)(15)}{8} = 16{,}875 \text{ lb-ft}$$

and the required section modulus is

$$S = \frac{M}{F_b} = \frac{16{,}875(12)}{1350} = 150 \text{ in.}^3$$

From Table A.8, use an 8 by 12 with $S = 165$ in.^{3}

Maximum shear = 9000/2 = 4500 lb. For the maximum shear stress

$$f_v = 1.5\frac{V}{A} = 1.5\frac{4500}{86.25} = 78.3 \text{ psi}$$

Since this is considerably less than the limit of 170 psi, it is not necessary to use the allowable reduction to the shear force at a distance from the end equal to the beam depth.

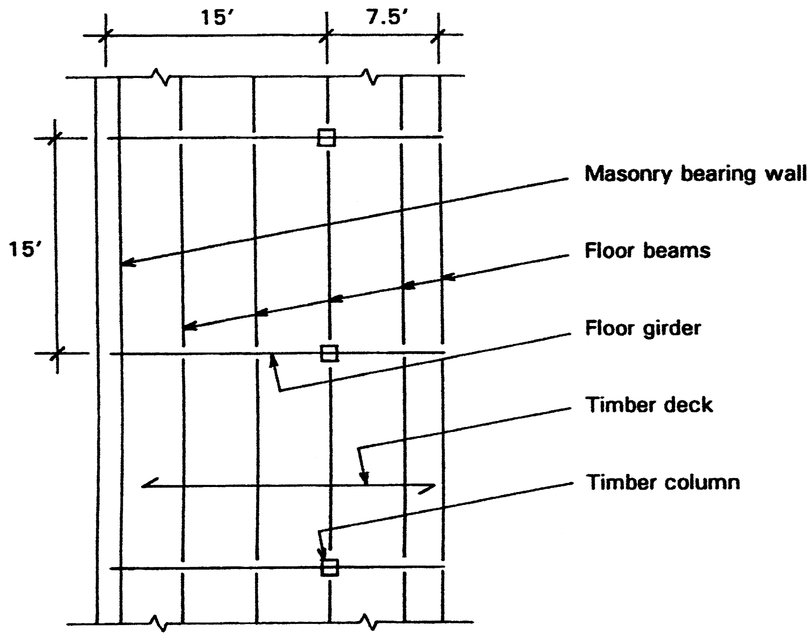


Figure 10.35 Framing for the timber floor structure.

For deflection, use Figure 4.13, which indicates that this is not a problem for the 12-in.-deep beam.

For the girder the supported area is $(15)(22.5) = 337.5\ \text{ft}^2$. The live load can thus be reduced, and the supported load of two beams will thus be used as 8000 lb (or 8 kips). The analysis for the girder is shown in Figure 10.36.

For the maximum girder moment of 71.4 kip-ft, the required section modulus is

$$S = \frac{71{,}400(12)}{1350} = 635\ \text{in.}^3$$

From Table A.8, use a 10 by 22 or a 12 by 20. For these deep sections, a reduction must be made of the section modulus based on the factor defined in Section 4.2 and given in Table 4.5. For the 22-in.-deep member, this factor is 0.937, and the effective section modulus becomes $0.937(732) = 686\ \text{in.}^3$, which is still greater than that required.

For the maximum shear value of 13,135 lb the shear stress for the 10 by 22 is

$$f_v = 1.5\frac{13{,}135}{204} = 97\ \text{psi}, \quad \text{not critical}$$

For the first-story column, assume a height of 10 ft. The load here is approximately twice the girder reaction, although a greater live load reduction is possible. Assume a design load of 60 kips, for which Table 4.18 indicates a 10 by 10 or an 8 by 12.

Connections for this system can be developed with steel elements much in the same manner as those used many years ago. Two special connection problems are those that occur at the splice joint of the multistory column and at the end support for the girders at the masonry wall. For the multistory column there are three possibilities for the splice joint:

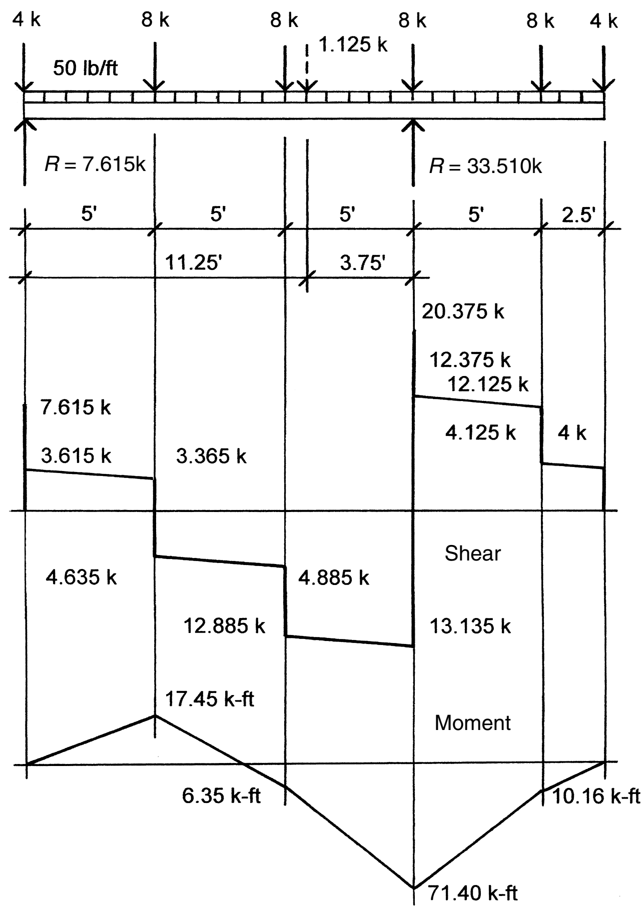


Figure 10.36 Analysis of the girder.

Continuous Columns. This means that the column is not jointed but rather is continuous through the second-floor joint. Girders would need to consist of spaced, double members straddling the columns.

End-Bearing Columns. This consists of having the upper column simply bear directly on top of the lower column using a steel device that is a single-piece combination of base and cap in one.

Pintles. This is a device that permits the single wide girder to pass through the joint while the column load is passed through a device inserted through the girder. See Figure 10.34*d*.

Pintles are not much used any more, but either of the other options is possible.

Timber Roof Truss

As shown in the building section in Figure 10.34, the roof structure utilizes clear-spanning timber trusses. There are several options for the construction of these trusses. The example illustrated here uses solid-sawn truss members with joints developed with steel bolts and steel plate gussets. This form is shown in Figure 10.37. Although the span is modest here, available lengths of timber elements will likely require some splicing of the chords for these trusses. One possible arrangement of the truss members is shown in Figure 10.38*b*, which allows for one splice in each of the two top chords and two splices in the bottom chord. The details for the connections in Figure 10.37 provide for these splices.

Figure 10.38 presents an investigation of the truss with results determined for a unit gravity load. These values for internal forces in the members can be used by simple multiplication for various specific loads, as determined by optional forms for the general roof construction. Based on the construction shown in Figure 10.34 and a minimum roof live load of 20 psf, the total loads on the individual roof purlins that are supported on the truss top chords will be approximately 1800 lb live load and 3600 lb dead load.

Figure 10.39 presents an investigation for wind load based on the criteria from ASCE 2005 (Ref. 1). The form of loading here is based on the roof slope and a minimum horizontally directed wind pressure of 20 psf at the roof level.

The results of the investigations in Figures 10.38 and 10.39 are summarized in Table 10.5. For design there are three combinations to consider:

Dead load plus live load
Dead load plus wind of the same force sense
Dead load plus wind of the opposite sense

Using the roof live load it is possible to use a stress increase. However, the live load in this case is already reduced due to the area supported by the truss. Thus, use is made of the full values of dead- and live-load forces with no consideration for stress modification.

When wind load is included, design values for members can be increased by a factor of 1.6. Therefore the combinations of dead load plus wind load in Table 10.5 have been reduced by a factor of $1/1.6 = 0.625$ for comparison with the gravity-load-only combination. In any event, it may be observed that the wind load for this example is not critical.

A summary of the design of the truss members for the loads in Table 10.5 is given in Table 10.6. Compression members are obtained from Table 4.10. These selections are conservative as a higher grade of wood will be used for the truss construction. A critical decision is for the thickness of the members in the direction perpendicular to the plane of the truss, as this must remain constant for all members. Member sizes are indicated for thicknesses of both 6 and 8 in. in Table 10.6.

A comparison of the trusses in Figures 4.12 and 10.37 will show some similarities and some differences. The basic truss

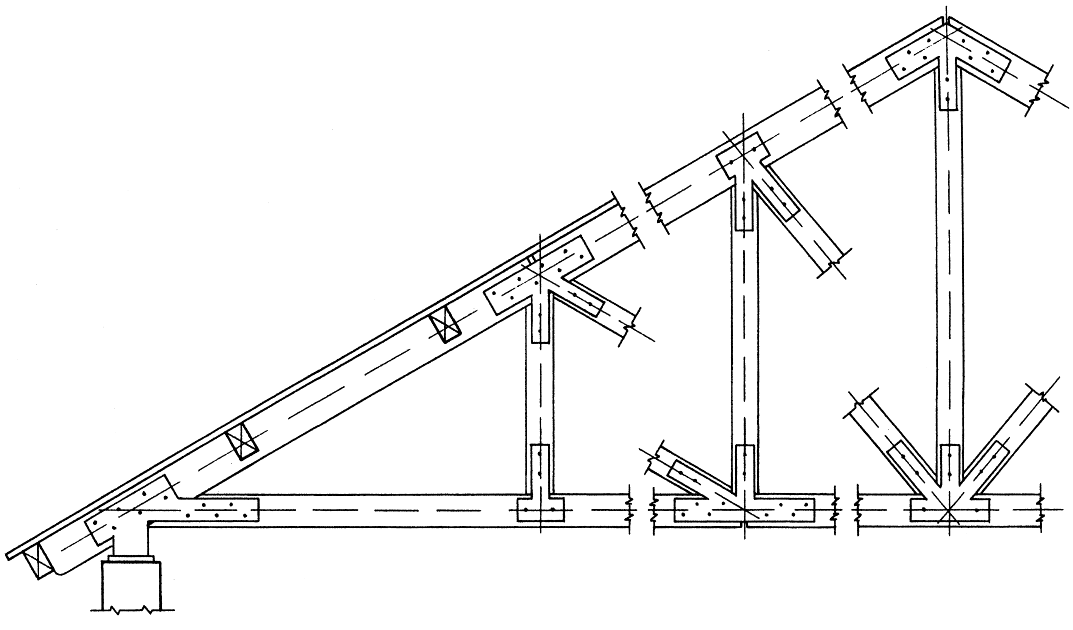


Figure 10.37 Form of the timber truss.

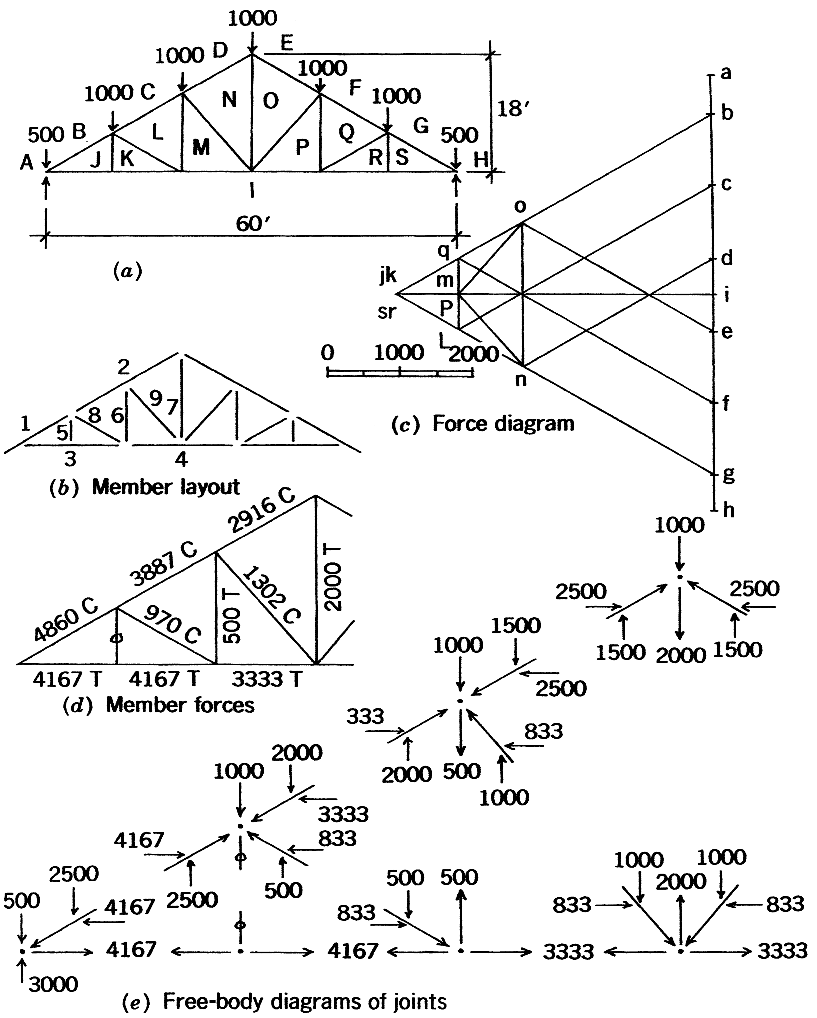


Figure 10.38 Analysis of the truss for gravity loads.

Table 10.5 Design Forces for the Truss (lb)

Member (See Fig. 10.38*b*)	Unit Gravity Load	Dead Load (3.6 × Unit)	Live Load (1.8 × Unit)	Wind Load	DL + LL
1	4860 C	17496 C	8748 C	3790 T	26244 C
2	3887 C	13994 C	6997 C	2450 T	20990 C
3	4167 T	15000 T	7500 T	1960 T/ 5600 C	22500 T
4	3333 T	12000 T	6000 T	3170 C	18000 T
5	0	0	0	0	0
6	500 T	1800 T	900 T	820 T/ 1460 C	2700 T
7	2000 T	7200 T	3600 T	1310 C	10800 T
8	970 C	3492 C	1746 C	1590 C/ 2830 T	5238 C
9	1302 C	4688 C	2344 C	2130 C/ 3800 T	7031 C

Note: C indicates compression, T indicates tension, 0 indicates that the member is not stressed for these loading conditions.

Table 10.6 Selection of the Truss Members

Member (See Fig. 10.38*b*)	Member Length (ft)	Design Force (kips)	Member Selection[a]: All with 6 in. Nominal Thickness	Member Selection[a]: All with 8 in. Nominal Thickness
1	11.7	26.3 C	6 × 10	8 × 8
2	11.7	21 C	6 × 8	8 × 8
3	10	22.5 T	6 × 6	8 × 8
4	10	18 T	6 × 6	6 × 8
5	6	0	6 × 6	6 × 8
6	12	2.7 T	6 × 6	6 × 8
7	18	10.8 T	6 × 6	6 × 8
8	11.7	5.24 C	6 × 6	6 × 8
9	15.6	7.03 C	6 × 6	8 × 8

[a]Minimum thickness for qualification as heavy timber is 6 in. Selections for top chords are without consideration for bending due to purlin loads not at joints.

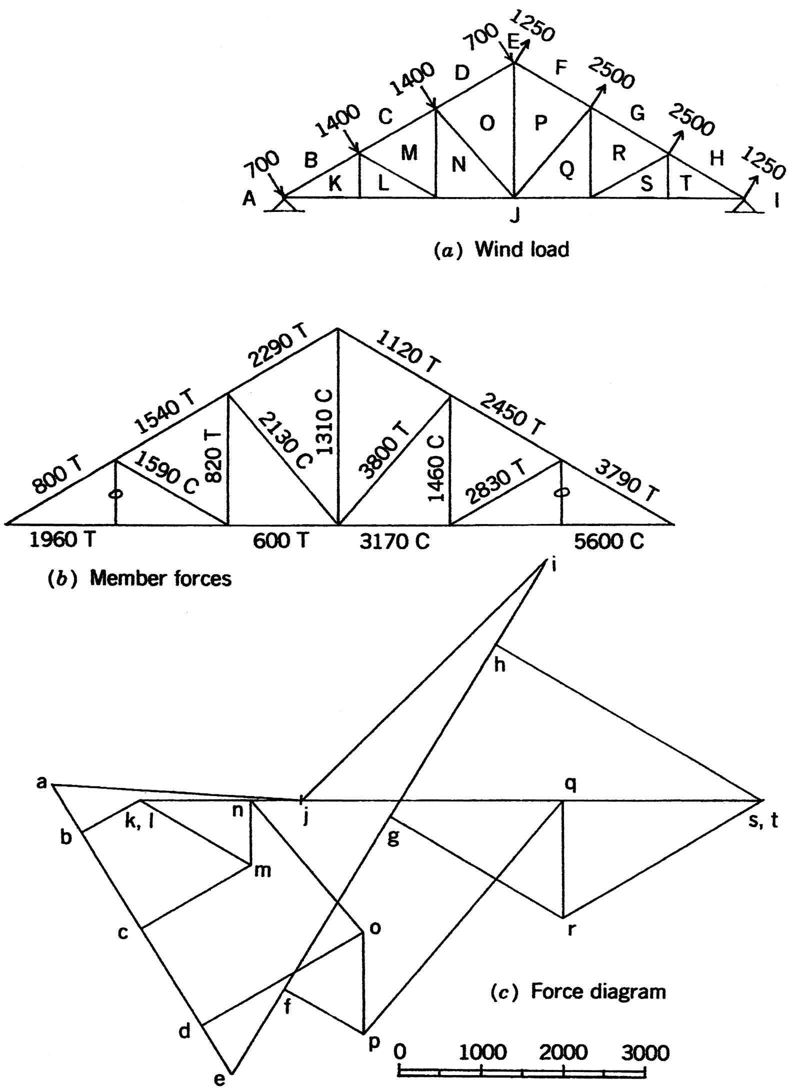


Figure 10.39 Analysis of the truss for wind load.

layout pattern is the same, but there are differences in the development of the roof overhang. There are also differences in the development of the truss joints; the joints in Figure 4.12 develop compression by bearing and tension members as steel rods.

Alternative Truss Construction

The truss in the preceding design is not likely to be used unless relationships to historical forms of construction are important to the general building design. A slightly more contemporary system for the truss is shown in Figure 10.40. This truss uses members of multiple elements with joints achieved with overlapping of the elements. This joint form permits the use of split-ring connectors which produce stronger and tighter joints. A major benefit of this construction is the reduction of joint deformations and the resulting reduction of truss deflection.

Feasibility of this scheme depends on the working out of joint layouts that permit the placing of the required split rings within the space defined by edge limits, spacing limits, and sizes of truss members. Also a bit tricky is the choice of members as single, double, or triple combinations. In Figure 10.40, achieving the lower chord joint with both a vertical and diagonal member requires the use of two outside splice pieces, since both the bottom chord and the diagonal are two-piece members.

Building Six: Alternative Two

Figure 10.41 shows construction details for an alternative construction for the interior of Building Six. Indicated here is the use of a steel roof truss and a steel framing system for the upper floor structures.

Figure 10.42 shows a framing plan for the upper floor. A possible choice for the floor deck is the same one used for Building Three, consisting of a formed sheet steel unit with 1.5-in.-high ribs and a concrete fill of 2.5 in. over the deck units. Units are securely attached to each other to develop a continuous deck and are sufficiently welded to the supporting beams to develop necessary diaphragm action for lateral loads.

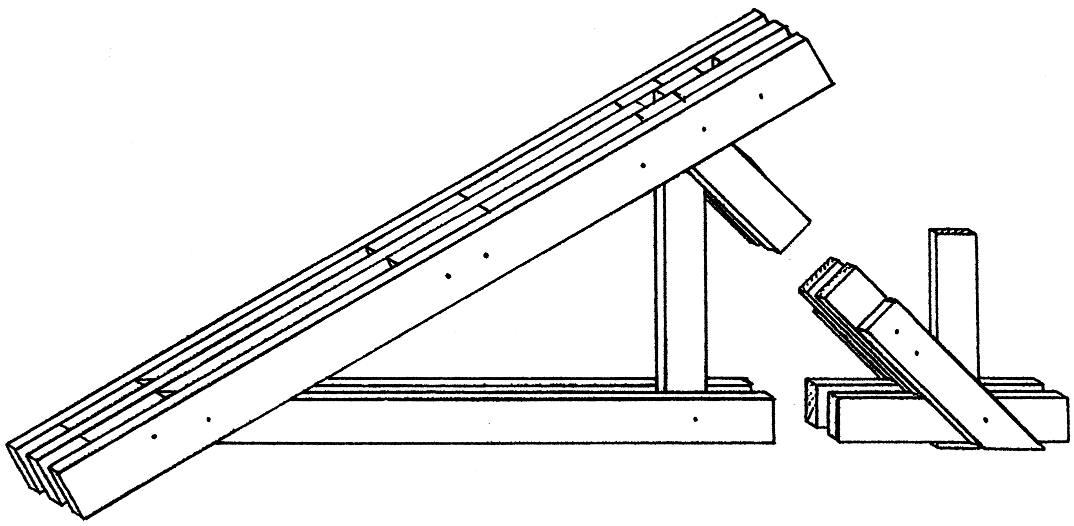


Figure 10.40 Form of the alternative truss construction.

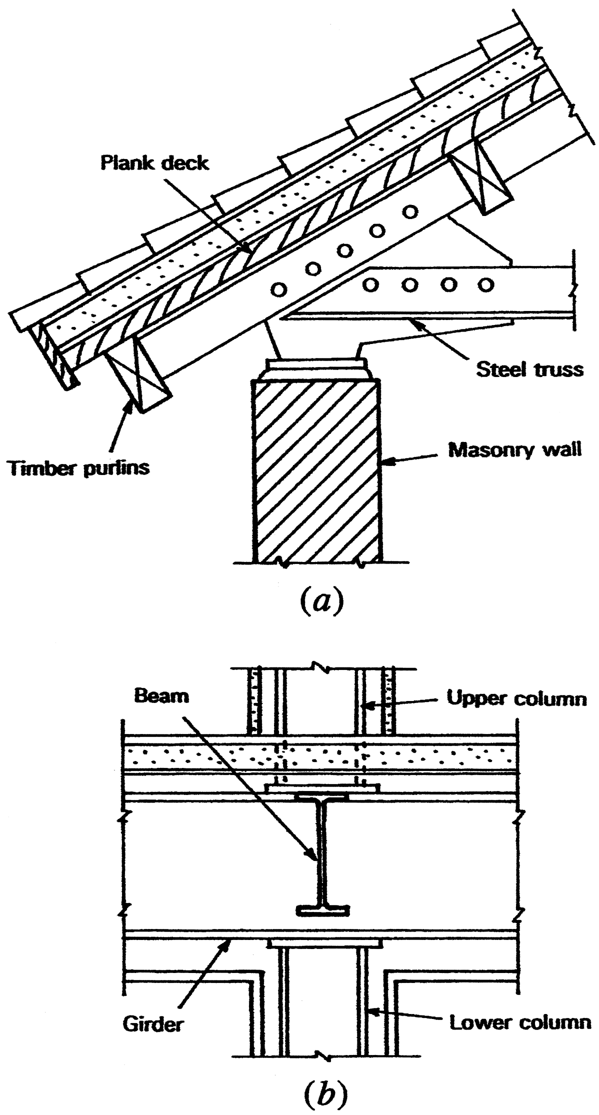


Figure 10.41 Details for the steel structure.

Two types of beams are shown in the plan in Figure 10.42. The interior beams carry a full 8-ft-wide strip of the deck. At the exterior wall and at the edge of the balcony are beams that carry a one-half-wide (4-ft-wide) strip of deck. The beam at the inside face of the exterior wall is used to prevent the wall from carrying the deck directly; thus, the only load transferred to the exterior wall is through the girder. W shapes are used for the girders and the interior beams and C, or channel, shapes for the edge beams.

For the interior beam, the ultimate load for the floor is determined as

$$w_u = 1.2(60) + 1.6(100) = 232\,\text{psf}$$

and the linear load on the beam is

$$w_u = 8(232) = 1860\,\text{plf}, \quad \text{or} \;\; 1.86\,\text{kips/ft}$$

For the simple beam action, the maximum bending moment is

$$M_u = \frac{wL^2}{8} = \frac{1.86(16)^2}{8} = 59.5\,\text{kip-ft}$$

Table 6.1 indicates the possibility for a modest shape for this short span. Choices include W 12 × 16 and W 10 × 19. Figure 4.5 indicates that the 12-in.-deep beam is OK for deflection.

The edge beams carry smaller loads, but the same shape is probably the lightest one acceptable, unless a different shape (e.g., C) is desired.

The supported area for the girder is $24(16) = 384\,\text{ft}^2$, which permits a 20% reduction of the live load. The total load on a single interior beam is thus

$$\begin{aligned} P_u &= (8 \times 16)[(1.2 \times 60) + (1.6 \times 80)] \\ &= 25{,}600\,\text{lb}, \quad \text{or} \;\; 25.6\,\text{kips} \end{aligned}$$

Adding a bit for the beam and girder weights, consider a design load of 26.5 kips. The girder loading and the resulting

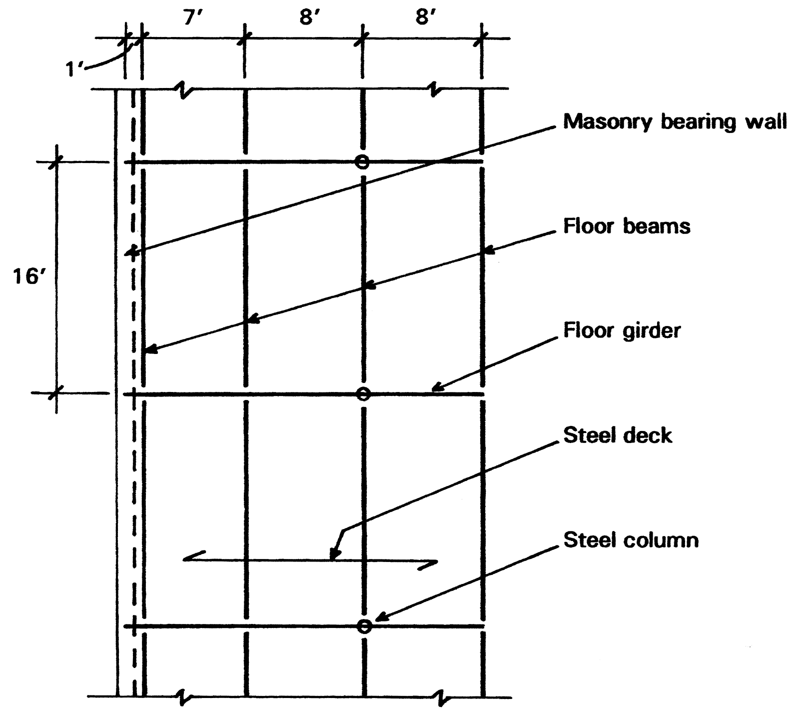


Figure 10.42 Framing plan for the steel structure.

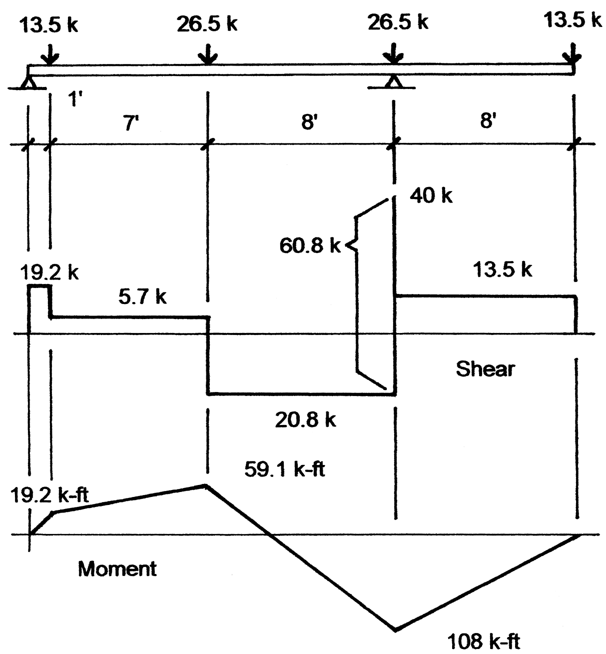


Figure 10.43 Analysis of the steel girder.

shear and moment values are thus as shown in Figure 10.43. Assuming a plastic yield failure, the lightest shape for this loading is a W 16 × 26. However, lateral bracing of 8 ft is critical for this shape, so a heavier shape is indicated. Investigation for lateral bracing will show a W 16 × 36 to be adequate.

The 16-in.-deep shape may be adequate for static deflection, but a deeper shape may be desirable for less bounciness of the cantilever.

For approximate sizes for the columns, assume loads of 60 kips at the second story and 120 kips at the first story. Possible choices are for a 4-in. standard-weight pipe at the second story and a 6-in. pipe at the first story.

While an exposed steel structure may have some visual appeal, codes will probably require covering up the steel for fire resistance. This will fatten up the columns; otherwise the sizes listed may appear a little skinny.

Building Six: Alternative Three

Another option for the interior structure for this building is for a sitecast concrete system. Figure 10.44 shows a partial framing plan for the upper floors, with a layout similar to that for the steel structure. A series of girders at 16-ft centers is supported by the pilasters in the masonry wall and by sitecast concrete columns at the interior. We will not show the computations for this system as the process is fully discussed for the concrete option for Building Seven.

The pilasters in the exterior wall that provide support for the girders are actually reinforced concrete columns cast within the CMU block system for the walls. These and other details for the construction of the walls are treated in the discussion that follows.

The masonry walls can work with any of the options illustrated here. For all three choices the walls may be developed essentially as separate structures with only minor attachment to the interior structure for the upper floors. This is most likely to be accomplished with the wood or steel interior structures. While it is also possible with the concrete structure, there is also the possibility for another level of involvement between the CMU construction and the sitecast concrete structure. This integrated masonry and concrete

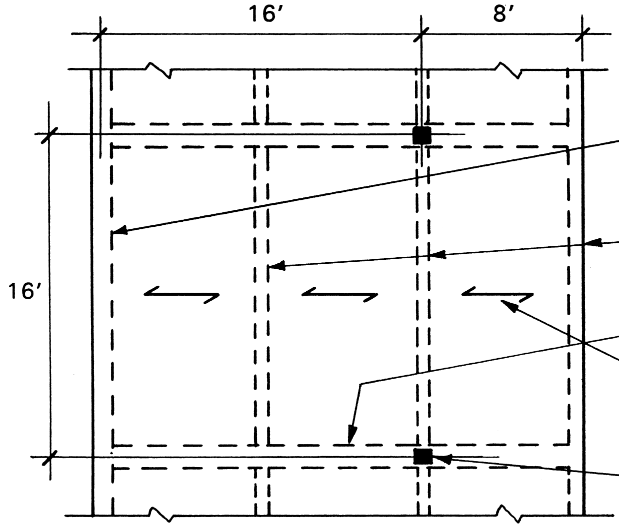


Figure 10.44 Framing plan for the concrete floor structure.

structure is illustrated in the following discussion together with other options for the exterior walls.

Composite Masonry and Concrete Walls

A partial elevation of one of the exterior walls is shown in Figure 10.45. The three-story-high pilasters are expressed as vertical ribs at 16-ft centers on the building exterior. A wide strip at the top of the wall is also thickened to match the pilasters.

Plan sections of the exterior walls that illustrate the formation of the CMU construction are shown in Figure 10.46. The two upper sections in the figure show the layouts for alternating courses of the masonry units as they occur at the level of the windows. The lower section in Figure 10.46 shows one of the courses in the solid-wall portion at the spandrel area between windows.

Although formed within the CMU construction, the pilasters actually consist of reinforced concrete columns of significant dimension, probably about 13 in. square within the nominal size 16-in. square masonry units. These columns may be visualized to be parts of the interior concrete structure if they are cast together with that system. This does indeed constitute an interacting, composite structure.

Vertical section details of the masonry walls are shown in Figure 10.47. Although the edge of the concrete floor structure appears to rest on the projected edge of the masonry, the load transfer from the floor structure to the wall is achieved only by the connection of the concrete girders to the pilasters.

Horizontal beams in the walls are formed by reinforced, concrete-filled, thickened sections. These beams will support the wall portions between pilasters, and thus the gravity load of the walls will not be carried by the concrete structure.

The infill walls of 8-in. CMUs, between pilasters, are essentially nonstructural in nature, although they will be developed in the usual form for reinforced construction. For lateral loads within the plane of the wall, these infill panels will so stiffen the wall that they will truly develop shear wall actions to brace the frame structure. For the size of

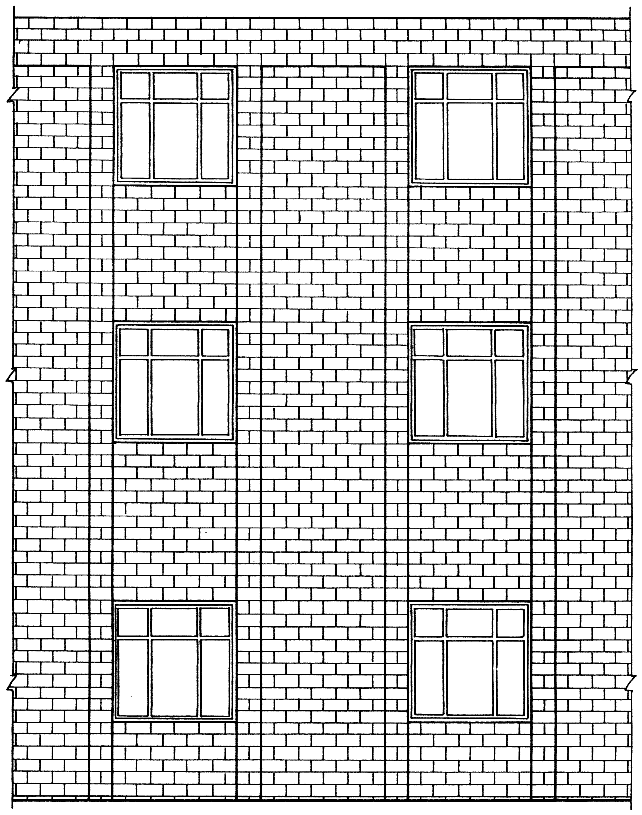


Figure 10.45 Partial elevation of the exterior wall with CMU construction.

this building and the amount of solid wall surface, as shown in Figure 10.45, this shear wall function can probably be provided with minimum CMU construction.

Building Six: Alternative Four

Many forms of construction can be used to produce a building with the appearance shown in Figure 10.45. The masonry exterior can be developed as an applied veneer over a variety of structures. One such variation is shown in the wall section

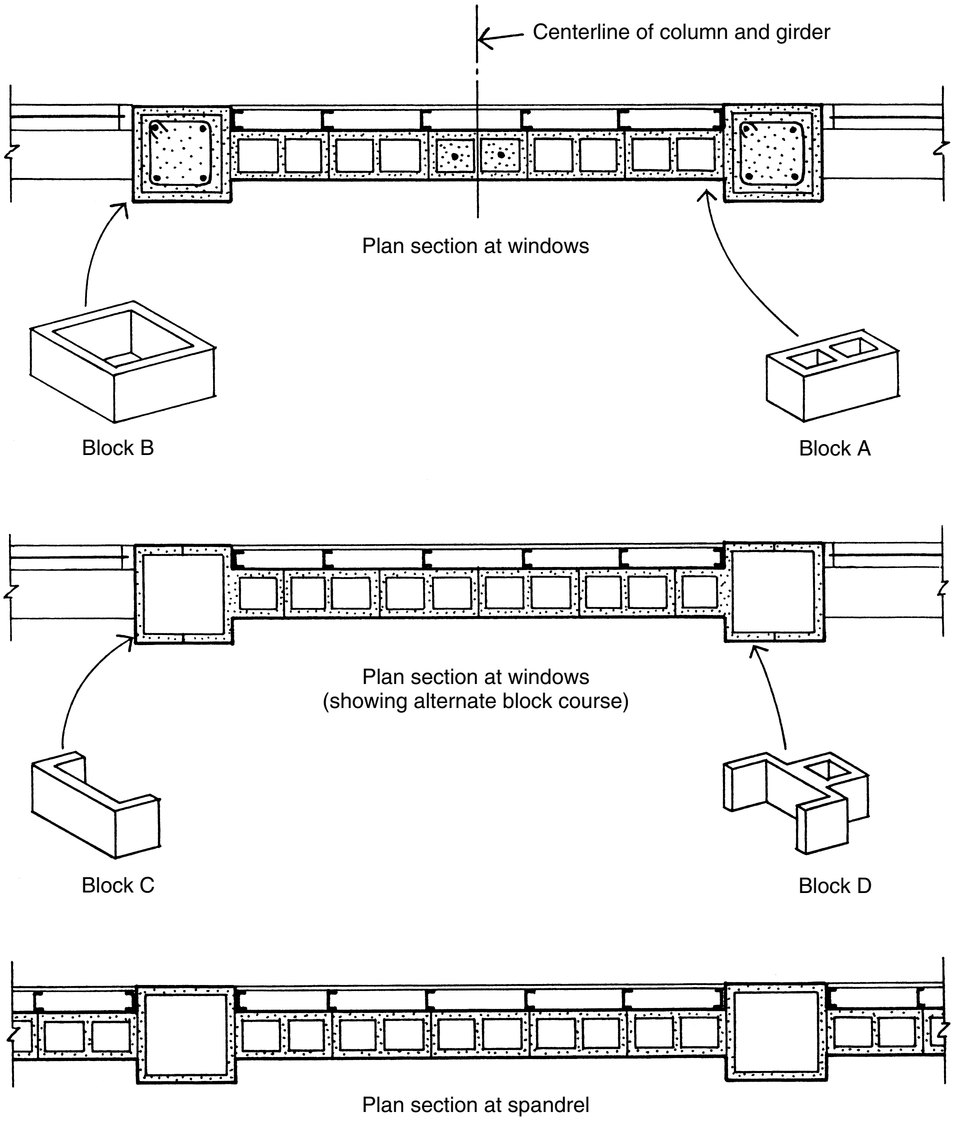


Figure 10.46 Plan section details of the exterior walls.

details in Figure 10.48. This is a modification of the wall section shown in Figure 10.47.

In Figure 10.48, the basic building structure—from the underside of the trusses to the bottom of the footings—is developed with sitecast, reinforced concrete. Actually, the modification consists essentially of differences in the exterior wall structure, because the interior columns and the framing for the upper floors would remain as shown previously.

The masonry veneer shown in Figure 10.48 could as well be installed over a steel frame and frequently is. Alternatively, a variety of exterior curtain wall systems could be placed on the concrete frame shown here. Some variations on this are illustrated for Building Seven.

Masonry surfaces are popular—or at least the *appearance* of masonry is. The appearance of masonry can be achieved in a variety of ways that vary in terms of their identity as real masonry. The construction shown in Figures 10.46 and 10.47 is quite close to that of real structural masonry, utilizing actual masonry units and mortar joints. The construction shown in Figure 10.48 uses real masonry for the building surface; it is simply not in the classification of structural masonry.

A variation on the opposite end of the spectrum of "reality" is that shown in Figure 10.49. This construction uses a form described as exterior fiber-reinforced insulation (EFI), which is quite popular and is reasonably energy smart because it keeps most of the construction mass inside. This is the ultimate Disneyland exterior, which can assume just about any form and appearance of its surface. The exterior shown in Figure 10.45 can easily be achieved by simply adhering thin, brick tiles to the outside surface of the construction.

Appearance issues are not very relative to the basic tasks of structural design, but they are of considerable concern to architects, so the structural designer needs to be aware of them.

The all-concrete structure here can be developed to perform all the necessary tasks of resisting gravity and lateral forces. Because of its natural fire-resistive character, it can also be exposed to view to any degree desired. It is not,

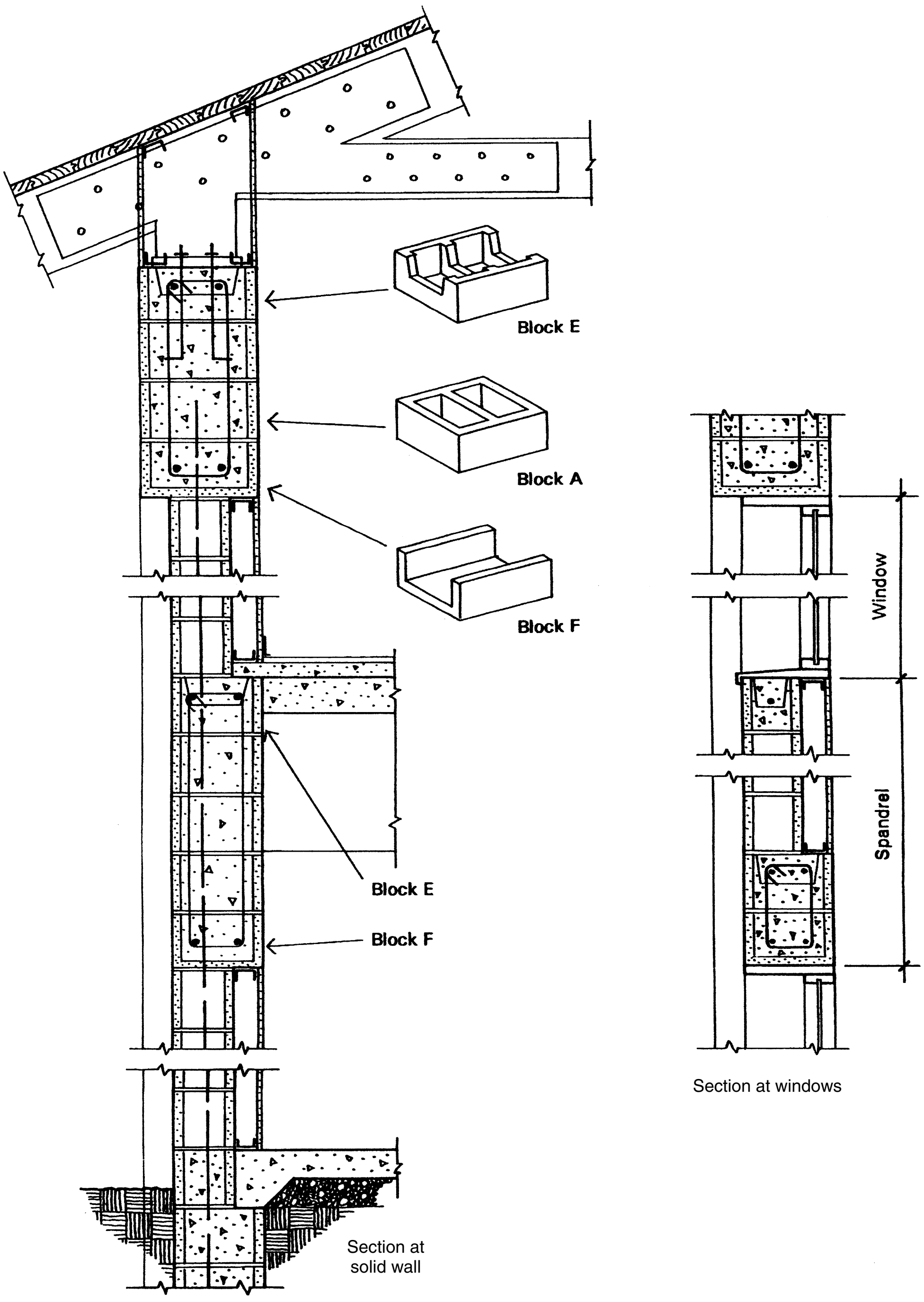


Figure 10.47 Vertical section details of the CMU walls.

however, very insulative, so it is not likely to be used as the single separating construction between the interior and exterior, except in very mild climates.

Design for Lateral Loads

For this building, design for lateral forces involves the following concerns:

Use of the masonry walls as shear walls
Use of elements of the roof and floor structures to serve necessary diaphragm collector, tie, and other lateral-load-resisting functions
Use of connections between the roof and floor framing and the masonry walls for transfer of forces due to lateral loads

Much of this work is illustrated in other examples in this chapter. What is not developed elsewhere has mostly to do with the problems of the hollowed-out building form (see Figure 10.34). The discussion here is therefore limited to this concern.

The floor structure at ground level and the roof structure generally present continuous surfaces. The slope of the roof makes it questionable to consider the roof deck as a horizontal

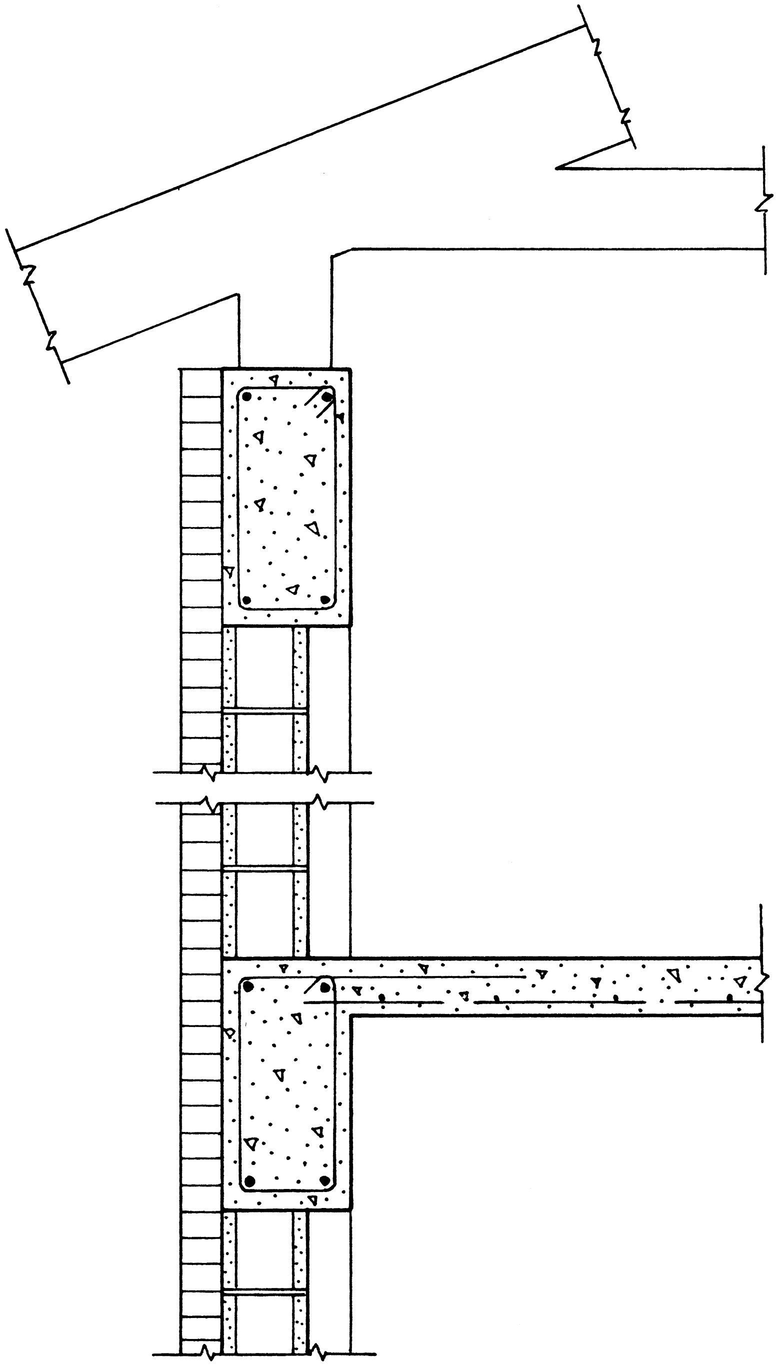


Figure 10.48 Concrete structure.

diaphragm. However, the lateral stability of the tops of the masonry walls will be much improved if horizontal trussing is developed at the level of the bottom chord of the trusses.

The floor structure at the second- and third-floor levels consist of donut-shaped surfaces. A small hole may be easily incorporated in either a vertical or horizontal diaphragm, but as the size of the hole increases, some different functioning of the structure must be considered. There is no clear line here, but the range of cases are shown in Figure 10.50.

One solution here is to treat the floor as a connected set of *subdiaphragms* consisting of smaller units within the whole surface. If these subdiaphragms are individually capable of their assigned tasks, the system as a whole should work. The case here is of this nature.

Although not often possible with a wood structure, the possibility exists for turning the entire floor into a stiff rigid frame. This is quite easily achieved with the sitecast slab-and-beam system.

Figure 10.51 shows a plan of the upper level floor deck. The central void extends for most of the building length, except at the ends and at a bridging point at the center of the building. With the concrete floor, it may be possible to use the reduced width diaphragms to span the entire building length, with only the end shear walls working as

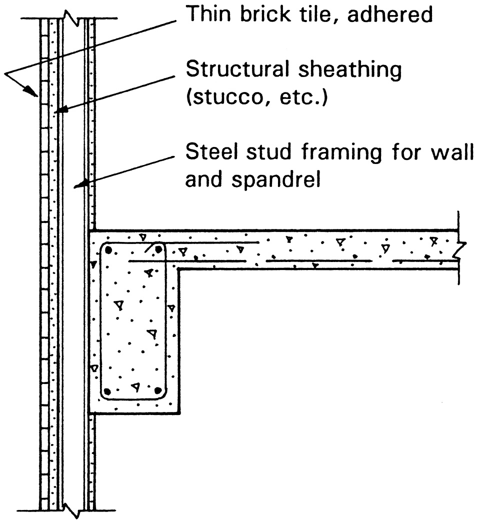


Figure 10.49 Masonry-appearing wall.

vertical bracing. However, another option is to develop the beams and columns at the sides of the bridge as rigid frame bents, thus considerably reducing the span length for the floor diaphragms.

As mentioned previously, there is no horizontal diaphragm at the level of the bottom of the roof trusses. The sloped roof decks may serve this purpose, but the angle of the surface here makes this questionable. The easiest solution is to develop a horizontal truss by adding diagonals and cross struts between the truss bottom chords. In this example, another possibility is the use of the pilaster columns as vertical cantilevers. This is only about an 8-ft cantilever here, so it seems feasible to achieve this work.

10.8 BUILDING SEVEN

This is a modest size office building, generally qualified as being low rise (see Figure 10.52). In this category, there is a considerable range of choice for the construction, although in a particular place, at a particular time, a few popular forms of construction tend to dominate the field.

General Considerations

Some modular planning is usually required for this type of building, involving the coordination of dimensions for spacing of columns, window mullions, and interior partitions in the building plan. This modular coordination may also be extended to the development of ceilings, general lighting, in-ceiling HVAC elements, and the systems for access to electrical power, phones, and other signal wiring.

For buildings built as investment properties, with speculative rental occupancies that may vary over the life of the building, it is usually desirable to accommodate future redevelopment of the building interior with some ease. For the basic construction, this means an emphasis on a design with as few permanent structural elements as possible. At a bare minimum, what is usually required is the construction of the major structure (columns, floors, and roof), the exterior walls, and the interior walls that enclose stairs, elevators, rest rooms, and risers for building services. Everything else should be nonstructural or demountable in nature, if possible.

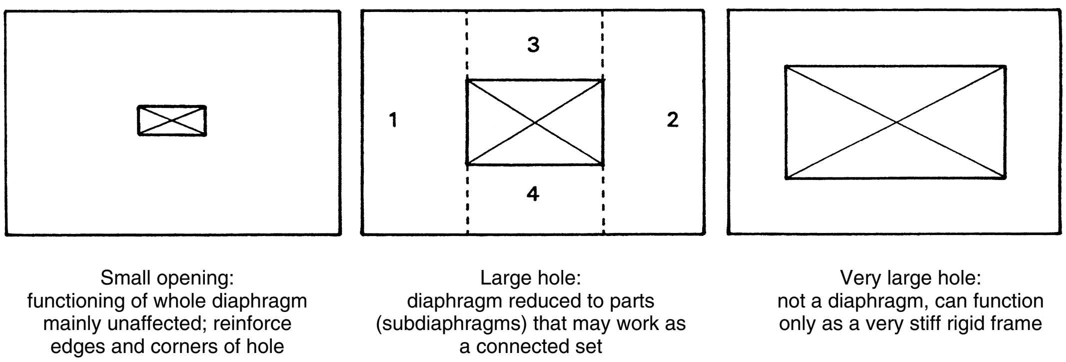


Figure 10.50 Range of effects of a hole in a horizontal diaphragm.

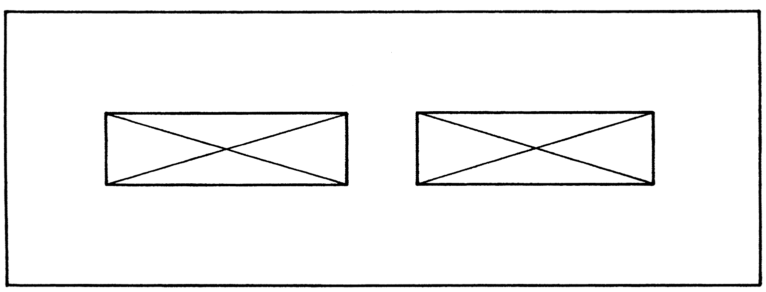


Figure 10.51 Form of the upper floor diaphragm.

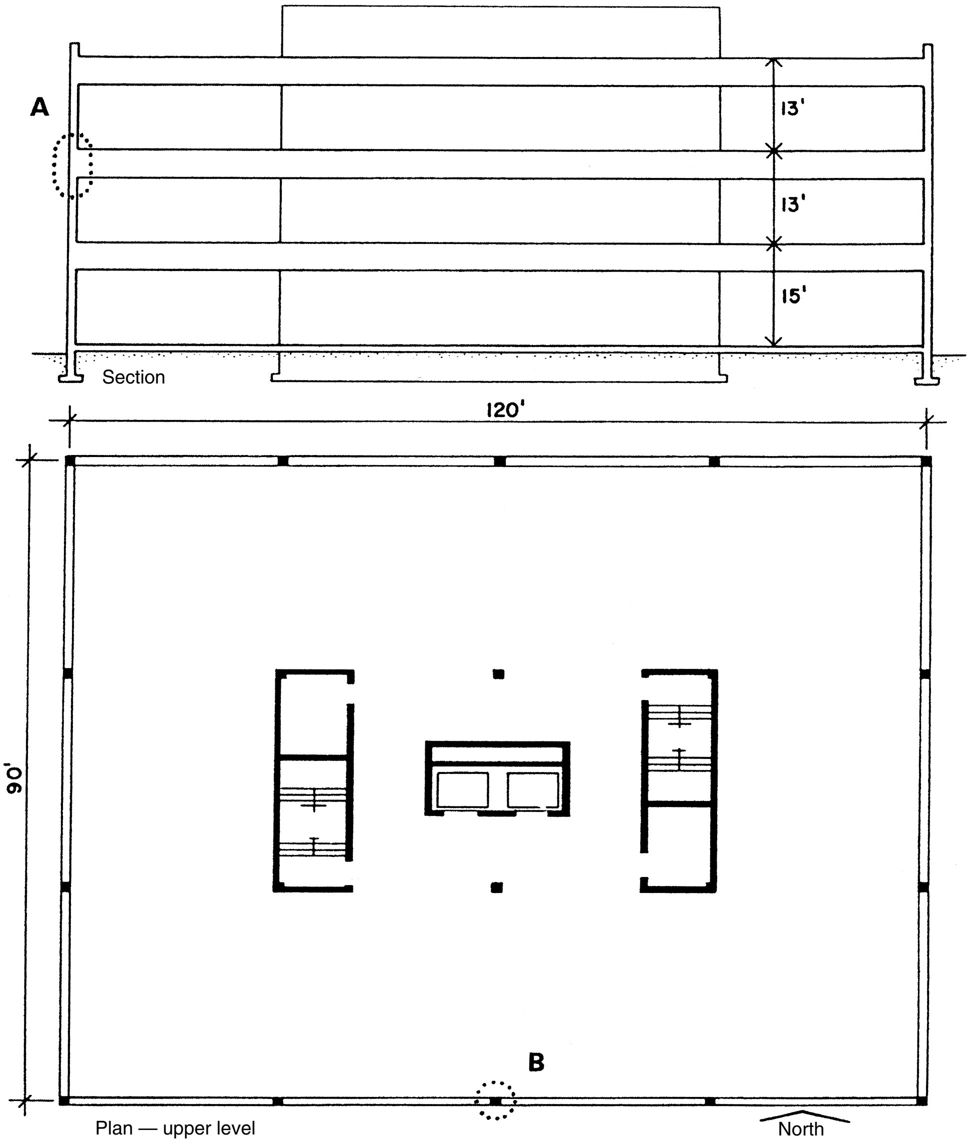


Figure 10.52 Building Seven: general form.

Spacing of columns on the building interior should be as wide as possible to reduce the number of freestanding columns in the rented portion of the building plan. A column-free interior may be possible if the distance from a central core (grouped permanent elements) to the outside walls is not too far for a single span. Spacing of columns at the building perimeter does not affect this issue, so additional columns are sometimes used here to reduce their size for gravity loads or to develop a stiff exterior rigid-frame bent for lateral loads.

The space between the underside of suspended ceilings and the top of the floor or roof construction above must contain many elements besides those of the basic construction. This usually represents a situation requiring major coordination for the integration of the space needs of the structural, HVAC, electrical, communications, lighting, and fire suppression systems. A major design decision that must be made early is that of the overall dimension of the space required for this collection of elements.

Depth required for the spanning structure and the general level-to-level story height will be established and will not be easily changed later if the detailed design of any of the enclosed systems indicates a need for more space. Generous provision of space for the building elements makes the work of the designers of the various building subsystems easier, but the overall effects on the building design must be considered. Extra height for exterior walls, stairs, elevators, and service risers will all result in additional cost, making tight control of the level-to-level distance very important.

A major architectural design issue for this building is the choice of a basic form of construction for the exterior walls. For the column-framed structure, two elements must be integrated: the columns and the nonstructural infill walls. The form of construction shown in Figure 10.53 shows the columns incorporated into the walls, with windows developed in strips between the columns and spandrels flush with the exterior column cover.

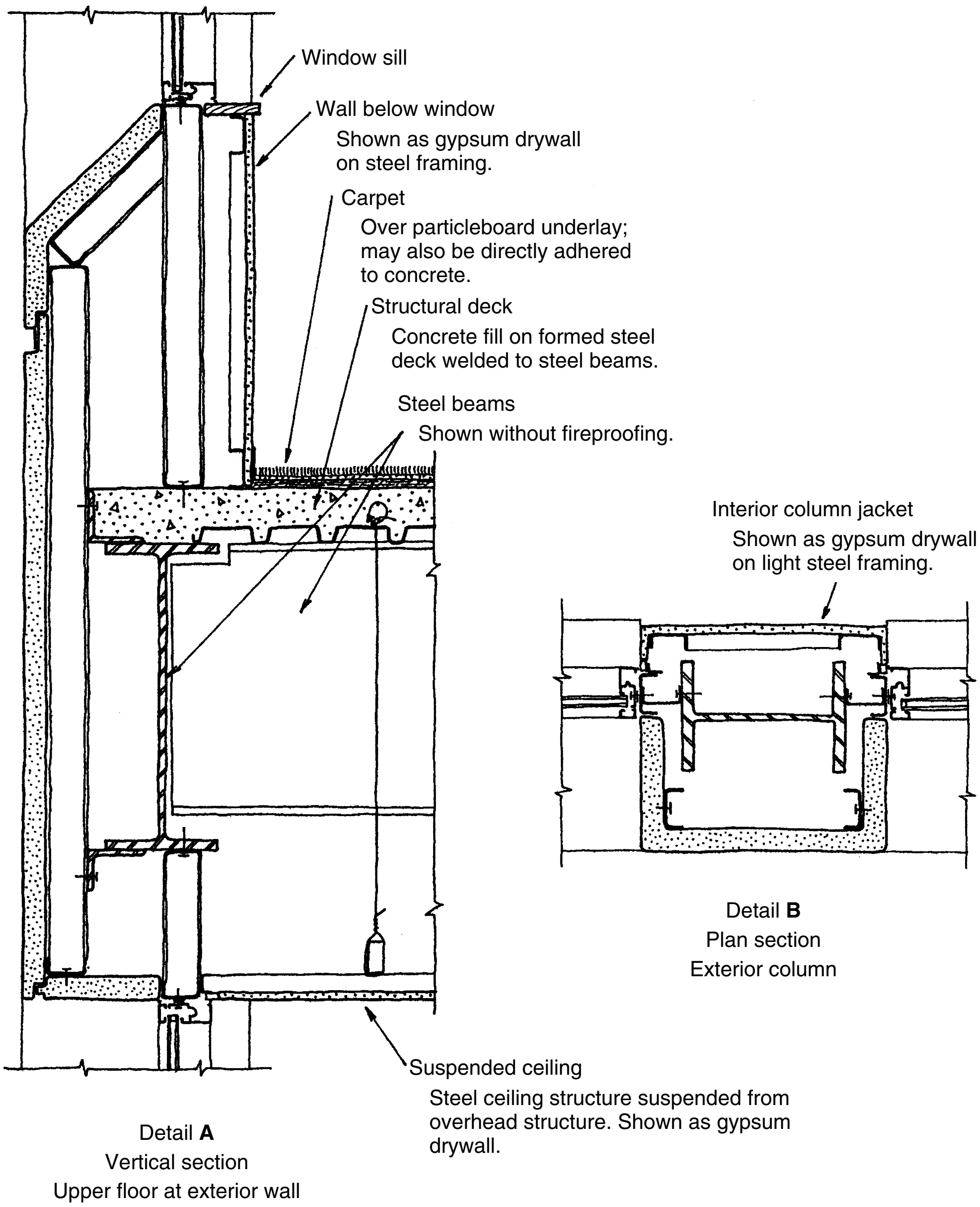


Figure 10.53 Construction details at the exterior wall on the upper floor.

The windows in this example do not exist as parts of a continuous curtain wall system. They are essentially single units placed in and supported by the general infill wall system. The exterior wall in general is developed as a stud-and-surfacing system, not unlike the typical light wood stud wall, except that the studs here are steel and the exterior surfacing consists of metal-faced sandwich panels. The interior wall surfaces consist of drywall attached to the metal studs.

Detailing of the wall construction here results in a considerable void space within the wall. This space may easily house elements for the building services.

Design Criteria

The following are used for the structural design work:

Codes: ASCE 2005 (Ref. 1) and *International Building Code* (IBC, Ref. 2)

Live loads
- Roof: 20 psf, reducible
- Floor: 50 psf for offices, 100 psf for lobbies and corridors, 20 psf for partitions

Wind: map speed 90 mph, exposure B

Assumed construction loads
- Floor finish: 5psf
- Ceilings, lights, ducts: 15 psf
- Walls, average surface weight
  - Interior, permanent: 15 psf
  - Exterior curtain wall: 25 psf
- Steel for rolled shapes: ASTM A36

Structural Alternatives

Structural options for this example are quite numerous, including possibly the light wood frame. Certainly, many

steel frame, concrete frame, and masonry wall systems are feasible. Choice of the structural elements will depend mostly on the desired plan form, the form of window arrangements, and the clear spans desired for the building interior.

At this height and taller, the basic structure must usually be steel, reinforced concrete, or masonry. Options illustrated here include versions in steel and concrete.

Design of the structural system must take into account both gravity and lateral loads. Gravity requires developing spanning systems for the roof and floors and the stacking of vertical supporting elements. The most common choices for the general lateral bracing system are the following (see Figure 10.54):

Core Shear Walls (Figure 10.54*a*). Use of solid walls around core elements (stairs, elevators, restrooms, duct shafts) produces a very rigid vertical structure; the rest of the construction may simply lean on this rigid core.

Truss-Braced Core. Similar to the shear wall core; trussed bents replace solid walls.

Perimeter Shear Walls (Figure 10.54*b*). Turn the building into a tubelike structure; walls may be structurally continuous and pierced with holes for windows and doors or may be built as individual linked piers between vertical strips of openings.

Mixed Exterior and Interior Shear Walls or Trussed Bent. For some building plans, the perimeter or core systems alone may not be feasible, requiring use of some mixture of walls and/or trussed bents.

Full Rigid-Frame Bents (see Figure 10.54*c*). Use all the available bents described by the vertical planes of columns and beams.

Perimeter Rigid-Frame Bents (see Figure 10.54*d*). Use only the columns and spandrel beams in the exterior wall planes, resulting in only two bracing bents in each direction for this building plan.

In the right circumstances, any of these schemes may be acceptable for this size building. Each has some advantages and disadvantages from both structural and architectural design points of view.

Presented here are schemes for use of three lateral bracing systems: a truss-braced core, a rigid-frame bent, and multistory shear walls. For the horizontal roof and floor structures, several schemes are also presented.

Design of the Steel Structure

Figure 10.55 shows a partial plan of a framing system for the typical upper floor that uses rolled steel beams spaced at a module related to the column spacing. As shown, the beams are 7.5 ft on center and the beams that are not on the column lines are supported by column-line girders. Thus, three-fourths of the beams are supported by the girders and the remainder are supported directly by the columns. The beams in turn support the one-way-spanning deck. Within this basic system, there are a number of variables:

Beam Spacing. Affects the deck span and the beam loading.

Deck. A variety available, as discussed later.

Beam/Column Relationship in Plan. As shown, permits possible development of vertical bents in both directions.

Column Orientation. The W shape has a strong axis and accommodates framing differently in different directions; it also reacts differently for rigid-frame actions in different directions.

Fire Protection. Columns, beams, and the undersides of decks need to be encased in fire-protective materials.

These issues are treated in the following discussions.

Inspection of the framing plan in Figure 10.55 reveals a few common elements of the system as well as several special beams required at the building core. The discussions

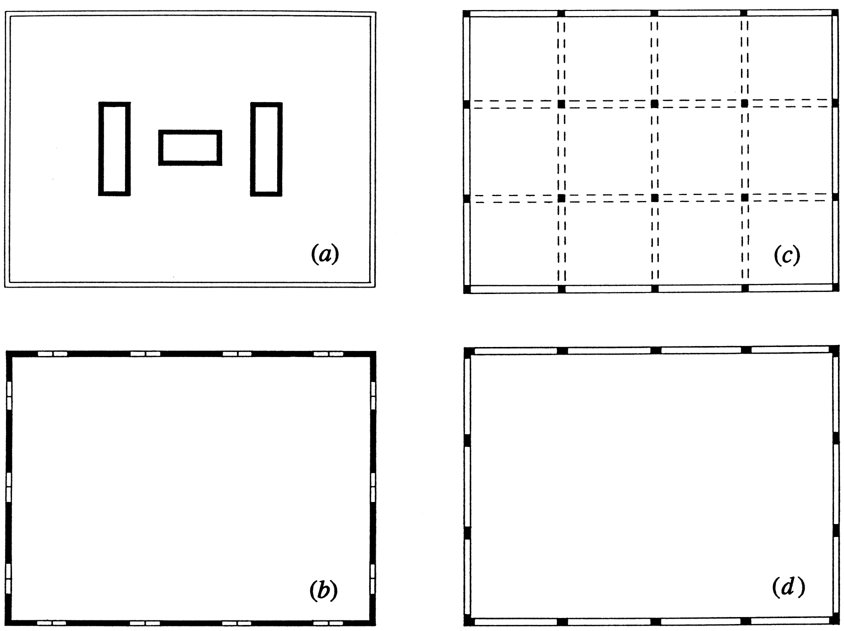


Figure 10.54 Options for the vertical elements of the lateral bracing system.

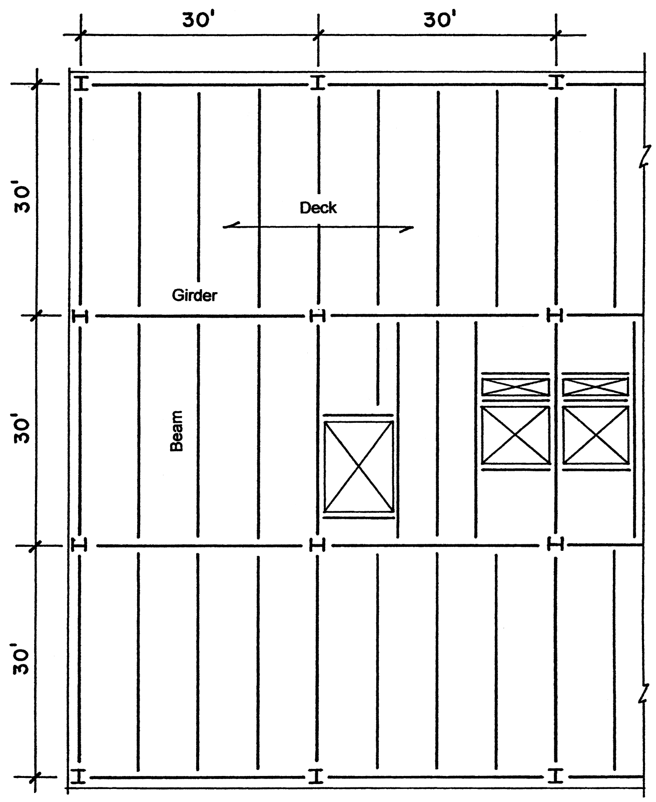


Figure 10.55 Framing plan for the upper floors.

that follow are limited to treatments of the common elements—that is, the members labeled "beam" and "girder" in Figure 10.55.

For speculative rental, it is assumed that different arrangements of the floors are possible. Thus, it is not possible to predict where there will be offices and where there will be corridors, each of which require different live loads. It is therefore not uncommon to design for the general system that relates to this issue. For the design work here, the following will be used:

For the deck, live load = 100 psf.
For the beams, live load = 80 psf, with 20 psf added to dead load for movable partitions.
For girders and columns, live load = 50 psf, with 20 psf added to dead load for partitions.

Structural Deck

Several options are possible for the floor deck. In addition to structural concerns—which include gravity loading and diaphragm action for lateral loading—considerations must be given to fire protection, to accommodation of wiring, piping, and ducts, and to attachment of finish flooring and ceiling construction.

Common Beam

As shown in Figure 10.55, this beam spans 30 ft and carries a load strip that is 7.5 ft wide. The total peripheral load support for one beam is thus $7.5 \times 30 = 225\ \text{ft}^2$. This allows for a reduced live load as follows:

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right) = 80\left(0.25 + \frac{15}{\sqrt{2(225)}}\right)$$
$$= 77\ \text{psf}$$

The beam loading is thus:

Live load = 7.5(77) = 578 lb/lineal ft (plf)
Dead load = 7.5(50 + 20) = 525 plf + beam weight, say 560 plf
Total factored unit load = 1.2(560) + 1.6(578) = 1597 plf
Total supported load = 1.597(30) = 48 kips

For this load and span, Table 5.1 yields the following possibilities: W 16 × 45, W 18 × 40, or W 21 × 44. Actual choice may be affected by various considerations. For example, the table used does not incorporate concerns for deflection or lateral bracing. The deeper shape will produce the least deflection, although in this case the live-load deflection for the 16-in. shape is within the usual limit (see Figure 5.5). For these beams, the deck will most likely provide adequate lateral bracing on a continuous basis. Other concerns may involve development of connections and accommodation of nonstructural elements of the construction.

This beam becomes the typical member, with other beams being designed for special circumstances, including the column-line beams, the spandrels, and beams at edges of openings.

Common Girder

Figure 10.56*a* shows the major loading condition for the girder as generated only by the supported beams. Although this ignores the effect of the weight of the girder as a uniformly distributed load, it is reasonable for use in an approximate design because the weight of the girder is a very minor loading.

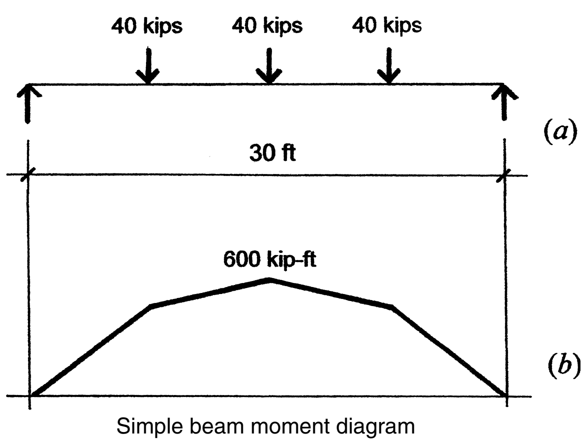


Figure 10.56 Loading condition for the girder.

Note that the girder carries three beams and thus has a total load periphery of $3(225) = 675\,\text{ft}^2$. The total reduced live load is thus

$$L = (80 \times 675)\left(0.25 + \frac{15}{\sqrt{2 \times 675}}\right)$$
$$= 35{,}100\,\text{lb}, \quad \text{or } 35.1\,\text{kips}$$

The unit beam load for design of the girder is thus:

Live load = 35.1/3 = 11.7 kips
Dead load = 0.560(30) = 16.8 kips
Factored unit load = 1.2(16.8) + 1.6(11.7) = 38.88 kips
To account for girder weight, use 40 kips.

This is the loading shown in Figure 10.56, from which the maximum moment is 600 kip-ft.

Selection of a member for this situation may be made using various data sources. Because this member is laterally braced at only 7.5-ft intervals, attention must be paid to this point. Table 5.1 may be used for selection, with the moment values from the table being correct as long as the laterally unsupported length does not exceed L_c. The lightest choices for the girder are W 18 × 106, W 21 × 101, W 24 × 94, W 27 × 94, and W 30 × 90. The deeper members will have less deflection, but the shallower ones are less critical for lateral buckling and will allow for greater room for building services in the floor/ceiling enclosed space.

Computations for deflection may be performed with formulas that recognize the true form of the loading. However, an approximate deflection can be found using an equivalent load derived from the maximum moment, as discussed in Chapter 5. The equivalent uniform load is obtained as

$$M = \frac{WL}{8} = 600\,\text{kip-ft}$$
$$W = \frac{8M}{L} = \frac{8(600)}{30} = 160\,\text{kips}$$

This hypothetical load may be used with simpler formulas to find an approximate deflection.

Although deflection of individual elements should be investigated, there are also wider issues relating to deflection, such as the following:

Bounciness of Floors. This involves the stiffness and the fundamental period of vibration of spanning elements and may relate to the deck and/or the beams. In general use of the static deflection limits usually ensures a reasonable lack of bounciness, but just about anything that increases stiffness improves the situation.

Transfer of Load to Nonstructural Elements. With the building construction completed, live-load deflections of the structure may result in bearing of spanning elements on nonstructural construction. Reducing deflections of the structure will help for this, but some special details may be required for attachment between the structure and the nonstructural construction.

Deflection During Construction. The deflection of the girders plus the deflection of the beams adds up to a cumulative deflection at the center of the column bay. This may be critical for live loads but can also create problems during construction. If the steel beams and steel deck are installed dead flat, then construction added later will cause deflection from the flat condition. In this example, that would include the concrete fill, which can cause a considerable deflection at the center of the column bay, and result in more thickness of the fill at this location. One response is to camber (bow upward) the beams in the shop so that they deflect to a flat position only after the dead load is applied.

Column Design for Gravity Loads

Design of the columns must include considerations for both gravity loads and any applicable lateral loads. Gravity loads for individual columns are based on the column's *periphery*, which is defined as the area of supported surface on each level supported. Loads are actually delivered to the columns by the beams and girders, but the peripheral area is used for load tabulation and determination of live-load reductions.

If beams are rigidly attached to columns with moment-resistive connections—as is done for development of rigid-frame bents—then gravity loads will also cause bending moments and shears in the columns. Otherwise, the gravity loads are essentially considered only as axial compression loads.

Involvement of the columns in development of resistance to lateral loads depends on the form of the lateral bracing system. If trussed bents are used, some columns will function as chords in the vertically cantilevered trussed bents, which will add some compressive forces and possibly cause some reversals with net tension in the columns. If columns are parts of a rigid frame, the chord actions will be involved, but the columns will also be subject to bending moments and shears from the rigid-frame actions. Whatever the lateral force actions may do, the columns must also work for gravity load effects alone.

Alternative Floor Construction with Trusses

A framing plan for the upper floor of Building Seven is shown in Figure 10.57, indicating the use of open-web steel joists and joist girders. Although this construction might be extended to the core and the exterior spandrels, it is more likely that rolled shapes will be used for these purposes. Although somewhat more applicable to longer spans and lighter loads, this system is reasonably applicable to this situation as well.

One potential advantage of using the all-truss framing for the horizontal structure is the high degree of freedom offered for passage of building service elements within the space enclosed by ceilings below and the finished floor above. A possible disadvantage is the need for greater depth of

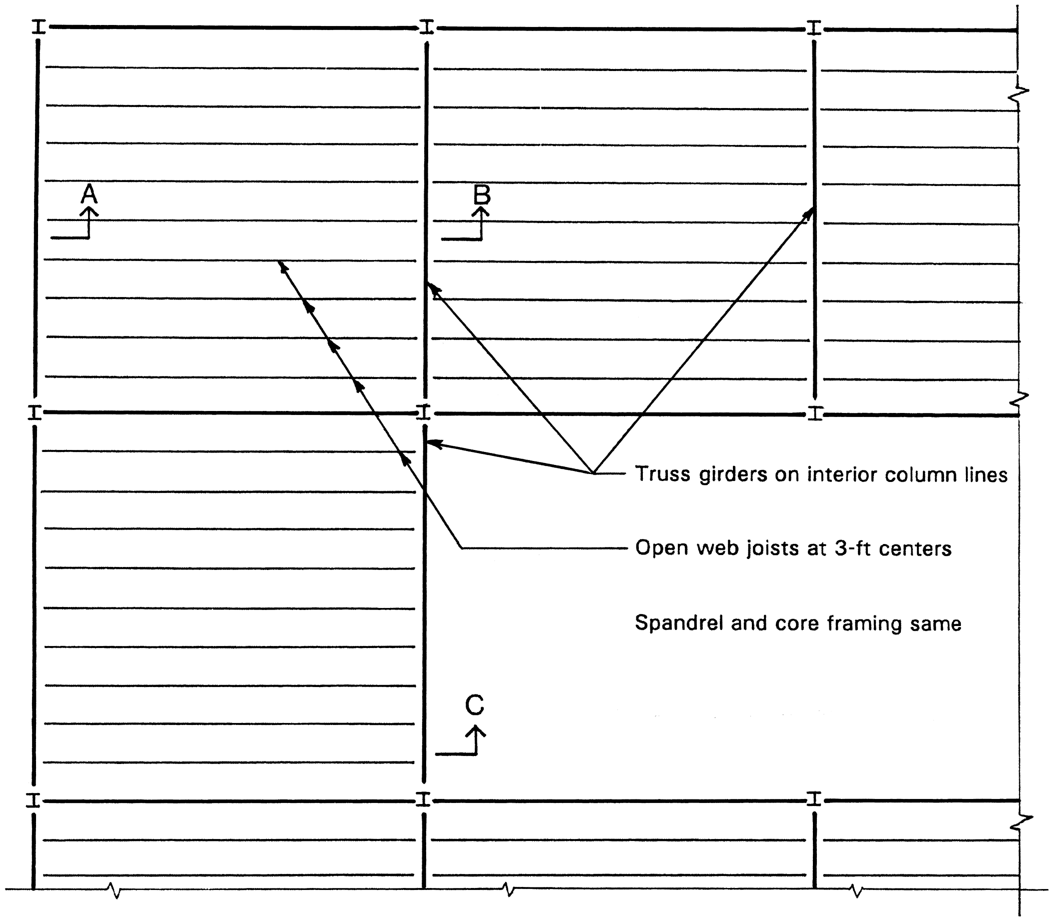


Figure 10.57 Partial framing plan for the all-truss floor.

the structure, adding to the building height, as discussed previously.

Design of the Open-Web Joists

General concerns and basic design for open-web joists are presented in Chapter 5. Using the data for this example, a joist design is as follows:

Joists at 3-ft centers, span 30 ft
Dead load = 3(70) = 210 lb/ft, without joists
Live load = 3(100) = 300 lb/ft
Total factored load = 1.2(210) + 1.6(300) = 732 lb/ft

Referring to Table 5.12, note that choices may be considered for any joist that will carry the total load of 732 lb/ft and a live load of 300 lb/ft on a 30-ft span. The following choices are possible:

24K9 at 12 lb/ft, safe load = 807 − 1.2(12) = 793 lb/ft
26K9, stronger and only 0.2 lb/ft heavier
28K8, stronger and only 0.7 lb/ft heavier
30K7, stronger and only 0.3 lb/ft heavier

A shallower depth joist means a shorter story height and total building height. A deep joist yields more space for service elements and also means less deflection and less floor bounce.

Design of the Joist Girder

Design of joist girders is discussed in Chapter 5. The pattern of the members is somewhat fixed by the spacing of the joists. To achieve reasonable proportions for the truss panels, diagonals should be at approximately 45°. Considerations for design are as follows:

An assumed depth of 3 ft is made. This should be considered a minimum (L/10) for the girder.
Use a live-load reduction of 40%. Thus live load per joist = (3)(30)(0.6)(50) = 2700 lb, or 2.7 kips.
Adding the partition load to an assumed dead load of 40 psf, dead load = (3)(30)(60) = 5400 lb + (30)(12 lb/ft for joist) = 5760 lb, or 5.76 kips.
Total factored panel point load for the girder is thus 1.2(5.76) + 1.6(2.7) = 11.23 kips.

Figure 10.58 shows the form of the girder. The girder designation is established as follows:

36G = 1 girder depth of 36 in.
10N = number of spaces between joists.
11.23K = design joist load.
Complete designation is thus 36G10N11.23K.

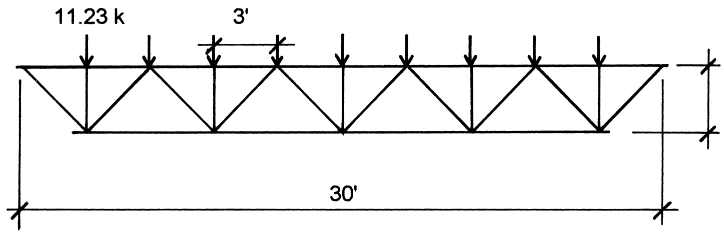


Figure 10.58 Form of the joist girder.

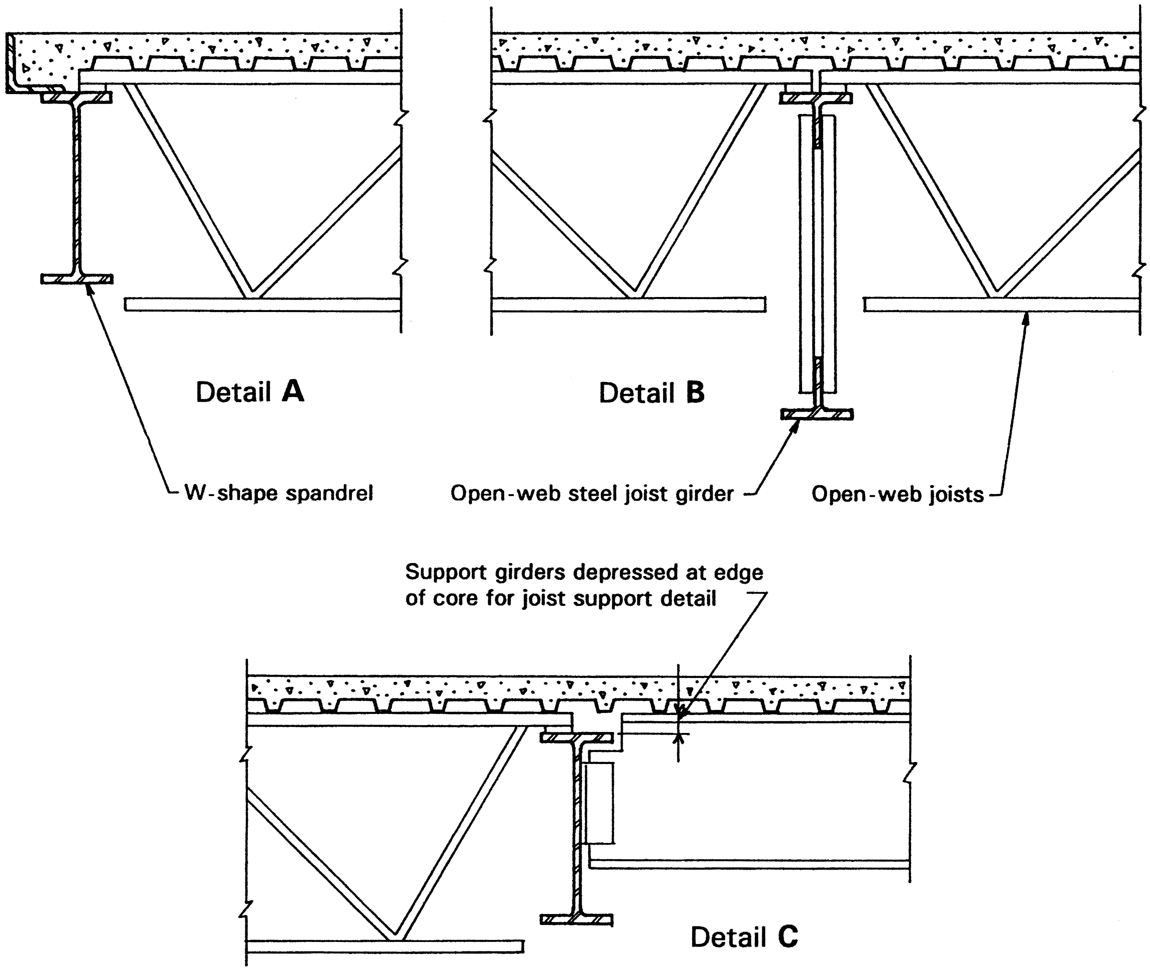


Figure 10.59 Details for the truss system. See plan in Figure 10.57 for locations.

Construction Details for the Truss Structure

Figure 10.59 shows some construction details for the truss system. The deck is the same as for the W-shape framing, although the shorter span will allow for a lighter gauge sheet deck. However, the deck must also be designed for diaphragm action.

Adding to the problem of height for this structure is the detail for the joist support, in which the joist ends sit on top of the girders.

With the closely spaced joists, ceiling construction may be directly supported by attachment to or by suspension from the bottom chords of the joists. However, it is also possible to suspend ceilings from the deck, as is generally done with widely spaced W shapes.

Another issue here is the usual necessity to have a fire-resistive ceiling construction, as it is not feasible to encase the joists and girders in fireproofing materials.

Design of the Trussed Bents for Wind

Figure 10.60 shows a partial framing plan for the core area, indicating the placement of some additional columns off of the 30-ft grid. These columns are used together with the regular columns and some of the horizontal framing to define a series of vertical trussed bents for development of the bracing system shown in Figure 10.61. There are four separate, but joined, bents in each direction of the building axes.

With the symmetrical building exterior form and the symmetrically placed core bracing, this is a reasonable system

for use in conjunction with the building's horizontal deck diaphragms for the wind bracing system.

The work that follows illustrates the design process using criteria for wind loading from the AISC standard (Ref. 1).

For the total wind force on the building, we will assume a base pressure of 15 psf, adjusted for height as described in the ASCE standard (Ref. 1). The design pressures and their zones of application are shown in Figure 10.62.

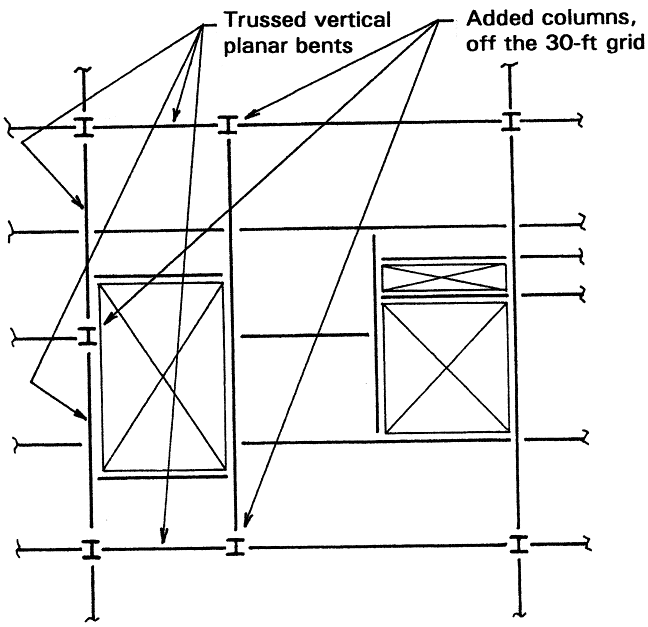


Figure 10.60 Modified framing plan for development of the trussed bents at the core.

For investigation of the lateral bracing system, the design wind pressures on the outside wall are distributed as edge loadings to the roof and floor diaphragms. These are shown as the forces H_1, H_2, and H_3 in Figure 10.62. The horizontal forces are next shown as loadings to one of the vertical bents in Figure 10.63*a*. For the bent loads, the total force per bent is determined by multiplying the unit edge diaphragm load by the building width and dividing by the number of bents for load in that direction. The bent loads are thus

$$H_1 = (165.5)(92/4) = 3807\text{ lb}$$
$$H_2 = (199.5)(92/4) = 4589\text{ lb}$$
$$H_3 = (210)(92/4) = 4830\text{ lb}$$

The truss loadings, together with the reaction forces at the supports, are shown in Figure 10.63*b*. The internal forces in the truss members resulting from this loading are shown in Figure 10.63*c*, with force values in pounds and sense indicated by C for compression and T for tension.

The forces in the diagonals may be used to design tension members using the load combination that includes wind (see Section 10.1). The compression forces in the columns may be used with gravity loads to see if this load combination is critical for the column design. The uplift forces in the columns should be compared with the gravity dead load to see if the column base must be designed for a tension anchorage force.

The horizontal forces should be added to the beams in the core framing and an investigation should be done for

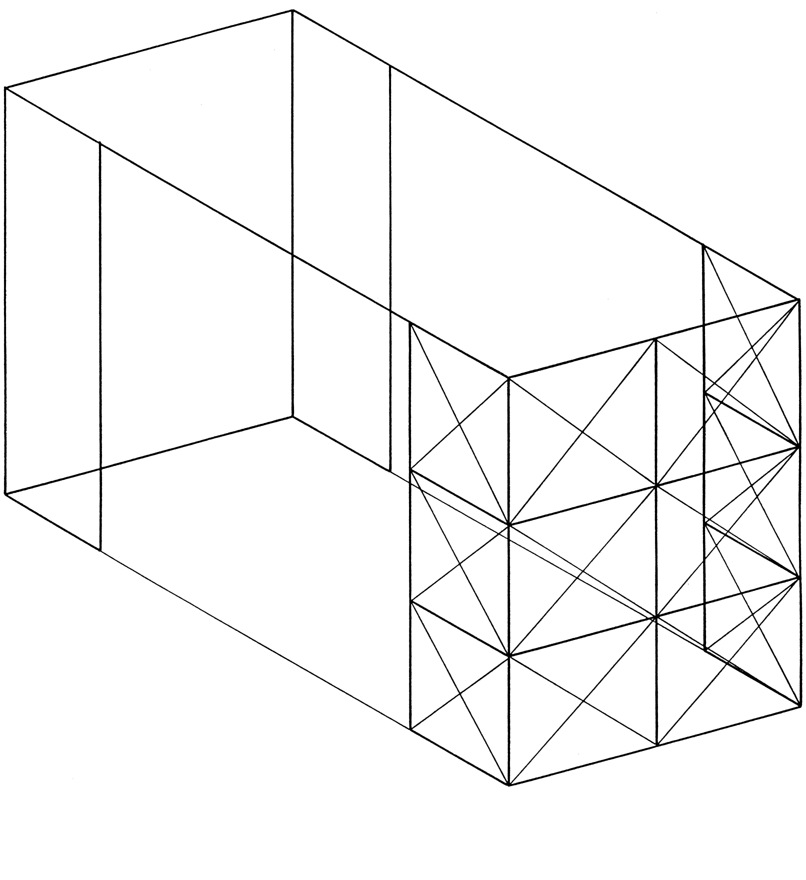


Figure 10.61 General form of the trussed bent bracing system at the building core.

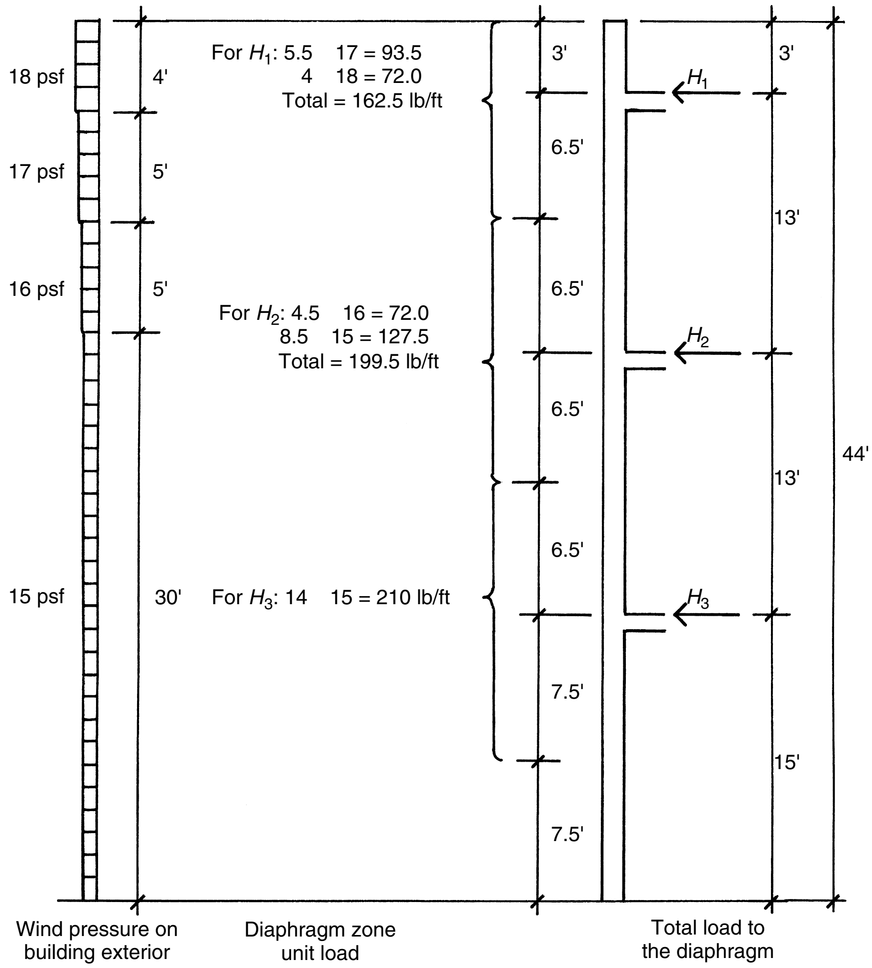


Figure 10.62 Development of the wind loads on the bracing system.

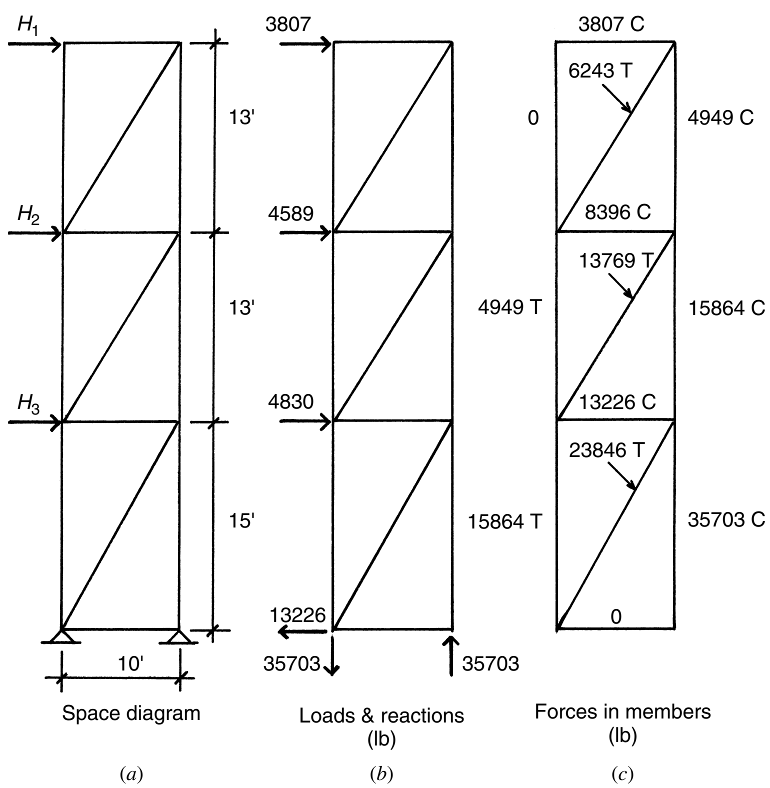


Figure 10.63 Investigation of the east–west bent.

the combined bending and compression. Because beams are often weak on their minor axis (*y* axis), it may be practical to add some bracing members to the framing at right angles to these beams to brace them against buckling.

Design of the diagonals and their connections to the beams and columns must be developed with consideration for the form of the members and some consideration for the wall construction in which they are imbedded.

A detail problem that must be solved is that of the crossing of the two diagonals at the middle of the bent. If double angles are used for the diagonals, the splice joint shown in Figure 10.64 is necessary. An option is to use either single angles or channels for the diagonals, allowing the members to pass each other back to back at the center.

Considerations for a Steel Rigid Frame

The general nature of rigid frames is discussed in Chapter 3. A critical issue with multistory rigid frames is the lateral strength and stiffness of the columns. Because the building must resist lateral forces in all directions, it becomes necessary to consider the shear and bending resistance of the columns in two directions. This presents a difficulty for the W-shape columns because they have considerably greater resistance on their major (x–x) axis versus their minor (y–y) axis.

Figure 10.65*a* shows a possible plan arrangement for column orientation for Building Seven, relating to the development of two major bracing bents in the east–west direction and five shorter and less stiff bents in the north–south direction.

The two stiff bents may well be approximately equal in resistance to the five shorter bents, giving the building a reasonably symmetrical response in the two directions.

Figure 10.65*b* shows a plan arrangement for columns designed to produce approximately symmetrical bents on the building perimeter. The form of such perimeter bracing is shown in Figure 10.66.

One advantage of perimeter bracing is the potential for using deeper (thus stiffer) spandrel beams, because their height dimension is not critical within the wall plane. The stiffness of the perimeter bents can also be increased with the

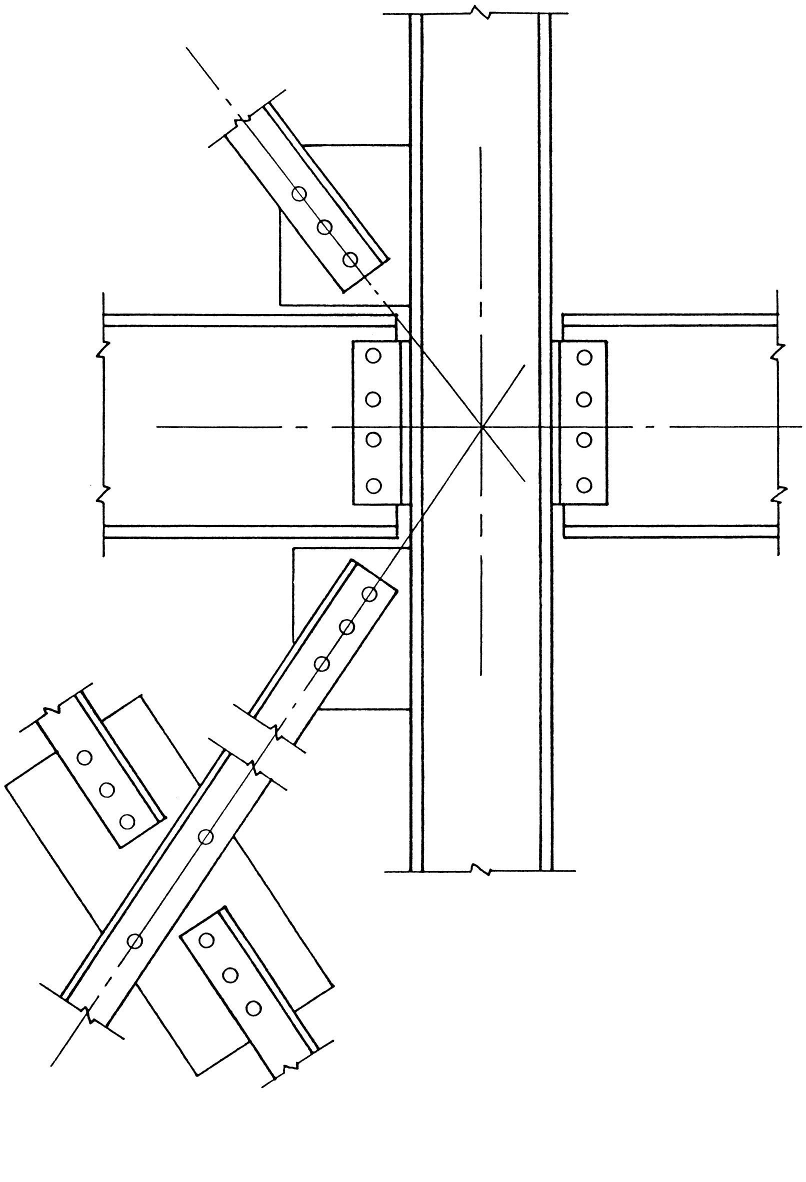


Figure 10.64 Details of the bent construction.

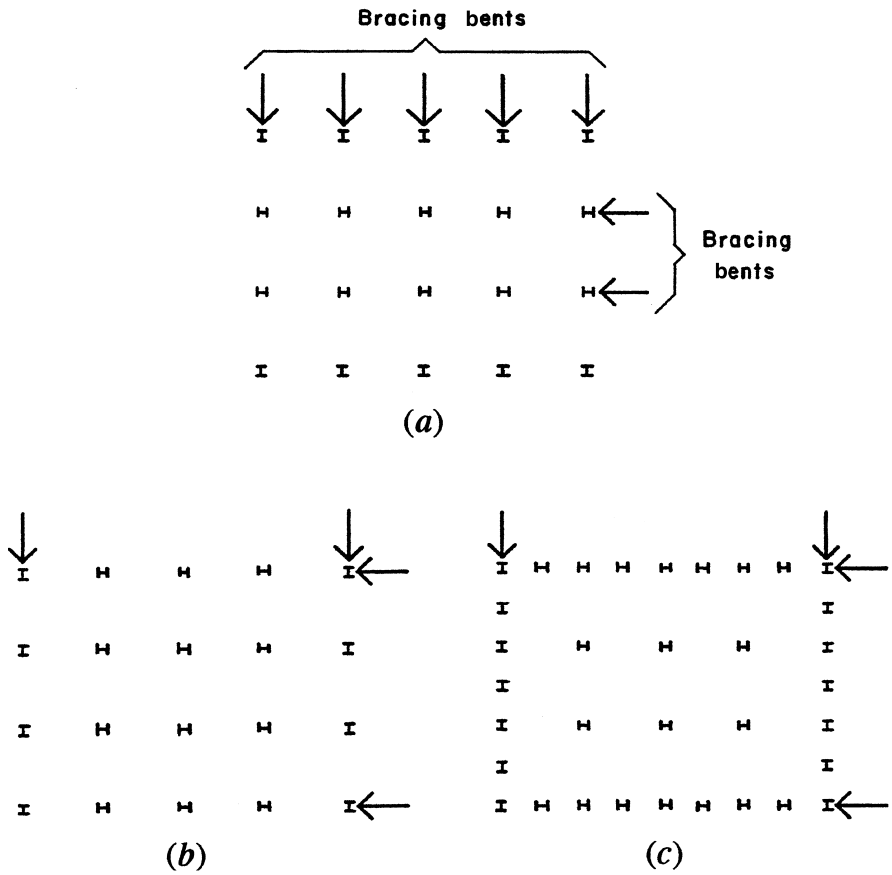


Figure 10.65 Optional arrangements for the columns in the rigid-frame bents.

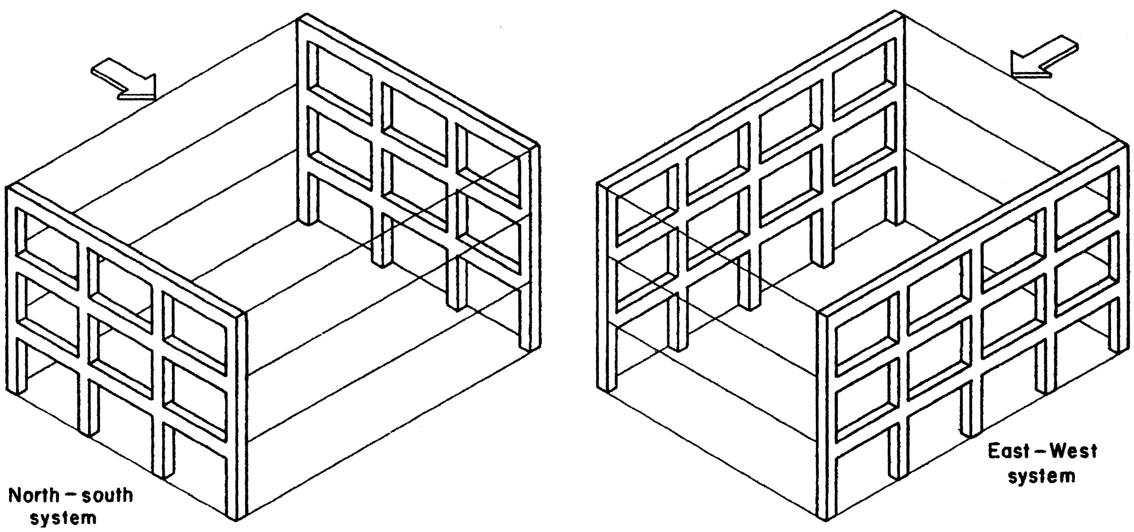


Figure 10.66 Form of the perimeter bent bracing system.

use of additional columns, as shown in Figure 10.65*c*. With a combination of closely spaced columns and deep spandrels, the perimeter bents may become very stiff, with the structure approaching the behavior of a pierced wall, rather than a flexing frame.

At the expense of requiring much stronger and stiffer columns and expensive moment-resistive connections, the rigid-frame bracing offers architectural planning advantages with the elimination of solid shear walls and diagonal framing.

Considerations for a Masonry Wall Structure

An option for Building Seven involves the use of structural masonry for the exterior walls. Figure 10.67 shows a partial elevation of the building and a partial framing plan for the upper floors. The construction details in Figure 10.68 show

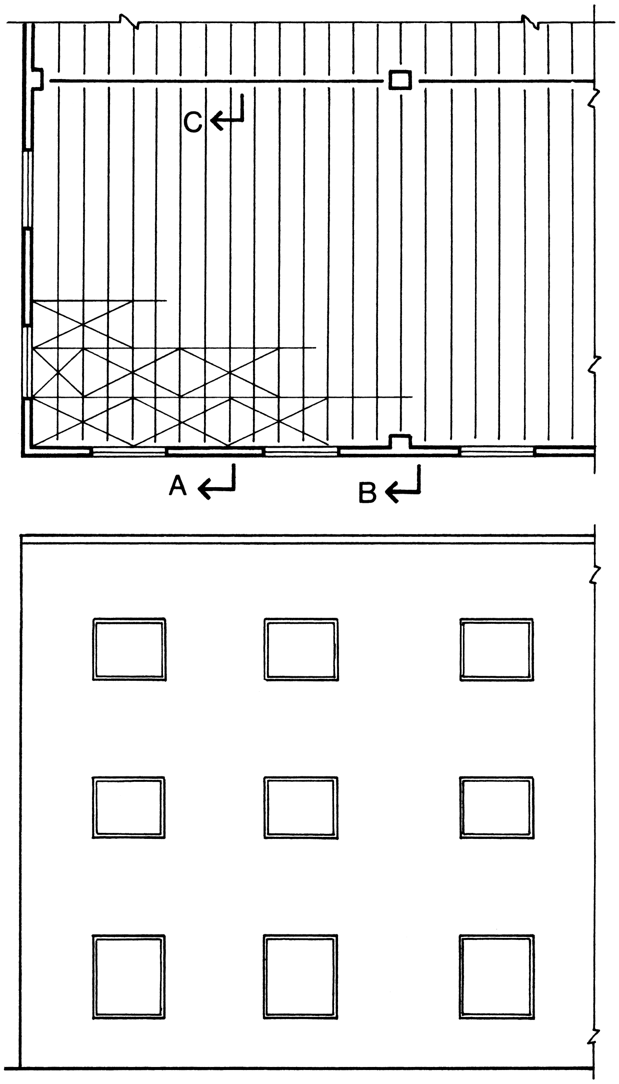


Figure 10.67 Partial framing plan and elevation for the masonry structure.

the use of reinforced CMU construction, as described in Chapter 7. One consideration for this construction involves the necessity for tight control of modular planning for wall openings, owing to the inability to cut the CMUs to fit tailored dimensions (see Figure 10.18*a*).

Typical Framed Floor

The floor framing system here uses column-line girders that support fabricated joists and a plywood deck. The girders could be glued-laminated timber but are shown here as rolled steel shapes. Supports for the steel girders consist of steel columns on the interior and masonry pilasters at the exterior walls.

The exterior masonry walls are used for direct support of the deck and joists through ledgers bolted to the wall. Load transfers for both gravity and lateral forces must be provided for in the connections provided here.

The development of the girder support at the exterior wall is a bit tricky with the pilaster, which must support not only the girder end but also the column above. There are two possible solutions here. The pilaster can be widened so that it can straddle the girder and continue vertically at its sides or a steel column can be used with the column encased by CMUs. The detail at B in Figure 10.68 is adequate for a one-story structure, but not for the multistory construction here.

Other issues to be resolved for this scheme are the framing at the building core and the lateral support for the steel girders. With the steel girders and columns already used, it is most likely that the core will be developed with ordinary steel framing here. As shown in Figure 10.68, detail C, the joists provide for lateral support for the girder top flange but are questionable for torsional bracing, so the girder must be designed for this condition.

Masonry Walls

Buildings much taller than this have been achieved with structural masonry, so the feasibility of the system is well established. The vertical loads and lateral loads increase in lower stories, so it is expected that some increases in structural capacity may be made in lower stories. The minimum code-required construction may be adequate for the top story, with increases achieved by more grout-filled voids and more reinforcement in lower stories. However, some codes require that all voids be filled-in shear walls, so the minimum construction will be much stronger in that case.

Design for Lateral Forces

A common solution for lateral bracing is the use of an entire masonry wall as a single pierced shear wall, with openings considered as producing the effect of a very stiff, nonflexing rigid frame. The basic approach for walls is to design for the total shear at each story, with variations of strength made within the range of change for the basic construction. However, it is also possible to make alterations of the wall form, as was done in times past for unreinforced masonry construction with stone and bricks.

It is also possible to design the wall as a series of spaced piers, with separation joints at selected locations. This may be a desirable option in climates with a high range of outdoor temperature fluctuation, where the joints would also work for control of thermal movements.

Construction Details

There are many concerns for the proper detailing of the masonry construction to fulfill structural and architectural functions. The general framing plan is shown in Figure 10.67. The locations of the details shown in Figure 10.68 and discussed here are shown on the framing plans.

Detail A. This shows the general exterior wall construction and the framing of the floor joists at the exterior wall. The wood ledger is used for vertical support of the joists, which are hung from metal framing devices fastened to the ledger. The plywood deck is

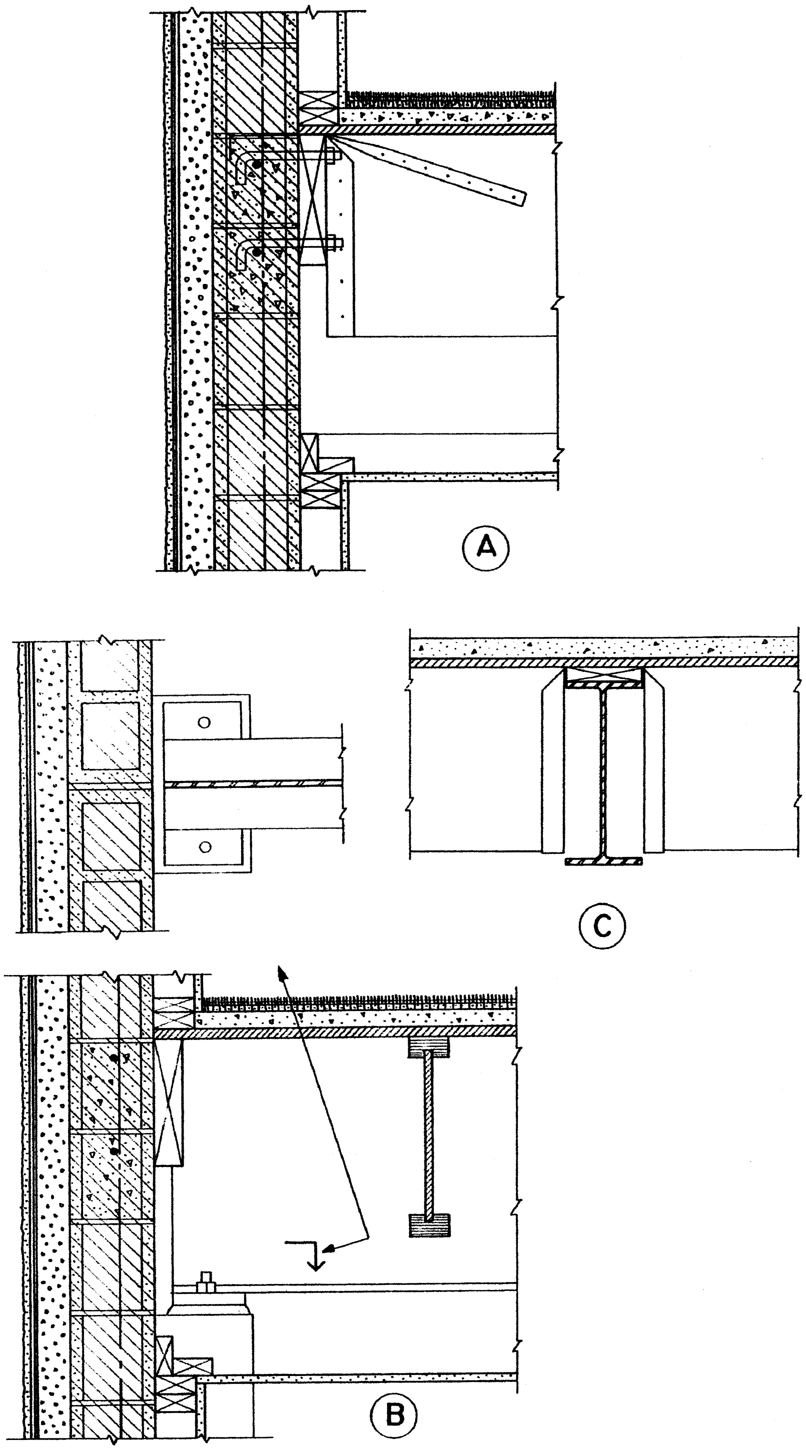


Figure 10.68 Construction details for the masonry structure.

nailed directly to the ledger to transfer its horizontal diaphragm shear loads to the wall. Outward forces on the wall must be resisted by anchorage directly between the wall and the joists. Ordinary hardware devices can be used for this, although the exact details depend on the type of forces (wind or seismic), their magnitude, the details of the joists, and the details of the wall construction. The anchor shown here is only symbolic. The complete construction indicates use of a concrete fill on the deck, an applied insulation system on the exterior, and a furred-out drywall finish on the interior.

Detail B. This shows the section and plan details at the joint between the girder and the pilaster. Issues dealing with the support of the girder and the upper column were discussed previously. If the large lump represented by the pilaster on the inside of the wall is undesirable and a steel column is used, the

pilaster could be eliminated or moved to the outside surface.

Detail C. This shows the use of the steel beam for the support of the joists and the deck. After the wood lumber piece is bolted to the top of the steel beam, the attachment of the joists and deck become essentially the same as it would be for a wood girder.

The Concrete Structure

A structural framing plan for the upper floors in Building Seven is presented in Figure 10.69, showing the use of a sitecast concrete slab-and-beam system. Support for the spanning structure is provided by concrete columns. The system for lateral bracing uses the exterior columns and spandrel beams to form rigid-frame bents at the building perimeter, as shown in Figure 10.66. This is a highly indeterminate structure for both gravity and lateral loads, and its precise engineering design would undoubtedly be done with a computer-aided design process. The work presented here treats the major issues and illustrates an approximate design using highly simplified methods.

Design of the Slab-and-Beam Floor Structure

For the floor structure, use $f_c' = 3\,\text{ksi}$ and $f_y = 40\,\text{ksi}$. As shown in Figure 10.69, the floor framing system consists of a series of parallel beams at 10-ft centers that support a continuous, one-way spanning slab and are in turn supported by column-line girders or directly by columns. Although special beams are required for the core framing, the system is made up largely of repeated elements. The discussion here will focus on three of these elements: the continuous slab, the four-span interior beam, and the three-span continuous spandrel girder.

Using the approximation method described in Chapter 6, the critical conditions are shown in Figure 10.70 for the three common elements. Use of these coefficients is reasonable for the slab and beam that support distributed loads. For the girder, however, the concentrated loads make use of the coefficients questionable. An adjusted method is described later for use with the girder.

Figure 10.71 shows a section of the exterior wall that illustrates the general form of the construction. The exterior columns and the spandrel beams are exposed to view. Use of the full available depth of the spandrel beam results in a much-stiffened bent on the building exterior. Combined with oblong-shaped exterior columns, this creates perimeter bents that will absorb most of the lateral force on the structure.

Design of the continuous slab is presented in the example in Section 6.2. The use of the 5-in. slab is based on assumed minimum requirements for fire protection. If a thinner slab is possible, the 9-ft clear span would not require this thickness based on limiting bending or shear conditions or recommendations for deflection. If the 5-in. slab is used, the result will be a slab with a low percentage of steel bar weight per square foot—a situation usually resulting in lower cost for the structure.

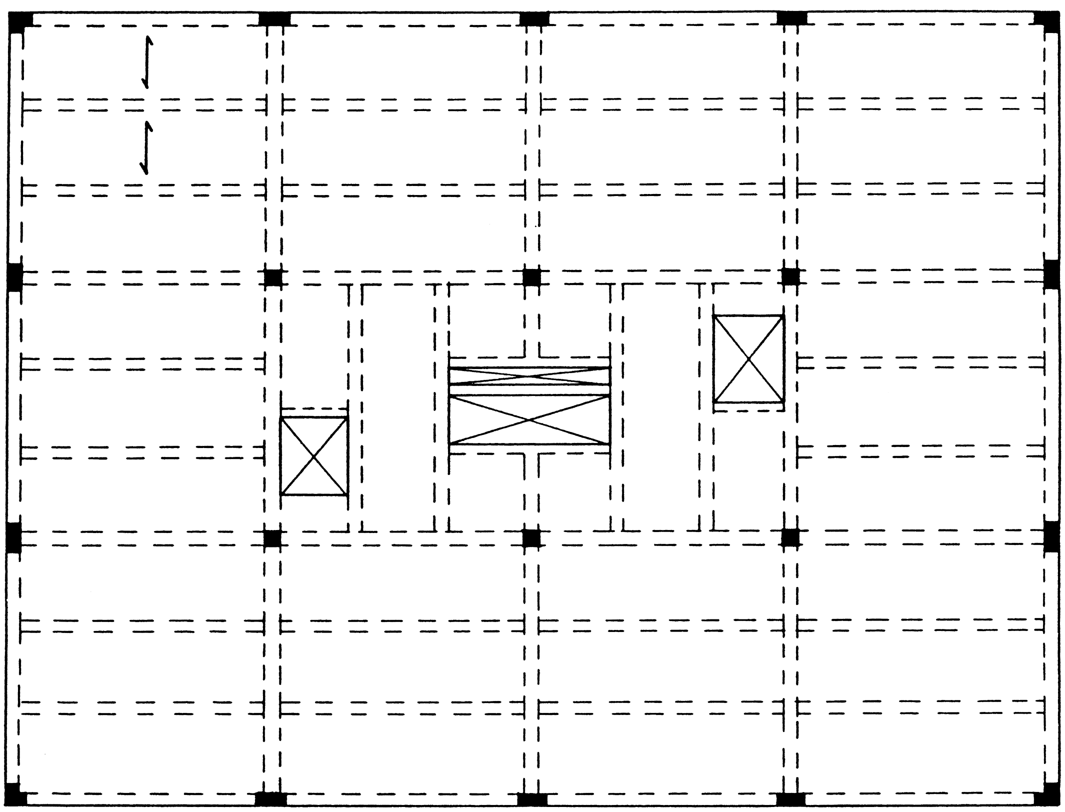


Figure 10.69 Framing plan for the concrete floor structure

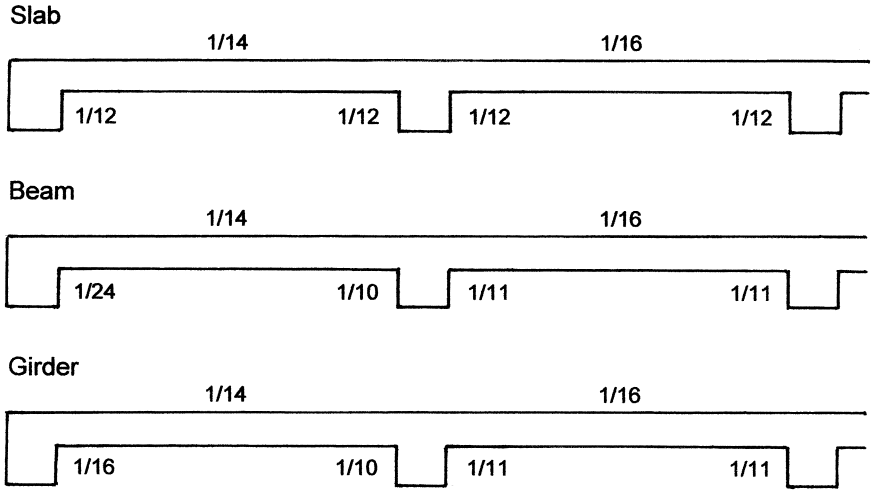


Figure 10.70 Approximate design factors for the slab-and-beam structure.

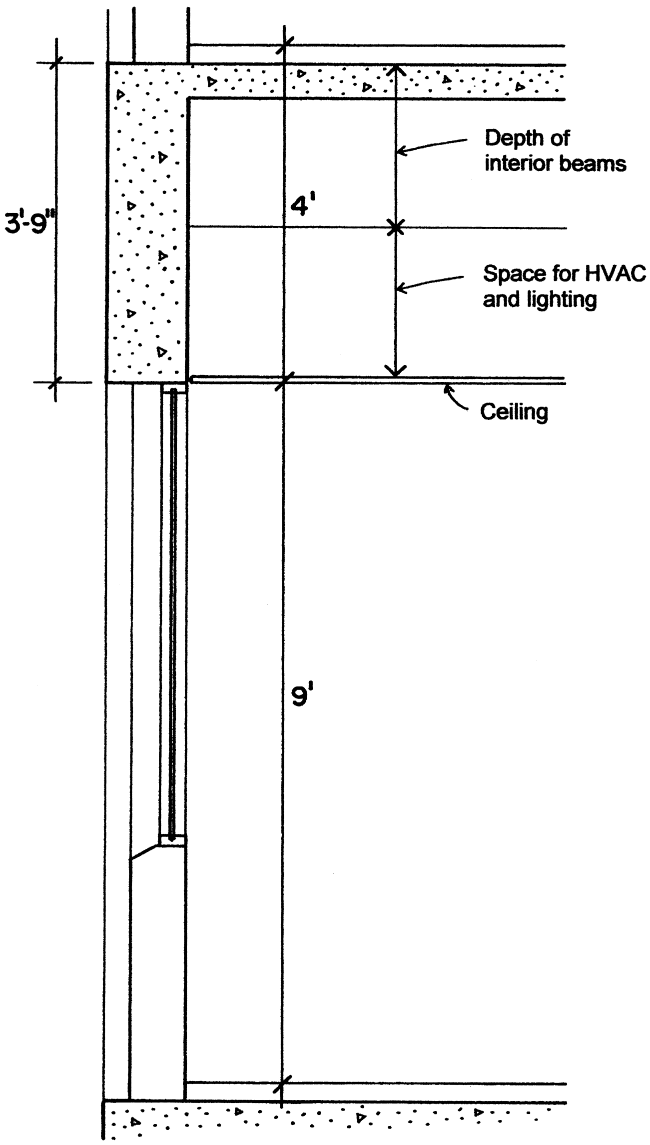


Figure 10.71 Section at the exterior wall.

The unit loads used for the slab design are determined as follows:

Floor live load: 100 psf [4.79 kPa] at the corridor
Floor dead load (see Table 10.2)
  Carpet and pad at 5 psf
  Ceiling, lights, and ducts at 15 psf
  2-in. lightweight concrete fill at 18 psf
  5-in. thick slab at 62 psf
  Total dead load: 100 psf [4.79 kPa]

With the slab determined, it is possible to consider the design of one of the typical beams, loaded by a 10-ft-wide strip of slab, as shown in Figure 10.69. The supports for these beams are 30 ft on center. If the girders and columns are 12 in. wide, the clear span for the beam becomes 29 ft, and its load periphery area is $29 \times 10 = 290\ \text{ft}^2$. Using the ASCE standard (Ref. 1), the reduced live load is determined as

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right)$$
$$= 100\left(0.25 + \frac{15}{\sqrt{2 \times 290}}\right) = 87\ \text{psf}$$

The beam load as a per-foot unit load is thus:

Live load = (87 psf)(10 ft) = 870 lb/ft [12.69 kN/m]
Dead load without the beam stem below the slab is (100 psf)(10 ft) = 1000 lb/ft [14.6 kN/m]
Estimating a 12-in.-wide, 20-in.-deep beam stem extending below the bottom of the slab, the added load is

$$\frac{12 \times 20}{12} \times 150\ \text{lb/ft}^3 = 250\ \text{lb/ft}$$

The total dead load for the beam is thus $1000 + 250 = 1250$ lb/ft, and the total factored load for the beam is

$$w_u = 1.2(1250) + 1.6(870) = 1500 + 1392$$
$$= 2892\ \text{lb/ft}$$

Consider now the four-span beam that is supported by the girders. The moment approximation factors for this beam are given in Figure 10.70, and a summary of design data is given in Figure 10.72.

Note that the beam design provides for tension reinforcement only, which is based on the assumption that the beam cross section is adequate to prevent a critical bending compressive stress in the concrete. Using the strength method, the basis for this is as follows.

From Figure 10.70 the maximum moment in the beam is

$$M_u = \frac{wL^2}{10} = \frac{2.89(29)^2}{10} = 243\,\text{kip-ft}\ [330\,\text{kM-m}]$$

$$M_r = \frac{M_u}{\phi} = \frac{243}{0.9} = 270\,\text{kip-ft}\ [366\,\text{kN-m}]$$

Then, using factors from Table 6.2 for a balanced section, the required value for bd^2 is determined as

$$bd^2 = \frac{M}{R} = \frac{270 \times 12}{1.149} = 2820\,\text{in.}^3$$

Various combinations of b and d may be derived from this relationship, as demonstrated in Section 6.2. For this example, assuming a beam width of 12 in.,

$$d = \sqrt{\frac{2820}{12}} = 15.3\,\text{in.}$$

With cover of 1.5 in., No. 3 stirrups, and moderate-size bars, an overall beam depth is obtained by adding approximately 2.5 in. to this dimension; thus any dimension of 17.8 in. or more will assure a lack of critical compressive stress in the concrete.

In most cases the specified overall depth dimension is rounded off to the nearest full inch. As discussed in Section 6.2, the balanced section is somewhat useful as a reference, but economy and better deflection control will be obtained with greater depth. We will therefore consider the use of an overall depth of 24 in., resulting in an approximate value for the effective beam depth of $24 - 2.5 = 21.5$ in.

For the beams the flexural reinforcement that is required in the top at the supports must pass either over or under the bars in the top of the girders. Figure 10.73 shows a section through the beam with an elevation of the girder in the background. It is assumed that the much heavier loaded girder will be deeper than the beams, so the bar intersection does not exist in the bottoms of the intersecting members. At the top, however, the beam bars are run under the girder bars, favoring the heavier loaded girder. For an approximate adjustment, 3.5 to 4 in. should be subtracted from the overall beam height to obtain an effective depth for design of the top reinforcement.

The beam cross section must also resist shear, and the beam dimensions should be verified to be adequate for this task before proceeding with design of the flexural reinforcement. Referring to Figure 6.18, the maximum shear force is approximated as 1.15 times the simple beam shear. For the beam this produces a maximum shear of

$$V_u = 1.15\frac{wL}{2} = 1.15\frac{2.89 \times 29}{2} = 48.2\,\text{kips}$$

As discussed in Section 6.2, this value may be reduced to the shear at a distance of the beam depth d from the support; thus

$$\text{Design } V = 48.2 - \left(\frac{20}{12} \times 2.89\right) = 43.4\,\text{kips}$$

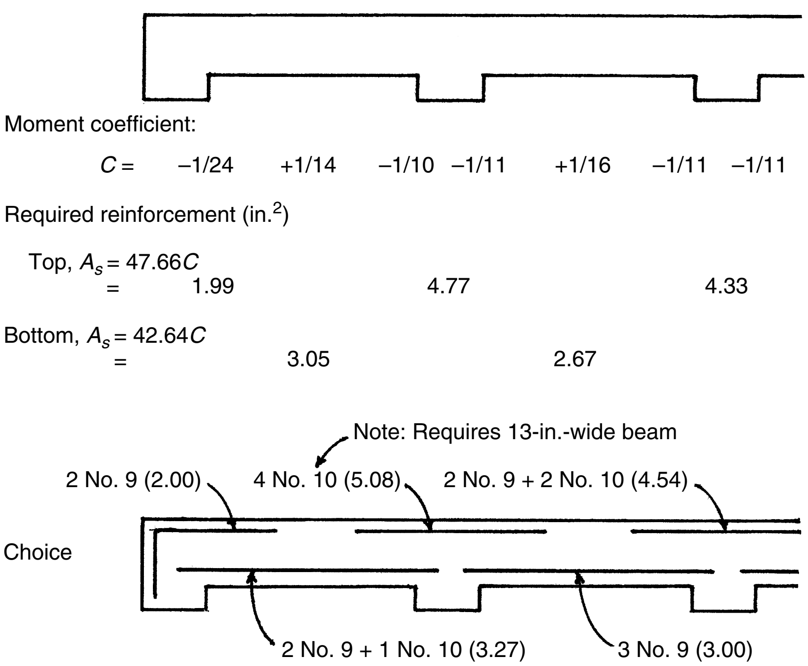


Figure 10.72 Summary of design for the four-span beam.

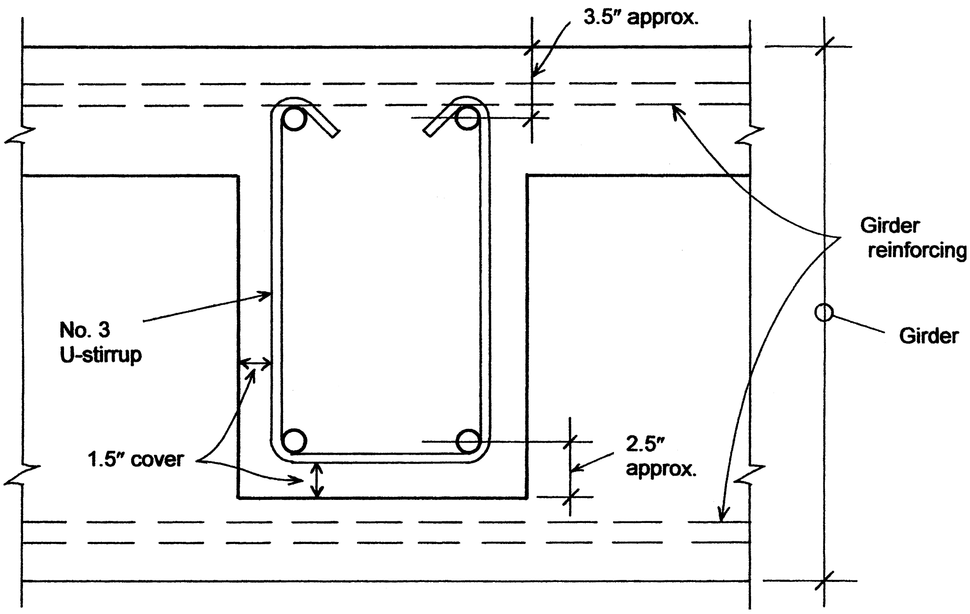


Figure 10.73 Intersection of the beam and girder.

And the maximum required shear capacity is

$$V = \frac{43.4}{\phi} = \frac{43.4}{0.75} = 57.9\,\text{kips}$$

Using $d = 20$ in. and $b = 12$ in., the limiting shear capacity of the concrete alone is

$$V = 2\sqrt{f_c'}bd = 2\sqrt{3000}(12 \times 20)$$
$$= 26{,}290\,\text{lb}, \quad \text{or } 26.3\,\text{kips}$$

This leaves a shear force to be developed by the steel equal to

$$V_s' = 57.9 - 26.3 = 31.6\,\text{kips}$$

and the closest stirrup spacing at the beam ends is

$$s = \frac{A_v f_y d}{V_s'} = \frac{0.22 \times 40 \times 20}{31.6} = 5.6\,\text{in.}$$

which is not an unreasonable spacing.

For the approximate design shown in Figure 10.72, the required area of steel at the points of support is determined as follows.

Assume $a = 6$ in.; thus $d - a/2 = 20 - 3 = 17$ in. Then, using $M_u = CwL^2$, in which C is the approximate moment factor from Figure 10.70,

$$A = \frac{M}{\phi f_y(d - a/2)} = \frac{C(2.89)(29)^2(12)}{0.9(40)(17)} = 47.66C$$

At midspan points the positive bending moments will be resisted by the slab and beam acting in T-beam action. For this condition, an approximate internal moment arm consists of $d - t/2$ and the required steel areas are approximated as

$$A_s = \frac{M}{\phi f_f(d - t/2)} = \frac{C(2.89)(29)^2(12)}{0.9(40)(21.5 - 5/2)} = 42.64C$$

Inspection of the framing plan in Figure 10.69 reveals that the girders on the north–south column lines carry the ends of the beams as concentrated loads at their third points (10 ft from each support). The spandrel girders at the building ends carry the outer ends of the beams plus their own dead weight. In addition, all spandrel beams support the weight of the exterior curtain walls. The form of the spandrels and the wall construction is shown in Figure 10.71. The exterior columns are widened to develop the perimeter bents that will be used for lateral bracing. Assuming a column width of 2 ft, the clear span of the spandrels is thus 28 ft. This span length will be used for the design of the spandrel girders and beams. Design of the lateral bracing bents is discussed later.

The beam loads represent a load periphery equal to that for a single beam (290 ft^2), so the live-load reduction for the spandrel girder is the same as that for one beam. From the beam loading, the total factored load is

$$P = 2.89(30/2) = 43.4, \quad \text{say } 44\,\text{kips}$$

The uniformly distributed load is all dead load, determined as:

Spandrel weight: [(12)(45)](150/144) = 563 lb/ft
Wall weight: (25 psf)(9 ft) = 225 lb/ft
Total = 563 + 225 = 788 lb/ft, say 0.8 kips/ft

The factored load is

$$w = 1.2(0.8) = 0.96, \quad \text{say } 1.0\,\text{kips/ft}$$

For the distributed load, approximate deign moments may be determined using the moment coefficients, as was done for the slab and beam. Values for this procedure are given in Figure 10.70. Thus

$$M_u = C(wL^2) = C(1.0)(28)^2 = 784C$$

The ACI code does not permit use of coefficients for concentrated loads. However, an approximate design may be performed using adjusted coefficients derived from tabulated loadings for beams with third-point placement of loads. Values for these loadings may be found in various references, including the AISC manual (Ref. 10). Using these coefficients, the moments are

$$M_u = C(PL) = C(44)(28) = 1232C$$

As in other cases involving multiple loadings, critical values (moment, shear, deflection, etc.) may be determined for individual loadings and then combined for the total effect of all the combined loadings. Care must be taken, however, to assure that the individual values occur at the same place on the member being investigated.

Figure 10.74 presents a summary of the approximation of moments for the spandrel girder. This is, of course, only the results of the gravity loading, which must also be combined with effects of lateral loading for complete design of the perimeter bents. The design of the spandrel girder is therefore deferred until after discussion of lateral loads for this structure. In any event, the gravity loading alone is one of the combined load cases that must be considered, and when gravity loads are of considerable magnitude and lateral loads are minor, the design for gravity alone may actually turn out to be the critical loading case.

Design of the Concrete Columns

The general cases for the concrete columns are as follows (see Figure 10.75):

Interior column, carrying primarily only gravity loads due to the stiffened perimeter bents

Corner columns, carrying the ends of the spandrel beams and functioning as the ends of the perimeter bents in both directions

Intermediate columns on the north and south sides, carrying the ends of the interior girders and functioning as members of the perimeter bents

Intermediate columns on the east and west sides, carrying the ends of the column-line interior beams and functioning as members of the perimeter bents

Summations of the design gravity loads for the columns may be done from the data given previously. As all columns will be subjected to combinations of axial load and bending moments, these loads represent only axial compression forces. Bending moments will be relatively low on interior columns, since they have beams on all sides. All concrete columns are designed for a minimum amount of bending, so design for axial load alone creates a minimum bending capacity. For an approximate design, therefore, it is reasonable to consider the interior columns for axial gravity loads only.

Figure 10.76 presents a summary of design for an interior column using loads determined from a column load summation with data given previously in this section. As discussed for the design of the steel structure, this is a hypothetical case, as there is actually no freestanding interior column. Note that a single size of 20 in. square is used for all three stories, a common practice permitting reuse of column forms and simplification of placement of reinforcement.

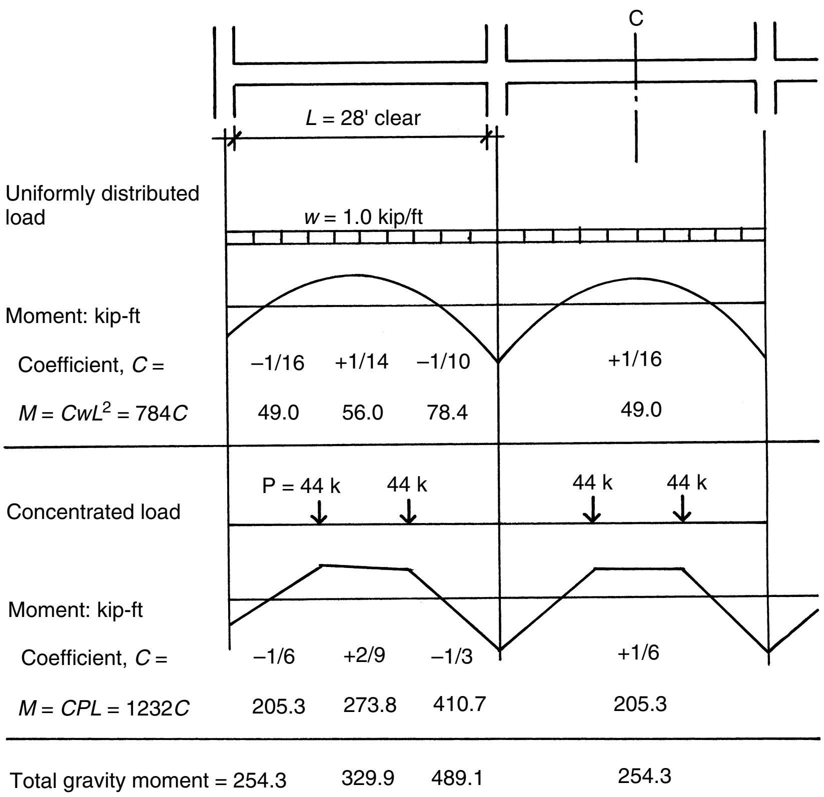


Figure 10.74 Investigation of the spandrel girder for gravity loading.

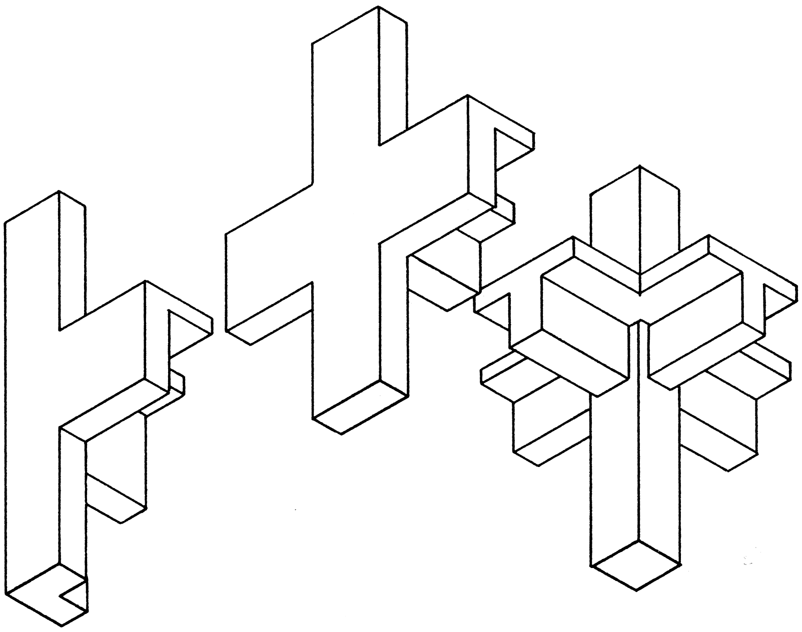


Figure 10.75 General form of the columns and the beams that frame into them.

Square column size: 20" $f'_c = 5$ ksi Grade 60 bars, $F_y = 60$ ksi

Level	Story height	Design ultimate load (kips)	Reinforcement: Bars	Reinforcement: %	Reinforcement: Layout	Reinforcement: Vertical arrangement	Actual factored load capacity with $e = 4"$ (kips)
Roof – 3rd Floor	13'	240	4 No. 10	1.27			680
3rd Floor – 2nd Floor	13'	450	4 No. 10	1.27			680
2nd Floor – 1st Floor		700	8 No. 9	2.00			720
1st Floor – Footing	5'						

Figure 10.76 Design of the interior column.

Column load capacities indicated in Figure 10.76 were obtained from the graphs in Section 6.3.

A general cost-savings factor is the use of relatively low percentages of steel reinforcement. An economical column, therefore, is one with a minimum percentage (usually a threshold of 1% of the gross section) of reinforcement. However, other factors often affect design choices, some common ones being the following:

Architectural planning of building interiors. Large columns are often difficult to plan around in developing of interior rooms, corridors, stair openings, and so on. Thus the *smallest* feasible column sizes—obtained with a maximum percentage of steel—are often preferred.

Ultimate load response of lightly reinforced columns borders on brittle fracture failure, whereas heavily

reinforced columns tend to have a yield form of ultimate failure.

A general rule of practice in rigid-frame design for lateral loadings is to prefer a form of ultimate response described as *strong column/weak beam failure*. In this example this relates more to the columns in the perimeter bents but may also somewhat condition the design choices for the interior columns, since they will take some of the total lateral load.

Where seismic loads are high, special detail requirements will often affect column design.

Column form may also be an issue that relates to architectural planning or to structural concerns. Round columns work well for some structural actions and may be quite economical for forming, but unless they are totally freestanding, they do not fit so well for planning of the rest of the building construction. Even square columns of large size may be difficult to plan around in some cases, an example being at the corners of stair wells and elevator shafts. T-shaped or L-shaped columns may be used in these situations.

Large bending moments in proportion to axial compression may also dictate some adjustments of column form or arrangement of reinforcement. When a column becomes essentially beamlike in its action, some of the practical considerations for beam design come into play. In this example these concerns apply to the exterior columns to some degree.

For the corner columns, the situation is similar to that for the intermediate exterior columns; that is, there is significant bending on both axes. Gravity loads will produce simultaneous bending on both axes, resulting in a net moment that is diagonal to the column. Lateral loads can cause the same effect, since neither wind nor earthquakes will always orient exactly to the major column axes, even though this is mostly how design investigation is performed.

Further discussion of the exterior columns is presented in the following discussion of the lateral bracing system.

Design for Lateral Forces

There are generally four options for the all-concrete structure with regard to design for lateral forces of wind or earthquakes. The basic systems for bracing are:

Exterior structural walls of concrete or masonry of sufficient strength and stiffness to resist the major portion of the lateral loads

Developed shear walls, forming a rigid building core or dispersed in separate panels on the exterior

Rigid-frame bents developed with all the planes of aligned beams and columns

Perimeter rigid-frame bents developed with the exterior columns and spandrel beams

While all of these offer acceptable solutions, the system chosen for this example is that using the perimeter rigid-frame bents, as shown in Figure 10.66. Other elements of the construction may well absorb some of the lateral load as the building deflects sideways. The interior column-line bents and any stiff walls at the building core will act in this manner. Successful design of the perimeter bents requires that they be formed with sufficient stiffness so as to be the primary resisters of lateral deflection.

As the building deflects sideways, any stiff elements of the construction may be subject to some lateral force. Glass tightly held in flexible window frames, stucco, and plaster on light structural frames and nonstructural masonry walls may thus be fractured by the building movements—as they frequently are. A major aspect of design for lateral forces is the development of connections between the bracing structure and the rest of the building construction to minimize these effects. It is a major task to develop the building construction so that everything is held in place but lateral loads—and gravity loads also for that matter—do not get applied to vulnerable elements of the construction.

With the same general building profile, the wind loads on this structure will be the same as those previously determined for the steel structure. As in the example in that part, the data given in Figure 10.62 is used to determine the horizontal forces on the bracing bents. The total horizontal force is applied to the structure through the horizontal elements—roof and floor decks—and then shared by the vertical elements of the bracing system. With two bents in each direction, the north–south bents will thus carry the following loads:

$$H_1 = 165.5(122)/2 = 10{,}096\text{ lb}, \quad \text{say } 10.1\text{ kips/bent}$$
$$H_2 = 199.5(122)/2 = 12{,}170\text{ lb}, \quad \text{say } 12.2\text{ kips/bent}$$
$$H_3 = 210(122)/2 = 12{,}810\text{ lb}, \quad \text{say } 12.8\text{ kips/bent}$$

Figure 10.77*a* shows a profile of the north–south bent with these loads applied. For an approximate analysis consider the individual stories of the bent to behave as shown in Figure 10.77*b*, with the columns developing an inflection point at their midheight points. Because the columns are all deflected the same sideways distance, the shear force in the columns may be assumed to be proportionate to the relative stiffness of the column. If the columns all have the same stiffness, the total load at each story would simply be divided by 4 to obtain the column shear forces.

Even if the columns are all the same size, however, they may not all have the same resistance to lateral deflection. The end columns in the bent are slightly less restrained at their ends (top and bottom) because they are framed on only one side by a beam. For this approximation, therefore, it is assumed that the relative stiffness of the end columns is one-half that of the intermediate columns. Thus the shear force in the end columns is one-sixth of the total bent shear force and that in the intermediate columns is one-third of the total force. The column shears for each of the three stories is thus as shown in Figure 10.77*c*.

The column shear forces produce bending moments in the columns. With the column inflection points (points of

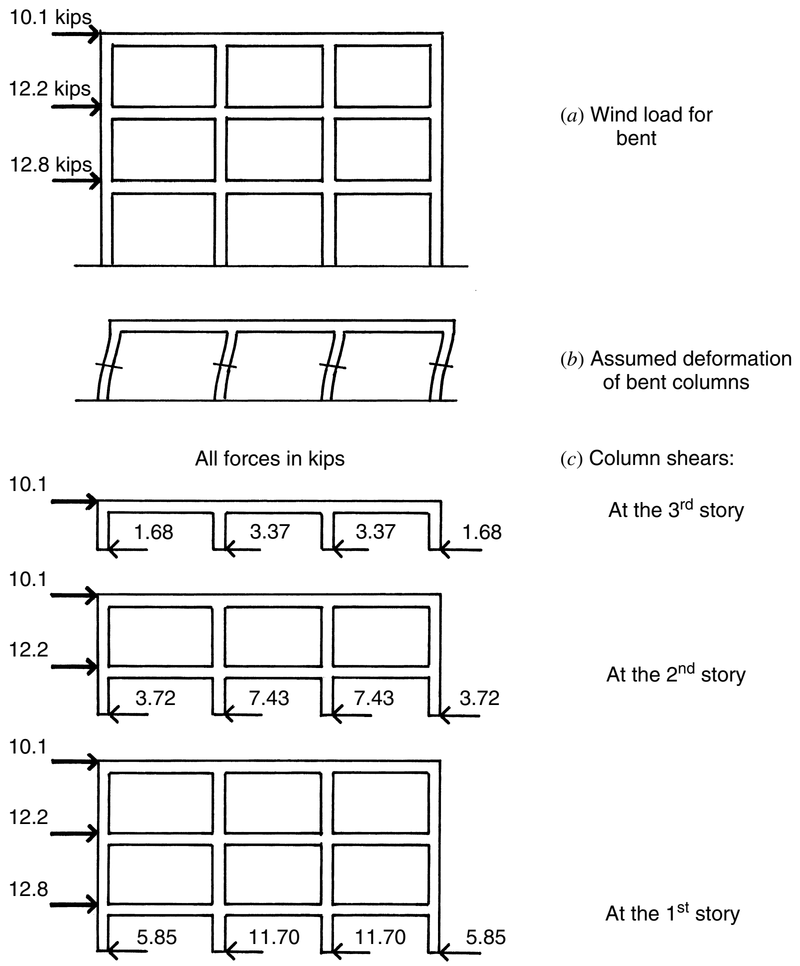


Figure 10.77 Investigation for shear in the bent columns.

zero moment) assumed to be at midheight, the moment produced by a single shear force is simply the product of the force and half the column height. These column moments must be resisted by the end moments in the rigidly attached beams, and the actions are as shown in Figure 10.78, At each column/beam intersection the sum of the column and beam moments must be balanced. Thus the total of the beam moments must be equated to the total of the column moments, and the beam moments may be determined once the column moments are known.

For example, at the second-floor level of the intermediate column, the sum of the column moments from Figure 10.78 is

$$M = 48.3 + 87.8 = 136.1 \text{ kip-ft}$$

Assuming the two beams framing the column to have equal stiffness at their ends, the beams will share this moment equally, and the end moment in each beam is thus

$$M = \frac{136.1}{2} = 68.05 \text{ kip-ft}$$

as shown in the figure.

The data displayed in Figure 10.78 may now be combined with that obtained from gravity load analysis for a combined load investigation.

Design of the Bent Columns

Axial load due to gravity is combined with any moments induced by gravity for a gravity-only analysis. Then the gravity load actions are combined with results from the lateral analysis using the adjustments for this combination.

Gravity-induced moments for the girders are taken from the girder analysis in Figure 10.75 and are assumed to produce the column moments shown in Figure 10.79. A summary of design for the intermediate column is given in Figure 10.80. Two conditions must be considered. The first is that with the gravity loads only. The second adds the lateral effects to the gravity effects. The dual requirements for the columns are given in the bottom of the table.

Note that a single column size is able to fulfill the requirements for all stories, not an unusual situation in many cases.

Design of the Bent Girders

The girders must be designed for the same conditions used for the columns. A summary of bending moments for the third-floor spandrel girder is shown in Figure 10.81. Values for the gravity moments are taken from Figure 10.74. Moments induced by wind are taken from Figure 10.78. Here the effects of gravity prevail and the wind loading is not a critical

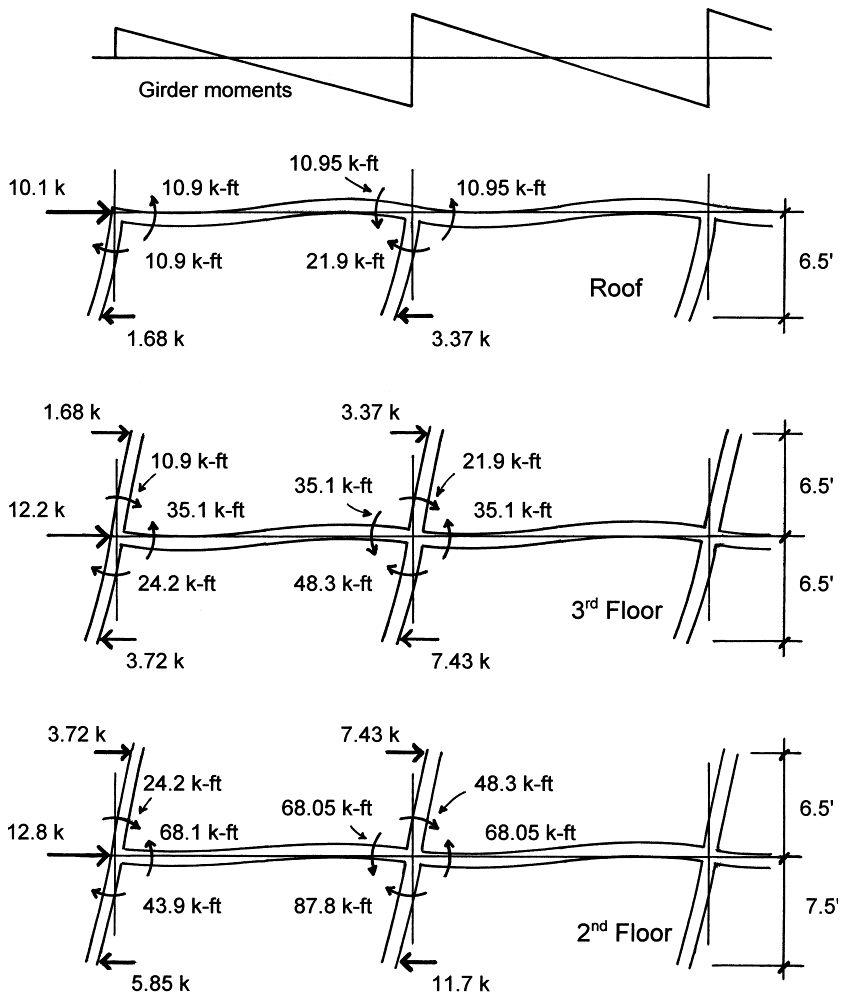


Figure 10.78 Investigation of the bent.

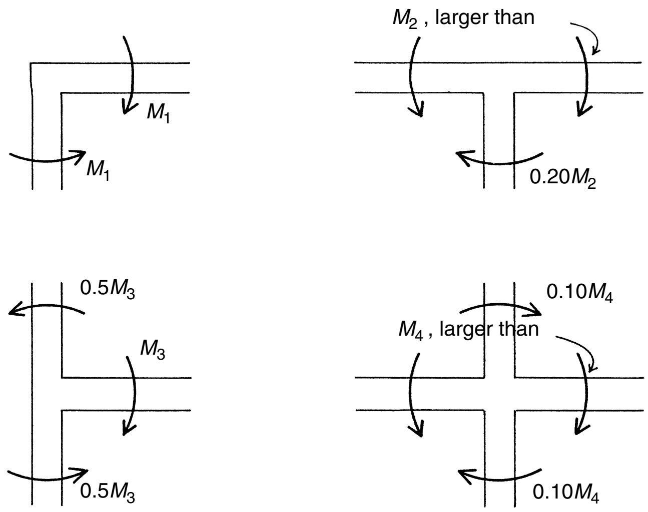


Figure 10.79 Gravity-induced column moments.

condition. This may not be the case in lower stories of taller buildings or possibly with seismic effects.

Figure 10.82 presents a summary of design work for the third-floor spandrel girder. The construction assumed is that shown in Figure 10.72. Some attention should be given to the relative stiffness of the columns and girders in the rigid-frame bent. Keep in mind, however, that the girder is almost three times as long as the column and thus may have a considerably stiffer cross section than the column without causing a disproportionate stiffness relationship to occur.

For computation of the required flexural reinforcement, the T-beam effect is ignored and an effective depth of 40 in. is assumed. Required areas of reinforcement may thus be derived as

$$A_s = \frac{M}{\phi f_s jd} = \frac{M \times 12}{0.9(40 \times 0.9 \times 40)} = 0.00926M$$

From Table 6.3, minimum reinforcement is

$$A_s = 0.005bd = 0.005(16 \times 40) = 3.2\,\text{in.}^2, \quad \text{not critical}$$

Values determined for the various critical locations are shown in Figure 10.82. It is reasonable to consider the stacking of bars in two layers in such a deep section, but it is not necessary for the selections shown in the figure.

The very deep and relatively thin spandrel should be treated somewhat as a slab/wall, and thus the section in Figure 10.82 shows some additional horizontal bars at

Note: Axial loads in kips, moments in kip-ft, dimensions in inches.

Story	Third	Second	First
Gravity load only:			
Axial live load	35	67	103
Axial dead load	83	154	240
Live – load moment	32	38	38
Dead load moment	128	90	90
Ultimate gravity load: $(1.2D + 1.6L)$			
Axial load	100+56 = 156	185+107 = 292	288+165 = 453
Moment	154+51 = 205	108+61 = 169	108+61 = 169
e	15.8 in.	7.0 in.	4.5 in.
Combined with wind:			
Wind load moment	21.9	48.3	87.8
Ultimate moment with wind $(1.2D+1.6W+L)$	154+35+32 = 221	108+77+38 = 223	108+141+38 = 287
Ultimate axial load with wind $(1.2D + L)$	100+35 = 135	185+67 = 252	288+103 = 391
e	19.6 in.	10.6 in.	8.8 in.
Choice of column from Figure 15.8	?? Design as a beam for M = 221	14 × 20 6 No. 9	14 × 24 6 N0. 10

3rd story: Use 14×24, d = 21 in., $A_s = M_u/\phi f_y jd = (221\times12)/0.9[60(0.9\times21)] = 2.6$ in.^{2}
Use same as 2nd story, 6 No. 9 bars (3 at each end). Use 14×24 for all stories.

Figure 10.80 Design of the intermediate bent column.

midheight points. In addition, the stirrups shown should be of a closed form to serve as ties, vertical reinforcement, and (with the extended top) negative-moment flexural reinforcement for the adjoining slab. In this situation it would be advisable to use stirrups for the entire girder span at a maximum spacing of 18 in. Closer stirrup spacing may be required near the supports if the end shear forces require it.

It is also advisable to use some continuous top and bottom reinforcement in spandrels. This relates to some of the following possible considerations:

- Miscalculation of lateral-force effects, giving some reserved reversal bending capacity to the girder
- A general capability for torsional resistance throughout the beam length (interior supported beam ends produce this effect)
- Something to hold up the stirrups during casting
- Some reduction to long-term creep deflection with all sections having double reinforcement

All of the reinforcement described in Figure 10.82 will tend to help reduce surface cracking in the exposed spandrels, a major consideration for the exposed concrete structure.

Design of the Foundations

Unless site conditions require the use of a more complex foundation system, it is reasonable to consider the use of simple shallow bearing foundations (footings) for this building. Column loads will vary, depending on which of the preceding structural schemes is selected. The heaviest loads are likely to occur with the all-concrete structure.

The most direct solution for a single column is a square footing centered on the column. A range of size for these footings is given in Table 6.10. For a freestanding column, the choice is relatively simple once an acceptable design pressure for the supporting soil is established.

Problems arise when conditions involve other than a freestanding column, which is actually the case for most of the columns in this building. On the plan in Figure 10.69 it

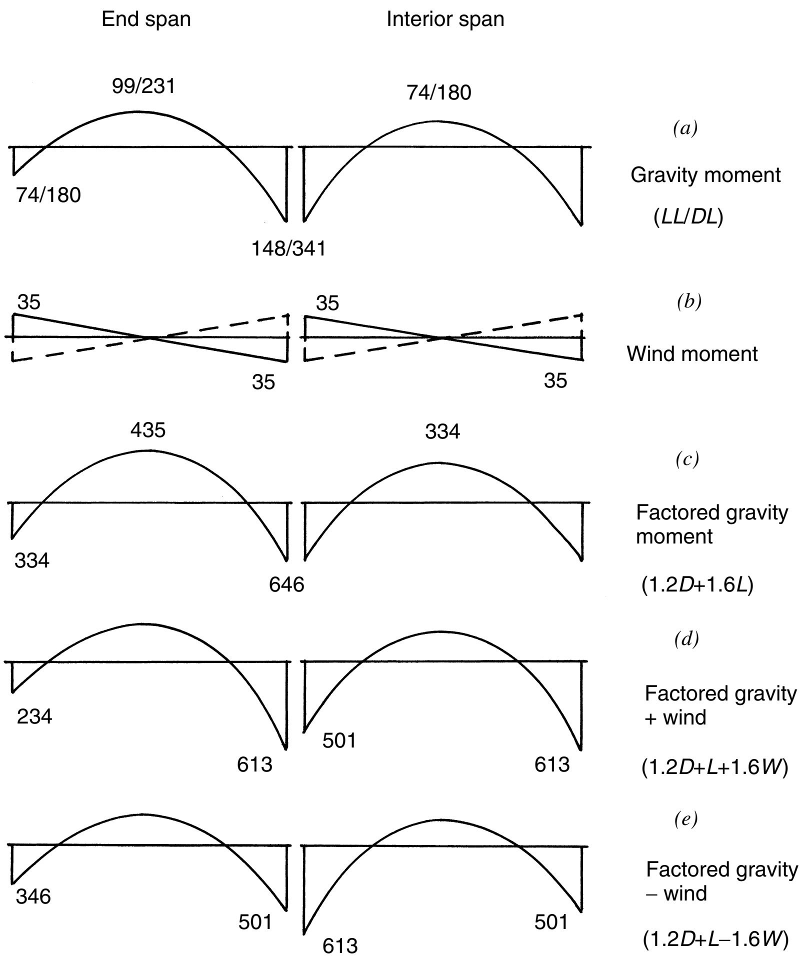


Figure 10.81 Moments for the girder.

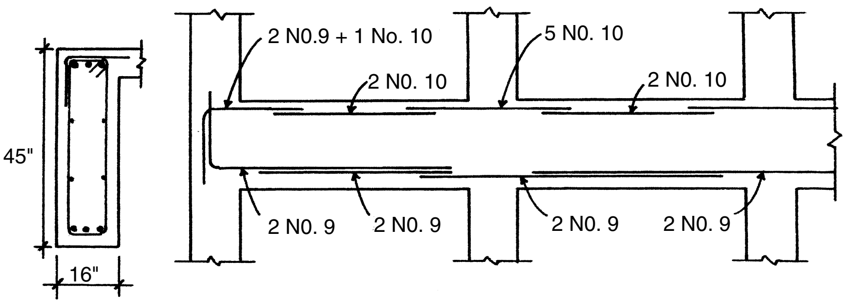


Figure 10.82 Design of the spandrel girder.

Design ultimate moment (see Figure 10.81)	+		435		334
	–	346		646	
Required reinforcement $A_s = 0.00926M$	Top	3.20		5.98	
	Bottom		4.03		3.09
Actual A_s	Top	3.27	2.54	6.35	2.54
	Bottom	2.00	4.00	2.00	4.00

may be observed that all but two of the columns are adjacent to other construction.

The stair tower may not be a problem, although in some buildings these may consist of relatively heavy masonry construction.

Assuming that the elevator serves the lowest level in the building, there will be a deep construction below this level to house the elevator pit. If the interior column footings are large, they may overlap the location of the elevator pit. In any case, these footings would need to be dropped to a level close to that of the bottom of the pit. If the plan layout results in a column right at the edge of the pit, this is a more complicated problem, as the elevator would need to be on top of the column footing. This would probably necessitate the design of a single combined footing for the columns and the elevator shaft and pit.

For the exterior columns there are two concerns. The first has to do with the support for the exterior building walls, which coincide with the location of the columns. If there is no basement and the exterior wall is light in construction—possibly a metal curtain wall supported at each building level—the exterior columns will likely get their own individual square footings and the short foundation wall will rest on the column footings and a small strip wall footing between the columns. This scheme is shown in the partial foundation plan in Figure 10.83*a*.

If the wall is very heavy, it may be reasonable to consider the use of a continuous wide wall footing that supports both the columns and the foundation wall, as shown in Figure 10.83*b*. If the strip footing is used and the column loads are not too great, it may be possible to use a tall foundation wall as a distributing beam in combination with the concentrated column loads and the distributed soil pressures. This continuous wall/beam could be simply designed for the required flexure and shear, with some concentrated area of continuous bars at the top and bottom and the usual wall reinforcement.

(*a*)

(*b*)

(*c*)

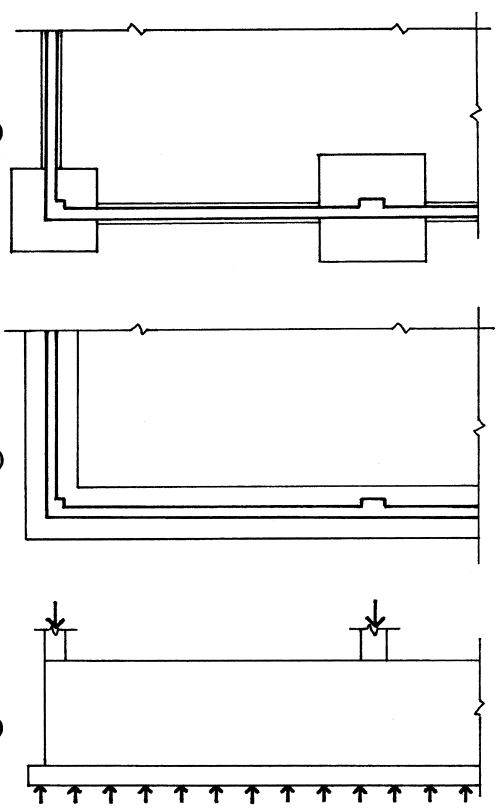


Figure 10.83 Foundations for Building Seven.

10.9 BUILDING EIGHT

Figure 10.84*a* shows a typical plan for the upper floor of a multistory apartment building that utilizes shear walls as the bracing system in both directions. In the north–south direction, the interior walls between the apartments are used together with the two end walls at the stairs. In the east–west direction, the long corridor walls are used. These long walls would be designed as continuous pierced walls, rather than as individual piers between the door openings. The presence of all of this permanent interior construction would not work well with most commercial occupancies but could be considered for apartments, dormitories, hotels, or jails.

The long east–west exterior wall could be developed with a conventional beam system using the ends of the shear walls as columns. Floors would most likely be developed as two-way-spanning slabs without beams, a scheme that considerably reduces the floor-to-floor distance and the overall building height.

There is a limit to the height or number of stories for the shear wall-braced building. One critical concern is for the aspect ratio of height to plan width; if the ratio is too high, overturn and lateral drift (deflection) may become critical. As shown in Figure 10.84*b*, the apartment-separating walls have an aspect ratio of 6 : 1, which is often considered as a practical limit. For the corridor walls the aspect ratio is quite low, so overturn is not likely a problem.

The concrete shear wall structure is successful for resisting wind but less successful for tall buildings in zones of high seismic risk. The combination of wall stiffness and very high construction weight generates a major lateral seismic force, whereas the weight is actually useful for wind resistance. For a building half this height, however, concrete or masonry shear walls may be equally practical for resistance of wind or seismic loads.

Change in this building's plan at the first story might create a special problem. If this level is developed with a relatively open plan, a soft- or weak-story condition may be created. All changes in the vertical bracing structure must be carefully studied for critical effects, especially for seismic loading.

Alternative Braced Steel Structure

If steel construction is a feasible solution for the apartment building, lateral resistance can be developed with either a braced frame (trussed frame) or a rigid frame. Some options for development of north–south trussing are shown in Figure 10.85. The schemes shown in Figures 10.85*a, b,*

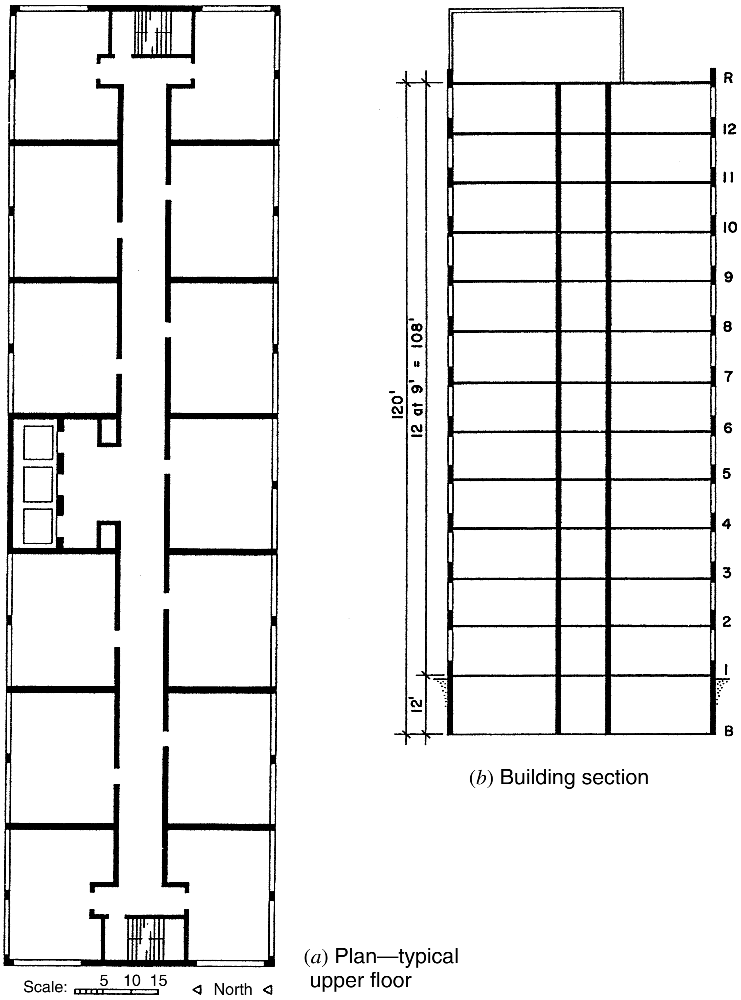


Figure 10.84 Plan and section for Building Eight.

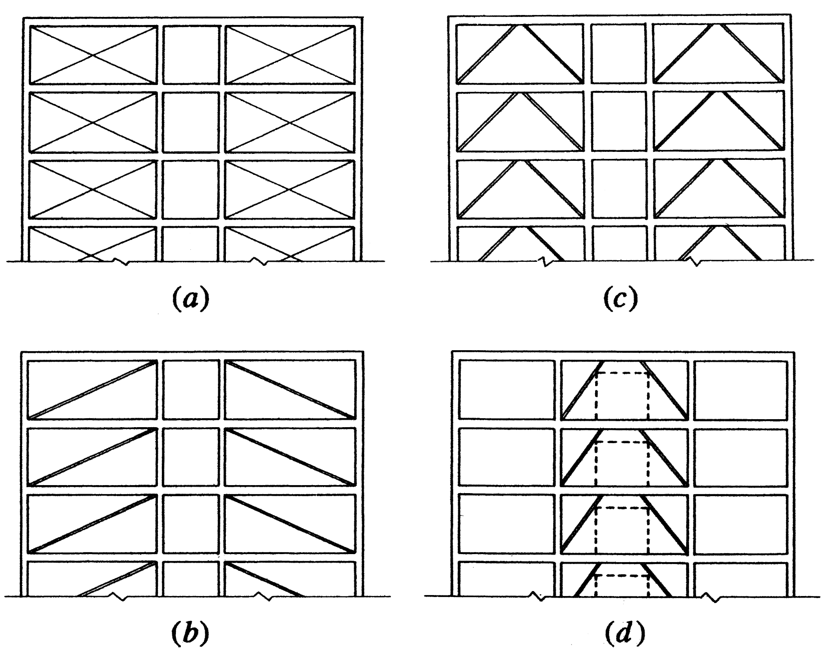


Figure 10.85 Trussed bracing for Building Eight.

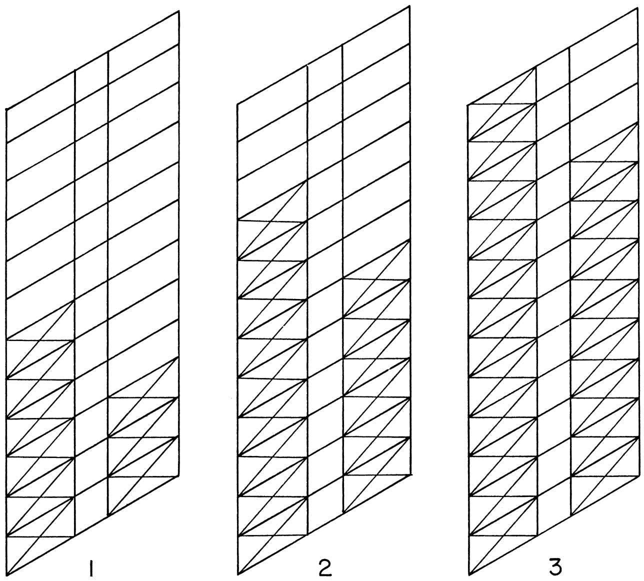


Figure 10.86 Form of the stepped bracing.

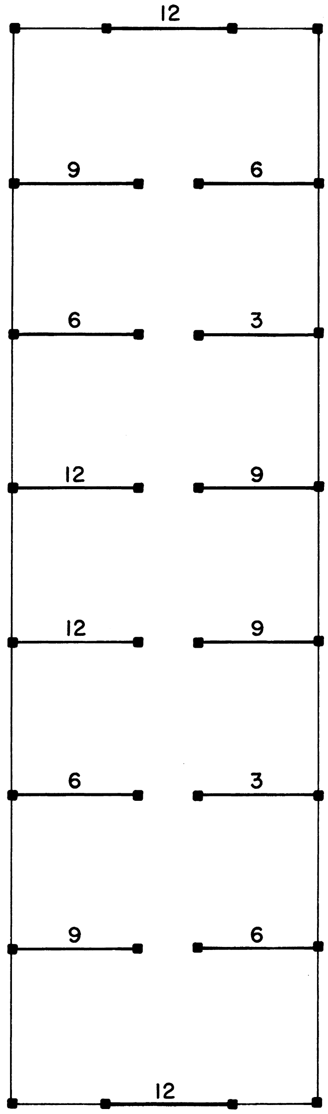


Figure 10.87 Plan arrangement of the stepped bracing.

and *c* utilize various forms of bracing in the same walls that were used for shear walls for the concrete structure. Thus, architectural planning for the braced frame is similar to planning for solid shear walls.

It is doubtful that it would be necessary to use bracing in every wall for the full height of the building. Figures 10.86 and 10.87 show the use of staggered or stepped bracing, in which some bays of bracing are reduced in upper floors. Figure 10.86 shows the different planes of bracing for this scheme and Figure 10.87 locates the different framed bents on the building plan. This form of graduated strength corresponds to the variation in lateral shear in the building height.

Selection of the bracing form and the arrangement in both plan and profile depend partly on the magnitude of loads. For seismic resistance, an eccentric bracing system may provide the energy capacity of a rigid frame with the relatively high resistance to drift of the trussed frame. The chevron-form system shown in Figure 10.85*d* is such a system, with trussing developed in the same location as the shear walls. An alternative system is shown in Figure 10.85*d*, with an arrangement that facilitates the placement of the corridor.

A steel rigid frame is also possible but probably not feasible due to the cost of the large number of rigid beam-to-column connections.

Alternative Concrete Rigid Frame

The plan shown in Figure 10.88 shows a scheme for the development of a concrete framed structure for the apartment building. East–west bracing is provided by the two long-perimeter bents. While more columns at a closer spacing might be used to stiffen these bents, the scheme shown is probably adequate for this building height.

For the north–south bracing, it is not feasible to expect the horizontal diaphragm to span from end to end. In addition, the north–south bents are much shorter in plan length. Therefore the scheme shown uses the two column bents on either side of the elevator core, with closely spaced columns emulating the two end bents.

Of course, there would be additional vertical supports for the floor and roof structures. The plan in Figure 10.87 indicates only the structure involved in the lateral bracing. While some additional vertical planar bents may be defined with the added interior columns and beams, their contributions to the lateral bracing can be controlled by the development of the stiffness of the bents. For example, in the north–south direction, the closer spacing of the columns as shown in Figure 10.88 will produce very stiff bents, while other bents are likely to have wider column spacing and offer less deflection resistance. Similarly, the elongated form of the exterior columns on the north and south sides together with deeper spandrel beams (as with Building Seven) will produce stiff bents.

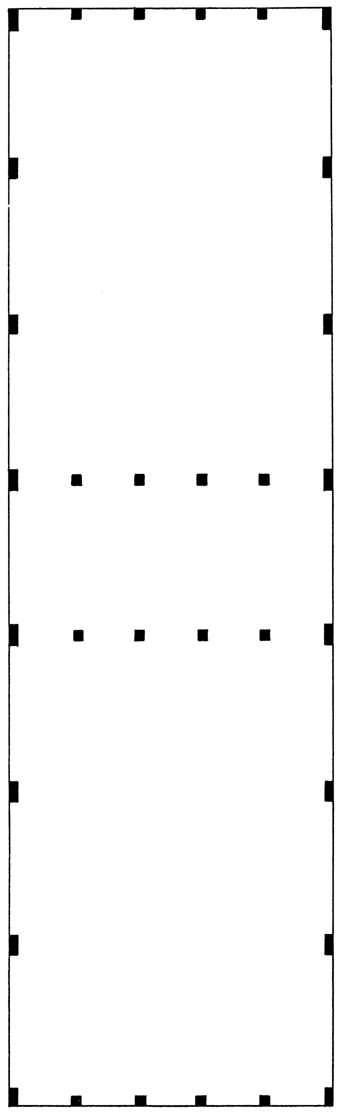


Figure 10.88 Plan of the rigid-frame bents.

As with any building, the choice of the lateral bracing system has implications for architectural planning. In this example, if the scheme shown in Figure 10.88 is used, the layout of rooms would be less constrained than it would be with the shear wall or braced frame schemes. It might be favored, for example, if apartments of differing sizes were to be placed on the same floor. It also generally frees the individual levels from concern for vertical alignment of walls, except for the need to acknowledge the aligned interior columns.

A special problem to consider with the rigid frame is the potential for modification of the frame behavior by nonstructural elements of the construction. Wall construction installed between the columns in the interior bents must be carefully detailed to avoid restricting normal rigid-frame deformations of the structure. The exterior walls must be carefully detailed as well. This is a two-way protection: permitting the structure to flex normally and preventing structural deformations from damaging nonstructural elements.

10.10 BUILDING NINE

This section presents some possibilities for developing the roof structure and the exterior walls for a sports arena (see Figure 10.89). Options for the structure are strongly related to the desired building form.

General Design Requirements

Functional planning requirements derive from specific activities to be housed and from seating, internal traffic, overhead clearance, and exit and entrance arrangements. Choice of the truss system shown here, for example, relates to a commitment to a square plan and a flat roof. Other choices may permit more flexibility in the building form or also limit it. Selection of a domed roof, for example, would pretty much require a round plan.

In addition to the long-span structure in this case, there is the problem of developing the 42-ft-high curtain wall. Braced laterally only at the top and bottom, the wall must achieve the 42-ft span in resisting wind or seismic forces.

In this example, the fascia of the roof trusses, the soffit of the overhang, and the curtain wall are all developed with ordinarily available construction products. The vertical span of the wall is achieved by a series of trusses that directly brace the vertical mullions.

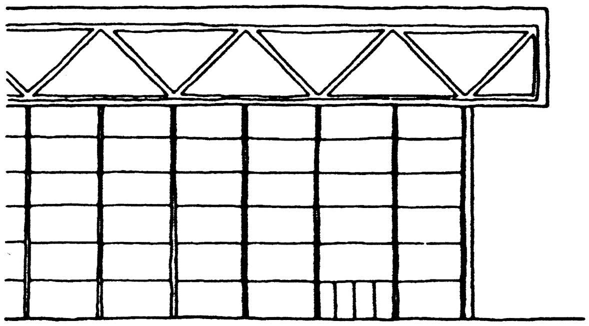
As an exposed structure, the truss system is a major visual element of the building interior. The truss members define a pattern that is orderly and pervading. There will, however, most likely be many additional items overhead—within and possibly below the trusses. Any coordination of the arrangement of these items with the truss modules will serve to produce a more orderly appearance. These items may include the following:

Elements of the roof drainage system
Ducts and registers for the HVAC system
A general lighting system
Signs, scoreboards, etc.
Elements of an audio system
Catwalks for access to the various equipment

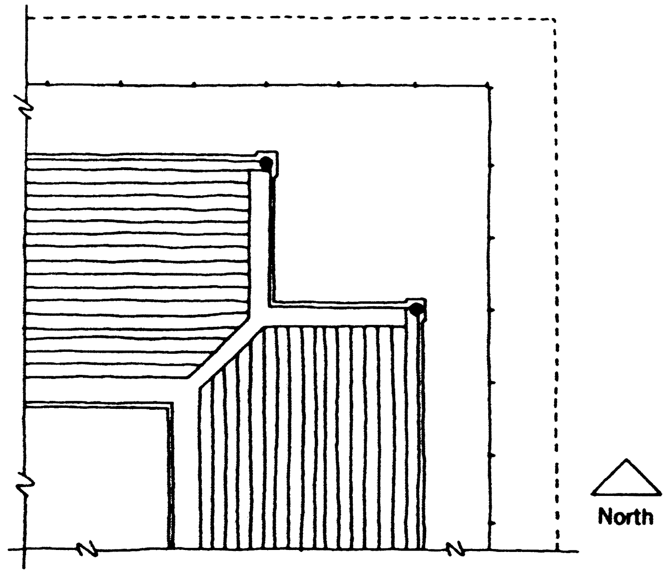
Structural Alternatives

The general form for the construction is shown in Figure 10.90. Discussion for this building will be limited to the development of the roof structure and the tall exterior wall. Some proposed details for the exterior wall are shown in Figure 10.91 using elements of a proprietary curtain wall system supported vertically by a framework of steel tubes and braced laterally by a series of trusses.

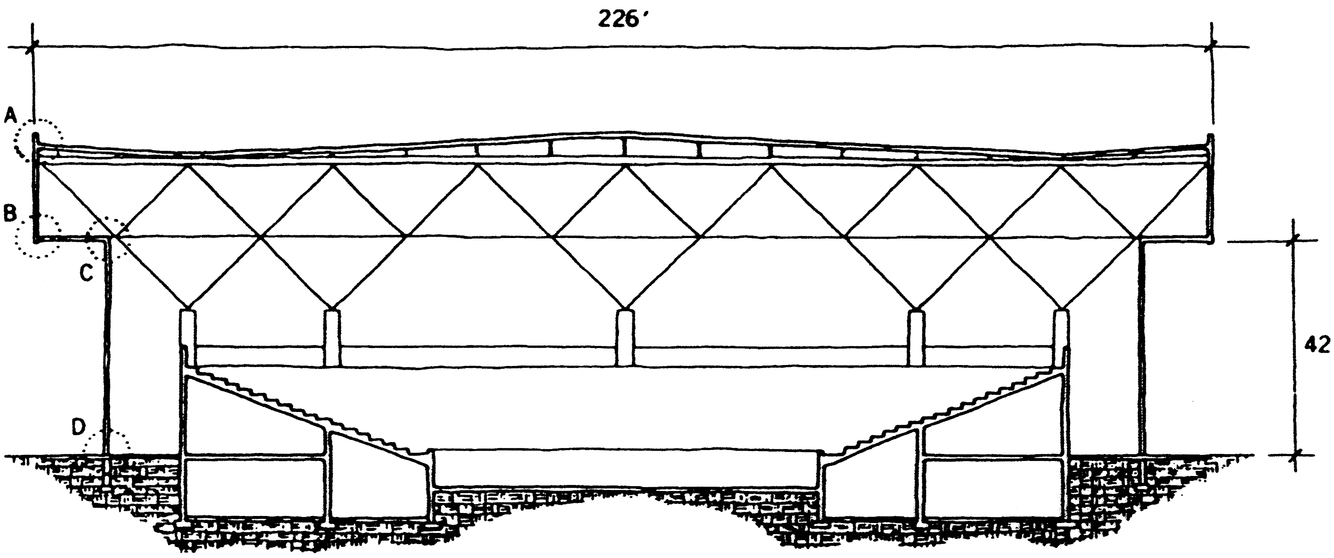
The 168-ft clear span used here is in the class of long-span structures, but not so great as to limit the options. A flat-spanning beam system is definitely out, but a one- or two-way truss system is feasible for the flat span. Most other options involve some form other than flat; this includes domes, arches, shells, folded plates, suspended cables, cable-stayed systems, and pneumatic systems.



Partial elevation



Partial plan



East–west section

Figure 10.89 General form of Building Nine.

The roof structure shown here uses a system described as an *offset grid*. The planning of this system requires the development of a module relating to the frequency of nodal points (joints) in the truss system. Supports for the truss must be provided at nodal points, and concentrated loads should be applied at nodal points. Although the nodal module relates basically to the truss, its ramifications in terms of supports and loads extend it to other parts of the building planning.

The basic module used here is 3.5 ft, or 42 in. Multiples and fractions of this basic dimension (X) throughout the building are in two and three dimensions. The truss nodal module is 8X, or 28 ft. The height of the exterior wall from ground to the underside of the soffit is 12X, or 42 ft.

Here, the roof construction on the top of the trusses, the roof edge fascia, the soffit, and the curtain wall use off-the-shelf products. The supporting columns, general seating, and other parts of the interior construction may also be of standard forms.

Window Wall Support Structure

As noted previously, the curtain wall itself consists of readily available products. The tubular steel supporting

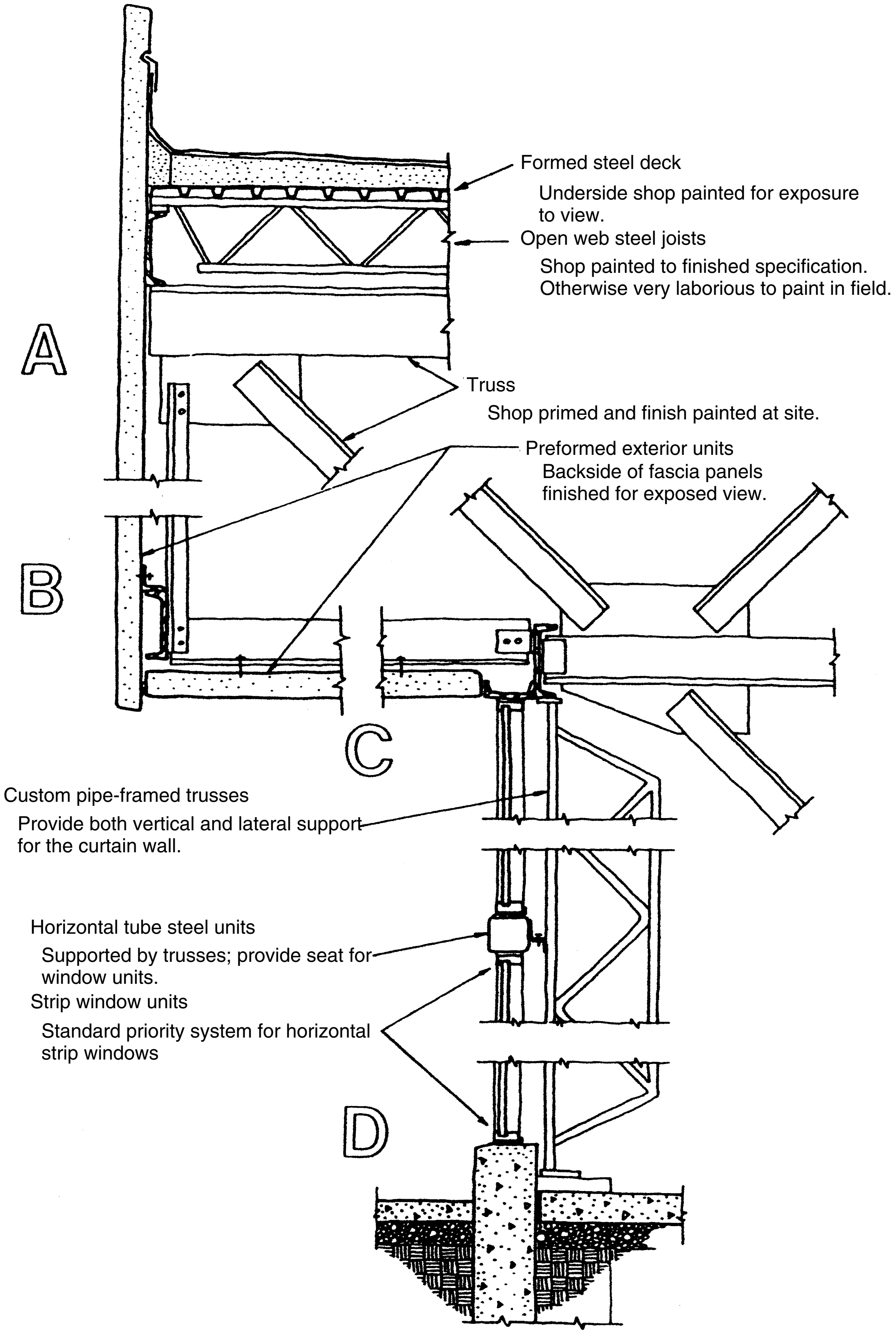


Figure 10.90 General construction details.

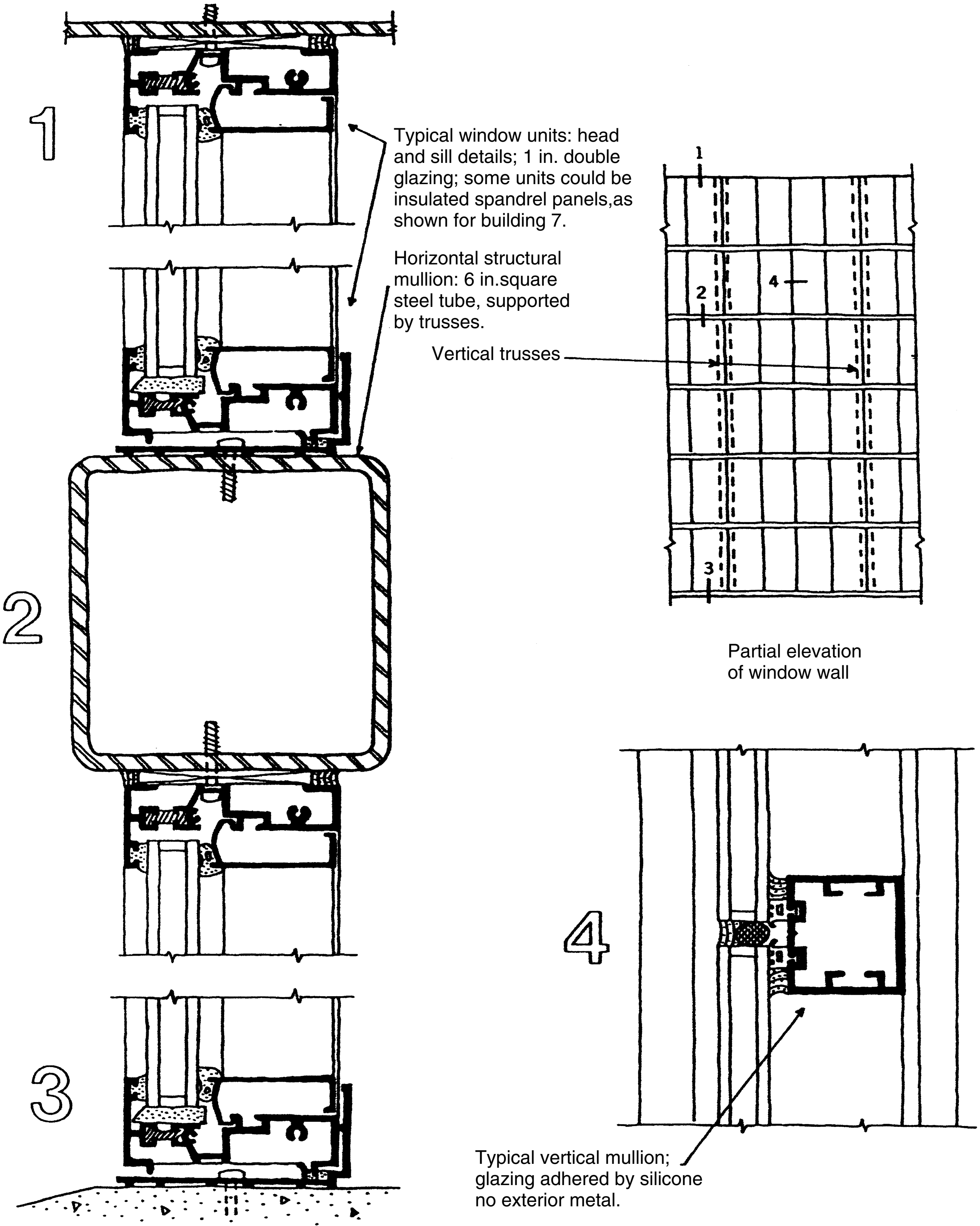


Figure 10.91 Wall construction details.

frame is itself self-supporting to a degree. When used for ordinary multistory construction, the window system is usually independently self-supporting if the clear story height is not more than 15 ft or so and the wall spans vertically.

In this situation, however, the 42-ft height is about three times the height of a normal single story. Thus, some additional support structure is indicated for both vertical gravity weight of the wall and lateral forces (probably from wind, because the wall is quite light in weight). The details in Figures 10.90 and 10.91 show the use of a two-component system consisting of trusses that span the 42-ft height and are spaced 14 ft on center and horizontal steel tubes that span the distance between trusses and support 7 ft of vertical height of the wall. The horizontal tubes may be supported by the trusses or may intersect corresponding vertical tubes to constitute a two-dimensional frame in front of the trusses.

It is possible, because of the relatively light weight of the wall, that the vertical loads may be carried by the closely spaced vertical mullions. In this case, the primary function of the horizontal tubes may be to span the 14 ft between trusses to resist wind loads. The tube shown here is certainly adequate for this task, although stiffness is probably more critical than bending strength in this situation. It is really not good for the wall construction to flip-flop during either seismic movements or fast reversals in wind direction.

Although not shown in the drawings, the form of the truss would probably be that of a so-called *delta truss*, with two chords opposing a single one, thus creating a triangular form in cross section (like the form of the Greek capital letter delta). A principal advantage of this form of truss is its relatively high lateral stability, permitting it to be used in a freestanding manner, without a need for additional lateral bracing.

The tall wall is not an unusual situation and there are many ways of achieving it. Most solutions, however, require some embellishment of the standard window wall products, especially to develop the horizontal (lateral) load conditions.

Truss System

The truss system shown here might be developed from various available proprietary systems produced by different manufacturers. In general, the designers should pursue the availability of these systems when beginning the design of such a structure. A completely custom-designed structure of this kind requires enormous investments of design time for basic development and planning of the structural form, development of the truss nodal point connections, and reliable investigation of the highly indeterminate structure. All of this special work is achievable, but the time and cost must most likely be justified by a strong desire for the visible form of the finished structure, as is often the case with long-span roof structures.

Aside from the considerations of the plan form, planning module, and development of the supports, there is the basic issue of the particular form and general nature of the truss structure. The square plan and biaxial plan symmetry seem to indicate the logic for a two-way spanning structure here. This is indeed what is shown in Figure 10.89. The offset grid form used here is characterized by the relationship between the layouts of the top and bottom chords of the truss system. The squares defined by the top chords are offset from those described by the bottom chords so that the top-chord nodal points (joints) lie directly over the center of the bottom-chord squares. As a result, there are no vertical web members and generally no vertical planar sets within the system.

The reader is referred here to the general discussion of two-way-spanning systems in Section 3.4. The remaining discussions of the truss system will make several references to issues and illustrations presented in that section.

Truss Supports

Location and form of the truss supports are critical issues for both the design of the truss system and the planning of the building. The support system shown here consists of a set of columns that are freestanding in the building interior, with the roof structure cantilevered between the line of supports and the edge of the roof. This system reduces the span of the truss system, which considerably assists the spanning structure.

The exact details of the connections between the columns and the truss system are discussed with regard to alternative solutions for the truss system form. Supporting the truss with columns at a single bottom-chord nodal point will usually produce some high concentrations of stress for the truss web members. A method used to alleviate this condition consists of placing a spreader between the top of the column and the bottom of the truss, thus permitting the distribution of the support force to a number of truss nodal points.

Development of the Roof Infill System

All of the schemes for the roof truss system shown here involve the provision of a grid of truss top chords at 28-ft centers in both directions at the general level of the surface. This provides a basic roof support system, but it does not develop a roof as such. Something else must be done to achieve the infilling, surface-developing construction whose surface can be covered with a roof membrane of some form. This is a common situation with roof framing structures—just a bit more challenging here.

Although some imaginative two-way spanning system might be developed for this structure, it is also possible to use various simpler systems merely to fill in the 28-ft-span voids. Figure 10.92 shows a very simple system consisting of steel open-web joists and formed sheet steel decking. For this construction the joists use the truss chords in one direction for support, thus creating a somewhat unsymmetrical loading condition for the truss top chords. However, this would most likely not affect the basic development of the larger truss system.

The steel could be exposed on its underside, along with the open-web joists. However, for extra thermal insulation and possibly for sound control, it may be desirable to provide a ceiling surface at the bottom of the joists.

A problem related to the roof infill system is that of the provision of drainage for the nominally "flat" roof. The roof

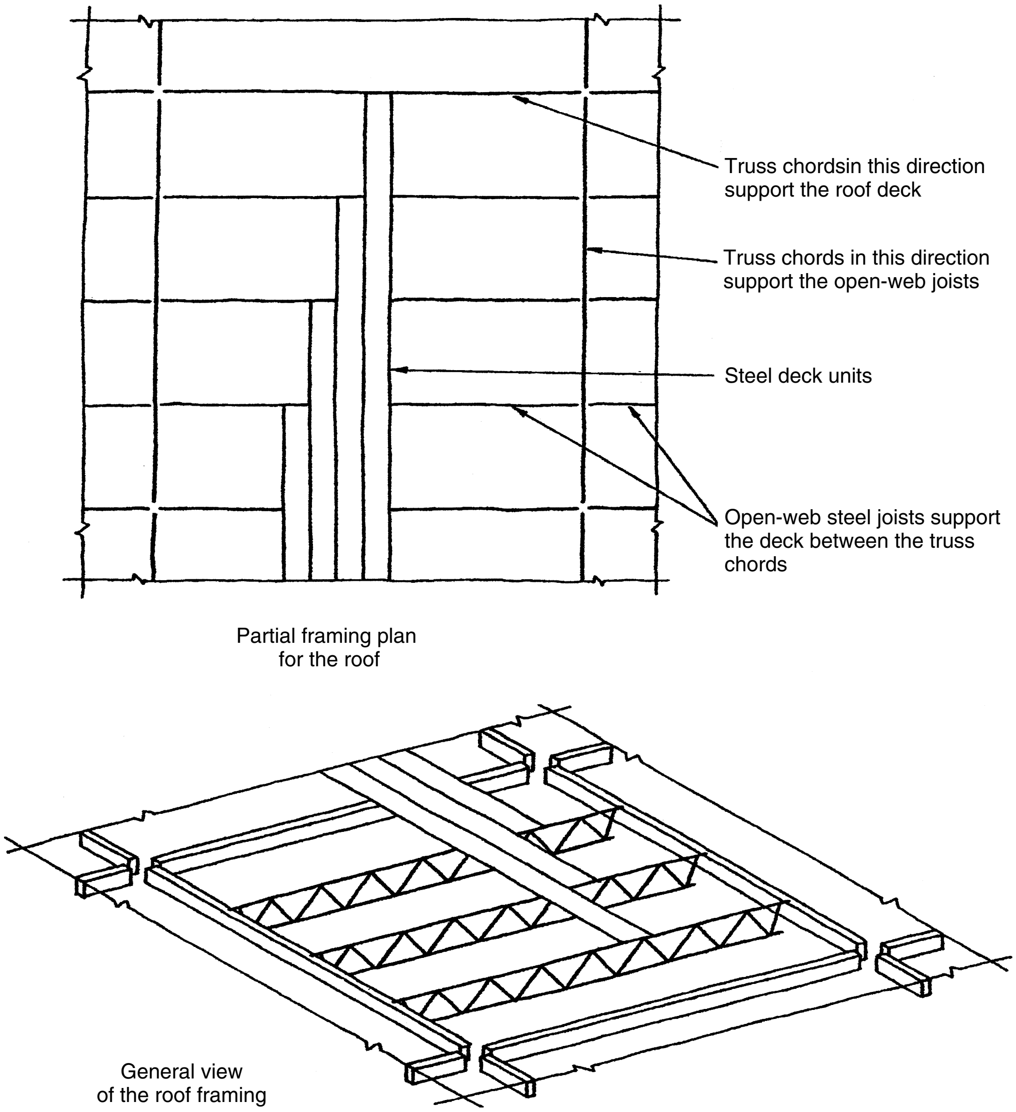


Figure 10.92 Development of the roof infill system.

surface would probably be sloped from a high point at the center of the roof to a low point near the roof edge. The roof drains would then be of the area drain form with vertical leaders carried down in the building interior, possibly at the location of the supporting columns. The building section in Figure 10.89 shows a roof profile developed on this basis.

The sloped roof surface could be achieved by sloping the tops of the trusses but is more likely to be accomplished by installing the trusses flat and building up the sloped surface on top of the trusses and possibly on top of the open-web joists.

Alternative One: Offset Grid System

Figures 10.93 and 10.94 show the general form and plan of the offset grid system. The plan shows the placement of the grid system resulting in the locations of the supports beneath top-chord joints.

The plan in Figure 10.94 indicates the use of three columns on each side of the structure, providing a total of 12 supports for the truss system. The tops of the columns are dropped below the trusses to permit the use of a pyramidal module of four struts between the top of the column and the joints in the bottom chords of the truss system. This reduces the magnitude of shear in the truss web members. While there are only 12 columns, there are a total of 48 struts. Thus a single strut carries only 1/48th of the full load of the truss system.

Except for the edges of the truss system, all truss joints consist of the joining of eight members, four chords and four diagonals. This makes for a complex joint, and its formation is a critical part of the truss design. For the joint design there are two critical aspects. The first concerns the size and form of the truss members. Each of these members is approximately 28 ft long, so they will not be small in cross-sectional size. Both the shape and size of the members must be considered in developing the connection method, which is the second critical aspect for the joint design.

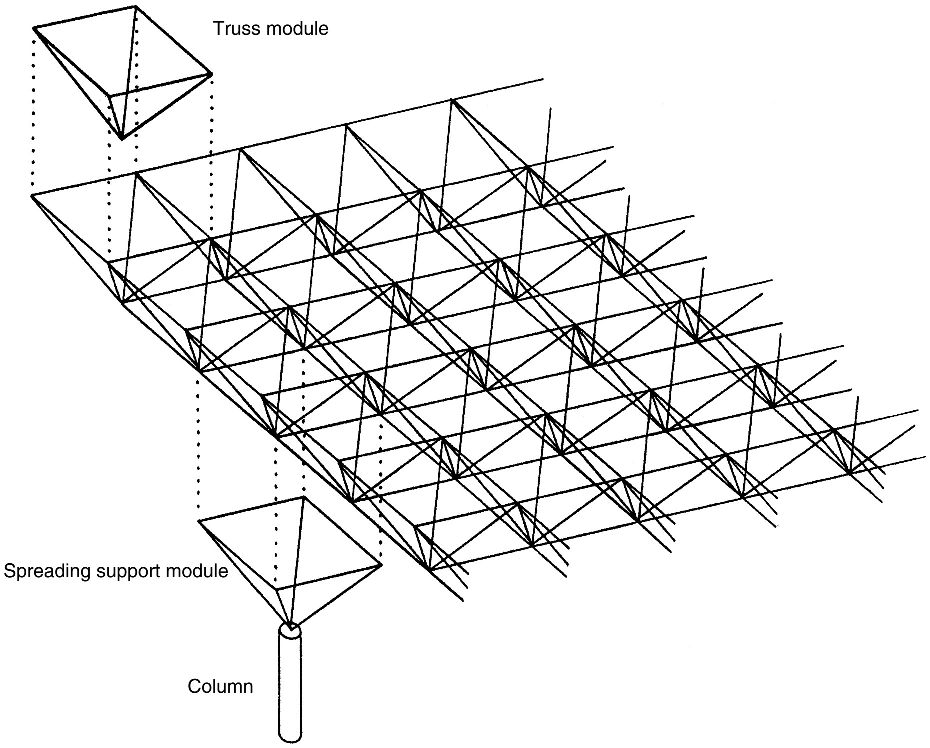


Figure 10.93 General form of the offset grid system.

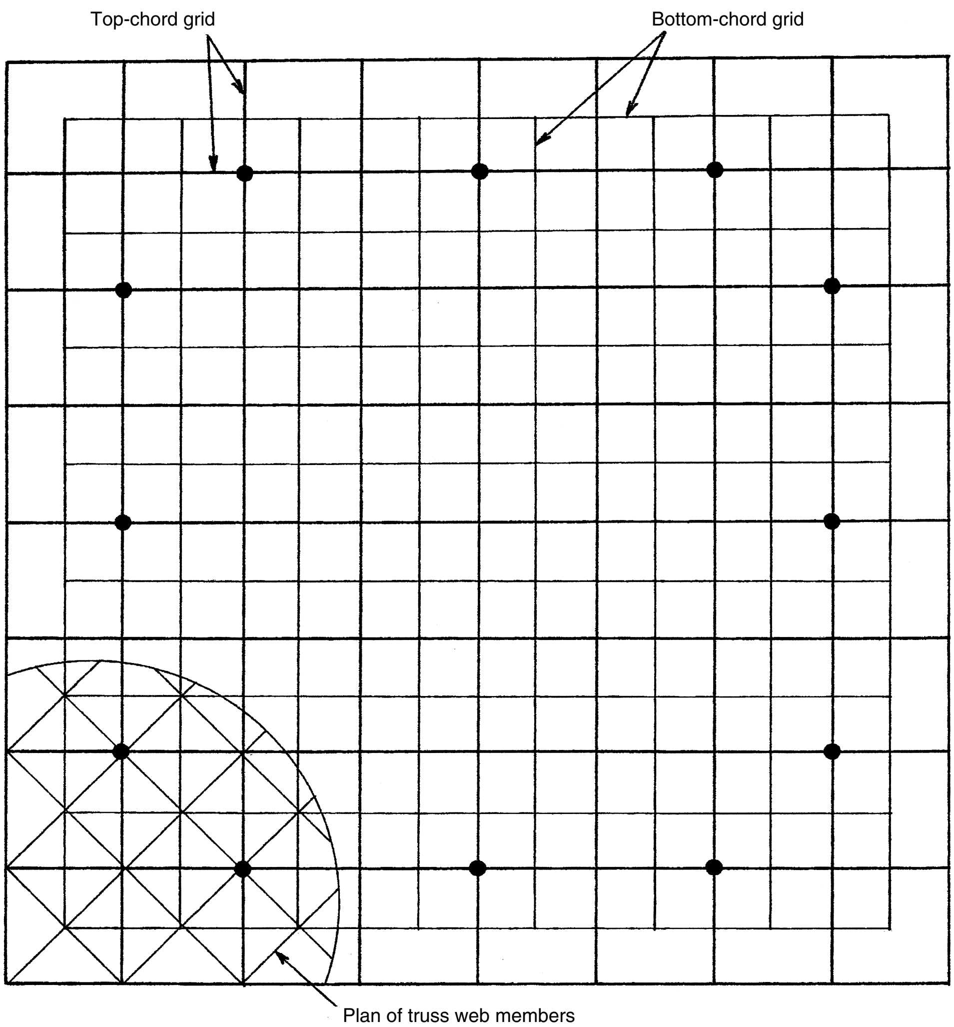


Figure 10.94 Plan of the offset grid system.

This joint occurs 145 times, including the edge joints. The joints must be relatively easy and economical to achieve for the structure to be cost effective. Development of the joints must be done with inputs from steel fabricators.

This truss form has been utilized many times with a range of sizes. At the scale here, the truss members are most likely to be single-rolled W shapes, steel pipe, or steel tubes. Because of the standard form of these shapes, plus the likelihood that only a few different sizes will be used, one possible solution for the joints is direct welding of the members to each other. For this assemblage, the member ends would be cut to fit against each other at the joints. While this sounds complex, it is a relatively routine job for experienced steel fabricators.

For assembly in the fabrication shop, transportation to the site, and erection, consideration must be given to breaking down the whole truss system into segments that are feasible to handle. This means that some of the joint connections will be achieved in the shop and some in the field. For economy, most assembly should be done in the shop, but the achieving of field connections must be accommodated.

Investigation of the forces in the members and joints of this structure involves highly complex mathematical procedures. However, with the use of computer-aided analyses, this is no longer an insurmountable task. Helping matters in this example is the simplification due to the biaxial symmetry of the structure in plan.

Resistance to lateral-force effects within the truss itself is quite easily achieved. The critical lateral-force problem here consists of the situation at the top of the supporting columns. The entire lateral force of the truss must be resolved at the top of these 12 columns—surely the most critical concern for the column design.

Since the columns are all likely to be integrated with the structure for the seating, the vertical cantilever distance for the columns can be reduced to the portion extending above the seating structure. (See the building section in Figure 10.89.) Although a round column shape is shown here, a square one would be more effective for moment.

Alternative Two: Two-Way Vertical Planar Truss System

Another possibility for the truss is shown in Figure 10.95. This consists of intersecting sets of vertical planar trusses. In this system, the top-chord grid squares are directly above the bottom-chord grid squares.

Figure 10.96 shows a plan layout for this system in which the columns occur at the center of the truss chord grids. Support of the truss could be achieved with a structural unit as shown in the previous example. However, what is shown

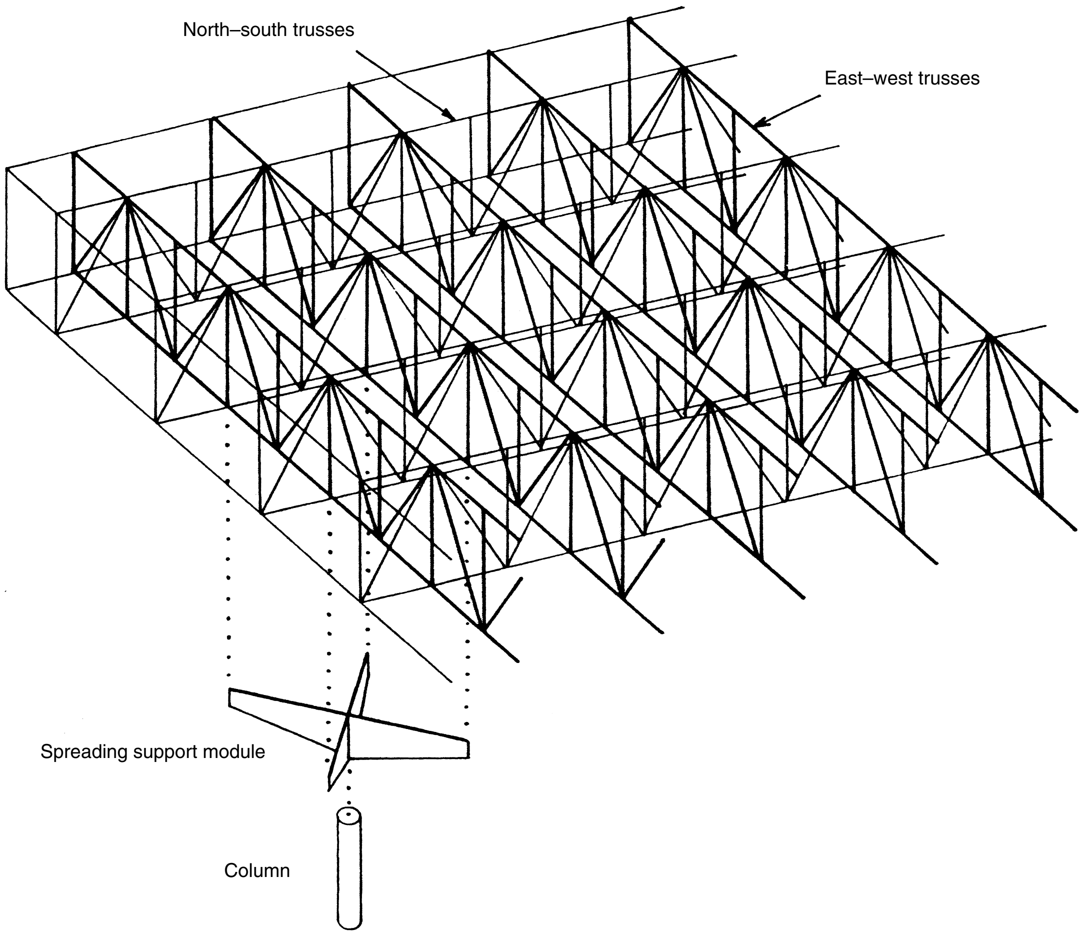


Figure 10.95 Form of the two-way truss with vertical planar trusses.

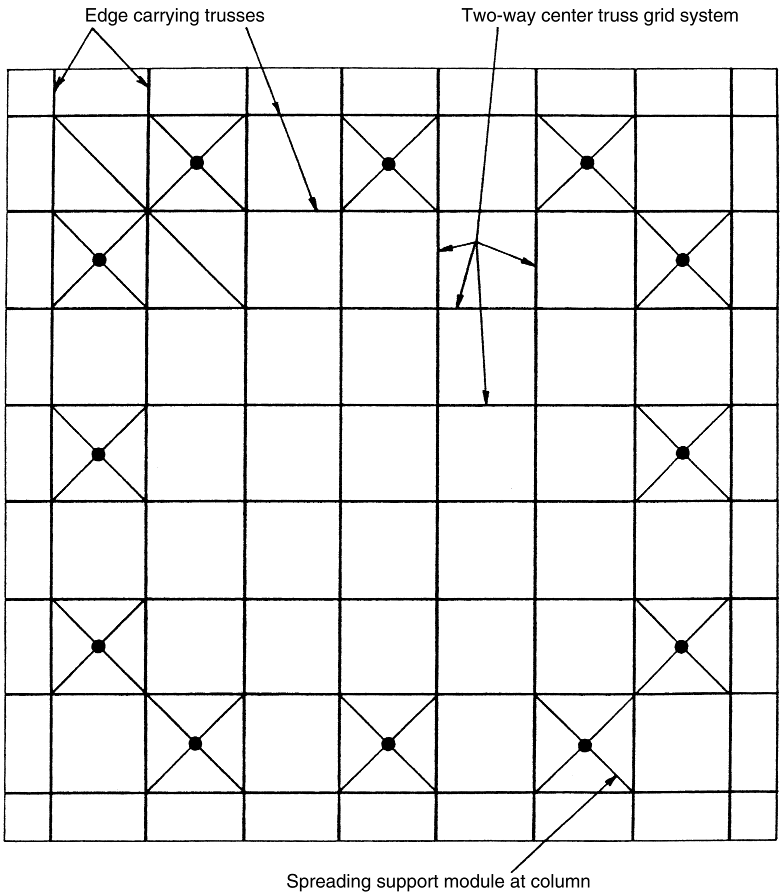


Figure 10.96 Plan of the system with vertical planar trusses.

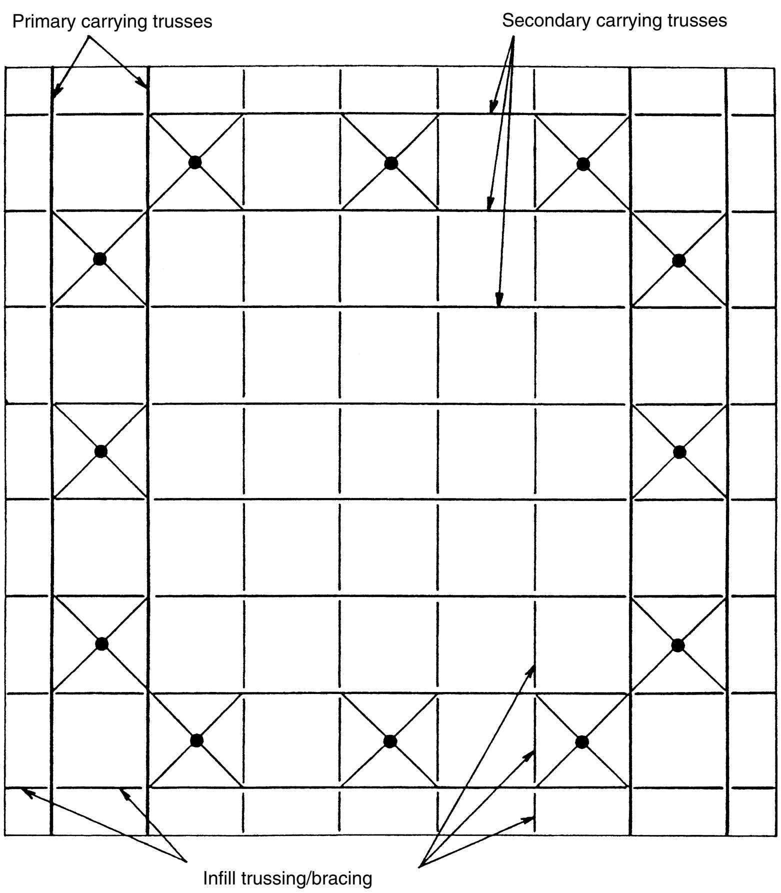


Figure 10.97 Plan of the system with one-way trusses.

here is a cross-shaped structure, most likely consisting of I-shaped members produced by welding of steel plates.

Since the top-chord grids are not offset, the cantilevered edges are created here by addition of half-module extensions of the spanning trusses.

The drawing in Figure 10.95 indicates the use of additional vertical members in the spanning trusses. These function to cut the horizontal span of the chords, thus offering a possibility for relief of the loads from the directly supported infill roof and any other supported elements attached to the chords.

For assembly and erection, it is possible with this system to fabricate the individual planar trusses in one direction in the shop and then attach the perpendicular truss members in the field.

A study of the plan of this system will reveal that there is a perimeter structure consisting of the planar trusses directly supported by the columns. This perimeter system then supports an interior two-way system of intersecting trusses as well as the cantilevered edges. This offers another approach to the problems of assembly and erection, utilizing the perimeter system.

Alternative Three: One-Way System

Figure 10.97 shows the plan for a system that is a variation on Alternative Two. Here, the perimeter trusses are used for support of one-way-spanning trusses in one direction and infill trussing between them. Although not different in basic form, there is some lack of symmetry in the behavior of this system, with the perimeter trusses that carry the interior trusses (designated the primary carrying trusses in the plan) working a little harder.

This system lends itself to some simplified design procedures, with most major units functioning as one-way-spanning elements. The rest of the system consists mostly of simple 28-ft-spanning infill. Assembly and erection would also be somewhat easier.

The lack of symmetry in function here will result in some corresponding lack in sizes of truss members and connections. The primary and secondary trusses will be considerably heavier than the remaining truss infill. For cost consideration, the infill will be correspondingly quite lighter; thus the total system cost may compare reasonably with that of the previous alternatives.

The simplicity of design of this structure is a definite advantage for the engineering designer. However, real justification for the choice of the system would depend on any cost advantages gained by simpler tasks of connecting, assembling, and erection of the structure.

Observation of the lack of symmetry in the construction may be disturbing to professionals but probably scarcely of note to the untrained public.

APPENDIX

A

Properties of Sections

This appendix deals with various geometric properties of planar (two-dimensional) areas. The areas referred to are the cross-sectional areas of structural members. These geometric properties are used in the analysis of stresses and deformations in the design of the structural members.

A.1 CENTROIDS

The *center of gravity* of a solid is the point at which all of its weight can be considered to be concentrated. Since a planar area has no weight, it has no center of gravity. The point in a planar area that corresponds to the center of gravity of a very thin plate of the same area and shape is called the *centroid* of the area. The centroid is a useful reference for various geometric properties of planar areas.

For example, when a beam is subjected to a bending moment, the materials in the beam above a certain plane in the beam are in compression and the materials below the plane are in tension. This plane is the *neutral-stress plane*, also called the neutral surface or the zero-stress plane (see Section 3.7). For a cross section of the beam the intersection of the neutral-stress plane is a line that passes through the centroid of the section and is called the *neutral axis* of the section. The neutral axis is very important for investigation of bending stresses in beams.

The location of the centroid for symmetrical shapes is located on the axis of symmetry for the shape. If the shape is bisymmetrical—that is, it has two axes of symmetry—the centroid is at the intersection of these axes. Consider the rectangular area shown in Figure A.1*a*; obviously its centroid is at its geometric center and is quite easily determined.

(*Note:* Tables A.3 through A.8 and Figure A.11, referred to in the discussion that follows, are located at the end of this appendix.)

For more complex forms, such as those of rolled steel members, the centroid will also be on any axis of symmetry. And, as for the simple rectangle, if there are two axes of symmetry, the centroid is readily located.

For simple geometric shapes, such as those shown in Figure A.1, the location of the centroid is easily established. However, for more complex shapes, the centroid and other properties may have to be determined by computations. One method for achieving this is by use of the *statical moment*, defined as the product of an area times its distance from some reference axis. Use of this method is demonstrated in the following examples.

Example 1. Figure A.2 is a beam cross section that is unsymmetrical with respect to a horizontal axis (such as X–X in the figure). The area is symmetrical about its vertical centroidal axis but the true location of the centroid requires locating the horizontal centroidal axis. Find the location of the centroid.

Solution. Using the statical moment method, first divide the area into units for which the area and location of the centroid are readily determined. The division chosen here is shown in Figure A.2*b* with the two parts labeled 1 and 2.

The second step is to choose a reference axis about which to sum statical moments and from which the location of the centroid is readily measured. A convenient reference axis for this shape is one at either the top or bottom of the shape. With the bottom chosen the distances from the centroids of the parts to this reference axis are shown in Figure A.2*b*.

The computation next proceeds to the determination of the unit areas and their statical moments. This work is summarized in Table A.1, which shows the total area to be

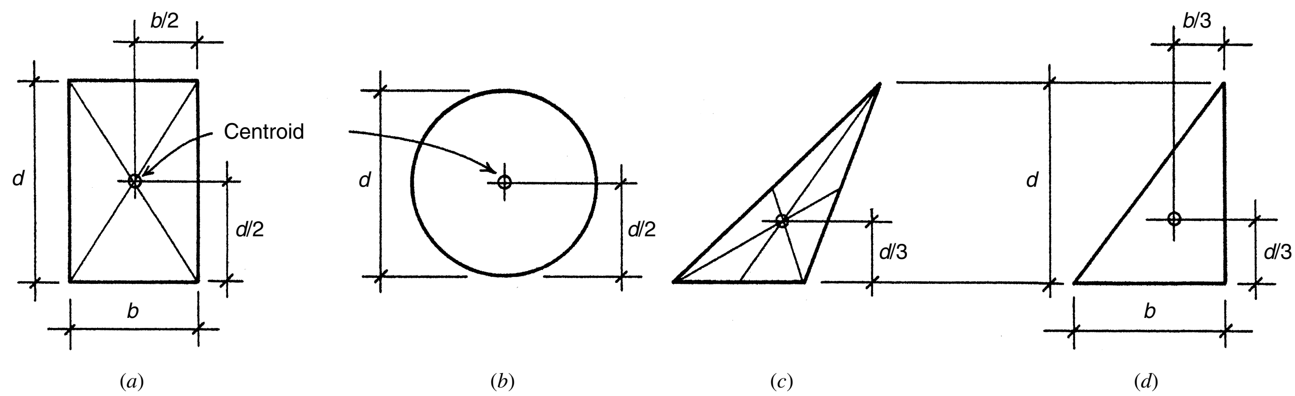


Figure A.1 Centroids of various shapes.

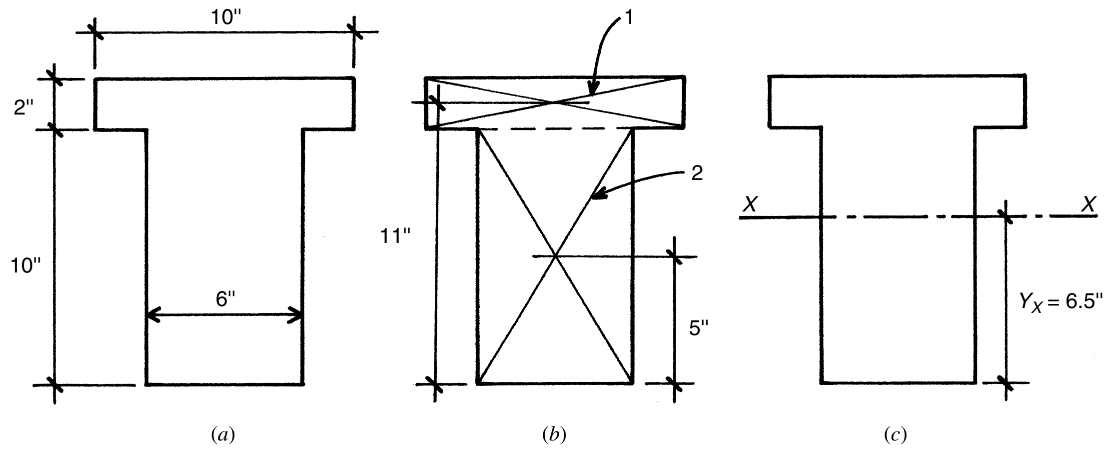


Figure A.2 Reference for Example 1.

Table A.1 Summary of Computations for Centroid: Example 1

Part	Area (in.^{2})	y (in.)	$A \times y$ (in.^{3})
1	$2 \times 10 = 20$	11	220
2	$6 \times 10 = 60$	5	300
Σ	80		520

$$y_x = \frac{520}{80} = 6.5 \text{ in.}$$

80 in.^{2} and the total statical moment to be 520 in.^{3} Dividing this moment by the total area produces the value of 6.5 in., which is the distance from the reference axis to the centroid of the whole shape, as shown in Figure A.2*c*.

A.2 MOMENT OF INERTIA

Consider the area enclosed by the irregular line in Figure A.3*a*. In this area, designated A, a small unit area a is indicated at z distance from the axis marked X–X. If this unit area is multiplied by the square of its distance from the reference axis, the result is the quantity az^2. If all of the units of the area are thus identified and the sum of these products is made, the result is defined as the *second moment* or the *moment of inertia* of the area, designated as I. Thus

$$\sum az^2 = I, \quad \text{or specifically } I_{X-X}$$

which is the moment of inertia of the area about the X–X axis.

The moment of inertia is a somewhat abstract item, less able to be visualized than area, weight, or center of gravity. It is nevertheless a real geometric property that becomes an essential factor for investigation of stresses and deformations due to bending. Of particular interest is the moment of inertia about a centroidal axis, and—most significantly—about a principal axis for the shape. Figures A.3*b, c, e,* and *f* indicate such axes for various shapes. An inspection of Tables A.3 through A.7 will reveal the properties of moment of inertia about the principal axes of the shapes in the tables.

Moment of Inertia of Geometric Figures

Values for moment of inertia can often be obtained from tabulations of structural properties. Occasionally it is necessary to compute values for a given shape. This may be a simple shape, such as a square, rectangle, circular, or triangular area. For such shapes simple formulas are derived to express the value for the moment of inertia.

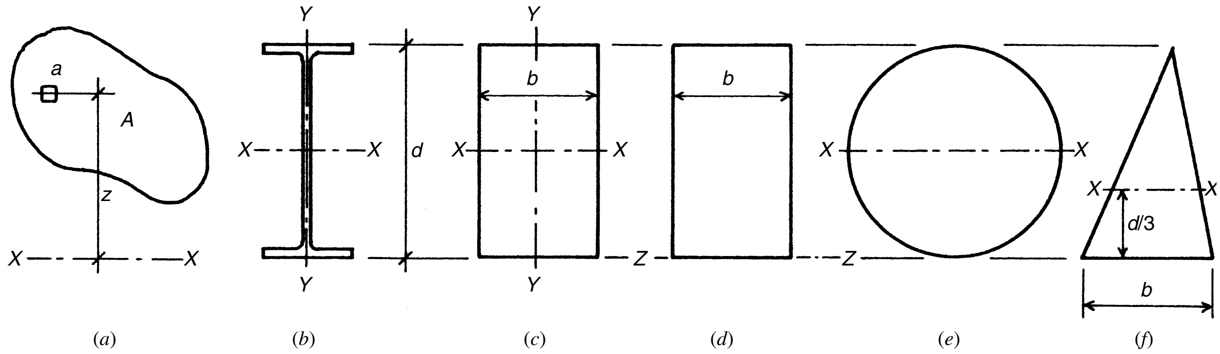


Figure A.3 Consideration of reference axes for the moment of inertia of various shapes of cross sections.

Rectangle. Consider the rectangle shown in Figure A.3*c*. Its width is b and its depth is d. The two principal axes are X–X and Y–Y, both passing through the centroid of the area. For this case the moment of inertia with respect to the centroidal axis X–X is

$$I_{X-X} = \frac{bd^3}{12}$$

and the moment of inertia with respect to the Y–Y axis is

$$I_{Y-Y} = \frac{db^3}{12}$$

Example 2. Find the value of the moment of inertia for a 6 × 12-in. wood beam about an axis through its centroid and parallel to the narrow dimension.

Solution. As listed in standard references for wood products, the actual dimensions of the section are 5.5 × 11.5 in. Then

$$I = \frac{bd^3}{12} = \frac{5.5(11.5)^3}{12} = 697.1\ \text{in.}^4$$

which is in agreement with the value for I_{X-X} in the references.

Circle. Figure A.3*e* shows a circular area with diameter d and axis X–X passing through its center. For the circular area the moment of inertia is

$$I = \frac{\pi d^4}{64}$$

Example 3. Compute the moment of inertia of a circular cross section 10 in. in diameter about its centroidal axis.

Solution. The moment of inertia is

$$I = \frac{\pi d^4}{64} = \frac{3.1416(10)^4}{64} = 490.9\ \text{in.}^4$$

Triangle. The triangle in Figure A.3*f* has a height h and a base width b. The moment of inertia about a centroidal axis parallel to the base is

$$I = \frac{bh^3}{36}$$

Example 4. If the base of the triangle in Figure A.3*f* is 12 in. wide and the height from the base is 10 in., find the value for the centroidal moment of inertia parallel to the base.

Solution. Using the given values in the formula,

$$I = \frac{bh^3}{36} = \frac{12(10)^3}{36} = 333.3\ \text{in.}^4$$

Open and Hollow Shapes. Values of moment of inertia for shapes that are open or hollow may sometimes be computed by a method of subtraction. The following examples demonstrate this process. Note that this is possible only for shapes that are symmetrical.

Example 5. Compute the moment of inertia for the hollow box section shown in Figure A.4*a* about a centroidal axis parallel to the narrow side.

Solution. Find first the moment of inertia of the shape defined by the outer limits of the box:

$$I = \frac{bd^3}{12} = \frac{6(10)^3}{12} = 500\ \text{in.}^4$$

Then find the moment of inertia for the shape defined by the void area:

$$I = \frac{4(8)^3}{12} = 170.7\ \text{in.}^4$$

The value for the hollow section is the difference; thus

$$I = 500 - 170.7 = 329.3\text{in}^4$$

Example 6. Compute the moment of inertia about the centroidal axis for the pipe section shown in Figure A.4*b*. The thickness of the shell is 1 in.

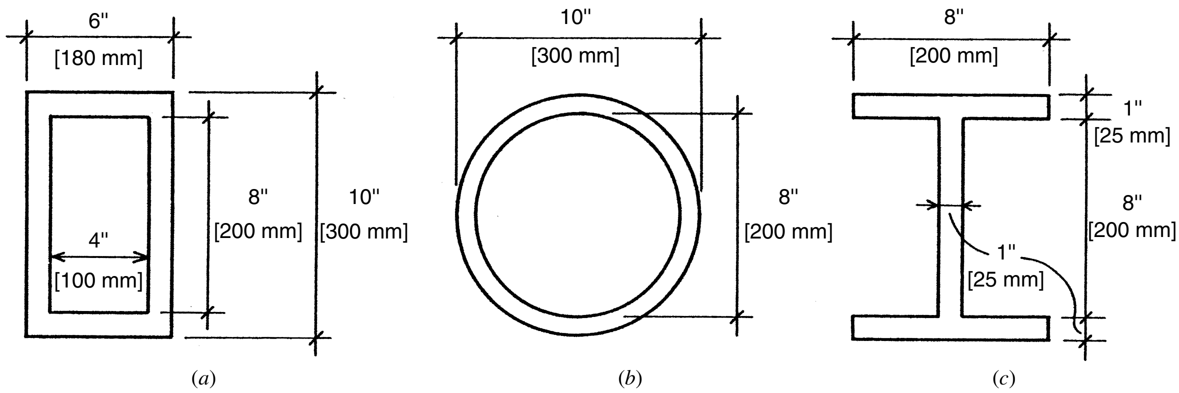


Figure A.4 Reference for Examples 5, 6, and 7.

Solution. As in the preceding example, the two values may be found and subtracted. Or a single computation may be made as follows:

$$I = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{3.1416}{64}(10^4 - 8^4) = 491 - 201 = 290 \text{ in.}^4$$

Example 7. Referring to Figure A.4*c*, compute the moment of inertia of the I-shape section about the centroidal axis parallel to the flanges.

Solution. This is essentially similar to the computation for Example 5. The two voids may be combined into a single one that is 7 in. wide. Thus

$$I = \frac{8(10)^3}{12} - \frac{7(8)^3}{12} = 667 - 299 = 368 \text{ in.}^4$$

Note that this method can only be used when the centroids of the outer shape and the void coincide. For example, it cannot be used to find the moment of inertia for the I shape about its vertical centroidal axis. For this computation the method discussed in the next section must be used.

A.3 TRANSFERRING MOMENTS OF INERTIA

Determination of the moment of inertia of unsymmetrical and complex shapes cannot be done by the simple processes illustrated in the preceding examples. An additional step that must be used is that involving the transfer of moment of inertia about a remote axis. The formula for achieving this transfer is as follows:

$$I = I_o + Az^2$$

where

I = moment of inertia of cross section about required reference axis

I_o = moment of inertia of cross section about its own centroidal axis, parallel to reference axis

A = area of cross section,

z = distance between two parallel axes

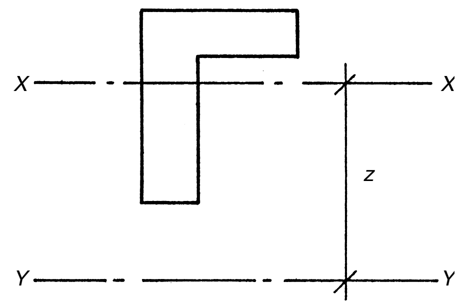


Figure A.5 Transfer of moment of inertia to a parallel axis.

These relationships are illustrated in Figure A.5, where X–X is the centroidal axis of the area and Y–Y is the reference axis for the transferred moment of inertia.

Application of this principle is illustrated in the following examples.

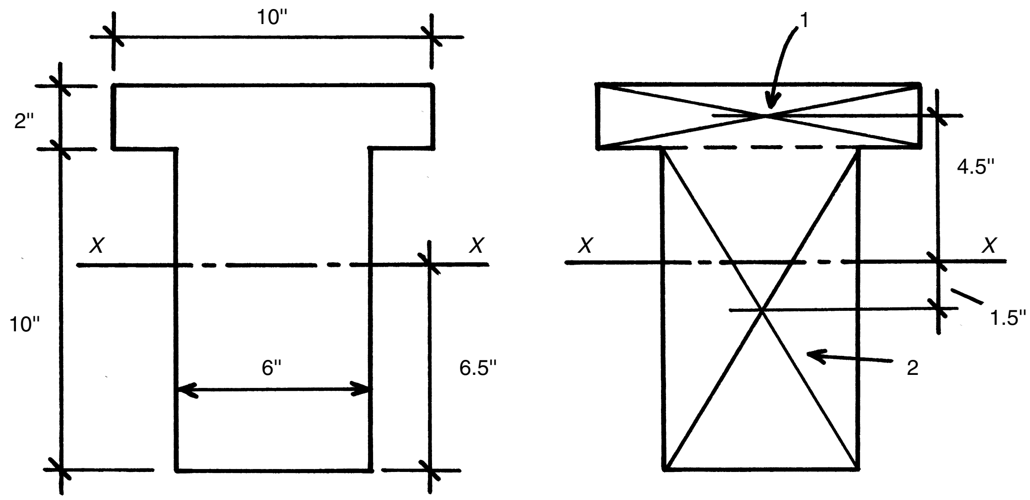
Example 8. Find the moment of inertia of the T-shaped area in Figure A.6 about its horizontal (X–X) centroidal axis. (*Note:* The location of the centroid for this section was solved as Example 1 in section A.1.)

Solution. A necessary first step in these problems is to locate the position of the centroidal axis if the shape is not symmetrical. In this case, the T shape is symmetrical about its vertical axis but not about the horizontal axis. Locating the position of the horizontal axis was the problem solved in Example 1 in section A.1.

The next step is to break the complex shape down into parts for which centroids, areas, and centroidal moments of inertia are readily found. As was done in Example 1, the shape here is divided between the rectangular flange part and the rectangular web part.

The reference axis to be used here is the horizontal centroidal axis. Table A.2 summarizes the process of determining the factors for the parallel-axis transfer process. The required value for I about the horizontal centroidal axis is determined to be 1046.7 in.^{4}.

A common situation in which this problem must be solved is in the case of structural members that are built



(*a*) (*b*)

Figure A.6 Reference for Example 8.

Table A.2 Summary of Computations for Moment of Inertia: Example 8

Part	Area (in.^{2})	y (in.)	I_o (in.^{4})	$A \times y^2$ (in.^{4})	I_x (in.^{4})
1	20	4.5	$10(2)^3/12 = 6.7$	$20(4.5)^2 = 405$	411.7
2	60	1.5	$6(10)^3/12 = 500$	$60(1.5)^2 = 135$	635
Σ					1046.7

up from distinct parts. One such section is that shown in Figure A.7, where a box-shaped cross section is composed by attaching two plates and two rolled channel sections. While this composite section is actually symmetrical about both its principal axes and the locations of these axes are apparent, the values for moment of inertia about both axes must be determined by the parallel-axis transfer process. The following example demonstrates the process.

Example 9. Compute the moment of inertia about the centroidal X–X axis of the built-up section shown in Figure A.7.

Solution. For this situation the two channels are positioned so that their centroids coincide with the reference axis. Thus the value of I_o for the channels is also their actual moment of inertia about the required reference axis, and their contributions to the required value here is simply two times their listed value for moment of inertia about their X–X axis, as given in Table A.4: $2(162) = 324\,\text{in.}^4$

The plates have simple rectangular cross sections, and the centroidal moment of inertia of one plate is thus

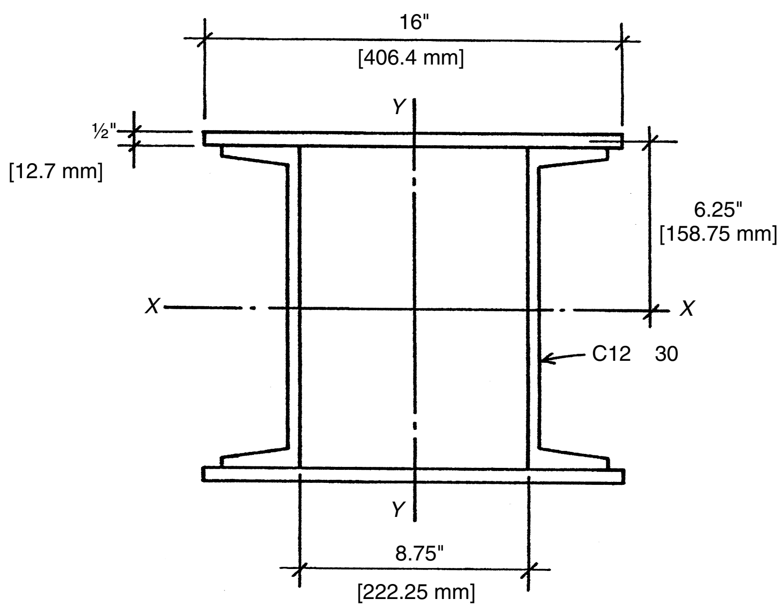


Figure A.7 Reference for Example 9.

determined as

$$I_o = \frac{bd^3}{12} = \frac{16 \times (0.5)^3}{12} = 0.1667\,\text{in.}^4$$

The distance between the centroid of the plate and the reference X–X axis is 6.25 in., and the area of one plate is 8 in.^{2} The moment of inertia for one plate about the reference axis is thus

$$I_o + Az^2 = 0.1667 + (8)(6.25)^2 = 312.7\,\text{in.}^4$$

and the value for the two plates is twice this, or 625.4 in.^{4}

Adding the contributions of the parts, the answer is $324 + 625.4 = 949.4$ in.^{4}

A.4 MISCELLANEOUS PROPERTIES

***Elastic Section Modulus,* S**

The term I/c in the formula for flexural stress is called the *section modulus.* Use of the section modulus permits a minor shortcut in the computations for flexural stress or the determination of the bending moment capacity of members. However, the real value of this property is in its measure of the relative bending strength of members. As a geometric property it is a direct index of bending strength for a given member cross section. Members of various cross section may thus be rank ordered in terms of their bending strength strictly on the basis of their S values. Because of its usefulness, the value of S is listed together with other significant properties in the tabulations for steel and wood members.

For members of standard form (structural lumber and rolled steel shapes) the value of S may be obtained from tables similar to those presented at the end of this chapter. For complex forms not of standard form, the value of S must be computed, which is readily done once the centroidal axes are located and moments of inertia about the centroidal axes are determined.

Example 10. Verify the tabulated value for the section modulus of a 6-by-12 wood beam about the centroidal axis parallel to its narrow side.

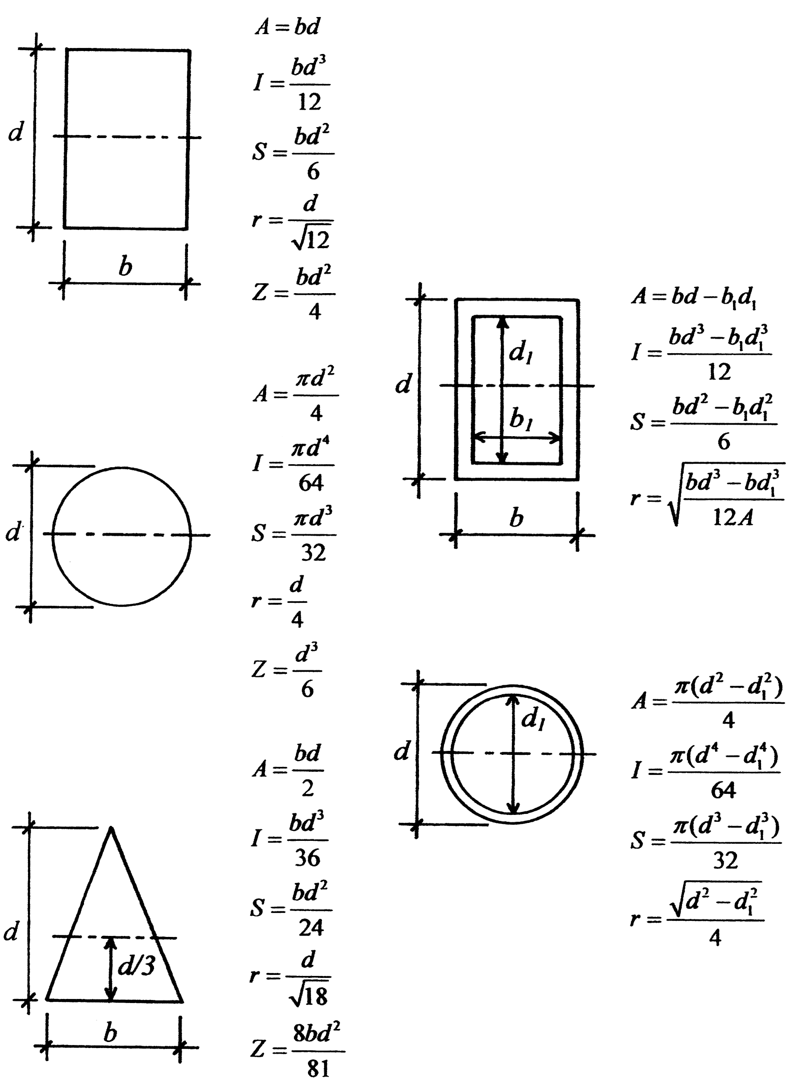
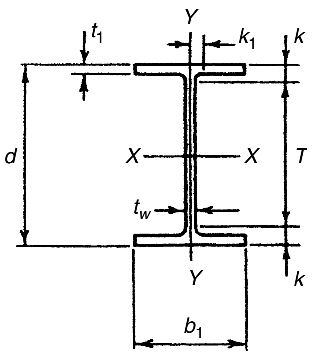


Figure A.8 Properties of various geometric shapes of cross sections.

Table A.3 Properties of W Shapes



			Web	Flange			Elastic Properties						Plastic
	Area	Depth	Thickness	Width	Thickness		Axis $X-X$			Axis $Y-Y$			Modulus
	A	d	t_w	b_f	t_f	k	I	S	r	I	S	r	Z_x
Shape	(in.^{2})	(in.)	(in.)	(in.)	(in.)	(in.)	(in.^{4})	(in.^{3})	(in.)	(in.^{4})	(in.^{3})	(in.)	(in.^{3})
W30 × 116	34.2	30.01	0.565	10.495	0.850	1.625	4930	329	12.0	164	31.3	2.19	378
× 108	31.7	29.83	0.545	10.475	0.760	1.562	4470	299	11.9	146	27.9	2.15	346
× 99	29.1	29.65	0.520	10.450	0.670	1.437	3990	269	11.7	128	24.5	2.10	312
W27 × 94	27.7	26.92	0.490	9.990	0.745	1.437	3270	243	10.9	124	24.8	2.12	278
× 84	24.8	26.71	0.460	9.960	0.640	1.375	2850	213	10.7	106	21.2	2.07	244
W24 × 84	24.7	24.10	0.470	9.020	0.770	1.562	2370	196	9.79	94.4	20.9	1.95	224
× 76	22.4	23.92	0.440	8.990	0.680	1.437	2100	176	9.69	82.5	18.4	1.92	200
× 68	20.1	23.73	0.415	8.965	0.585	1.375	1830	154	9.55	70.4	15.7	1.87	177
W21 × 83	24.3	21.43	0.515	8.355	0.835	1.562	1830	171	8.67	81.4	19.5	1.83	196
× 73	21.5	21.24	0.455	8.295	0.740	1.500	1600	151	8.64	70.6	17.0	1.81	172
× 57	16.7	21.06	0.405	6.555	0.650	1.375	1170	111	8.36	30.6	9.35	1.35	129
× 50	14.7	20.83	0.380	6.530	0.535	1.312	984	94.5	8.18	24.9	7.64	1.30	110
W18 × 86	25.3	18.39	0.480	11.090	0.770	1.437	1530	166	7.77	175	31.6	2.63	186
× 76	22.3	18.21	0.425	11.035	0.680	1.375	1330	146	7.73	152	27.6	2.61	163
× 60	17.6	18.24	0.415	7.555	0.695	1.375	984	108	7.47	50.1	13.3	1.69	123
× 55	16.2	18.11	0.390	7.530	0.630	1.312	890	98.3	7.41	44.9	11.9	1.67	112
× 50	14.7	17.99	0.355	7.495	0.570	1.250	800	88.9	7.38	40.1	10.7	1.65	101
× 46	13.5	18.06	0.360	6.060	0.605	1.250	712	78.8	7.25	22.5	7.43	1.29	90.7
× 40	11.8	17.90	0.315	6.015	0.525	1.187	612	68.4	7.21	19.1	6.35	1.27	78.4
W16 × 50	14.7	16.26	0.380	7.070	0.630	1.312	659	81.0	6.68	37.2	10.5	1.59	92.0
× 45	13.3	16.13	0.345	7.035	0.565	1.250	586	72.7	6.65	32.8	9.34	1.57	82.3
× 40	11.8	16.01	0.305	6.995	0.505	1.187	518	64.7	6.63	28.9	8.25	1.57	72.9
× 36	10.6	15.86	0.295	6.985	0.430	1.125	448	56.5	6.51	24.5	7.00	1.52	64.0
W14 × 211	62.0	15.72	0.980	15.800	1.560	2.250	2660	338	6.55	1030	130	4.07	390
× 176	51.8	15.22	0.830	15.650	1.310	2.000	2140	281	6.43	838	107	4.02	320
× 132	38.8	14.66	0.645	14.725	1.030	1.687	1530	209	6.28	548	74.5	3.76	234
× 120	35.3	14.48	0.590	14.670	0.940	1.625	1380	190	6.24	495	67.5	3.74	212
× 74	21.8	14.17	0.450	10.070	0.785	1.562	796	112	6.04	134	26.6	2.48	126

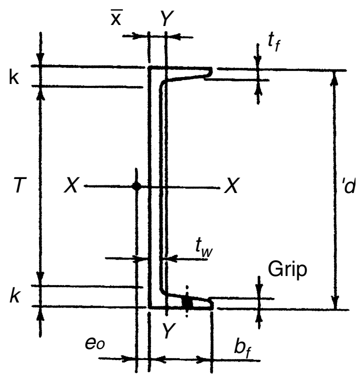
(*Continued*)

Table A.3 (*Continued*)

	Area	Depth	Web Thickness	Flange Width	Flange Thickness		Elastic Properties Axis X–X			Axis Y–Y			Plastic Modulus
Shape	A (in.^{2})	d (in.)	t_w (in.)	b_f (in.)	t_f (in.)	k (in.)	I (in.^{4})	S (in.^{3})	r (in.)	I (in.^{4})	S (in.^{3})	r (in.)	Z_x (in.^{3})
× 68	20.0	14.04	0.415	10.035	0.720	1.500	723	103	6.01	121	24.2	2.46	115
× 48	14.1	13.79	0.340	8.030	0.595	1.375	485	70.3	5.85	51.4	12.8	1.91	78.4
× 43	12.6	13.66	0.305	7.995	0.530	1.312	428	62.7	5.82	45.2	11.3	1.89	69.6
× 34	10.0	13.98	0.285	6.745	0.455	1.000	340	48.6	5.83	23.3	6.91	1.53	54.6
× 30	8.85	13.84	0.270	6.730	0.385	0.937	291	42.0	5.73	19.6	5.82	1.49	47.3
W12 × 136	39.9	13.41	0.790	12.400	1.250	1.937	1240	186	5.58	398	64.2	3.16	214
× 120	35.3	13.12	0.710	12.320	1.105	1.812	1070	163	5.51	345	56.0	3.13	186
× 72	21.1	12.25	0.430	12.040	0.670	1.375	597	97.4	5.31	195	32.4	3.04	108
× 65	19.1	12.12	0.390	12.000	0.605	1.312	533	87.9	5.28	174	29.1	3.02	96.8
× 53	15.6	12.06	0.345	9.995	0.575	1.250	425	70.6	5.23	95.8	19.2	2.48	77.9
× 45	13.2	12.06	0.335	8.045	0.575	1.250	350	58.1	5.15	50.0	12.4	1.94	64.7
× 40	11.8	11.94	0.295	8.005	0.515	1.250	310	51.9	5.13	44.1	11.0	1.93	57.5
× 30	8.79	12.34	0.260	6.520	0.440	0.937	238	38.6	5.21	20.3	6.24	1.52	43.1
× 26	7.65	12.22	0.230	6.490	0.380	0.875	204	33.4	5.17	17.3	5.34	1.51	37.2
W10 × 88	25.9	10.84	0.605	10.265	0.990	1.625	534	98.5	4.54	179	34.8	2.63	113
× 77	22.6	10.60	0.530	10.190	0.870	1.500	455	85.9	4.49	154	30.1	2.60	97.6
× 49	14.4	9.98	0.340	10.000	0.560	1.312	272	54.6	4.35	93.4	18.7	2.54	60.4
× 39	11.5	9.92	0.315	7.985	0.530	1.125	209	42.1	4.27	45.0	11.3	1.98	46.8
× 33	9.71	9.73	0.290	7.960	0.435	1.062	170	35.0	4.19	36.6	9.20	1.94	38.8
× 19	5.62	10.24	0.250	4.020	0.395	0.812	96.3	18.8	4.14	4.29	2.14	0.874	21.6
× 17	4.99	10.11	0.240	4.010	0.330	0.750	81.9	16.2	4.05	3.56	1.78	0.844	18.7

Source: Adapted from data in the *Manual of Steel Construction*, Load and Resistance Factor Design, 3rd ed. (Ref. 10), with permission of the publisher, American Institute of Steel Construction. This table is a sample from an extensive set of tables in the reference document.

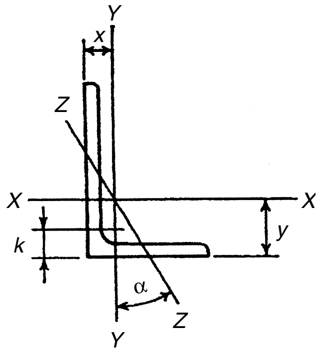
Table A.4 Properties of American Standard Channels



Shape	Area A (in.^{2})	Depth d (in.)	Web Thickness t_w (in.)	Flange Width b_f (in.)	Flange Thickness t_f (in.)	k (in.)	Axis $X-X$ I (in.^{4})	Axis $X-X$ S (in.^{3})	Axis $X-X$ r (in.)	Axis $Y-Y$ I (in.^{4})	Axis $Y-Y$ S (in.^{3})	Axis $Y-Y$ r (in.)	x^a (in.)	$e_o{}^b$ (in.)
C15 × 50	14.7	15.0	0.716	3.716	0.650	1.44	404	53.8	5.24	11.0	3.78	0.867	0.798	0.583
× 40	11.8	15.0	0.520	3.520	0.650	1.44	349	46.5	5.44	9.23	3.37	0.886	0.777	0.767
× 33.9	9.96	15.0	0.400	3.400	0.650	1.44	315	42.0	5.62	8.13	3.11	0.904	0.787	0.896
C12 × 30	8.82	12.0	0.510	3.170	0.501	1.13	162	27.0	4.29	5.14	2.06	0.763	0.674	0.618
× 25	7.35	12.0	0.387	3.047	0.501	1.13	144	24.1	4.43	4.47	1.88	0.780	0.674	0.746
× 20.7	6.09	12.0	0.282	2.942	0.501	1.13	129	21.5	4.61	3.88	1.73	0.799	0.698	0.870
C10 × 30	8.82	10.0	0.673	3.033	0.436	1.00	103	20.7	3.42	3.94	1.65	0.669	0.649	0.369
× 25	7.35	10.0	0.526	2.886	0.436	1.00	91.2	18.2	3.52	3.36	1.48	0.676	0.617	0.494
× 20	5.88	10.0	0.379	2.739	0.436	1.00	78.9	15.8	3.66	2.81	1.32	0.692	0.606	0.637
× 15.3	4.49	10.0	0.240	2.600	0.436	1.00	67.4	13.5	3.87	2.28	1.16	0.713	0.634	0.796
C9 × 20	5.88	9.0	0.448	2.648	0.413	0.94	60.9	13.5	3.22	2.42	1.17	0.642	0.583	0.515
× 15	4.41	9.0	0.285	2.485	0.413	0.94	51.0	11.3	3.40	1.93	1.01	0.661	0.586	0.682
× 13.4	3.94	9.0	0.233	2.433	0.413	0.94	47.9	10.6	3.48	1.76	0.962	0.669	0.601	0.743
C8 × 18.75	5.51	8.0	0.487	2.527	0.390	0.94	44.0	11.0	2.82	1.98	1.01	0.599	0.565	0.431
× 13.75	4.04	8.0	0.303	2.343	0.390	0.94	36.1	9.03	2.99	1.53	0.854	0.615	0.553	0.604
× 11.5	3.38	8.0	0.220	2.260	0.390	0.94	32.6	8.14	3.11	1.32	0.781	0.625	0.571	0.697
C7 × 14.75	4.33	7.0	0.419	2.299	0.366	0.88	27.2	7.78	2.51	1.38	0.779	0.564	0.532	0.441
× 12.25	3.60	7.0	0.314	2.194	0.366	0.88	24.2	6.93	2.60	1.17	0.703	0.571	0.525	0.538
× 9.8	2.87	7.0	0.210	2.090	0.366	0.88	21.3	6.08	2.72	0.968	0.625	0.581	0.540	0.647
C6 × 13	3.83	6.0	0.437	2.157	0.343	0.81	17.4	5.80	2.13	1.05	0.642	0.525	0.514	0.380
× 10.5	3.09	6.0	0.314	2.034	0.343	0.81	15.2	5.06	2.22	0.866	0.564	0.529	0.499	0.486
× 8.2	2.40	6.0	0.200	1.920	0.343	0.81	13.1	4.38	2.34	0.693	0.492	0.537	0.511	0.599

Source: Adapted from data in the *Manual of Steel Construction* Load and Resistance Factor Design, 3rd ed. (Ref. 10), with permission of the publisher, American Institute of Steel Construction. This table is a sample from an extensive set of tables in the reference document.
[a]Distance to centroid of section.
[b]Distance to shear center of section.

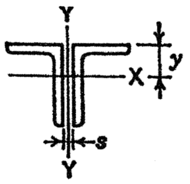
Table A.5 Properties of Single-Angle Shapes



Size and Thickness (in.)	k (in.)	Weight per ft (lb)	Area A (in.^{2})	Axis X–X I (in.^{4})	S (in.^{3})	r (in.)	y (in.)	Axis Y–Y I (in.^{4})	S (in.^{3})	r (in.)	x (in.)	Axis Z–Z r (in.)	Tan a
$8 \times 8 \times 1\frac{1}{8}$	1.75	56.9	16.7	98.0	17.5	2.42	2.41	98.0	17.5	2.42	2.41	1.56	1.000
$\times 1$	1.62	51.0	15.0	89.0	15.8	2.44	2.37	89.0	15.8	2.44	2.37	1.56	1.000
$8 \times 6 \times \frac{3}{4}$	1.25	33.8	9.94	63.4	11.7	2.53	2.56	30.7	6.92	1.76	1.56	1.29	0.551
$\times \frac{1}{2}$	1.00	23.0	6.75	44.3	8.02	2.56	2.47	21.7	4.79	1.79	1.47	1.30	0.558
$6 \times 6 \times \frac{5}{8}$	1.12	24.2	7.11	24.2	5.66	1.84	1.73	24.2	5.66	1.84	1.73	1.18	1.000
$\times \frac{1}{2}$	1.00	19.6	5.75	19.9	4.61	1.86	1.68	19.9	4.61	1.86	1.68	1.18	1.000
$6 \times 4 \times \frac{5}{8}$	1.12	20.0	5.86	21.1	5.31	1.90	2.03	7.52	2.54	1.13	1.03	0.864	0.435
$\times \frac{1}{2}$	1.00	16.2	4.75	17.4	4.33	1.91	1.99	6.27	2.08	1.15	0.987	0.870	0.440
$\times \frac{3}{8}$	0.87	12.3	3.61	13.5	3.32	1.93	1.94	4.90	1.60	1.17	0.941	0.877	0.446
$5 \times 3\frac{1}{2} \times \frac{1}{2}$	1.00	13.6	4.00	9.99	2.99	1.58	1.66	4.05	1.56	1.01	0.906	0.755	0.479
$\times \frac{3}{8}$	0.87	10.4	3.05	7.78	2.29	1.60	1.61	3.18	1.21	1.02	0.861	0.762	0.486
$5 \times 3 \times \frac{1}{2}$	1.00	12.8	3.75	9.45	2.91	1.59	1.75	2.58	1.15	0.829	0.750	0.648	0.357
$\times \frac{3}{8}$	0.87	9.8	2.86	7.37	2.24	1.61	1.70	2.04	0.888	0.845	0.704	0.654	0.364
$4 \times 4 \times \frac{1}{2}$	0.87	12.8	3.75	5.56	1.97	1.22	1.18	5.56	1.97	1.22	1.18	0.782	1.000
$\times \frac{3}{8}$	0.75	9.8	2.86	4.36	1.52	1.23	1.14	4.36	1.52	1.23	1.14	0.788	1.000
$4 \times 3 \times \frac{1}{2}$	0.94	11.1	3.25	5.05	1.89	1.25	1.33	2.42	1.12	0.864	0.827	0.639	0.543
$\times \frac{3}{8}$	0.81	8.5	2.48	3.96	1.46	1.26	1.28	1.92	0.866	0.879	0.782	0.644	0.551
$\times \frac{5}{16}$	0.75	7.2	2.09	3.38	1.23	1.27	1.26	1.65	0.734	0.887	0.759	0.647	0.554
$3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$	0.75	8.5	2.48	2.87	1.15	1.07	1.01	2.87	1.15	1.07	1.01	0.687	1.000
$\times \frac{5}{16}$	0.69	7.2	2.09	2.45	0.976	1.08	0.990	2.45	0.976	1.08	0.990	0.690	1.000
$\times \frac{5}{16}$	0.75	6.1	1.78	2.19	0.927	1.11	1.14	0.939	0.504	0.727	0.637	0.540	0.501
$3 \times 3 \times \frac{3}{8}$	0.69	7.2	2.11	1.76	0.833	0.913	0.888	1.76	0.833	0.913	0.888	0.587	1.000
$\times \frac{5}{16}$	0.62	6.1	1.78	1.51	0.707	0.922	0.865	1.51	0.707	0.922	0.865	0.589	1.000
$3 \times 2\frac{1}{2} \times \frac{3}{8}$	0.75	6.6	1.92	1.66	0.810	0.928	0.956	1.04	0.581	0.736	0.706	0.522	0.676
$\times \frac{5}{16}$	0.69	5.6	1.62	1.42	0.688	0.937	0.933	0.898	0.494	0.744	0.683	0.525	0.680
$3 \times 2 \times \frac{3}{8}$	0.69	5.9	1.73	1.53	0.781	0.940	1.04	0.543	0.371	0.559	0.539	0.430	0.428
$\times \frac{5}{16}$	0.62	5.0	1.46	1.32	0.664	0.948	1.02	0.470	0.317	0.567	0.516	0.432	0.435
$2\frac{1}{2} \times 2\frac{1}{2} \times \frac{3}{8}$	0.69	5.9	1.73	0.984	0.566	0.753	0.762	0.984	0.566	0.753	0.762	0.487	1.000
$\times \frac{5}{16}$	0.62	5.0	1.46	0.849	0.482	0.761	0.740	0.849	0.482	0.761	0.740	0.489	1.000
$2\frac{1}{2} \times 2 \times \frac{3}{8}$	0.69	5.3	1.55	0.912	0.547	0.768	0.831	0.514	0.363	0.577	0.581	0.420	0.614
$\times \frac{5}{16}$	0.62	4.5	1.31	0.788	0.466	0.776	0.809	0.446	0.310	0.584	0.559	0.422	0.620

Source: Adapted from data in the *Manual of Steel Construction* Load and Resistance Factor Design, 3rd ed. (Ref. 10), with permission of the publishers American Institute of Steel Construction. This table is a sample from an extensive set of tables in the reference document.

Table A.6 Properties of Double-Angle Shapes with Long Legs Back to Back



Size and Thickness (in.)	Weight per ft (lb)	Area A (in.^{2})	Axis X–X I (in.^{4})	S (in.^{3})	r (in.)	y (in.)	Axis Y–Y Radii of Gyration Back to Back of Angles, in. 0	3/8	3/4
$8 \times 6 \times 1$	88.4	26.0	161.0	30.2	2.49	2.65	2.39	2.52	2.66
$\times 3/4$	67.6	19.9	126.0	23.3	2.53	2.56	2.35	2.48	2.62
$\times 1/2$	46.0	13.5	88.6	16.0	2.56	2.47	2.32	2.44	2.57
$6 \times 4 \times 3/4$	47.2	13.9	49.0	12.5	1.88	2.08	1.55	1.69	1.83
$\times 1/2$	32.4	9.50	34.8	8.67	1.91	1.99	1.51	1.64	1.78
$\times 3/8$	24.6	7.22	26.9	6.64	1.93	1.94	1.50	1.62	1.76
$5 \times 3\frac{1}{2} \times 1/2$	27.2	8.00	20.0	5.97	1.58	1.66	1.35	1.49	1.63
$\times 3/8$	20.8	6.09	15.6	4.59	1.60	1.61	1.34	1.46	1.60
$5 \times 3 \times 1/2$	25.6	7.50	18.9	5.82	1.59	1.75	1.12	1.25	1.40
$\times 3/8$	19.6	5.72	14.7	4.47	1.61	1.70	1.10	1.23	1.37
$\times 5/16$	16.4	4.80	12.5	3.77	1.61	1.68	1.09	1.22	1.36
$4 \times 3 \times 1/2$	22.2	6.50	10.1	3.78	1.25	1.33	1.20	1.33	1.48
$\times 3/8$	17.0	4.97	7.93	2.92	1.26	1.28	1.18	1.31	1.45
$\times 5/16$	14.4	4.18	6.76	2.47	1.27	1.26	1.17	1.30	1.44
$3\frac{1}{2} \times 2\frac{1}{2} \times 3/8$	14.4	4.22	5.12	2.19	1.10	1.16	0.976	1.11	1.26
$\times 5/16$	12.2	3.55	4.38	1.85	1.11	1.14	0.966	1.10	1.25
$\times 1/4$	9.8	2.88	3.60	1.51	1.12	1.11	0.958	1.09	1.23
$3 \times 2 \times 3/8$	11.8	3.47	3.06	1.56	0.940	1.04	0.777	0.917	1.07
$\times 5/16$	10.0	2.93	2.63	1.33	0.948	1.02	0.767	0.903	1.06
$\times 1/4$	8.2	2.38	2.17	1.08	0.957	0.993	0.757	0.891	1.04
$2\frac{1}{2} \times 2 \times 3/8$	10.6	3.09	1.82	1.09	0.768	0.831	0.819	0.961	1.12
$\times 5/16$	9.0	2.62	1.58	0.932	0.776	0.809	0.809	0.948	1.10
$\times 1/4$	7.2	2.13	1.31	0.763	0.784	0.787	0.799	0.935	1.09

Source: Adapted from data in the *Manual of Steel Construction* Load and Resistance Factor Design, 3rd ed. (Ref. 10), with permission of the publishers American Institute of Steel Construction. This table is a sample from an extensive set of tables in the reference document.

Table A.7 Properties of Standard Weight Steel Pipe

Dimensions					Properties			
Nominal Diameter (in.)	Outside Diameter (in.)	Inside Diameter (in.)	Wall Thickness (in.)	Weight per ft (lb)	A (in.^{2})	I (in.^{4})	S (in.^{3})	r (in.)
3	3.500	3.068	0.216	7.58	2.23	3.02	1.72	1.16
3½	4.000	3.548	0.226	9.11	2.68	4.79	2.39	1.34
4	4.500	4.026	0.237	10.79	3.17	7.23	3.21	1.51
5	5.563	5.047	0.258	14.62	4.30	15.2	5.45	1.88
6	6.625	6.065	0.280	18.97	5.58	28.1	8.50	2.25
8	8.625	7.981	0.322	28.55	8.40	72.5	16.8	2.94
10	10.750	10.020	0.365	40.48	11.9	161	29.9	3.67
12	12.750	12.000	0.375	49.56	14.6	279	43.8	4.38

Source: Adapted from data in the *Manual of Steel Construction* Load and Resistance Factor Design, 3rd ed. (Ref. 10), with permission of the publishers American Institute of Steel Construction. This table is a sample from an extensive set of tables in the reference document.

Solution. From Table A.8 the actual dimensions of this member are 5.5×11.5 in., And the value for the moment of inertia is 697.068 in.^{4} Then

$$S = \frac{I}{c} = \frac{697.068}{5.75} = 121.229\ \text{in.}^3$$

which agrees with the value in Table A.8.

Plastic Section Modulus, Z

The plastic section modulus, designated Z, is used in a similar manner to the elastic stress section modulus S. The plastic modulus is used to determine the fully plastic stress moment capacity of a steel beam. Thus

$$M_p = F_y \times Z$$

The use of the plastic section modulus is discussed in Section 9.2.

Radius of Gyration, r

For design of slender compression members an important geometric property is the *radius of gyration*, defined as

$$r = \sqrt{\frac{I}{A}}$$

Just as with moment of inertia and section modulus values, the radius of gyration has an orientation to a specific axis in the planar cross section of a member. Thus if the I used in the formula for r is that with respect to the X–X centroidal axis, then that is the reference for the specific value of r.

A value of r with particular significance is that designated as the *least radius of gyration*. Since this value will be related to the least value of I for the cross section and since I is an index of the bending stiffness of the member, then the least value for r will indicate the weakest response of the member to bending. This relates specifically to the resistance of slender compression members to buckling. Buckling is essentially a sideways bending response, and its most likely occurrence will be on the axis identified by the least value of I or r.

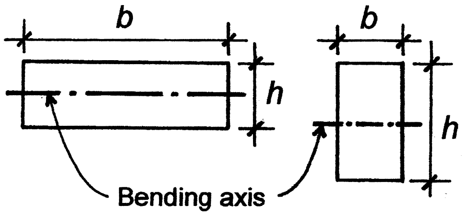
A.5 TABLES OF PROPERTIES OF SECTIONS

Figure A.8 presents formulas for obtaining geometric properties of various simple plane sections. Some of these may be used for single-piece structural members or for the building up of complex members.

Tables A.3 through A.8 present the properties of various plane sections. These are sections identified as those of standard industry-produced sections of wood and steel. Standardization means that the shapes and dimensions of the sections are fixed and each specific section is identified in some way.

Structural members may be employed for various purposes, and thus they may be oriented differently for some structural uses. Of note for any plane section are the *principal axes* of the section. These are the two mutually perpendicular, centroidal axes for which the values will be greatest and least respectively for the section; thus the axes are identified as the major and minor axes. If sections have an axis of symmetry, it will always be a principal axis—either major or minor.

Table A.8 Properties of Structural Lumber



Dimenions (in.)		Area	X–X Axis		Y–Y Axis		Weight at 35 lb/ft^3 Density
			Section Modulus	Moment of Inertia	Section Modulus	Moment of Inertia	
Nominal	Actual	A	S	I	S	I	
$b \times h$	$b \times h$	(in.^{2})	(in.^{3})	(in.^{4})	(in.^{3})	(in.^{4})	(lb/ft)
2 × 3	1.5 × 2.5	3.75	1.563	1.953	0.938	0.703	0.911
2 × 4	1.5 × 3.5	5.25	3.063	5.359	1.313	0.984	1.276
2 × 6	1.5 × 5.5	8.25	7.563	20.80	2.063	1.547	2.005
2 × 8	1.5 × 7.25	10.88	13.14	47.63	2.719	2.039	2.643
2 × 10	1.5 × 9.25	13.88	21.39	98.93	3.469	2.602	3.372
2 × 12	1.5 × 11.25	16.88	31.64	178.0	4.219	3.164	4.102
2 × 14	1.5 × 13.25	19.88	43.89	290.8	4.969	3.727	4.831
3 × 4	2.5 × 3.5	8.75	5.104	8.932	3.646	4.557	2.127
3 × 6	2.5 × 5.5	13.75	12.60	34.66	5.729	7.161	3.342
3 × 8	2.5 × 7.25	18.13	21.90	79.39	7.552	9.440	4.405
3 × 10	2.5 × 9.25	23.13	35.65	164.9	9.635	12.04	5.621
3 × 12	2.5 × 11.25	28.13	52.73	296.6	11.72	14.65	6.836
3 × 14	2.5 × 13.25	33.13	73.15	484.6	13.80	17.25	8.051
3 × 16	2.5 × 15.25	38.13	96.90	738.9	15.89	19.86	9.266
4 × 4	3.5 × 3.5	12.25	7.146	12.51	7.146	12.51	2.977
4 × 6	3.5 × 5.5	19.25	17.65	48.53	11.23	19.65	4.679
4 × 8	3.5 × 7.25	25.38	30.66	111.1	14.80	25.9	6.168
4 × 10	3.5 × 9.25	32.38	49.91	230.8	18.89	33.05	7.869
4 × 12	3.5 × 11.25	39.38	73.83	415.3	22.97	40.20	9.570
4 × 14	3.5 × 13.25	46.38	102.4	678.5	27.05	47.34	11.27
4 × 16	3.5 × 15.25	53.38	135.7	1034	31.14	54.49	12.97
5 × 5	4.5 × 4.5	20.25	15.19	34.17	15.19	34.17	4.922
6 × 6	5.5 × 5.5	30.25	27.73	76.26	27.73	76.26	7.352
6 × 8	5.5 × 7.5	41.25	51.56	193.4	37.81	104.0	10.03
6 × 10	5.5 × 9.5	52.25	82.73	393.0	47.90	131.7	12.70
6 × 12	5.5 × 11.5	63.25	121.2	697.1	57.98	159.4	15.37
6 × 14	5.5 × 13.5	74.25	167.1	1128	68.06	187.2	18.05
6 × 16	5.5 × 15.5	85.25	220.2	1707	78.15	214.9	20.72
6 × 18	5.5 × 17.5	96.25	280.7	2456	88.23	242.6	23.39
6 × 20	5.5 × 19.5	107.3	348.6	3398	98.31	270.4	26.07
6 × 22	5.5 × 21.5	118.3	423.7	4555	108.4	298.1	28.74
6 × 24	5.5 × 23.5	129.3	506.2	5948	118.5	325.8	31.41
8 × 8	7.5 × 7.5	56.25	70.31	263.7	70.31	263.7	13.67
8 × 10	7.5 × 9.5	71.25	112.8	535.9	89.06	334.0	17.32
8 × 12	7.5 × 11.5	86.25	165.3	950.5	107.8	404.3	20.96
8 × 14	7.5 × 13.5	101.3	227.8	1538	126.6	474.6	24.61
8 × 16	7.5 × 15.5	116.3	300.3	2327	145.3	544.9	28.26

Table A.8 (*Continued*)

Dimenions (in.)			*X–X* Axis		*Y–Y* Axis		Weight at 35 lb/ft^3 Density
Nominal $b \times h$	Actual $b \times h$	Area A (in.^{2})	Section Modulus S (in.^{3})	Moment of Inertia I (in.^{4})	Section Modulus S (in.^{3})	Moment of Inertia I (in.^{4})	(lb/ft)
8 × 18	7.5 × 17.5	131.3	382.8	3350	164.1	615.2	31.90
8 × 20	7.5 × 19.5	146.3	475.3	4634	182.8	685.5	35.55
8 × 22	7.5 × 21.5	161.3	577.8	6211	201.6	755.9	39.19
8 × 24	7.5 × 23.5	176.3	690.3	8111	220.3	826.2	42.84
10 × 10	9.5 × 9.5	90.25	142.9	678.8	142.9	678.8	21.94
10 × 12	9.5 × 11.5	109.3	209.4	1204	173.0	821.7	26.55
10 × 14	9.5 × 13.5	128.3	288.6	1948	203.1	964.5	31.17
10 × 16	9.5 × 15.5	147.3	380.4	2948	233.1	1107	35.79
10 × 18	9.5 × 17.5	166.3	484.9	4243	263.2	1250	40.41
10 × 20	9.5 × 19.5	185.3	602.1	5870	293.3	1393	45.03
10 × 22	9.5 × 21.5	204.3	731.9	7868	323.4	1536	49.64
10 × 24	9.5 × 23.5	223.3	874.4	10270	353.5	1679	54.26
12 × 12	11.5 × 11.5	132.3	253.5	1458	253.5	1458	32.14
12 × 14	11.5 × 13.5	155.3	349.3	2358	297.6	1711	37.73
12 × 16	11.5 × 15.5	178.3	460.5	3569	341.6	1964	43.32
12 × 18	11.5 × 17.5	201.3	587.0	5136	385.7	2218	48.91
12 × 20	11.5 × 19.5	224.3	728.8	7106	429.8	2471	54.51
12 × 22	11.5 × 21.5	247.3	886.0	9524	473.9	2725	60.10
12 × 24	11.5 × 23.5	270.3	1058	12440	518.0	2978	65.69
14 × 14	13.5 × 13.5	182.3	410.1	2768	410.1	2768	44.30
14 × 16	13.5 × 15.5	209.3	540.6	4189	470.8	3178	50.86
14 × 18	13.5 × 17.5	236.3	689.1	6029	531.6	3588	57.42
14 × 20	13.5 × 19.5	263.3	855.6	8342	592.3	3998	63.98
14 × 22	13.5 × 21.5	290.3	1040	11180	653.1	4408	70.55
14 × 24	13.5 × 23.5	317.3	1243	14600	713.8	4818	77.11
16 × 16	15.5 × 15.5	240.3	620.6	4810	620.6	4810	58.39
16 × 18	15.5 × 17.5	271.3	791.1	6923	700.7	5431	65.93
16 × 20	15.5 × 19.5	302.3	982.3	9578	780.8	6051	73.46
16 × 22	15.5 × 21.5	333.3	1194	12840	860.9	6672	81.00
16 × 24	15.5 × 23.5	364.3	1427	16760	941.0	7293	88.53
18 × 18	17.5 × 17.5	306.3	893.2	7816	893.2	7816	74.44
18 × 20	17.5 × 19.5	341.3	1109	10810	995.3	8709	82.94
18 × 22	17.5 × 21.5	376.3	1348	14490	1097	9602	91.45
18 × 24	17.5 × 23.5	411.3	1611	18930	1199	10500	99.96
20 × 20	19.5 × 19.5	380.3	1236	12050	1236	12050	92.42
20 × 22	19.5 × 21.5	419.3	1502	16150	1363	13280	101.9
20 × 24	19.5 × 23.5	458.3	1795	21090	1489	14520	111.4
22 × 22	21.5 × 21.5	462.3	1656	17810	1656	17810	112.4
22 × 24	21.5 × 23.5	505.3	1979	23250	1810	19460	122.8
24 × 24	23.5 × 23.5	552.3	2163	25420	2163	25420	134.2

Source: Compiled from data in the *National Design Specification for Wood Construction* (Ref. 3), with permission of the publishers American Forest and Paper Association.

For sections with two perpendicular axes of symmetry (rectangle, H, I, etc.), one axis will be the major axis and the other the minor axis. In the tables of properties the listed values for I, S, and r are all identified as to a specific axis, and the reference axes are identified in a figure for the table.

Other values given in the tables are for significant dimensions, total cross-sectional area, and weight of a 1-ft-long piece of the member. The weight of wood members is given in the table, assuming an average density for structural softwood of 35 lb/ft^3. The weight of steel members is given for W and channel shapes as part of their designation; thus a W8 × 67 member weighs 67 lb/ft. For steel angles and pipes the weight is given in the table, as determined from the density of steel at 490 lb/ft^3.

The designation of some members indicates their true dimensions. Thus a 10-in. channel and a 6-in. angle have true dimensions of 10 and 6 in. For W shapes and pipe, the designated dimensions are *nominal*, and the true dimensions must be obtained from the tables.

APPENDIX

B

Glossary

The material presented in this glossary constitutes a brief dictionary of words and terms frequently encountered in discussions of the design of building structures. Many of the words and terms have reasonably well-established meanings; in those cases we have tried to be consistent with the accepted usage. In some cases, however, words and terms are given different meanings by different authors or by groups that work in different fields, in which case the definition here is that used for the work in this book.

In some cases words and terms are commonly misused with regard to their precise meaning, an example being "unreinforced," which would imply something from which reinforcing has been removed, whereas it is commonly used to refer to something that was never reinforced in the first place. Where such is the case, we have given the commonly used meaning here.

To be clear in its requirements, a legal document such as a building code often defines some words and terms. Care should be exercised when reading such documents to be sure of these precise meanings.

Abutment. Originally, the end support of an arch or vault. Now, any support that receives both vertical and lateral loading.

Acceleration. Rate of change of the velocity. Acceleration of the ground surface is more significant than its displacement in determining the dynamic earthquake effect on the building structure.

Accidental Torsion. Torsional effect on buildings, due to minimum accidental eccentricity required by codes, even when there is no actual computed eccentricity.

Active Lateral Soil Pressure (of Soil). See *Lateral Soil Pressure.*

Adequate. Just enough; sufficient. Indicates a quality of bracketed acceptability—on the one hand, not insufficient, but on the other hand, not superlative or excessive.

Aggregate. Inert, loose material that makes up the largest part (typically two-thirds to three-fourths) of the concrete; what the water and cement paste holds together; ordinarily consists of stone—ranging in size from medium sand to coarse gravel.

Allowable Stress. See *Stress.*

Allowable Stress Design (ASD). Structural design method that employs limits based on allowable stresses and responses to service (actual usage) load conditions. See *Strength Design.*

Amplitude. See *Vibration.*

Analysis. Separation into constituent parts. In engineering, the investigative determination of the detail aspects of a particular phenomenon. May be qualitative, meaning a general evaluation of the nature of the phenomenon, or quantitative, meaning the numerical determination of the magnitude of the phenomenon. See also *Synthesis.*

Anchorage. Attachment for resistance to movement; usually opposing uplift, overturn, sliding, or horizontal separation. *Tiedown*, or *holddown*, refers to anchorage against uplift or overturn. *Positive anchorage* refers to fastening that does not easily loosen under a condition of repeated, reversing loading.

Aspect Ratio. Proportionate ratio of the dimensions of an object, such as height-to-width ratio of a shear wall.

Base. Level at which earthquake motions are considered to be delivered to a building.

Base Shear. Total design lateral force (horizontal shear) at the building base.

Beam. Structural element that sustains transverse loading and develops internal forces of bending and shear in resisting loads. Also called a *girder* if very large, a *joist* if small or in

closely spaced sets, a *rafter* if used for a roof, and a *header* or *lintel* if used over an opening in a wall.

Bearing Foundation. Foundation that transfers loads to soil by direct vertical contact pressure (bearing). Usually refers to a *shallow bearing foundation*, which is placed directly beneath the lowest part of the building and not very far from the ground surface. See also *Footing*.

Bending. Turning action that causes change in the curvature of linear elements; characterized by the development of opposed internal stresses of compression and tension. See also *Moment*.

Bent. Planar framework, or some defined portion of one, that is intended for resistance to both horizontal and vertical loads in the plane of the frame.

Box System. Lateral bracing system in which horizontal loads are resisted not by a column and beam system but rather by planar elements (shear walls and horizontal diaphragms) or braced frames (trusses).

Braced Frame. Building code term for a trussed frame used for lateral bracing.

Bracing. General term used for elements that provide support against sideways movements due to lateral loads or to the buckling of slender elements.

Brittle Fracture. Sudden failure, usually due to tension or shear; the usual failure of brittle materials, such as glass, plaster, and concrete.

Buckling. Collapse, in the form of sudden sideways deflection or of torsional rotation (twisting).

Building Code. Legal document for regulation of building form, features, and construction. Model codes are developed by recommending organizations; real codes are enacted as ordinances by some governmental unit (city, county, state).

Built-Up Member. Structural member assembled from two or more parts in a manner that results in the combined parts working as a single unit.

Calculation. Ordered, rational determination, usually by mathematical computations.

Centroid. Geometric center of an object, usually analogous to the center of gravity. The point at which the entire mass of the object may be considered to be concentrated when considering the moment of the mass.

Cold-Formed Element. Structural element produced from sheet steel by bending, rolling, or stamping without heating of the steel.

Collector. Force transfer element that functions to collect loads from a horizontal diaphragm and distribute them to the vertical elements of the lateral resistive system.

Composite Panel. Structural panel with wood veneer faces and a fiberboard core. In thick panels there is also a center wood veneer.

Compression. Force action that tends to press adjacent particles of a material together and to cause shortening of objects in the direction of the compressive force.

Concrete Masonry Unit (CMU). Precast concrete unit; or, good old concrete block.

Connection. Union or joining of two or more distinct elements. In a structural assemblage, a connection device itself becomes an entity, with interactions of the connected elements visualized in terms of their actions on the connecting device.

Continuity. Character of continuous, monolithic structural elements; wherein actions of adjacent elements are influenced by their continuous nature; such as with multistory columns, multispan beams, and multielement rigid frames.

Core Bracing. Concentration of the vertical elements of a lateral bracing system at a central location in the building; usually at the location of elevators, stairs, and vertical service elements.

Creep. Plastic deformation at constant stress levels that occurs over time (basically under dead load); a common effect in structures of concrete.

Critical Damping. Damping that will result in a return from initial deformation to the neutral position within one cycle of vibration.

Curtain Wall. Exterior wall of a building that is supported entirely by the building structure, rather than being self-supporting or a bearing wall.

Damping. See *Vibration.*

Dead Load. See *Load*.

Deflection. Lateral movement of a structure under loading, such as the sag of a beam.

Determinate Structure. Structure having the exact sufficiency for stability and therefore being subject to investigation by consideration of the equilibrium of simple static forces alone. See also *Indeterminate Structure.*

Diaphragm. Planar element (wall, floor deck, etc.) used to resist forces in its own plane by shear action. See also *Horizontal Diaphragm* and *Shear Wall*.

Doubly Reinforced. Concrete member with both tension and compression reinforcement, usually opposed in bending.

Drag. Refers to wind effect on surfaces parallel to the wind direction. *Ground drag* refers to the effect of the ground surface in slowing the wind velocity near ground level.

Drag Strut. Structural member that transfers load across a building and into some part of the vertical bracing system. See also *Collector.*

Drift. Lateral deflection. *Story drift* refers to lateral deflection of one level of a structure with respect to the level below.

Ductility. Stress–strain (load–deformation) behavior that results from the plastic yielding of materials. To be significant—qualifying a material as ductile—the plastic yield before failure should be several times the elastic strain up to the point of plastic yield.

Dynamic. Load effects or structural responses that are not static in nature. That is, they involve time-related considerations such as momentum, vibration, and energy effects, versus simple force.

Eccentric Bracing. Braced frame in which the diagonal members do not connect to the joints of the beam-and-column frame, thus resulting in bending and shear in the frame members.

Elastic Behavior. Used to describe two aspects of stress–strain behavior. The first is a constant stress–strain proportionality, or constant modulus of elasticity, as represented by a straight-line form of the stress–strain graph. The second is the stress level limit within which all strain is recoverable; that is, there is no permanent deformation. The latter phenomenon may occur even though the stress–strain relationship is nonlinear (as it is for wood, for example).

Engineered Wood. General term for products produced from wood other than single pieces of sawn wood.

Equilibrium. Balanced state or condition, usually used to describe a situation in which opposed force effects neutralize each other to produce a net effect of zero.

Equivalent Static Force Analysis. Technique by which a dynamic effect is translated into a hypothetical (equivalent) static effect that produces a similar result.

Factored Load. Service load multiplied by a factor to produce an adjusted load for strength design.

Field Assemblage. Construction work performed at the construction site (the field). Refers mostly to production and erection of steel frames.

Flexible. See *Stiffness*.

Footing. Shallow bearing–type foundation element consisting of a concrete pad cast directly into an excavation.

Freestanding Wall. See *Wall*.

Function. Capability; intended use.

Fundamental Period. See *Period.*

Grade. 1. Level of the ground surface. 2. Rated quality (capability, capacity, refinement, etc.) of material.

Grade Beam. Foundation element at or near the finished ground level that acts as a footing, a tie, or a spanning element.

Grain. 1. Discrete particle of material that constitutes a loose material, such as soil. 2. Fibrous orientation of wood.

Grout. Lean concrete (predominantly water, cement, and sand) used as a filler in the voids of masonry units, under steel bearing plates, and so on.

Gust. Increase, or surge, of short duration in a sustained wind velocity.

Header. Beam at the edge of an opening in a roof or floor or at the top of an opening in a wall.

Horizontal Bracing System. Diaphragm or truss in a horizontal plane that collects lateral loads and distributes them to vertical bracing elements.

Horizontal Diaphragm. Usually a roof or floor deck used as part of a lateral bracing system. See *Diaphragm*.

Hot Rolling. Industrial process in which an ingot (lump) of steel is heated to the softening point and then repeatedly squeezed between rollers to produce a linear element with a constant cross section.

Indeterminate Structure. In general, any structure whose load-resisting behavior cannot be determined by simple consideration of static equilibrium.

Inelastic. See *Stress–Strain Behavior*.

Inertia. See *Mass.*

Irregular Structure. See *Regular Structure.*

Joist. See *Beam*.

Kern Limit. Limiting dimension for the eccentricity (off-center condition) of a compression force if tension stress is to be avoided.

Lateral. Sideways. Used to describe something that is perpendicular to a major axis or direction. With respect to the vertical direction of gravity forces, primary effects of wind, earthquakes, and horizontal soil pressures are called *lateral effects*. Horizontal buckling of beams is called lateral buckling.

Lateral Resistive System. Combination of elements of a structure that contributes to the general bracing against lateral forces.

Lateral Soil Pressure. Horizontal soil pressure of two kinds: 1. *Active* pressure is that exerted by a retained soil on a restraining structure. 2. *Passive pressure* is that exerted by soil against an object that is attempting to move sideways.

Lateral Unsupported Length. For a linear structural element (beam, column), the distance between points of assured lateral bracing.

Liquefaction. Action in which a soil deposit temporarily loses its shear resistance and takes on the character of a liquid; usually resulting from some dynamic vibration, such as that occurring during an earthquake.

Live Load. See *Load*.

Load. Active force (or combination of forces) exerted on a structure. *Dead load* is permanent gravity load, including the weight of the structure itself. *Live load* is literally any load which is not permanent, although the term is ordinarily applied to distributed surface loads on roofs and floors. *Service load* is that to which the structure is expected to be subjected. *Factored load* is the service load modified by amplification factors for use in strength design.

Load and Resistance Factor Design (LRFD). See *Strength Design*.

Mass. Dynamic property of an object that causes it to resist change in its state of motion; this resistance is called *inertia*. The magnitude of the mass per unit volume of the object is called *density*. Dynamic force is defined as $F = ma$, or force equals mass times acceleration. Weight is defined as the force produced by the acceleration of gravity; thus, $W = mg$.

Member. One of the distinct elements of an assemblage.

Moment. Action tending to produce turning or rotation. Product of a force and a distance (lever arm); yields a measurement unit of force times distance: foot-pounds, kilonewton-meters, and so on. Bending moment causes curvature of linear elements; torsional moment causes twisting rotation.

Moment of Inertia. Second moment of an area about a fixed line (axis) in the plane of the area. A purely mathematical property, not subject to direct physical measurement. Has significance in that it can be quantified for any geometric shape and is a measurement of certain structural responses, such as deflection of beams.

Natural Period. See *Period.*

Net Section. Cross-sectional area of a structural member reduced by holes, notches, and so on. Most significant in determination of tension response.

Normal. 1. Ordinary, usual, unmodified state of something. 2. Perpendicular, such as pressure on a surface.

Occupancy Importance Factor, *I*. Code term used in basic equations for lateral force. Expresses potential for increased concern for certain building occupancies.

Open Web Joist. Light steel truss, usually with parallel chords, commonly used in closely spaced sets, as with wood floor joists. A manufactured product.

Optimal. Best; most satisfying. The best solution to a set of criteria is the optimal one. When the criteria have opposed values, there may be no single optimal solution, except by the superiority of a single criterion, such as the lightest, the strongest, the cheapest, and so on.

Overturn. Rotational effect consisting of toppling or tipping over; an effect of lateral loads on vertical elements.

Passive Soil Pressure. See *Lateral Soil Pressure.*

P–delta Effect. Secondary bending effect on vertical members of a frame, induced by the vertical loads acting on the laterally displaced (deflected) members.

Pedestal. Short pier or upright compression member. A column qualified by a ratio of unsupported (unbraced) height to least lateral dimension of 3 or less.

Perimeter Bracing. Vertical elements of a lateral bracing system located at the building perimeter.

Period (of Vibration). Elapsed time for one full cycle of vibration. For an elastic structure in simple vibration, the period is a constant (called the *natural* or *fundamental period*) and is independent of the magnitude of the amplitude of the vibration, of the number of cycles, and of most damping or resonance effects. See also *Vibration*.

Pier. 1. Short, stocky column with a height not greater than three times its least lateral dimension. 2. Deep foundation element that is placed in an excavation rather than being forcefully driven as a pile. Although it actually refers to a particular method of excavation, the term *caisson* is commonly used to describe a pier foundation.

Pile. Deep foundation element, consisting of a linear, shaftlike member, that is placed by being driven dynamically into the ground. *Friction piles* develop resistance to both downward load and upward load (pullout) through friction between the soil and the pile surface. *End-bearing piles* are driven so that their ends are seated in low-lying strata of rock or very hard soil.

Plain Concrete. Concrete cast without reinforcement or prestressing.

Plastic. In structural investigation, the type of stress–strain response that occurs in ductile behavior, beyond the yield stress point; usually results in permanent deformation.

Plastic Hinge. Rotational effect that occurs in steel members when the entire cross section is subjected to yield stress.

Plastic Moment. Resisting moment produced at the point of development of a plastic hinge.

Positive Anchorage. See *Anchorage*.

Poured-in-Place Concrete. Concrete cast where it is intended to stay; also called *sitecast*.

Precast Concrete. Concrete members cast at a location other than that at which they are to be used.

Preconsolidation. Condition of a highly compressed soil, usually referring to a condition produced by the weight of soil above on some lower soil strata. May also refer to a condition produced by other than natural causes—piling up of soil on the site, vibration, or saturation that dissolves soil bonding, for example.

Presumptive Soil Pressure. Value for allowable vertical soil pressure that is used in the absence of intensive investigation and testing. Requires a minimum of soil sampling and identification and is usually quite conservative.

Principal Axes. Set of mutually perpendicular axes through the centroid of an area, about which the moments of inertia will be maximum and minimum. Called individually the *major axis* and the *minor axis*.

Radius of Gyration. Defined mathematical property: the square root of the moment of inertia divided by the area of a section.

Reaction. Response. In structural investigation, the response of the structure to the loads, or the response of the supports to the loaded structure. Mostly used to describe the latter.

Redundancy (in Structures). Refers to the existence of multiple load paths or to multistage response to loads.

Reentrant Corner. Exterior corner in a building plan having a form that is indented.

Reference Design Values. Values for allowable stress and modulus of elasticity for wood with no modification for usage conditions.

Regular Structure. With reference to lateral loading, a structure that is symmetrical and has an ordered regular form without abrupt changes affecting dynamic response.

Reinforce. To strengthen, usually by adding something.

Relative Stiffness. See *Rigidity*.

Resistance Factor. Reduction factor for adjustment of the ultimate resistance of a structural element to a force action: bending, compression, shear, and so on.

Restoring Moment. Resistance to overturn due to the weight of the laterally loaded element.

Rigid Bent. See *Rigid Frame*.

Rigid Frame. Common term for a framework in which members are connected by joints that are capable of transmitting bending moments to the ends of the members. The term "rigid" derives not so much from

the character of the frame as from that of the rigid joints. Now more accurately described as a *moment-resisting space frame*—a mouthful, but more accurate.

Rigidity. Degree of resistance to deformation; highly resistive elements are *stiff* or *rigid*, elements with low resistance are *flexible*.

Risk. Degree of probability of loss due to some potential hazard.

Rolled Shape. Steel member with cross section produced by *hot rolling*.

Safety. Relative unlikelihood of failure; absence of danger. The *safety factor* is the ratio of the structure's ultimate resistance to the actual demand (service load) on the structure.

Section. Two-dimensional area or profile obtained by passing a plane through a form. *Cross section* usually implies a section at right angles to another section or to the linear axis of an object (such as a vertical cross section of a horizontal beam).

Sense. See *Sign*.

Separation Joint. Joint between adjacent parts of a building that allows for independent movement of the parts.

Service Conditions. Situations arising from the usage of a structure. See *Load*.

Service Load. See *Load*.

Shear. Force effect that is lateral (perpendicular) to a structure, or one that involves a slipping effect, as opposed to a push–pull effect on a cross section.

Shear Wall. Vertical diaphragm; acts as a bracing element for horizontal force (shear) by developing shear stress in the plane of the wall.

Shop Assemblage. Refers to construction work performed at a production facility (the shop), as opposed to work done at the construction site (the field). Refers mostly to production and erection of steel frames.

Sign. Algebraic notation of sense: positive, negative, or neutral. Relates to direction of forces—if up is positive, down is negative; to stress—if tension is positive, compression is negative; to rotation—if clockwise is positive, counterclockwise is negative.

Sitecast Concrete. See *Poured-in-Place Concrete*.

Slab. Horizontal, planar element of concrete. Occurs as a roof or floor deck in a framed structure (called a *supported slab*) or as a pavement poured directly on the ground surface (called a *slab on grade*).

Slenderness. Relative thinness; a measurement of resistance to buckling.

Soft Story. In a multistory structure, a story level whose lateral stiffness is significantly less than that of stories above or below it.

Stability. Refers to the inherent capability of a structure to develop force resistance as a result of its form, orientation, articulation of its parts, type of connections, methods of support, and so on. Is not related to quantified strength or stiffness, except when actions involve buckling of slender elements.

Static. State that exists when acceleration is zero; thus the state of motion is unchanging. Generally refers to conditions in which no motion is occurring.

Stiffness. See *Rigidity*.

Strain. Deformation resulting from stress; measured as a percentage change and thus dimensionless.

Strength. Capacity to resist force.

Strength Design. One of two fundamental design methods for assuring a margin of structural safety. *Allowable stress design* (ASD) is performed by analyzing stresses produced by *service loads* and comparing them to established limits. *Strength design*, also called *ultimate strength design*, is performed by using a design ultimate load (a magnification of the service load) and comparing it to the ultimate resistance of the structure. When strength design is performed with both factored loads and factored resistances, it is called *load and resistance factor design* (LRFD).

Stress. Mechanism of force within a material of a structure, visualized as a pressure effect (tension or compression) or a shear effect on the surface of a unit of material and quantified as force per unit area. *Allowable stress* is a limit established for design by stress methods; *ultimate stress* is that developed at a failure condition.

Stress Design. See *Strength Design.*

Stress–Strain Behavior. Relation of stress to strain in a material or structure; usually visualized by a stress–strain graph covering the range from no load to failure. Various aspects of the form of the graph define particular behavior characteristics of the material. A straight line indicates an *elastic* relationship; a curve indicates *inelastic* behavior. A sudden bend in the graph usually indicates a plastic strain or *yield* that results in some permanent deformation. The slope of the graph (if straight), or of a tangent to the curve, indicates the relative stiffness of the material; measured by the tangent of the angle (stress/strain) and called the *modulus of elasticity*.

Structure. That which gives form to something and works to resist changes in the form due to the actions of various forces.

Stud. One of a set of closely spaced columns used to produce a framed wall.

Synthesis. Process of combining a set of components into a whole; opposite of analysis.

System. Set of interrelated elements; an ordered assemblage; an organized procedure or method.

Tension. Force action that tends to separate adjacent particles of a material or pull elements apart. Produces straightening effects and elongation.

Tiedown. See *Anchorage*.

Torsion. Rotational (moment) effect involving twisting in a plane perpendicular to the linear axis of an element.

Truss. Framework of linear elements that achieves stability through triangular formations of the elements.

Unreinforced. Grammatically incorrect but commonly used term referring to concrete or masonry structures without

reinforcement. Unreinforced concrete is also called *plain concrete*.

Uplift. Net upward (lifting) force effect; may be due to wind, overturning moment, or an upward seismic acceleration.

Vector. Mathematical quantity having direction as well as magnitude and sense (sign). Comparison is made to *scalar* quantities having only magnitude and sense, such as time and temperature. A vector may be represented by an arrow with its length proportional to the magnitude, the angle of its line indicating the direction, and the arrowhead representing the sign.

Velocity. Time rate of motion; also commonly called *speed.*

Vertical Diaphragm. See *Shear Wall.*

Vibration. Cyclic, rhythmic motion of an object. Occurs when the object is displaced from some neutral position and seeks to restore itself to a state of equilibrium when released. In its pure form it occurs as a harmonic motion with a characteristic behavior described by the cosine form of the displacement–time graph of the motion. The magnitude of the displacement from the neutral position is called the *amplitude.* The time elapsed for one full cycle of vibration is called the *period*. The number of cycles occurring in 1 second is called the *frequency.* Effects that tend to reduce the amplitude of succeeding cycles are called *damping.* The increase of amplitude in successive cycles is called a *resonant effect.*

Void Ratio. Term commonly used to indicate the amount of void in a soil, expressed as the ratio of the volume of the void to the volume of the solids.

Wall. Vertical, usually planar building element. *Foundation walls* are those partly or totally below ground. *Bearing walls* are used to carry vertical loads. *Shear walls* are used as bracing elements for horizontal forces in the plane of the wall. *Freestanding walls* are walls whose tops are not laterally braced. *Retaining walls* resist horizontal soil pressures perpendicular to the wall plane. *Curtain walls* are nonstructural exterior walls. *Partition walls* are nonstructural interior walls.

Weak Story. In a multistory structure, a story level whose lateral strength is significantly less than stories above or below.

Wet Concrete. Freshly mixed concrete before hardening.

Wind Stagnation Pressure. Reference wind pressure established by the basic wind speed for the region; used in determining design wind pressures.

Yield. See *Stress–Strain Behavior.*

APPENDIX

C

Exercise Problems

The following materials are provided for readers who use this book for individual study or for use as a course text. For problems that involve numerical computations, answers are given following the problems. However, the reader is advised to attempt to work the problems without referring to the answers except for a check. This procedure will serve better to test the reader's understanding of the work.

Problems are grouped by the chapter to which the work corresponds. Numbering is given for reference purposes only. In many cases the units used are arbitrary, as the relationships and procedures are more significant; in these cases metric units have been omitted for simplicity.

Chapter 1

1. For each of the basic structural materials (wood, steel, concrete, masonry) list both limitations and advantages in their uses for building structures.
2. For each of the limitations listed in Problem 1, describe what measures (if any) can be taken to overcome them.
3. Find a building that is just beginning to be constructed. Visit the site periodically and photograph the progress of the construction. Take pictures from the same locations on successive visits. Organize the pictures to illustrate the growth of the building structure.
4. Find a building that has been built recently. Contact people who were involved in the design and construction. Interview them and write a report on the design and construction of the structure for the building.
5. *The Make and Break.* This assignment involves the actual construction of a structure to perform a specific task. The following is an example; variations are possible.
   Design and build a structure to span 4 ft on a simple, horizontal span and to carry a single concentrated load at the center of the span. End support is limited to vertical reactions only. Materials for the structure are limited to wood and paper. Any means may be used for attachment of parts. The efficiency of the structure on a strength-to-weight basis is critical. The structure will be weighed, load tested to destruction, and a score will be determined using the graph shown.

Demonstration Projects—for assignment or for classroom demonstration

6. *Involvement.* Buckling of a vertical element as related to slenderness.
   Procedure. Select a slender linear element (thin strip of wood, plastic, or metal) and find its total compression resistance for various increments of length. Start with a ratio of length to thickness of at least 200 for the longest specimen.
   Find. Relation of load capacity to length (or to length-to-thickness ratio).
7. *Involvement.* Bending resistance related to shape.
   Procedure. Test the bending resistance of a linear element on a single span when subjected to load at the span center. Test elements of the same type of material and same total cross-sectional area, but with different shapes and different orientations to the load. Both bending strength (load capacity) and stiffness (deflection) may be tested.
   Find. Correlation of bending resistance and shape in beams.
8. *Involvement.* Bending and span.
   Procedure. Test a linear element for bending as in Problem 7. Test various specimens of the same material and cross section but of increasing span length.
   Find. Relation of total load capacity (and/or deflection) to span length.

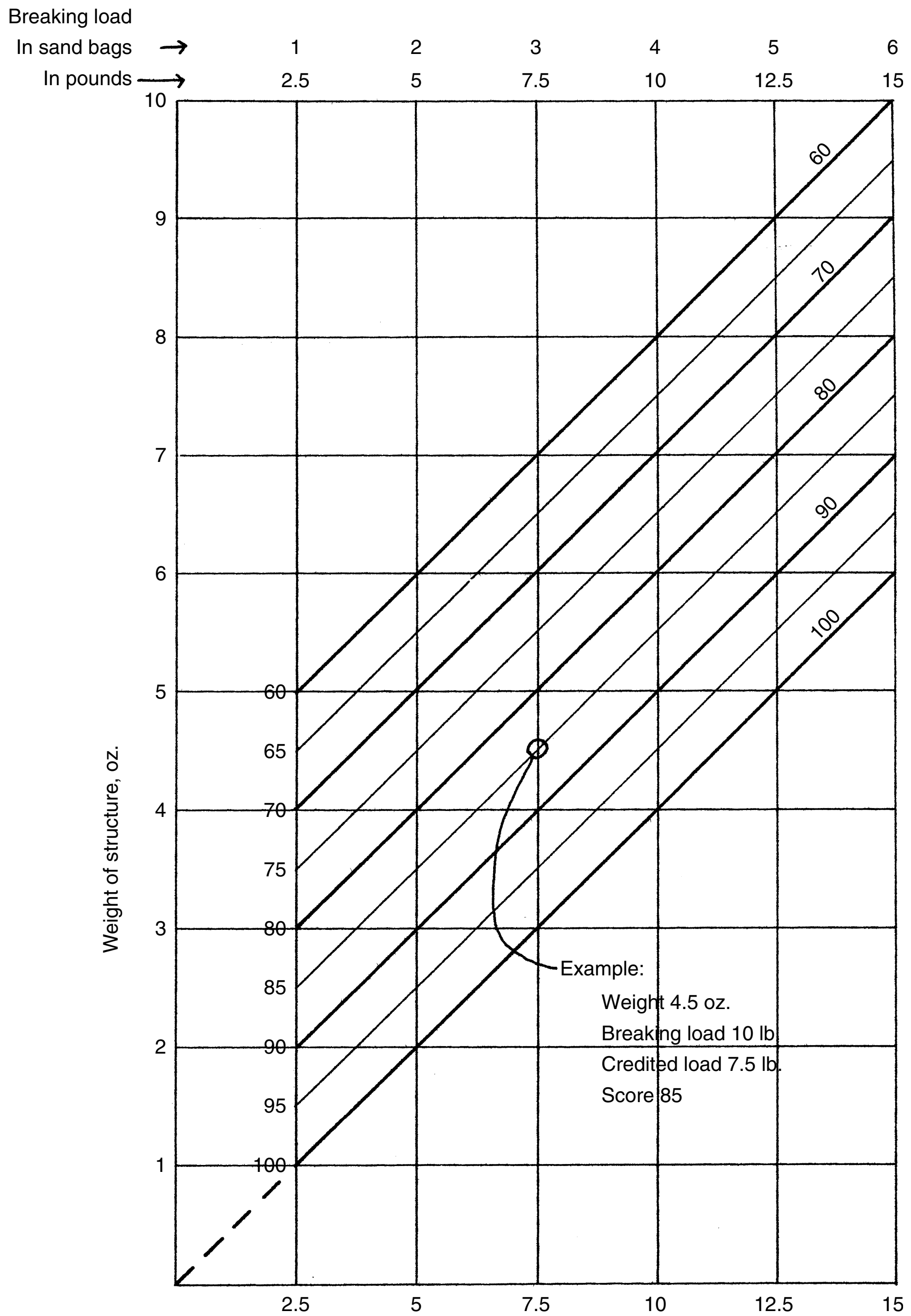


Figure C.1 Problem 5.

9. *Involvement.* Bending resistance and support restraint.
Procedure. Test the bending resistance of a linear element as in Problem 7. Test the same element on the same span, but with three different conditions at the support, as follows: (a) both ends free to turn (rotate), (b) one end clamped to prevent turning, (c) both ends clamped.
Find. Effect of end restraint on load capacity (and/or deflection).
10. *Involvement.* One-way versus two-way spanning. Effect of ratio of span lengths in rectangular two-way spans.
Procedure. Test a thin planar element in bending (a sheet of cardboard, glass, plastic, metal) with a single load at the center of the span. Test specimens with the following shapes and support conditions at the edges: (a) square sheet, two opposite edges supported; (b) square sheet, three edges supported; (c) square sheet, four edges supported; (d) rectangular sheet with the small dimension the same as in (c) and the long dimension a multiple of the short with increasing magnitude in successive specimens: 1.25, 1.5, 1.75, 2.0.
Find. Comparison of one-way and two-way spanning. Relation of panel dimension ratio to effectiveness of two-way action in rectangular panels.

11. *Involvement.* Torsion resistance of various cross-sectional shapes.
Procedure. Test various linear elements of the same material and the same total cross-sectional area but with different shapes. Fix one end and twist the other without causing bending. Measure the twisting force for total load capacity or for some constant increment of rotation.
Find. Effectiveness of various cross-sectional shapes in torsion.
12. *Involvement.* Sag ratio of cables.
Procedure. Test a tension element (string, wire, chain) for its total load resistance with various ratios of sag to span. Test by supporting the two ends and loading at the center.
Find: Relation of sag ratio (sag to span) and load capacity.
13. *Involvement.* Span-to-rise ratio in arches.
Procedure. Test a flexible sheet (cardboard, plastic, aluminum) for its resistance to load in arch action. Attach two blocks to a base and bend the sheet to form an arch, kicking against the blocks. Load with weights at the center of the arch. Test specimens with various span-to-rise ratios.
Find. Mode of failure and effectiveness of arch for various span-to-rise ratios.

Chapter 2

14. Using both algebraic and graphic techniques, find the horizontal and vertical components for the force: (a) $F = 100$ lb, angle $= 45°$; (b) $F = 200$ lb, angle $= 30°$; (c) $F = 200$ lb, angle $= 27°$; (d) $F = 327$ lb, angle $= 40°$.

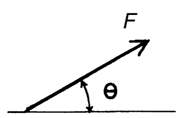


Figure C.2 Problem 14.

15. Using both algebraic and graphic techniques, find the resultant (magnitude and direction) for the following force combinations: (a) $H = 100$ lb, $V = 100$ lb; (b) $H = 50$ lb, $V = 100$ lb; (c) $H = 43$ lb, $V = 61$ lb; (d) $H = 127$ lb, $V = 47$ lb.

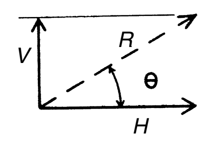


Figure C.3 Problem 15.

16. Using both algebraic and graphic techniques, find the resultant (magnitude and direction) for the following force combinations: (a) $F_1 = 100$ lb, $F_2 = 100$ lb, $F_3 = 100$ lb, $\theta = 45°$; (b) $F_1 = 100$ lb, $F_2 = 200$ lb, $F_3 = 150$ lb, $\theta = 30°$; (c) $F_1 = 47$ lb, $F_2 = 63$ lb, $F_3 = 112$ lb, $\theta = 40°$; (d) $F_1 = 58$ lb, $F_2 = 37$ lb, $F_3 = 81$ lb, $\theta = 28°$.

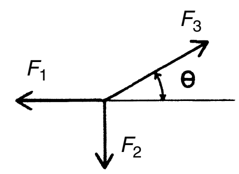


Figure C.4 Problem 16.

17. Using both algebraic and graphic techniques, find the tension in the rope: (a) $x = 0$; (b) $x = 10$ ft; (c) $x = 5$ ft; (d) $x = 4$ ft 2 in.

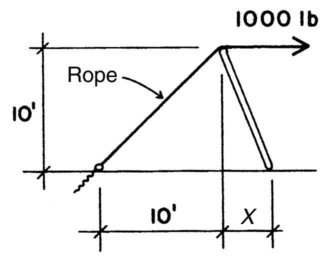


Figure C.5 Problem 17.

18. Using a Maxwell diagram, find the internal forces in the members of the truss.

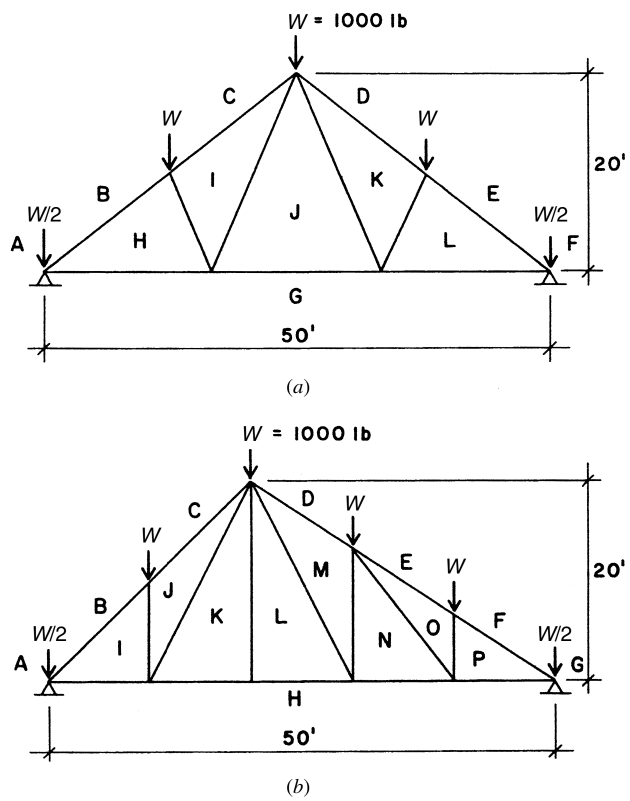


Figure C.6 Problem 18.

19. Using the algebraic method, find the internal forces in the truss in Problem 18. Draw the separated joint diagram, showing all components as well as the actual member forces.
20. Find the reactions for the beams: (a) $x = 6$ ft; (b) $x = 4$ ft; (c) $x = 5$ ft; (d) $x = 7$ ft 2 in.

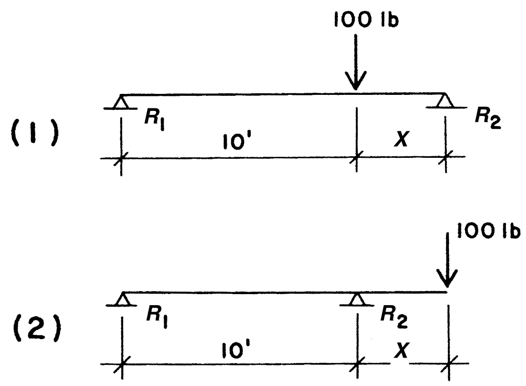


Figure C.7 Problem 20.

21. Find the magnitude and direction of both reactions: (a) $x = 10$ ft, $H_1 = H_2$; (b) $x = 13$ ft, $H_1 = H_2$; (c) $x = 10$ ft, $H_2 = 0$; (d) $x = 13$ ft, $H_2 = 0$.

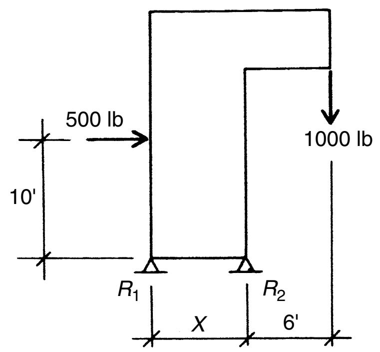


Figure C.8 Problem 21.

22. What axial load may be placed on a short timber post whose actual cross-sectional dimensions are 9.5 × 9.5 in. [241.3 mm] if the allowable unit compressive stress in the wood is 1100 psi [7585 kPa]?
23. The allowable bearing capacity of a soil is 8000 psf [383 kPa]. What should be the length of the side of a square footing if the total load (including the weight of the footing) is 240 kips [1068 kN]?
24. Determine the minimum cross-sectional area of a steel bar required to support a tensile force of 50 kips [222.4 kN] if the allowable tensile stress is 20 ksi [137.9 MPa].
25. A short square timber post supports a load of 115 kips [511.5 kN]. If the allowable compressive stress is 1000 psi [6895 kPa], what nominal-size square timber should be used? (See Table A.8.)
26. A joint similar to that shown in Figure 2.26*a* is to be used to connect a concrete floor slab to a supporting concrete wall. What is the minimum dimension for the width of the tongue in the slab if the maximum unit stress in shear is 60 psi [414 kPa] and the load is 2500 lb [11.12 kN] per running foot of the joint?
27. A steel bolt with a diameter of 0.75 in. [19 mm] is to be used in a joint similar to that in Figure 2.26*b*. What is the maximum permissible load on the joint if the allowable shear stress in the bolt is 14 ksi [96.5 MPa]?
28. A wood beam has a nominal cross section of 8 × 12 in. If the beam is subjected to a bending moment about its strong axis and the maximum allowable bending stress is 1400 psi [9.65 MPa], what is the maximum bending moment permitted? (See Table A.8 for properties of wood sections.)
29. How much will a nominal 8-by-8 Douglas fir post shorten under an axial compression load of 45 kips [200 kN] if the post is 12 ft [3.66 m] high? Use $E =$ 1500 ksi [10.34 GPa]; actual post dimensions are 7.5 × 7.5 in.
30. What force must be applied to a steel bar that is 1 in. [25 mm] square and 2 ft [610 mm] long to produce an elongation of 0.016 in. [0.4064 mm]?
31. A concrete structure 200 ft long is subjected to a temperature change of 140°F. Assume a value of $E =$ 4000 ksi for the concrete and find (a) the total length change if movement is not prevented and (b) the stress in the concrete if movement is prevented.
32. A 16-in.-diameter round concrete column with $E =$ 6000 ksi is reinforced with six 1-in.-diameter round steel rods and sustains an axial compression force of 200 kips. Find the stresses in the steel and concrete.
33. A 10-in.-diameter round column has an axial compression force of 20 kips. Find the direct compressive stress and the shear stress on a section that is inclined 60° from the axis of the column.
34. A roof framing member similar to that shown in Figure 2.47 consists of a nominal 6 by 8 with $S_x =$ 51.6 in.^{3} and $S_y = 37.81$ in.^{3} and sustains a bending moment of 4 kip-ft in a vertical plane. If the roof slope is 22°, find the maximum bending stress and the net stress at the four corners of the section.
35. A 16 × 24-in. column sustains a compression force of 100 kips that is 3 in. eccentric from both centroidal axes of the section. Find the stress at the four corners of the section and the location of the neutral axis.
36. A beam has an I-shaped cross section with an overall depth of 16 in., web thickness of 2 in., and flanges that are 8 in. wide and 3 in. thick. Compute the critical shear stresses and plot the distribution of shear on the cross section if the beam sustains a shear force of 20 kips.
37. Find the angle at which the block shown in Figure 2.50 will slip if the coefficient of friction is 0.35.
38. For the wall and footing shown, make separate determinations of the possibilities for slipping and overturn: $H = 1000$ lb and $W = 3000$ lb.

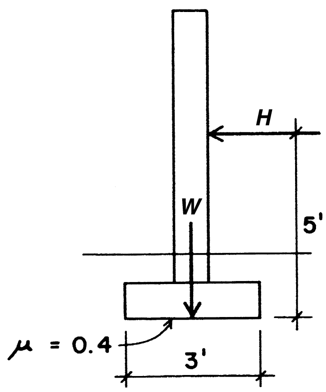


Figure C.9 Problem 38.

Chapter 3

39. For each of the beams shown, find the reactions; draw complete shear and moment diagrams indicating all critical values; sketch the deflected shape; indicate significant relationships between diagrams.

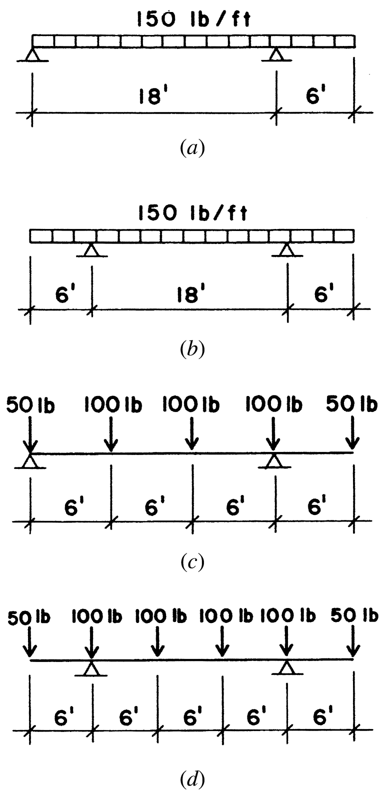


Figure C.10 Problem 39.

40. Find the maximum tension in the cable if $T = 10$ kips and: (a) $x = y = 10$ ft, $s = t = 4$ ft; (b) $x = 12$ ft, $y = 16$ ft, $s = 8$ ft, $t = 12$ ft.

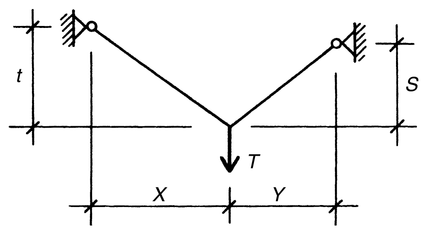


Figure C.11 Problem 40.

41. A spanning cable is subjected to a load of 1 kip/ft distributed uniformly with respect to the horizontal span of 120 ft. Find the maximum tension force in the cable at the support if the sag is 16 ft.
42. The compression force at the bottom of a square footing is 40 kips [178 kN] and the bending moment is 30 kip-ft [40.7 kN-m]. Find the maximum soil pressure for footing widths of (a) 5 ft [1.5 m]; (b) 4 ft [1.2 m].
43. For the frames shown, find the components of the reactions, draw the free-body diagrams of the whole frame and the individual members, draw the shear and moment diagrams for the members, and sketch the deflected shape of the loaded structure.

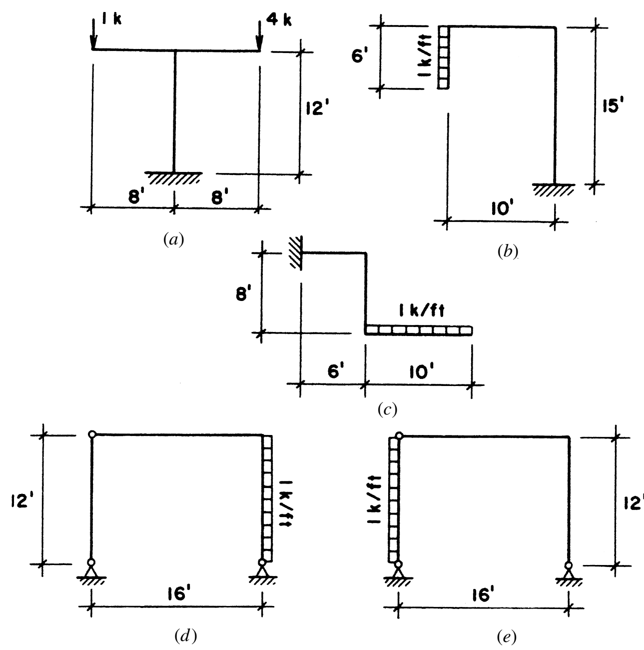


Figure C.12 problem 43.

Chapter 4

44. If the maximum bending moment on an 8-by-10 beam is 16 kip-ft [22 kN-m], what is the value of the maximum bending stress?

45. Find the required standard lumber shape with the least cross-sectional area from Table A.8 for a beam in which the maximum bending moment is 20 kip-ft [27 kN-m] if the allowable bending stress is 1600 psi [11.0 MPa].
46. Find the maximum resisting moment for a beam consisting of a wood 6 by 10 if the allowable bending stress is 1400 psi [9.65 MPa].
47. A 10 × 14-in. beam has a span of 15 ft [4.5 m] with a single concentrated live load of 9 kips [40 kN] located 5 ft [1.5 m] from one end. Compute the maximum shear stress.
48. A 6 × 12-in. beam has a total uniformly distributed live load of 3600 lb [16.0 kN] and a dead load of 3000 lb [13.3 kN] on a span of 10 ft [3 m]. If the maximum allowable shear stress is 170 psi, is the beam adequate for shear?
49. A 10 × 12-in. beam of Douglas fir-larch, No. 1 grade, is used on a simple span of 12 ft [3.6 m]. It supports a uniformly distributed live load of 800 lb/ft [11.67 kN/m] and a total uniformly distributed dead load of 400 lb/ft [5.84 kN/m]. Is the beam size adequate for shear?
50. A 6 × 12-in. beam of Douglas fir-larch, No. 1 grade, has 3 in. [75 mm] of end bearing to develop a reaction force of 5000 lb [22.2 kN]. Is the situation adequate for bearing?
51. A 3 × 16-in. rafter cantilevers over a 3 × 16-in. support beam. If both members are of Douglas fir-larch, No. 1 grade, is the situation adequate for bearing? The rafter load on the support beam is 3000 lb [13.3 kN].
52. A 6 × 14-in. beam of Douglas fir-larch, No. 1 grade is 16 ft [4.88 m] long and supports a total uniformly distributed load of 6000 lb [26.7 kN]. Investigate the deflection.
53. An 8 × 12-in. beam of Douglas fir-larch, dense No. 1 grade is 12 ft [3.66 m] in length and has a concentrated load of 5 kips [22.2 kN] at the center of the span. Investigate the deflection.
54. Two concentrated loads of 3500 lb [15.6 kN] each are located at the third points of a 15-ft-span [4.57-m] beam. The 10 × 14-in. beam is of Douglas fir-larch, select structural grade. Investigate the deflection.
55. An 8 × 14-in. beam of Douglas fir-larch, select structural grade, has a span of 16 ft [4.88 m] and a total uniformly distributed load of 8 kips [35.6 kN]. Investigate the deflection.
56. Find the least weight Douglas fir-larch, No. 1 grade, section that can be used for a span of 18 ft [5.49 m] with a total uniformly distributed load of 8 kips [44.5 kN]. Investigate the deflection.
57. Using Douglas fir-larch, No. 2 grade, pick the floor joist size required from Table 4.7 for the stated conditions. Live load is 40 psf, dead load is 10 psf, and deflection is limited to $L/360$ under live load only.

	Joist Spacing (in.)	Joist Span (ft)
(a)	16	14
(b)	12	14
(c)	16	16
(d)	12	20

58. Using Douglas fir-larch, No. 2 grade, pick the rafter size required from Table 4.8 for the stated conditions. Live load is 20 psf, dead load is 20 psf, and deflection is limited to $L/240$ under live load only.

	Rafter Spacing (in.)	Rafter Span (ft)
(a)	16	12
(b)	24	12
(c)	16	18
(d)	24	18

59. Find the allowable axial compression load for the following wood columns. Use Douglas fir-larch, No. 2 grade.

	Nominal Size	Unbraced Height	
	(in.)	ft	m
(a)	4 × 4	8	2.44
(b)	6 × 6	10	3.05
(c)	8 × 8	18	5.49
(d)	10 × 10	14	4.27

60. Select square column sections of Douglas fir-larch, No. 1 grade, for the following data.

	Required Axial Load		Unbraced Height	
	kips	kN	ft	m
(a)	20	89	8	2.44
(b)	50	222	12	3.66
(c)	50	222	20	6.10
(d)	100	445	16	4.88

61. Nine-feet-high 2-by-4 studs of Douglas fir-larch, No. 1 grade, are used in an exterior wall. Wind load is 17 psf on the wall surface; studs are 24 in. on center; the gravity load on the wall is 400 lb/ft of wall length. Investigate the studs for combined action of compression plus bending.

62. Ten-feet-high 2-by-4 studs of Douglas fir-larch, No. 1 grade, are used in an exterior wall. Wind load is 25 psf on the wall surface; studs are 16 in. on center; the gravity load on the wall is 500 lb/ft of wall length. Investigate the studs for combined action of compression plus bending.
63. A 10-by-10 column of Douglas fir-larch, No. 1 grade, is 9 ft high and carries a compression load of 20 kips that is 7.5 in. eccentric from the column's centroidal axis. Investigate the column for combined compression plus bending.
64. A 12-by-12 column of Douglas fir-larch, No. 1 grade, is 12 ft high and carries a compression load of 24 kips that is 9.5 in. eccentric from the column's centroidal axis. Investigate the column for combined compression plus bending.
65. A joint similar to that in Figure 4.34 is formed with outer members of 1 in. nominal thickness ($\frac{3}{4}$ in. actual thickness) and 10d common wire nails. Find the compression force that can be transferred to the two side members.
66. Same as Problem 67, except outer members are 2 by 10, middle member is 4 by 10, and nails are 20d.
67. A truss heel joint similar to that in Figure 4.35 is made with gusset plates of 1/2-in. plywood and 8d nails. Find the tension force limit for the bottom chord.
68. Same as Problem 69, except plywood is 3/4 in. and nails are 10d.

Chapter 5

69. A simple-span, uniformly loaded beam consists of a W18 × 50 of A36 steel. Find the percentage of gain in the limiting moment if a fully plastic condition is assumed, instead of a condition limited by elastic stress.
70. Same as Problem 69, except the shape is a W16 × 45.
71. Determine the nominal moment capacity (M_n) for a W30 × 90 made of A36 steel with the following unbraced lengths: (a) 5 ft; (b) 15 ft; (c) 30 ft.
72. Determine the nominal moment capacity (M_n) for a W16 × 36 made of A36 steel with the following unbraced lengths: (a) 5 ft; (b) 10 ft; (c) 20 ft.
73. Design for flexure, assuming a plastic failure mode, a beam of A36 steel that spans 14 ft [4.3 m] and has a total superimposed, uniformly distributed, dead load of 13.2 kips [59 kN] and a total uniformly distributed live load of 26.4 kips [108 kN].
74. Same as Problem 73, except the span is 16 ft [4.9 m] and superimposed load is a single concentrated live load of 40 kips [178 kN] located at midspan.
75. A W shape of A36 steel is to be used for a uniformly loaded simple beam carrying a total superimposed dead load of 27 kips [120 kN] and a total live load of 50 kips [222 kN] on a 45-ft [13.7-m] span. Select the lightest weight shape for laterally unbraced lengths of (a) 10 ft [3.05 m]; (b) 15 ft [4.57 m}; (c) 22.5 ft [6.90 m].
76. A W shape of A36 steel is to be used for a uniformly loaded simple beam carrying a total superimposed dead load of 30 kips [133 kN] and a total live load of 40 kips [178 kN] on a 24-ft [7.32-m] span. Select the lightest weight shape for laterally unbraced lengths of (a) 6 ft [1.83 m]; (b) 8 ft [2.44 m}; (c) 12 ft [3.66 m].
77. Compute the shear capacity ($\phi_v V_n$) for the following simple-span beams of A36 steel: (a) W24 × 84; (b) W12 × 45; (c) W10 × 33.
78. Find the maximum deflection in inches for the following simple beams of A36 steel with uniformly distributed load. Find values using (1) the equation for deflection and (2) the curves in Figure 5.7.
    (a) W10 × 33, span 18 ft, total service load 1.67 kips/ft
    (b) W16 × 36, span 20 ft, total service load 2.5 kips/ft
    (c) W18 × 46, span 24 ft, total service load 2.29 kips/ft
    (d) W21 × 57, span 27 ft, total service load 2.5 kkips/ft
79. For each of the following conditions find (1) the lightest permitted shape and (2) the shallowest permitted shape of A36 steel:

	Span (ft)	Live Load (kips/ft)	Superimposed Dead Load (kips/ft)
(a)	16	3	3
(b)	20	1	0.5
(c)	36	0.833	0.278
(d)	40	1.25	1.25

80. Open-web steel joists are to be used for a roof with a live load of 30 psf and a superimposed dead load of 20 psf (not including the joist weight) on a span of 48 ft. Joists are 4 ft on center and deflection under live load is limited to 1/360 of the span. Select the lightest joist.
81. Open-web steel joists are to be used for a roof with a live load of 25 psf and a superimposed dead load of 18 psf (not including the joist weight) on a span of 44 ft. Joists are 5 ft on center and deflection under live load is limited to 1/360 of the span. Select the lightest joist.
82. Open-web steel joists are to be used for a floor with a live load of 50 psf and a superimposed dead load of 45 psf (not including the joist weight) on a span of 36 ft. Joists are 2 ft on center and deflection under live load is limited to 1/360 of the span. Select the lightest joist.
83. Open-web steel joists are to be used for a floor with a live load of 100 psf and a superimposed dead load of 35 psf (not including the joist weight) on a span of 26 ft. Joists are 2 ft on center and deflection under live load is limited to 1/360 of the span. Select the lightest joist.

84. Using data from Table 5.7, select the lightest steel deck for the following conditions:
    (a) Simple span of 7 ft, total load of 45 psf
    (b) Simple span of 5 ft, total load of 50 psf
    (c) Two-span condition, span of 8.5 ft, total load of 45 psf
    (d) Two-span condition, span of 6 ft, total load of 50 psf
    (e) Three-span condition, span of 6 ft, total load of 50 psf
    (f) Three-span condition, span of 8 ft, total load of 50 psf
85. Determine the maximum factored axial compression load for an A36 W10 × 49 column with an unbraced height of 15 ft. Assume $K = 1.0$.
86. Determine the maximum factored axial compression load for an A36 W12 × 120 column with an unbraced height of 22 ft if both ends are fixed against rotation and horizontal movement.
87. Determine the maximum factored axial compression load for the column in Problem 85 if the conditions are as shown in Figure 5.37 with $L_1 = 15$ ft and $L_2 = 8$ ft.
88. Determine the maximum factored axial compression load for the column in Problem 86 if the conditions are as shown in Figure 5.37 with $L_1 = 40$ ft and $L_2 = 22$ ft.
89. Using Table 5.9, select an A36 column shape for an axial dead load of 60 kips and an axial live load of 88 kips if the unbraced height about both axes is 12 ft. Assume $K = 1.0$.
90. Select a column shape using the same data as in Problem 89, except the dead load is 103 kips and live load is 155 kip. The unbraced height about the x axis is 16 ft and the unbraced height about the y axis is 12 ft.
91. Using Table 5.10, select the minimum-size standard weight pipe column for an axial dead load of 20 kips, a live load of 30 kips, and the following unbraced heights: (a) 8 ft; (b) 12 ft; (c) 18 ft; (d) 25 ft.
92. A structural tubing steel column, designated as $HSS4 \times 4 \times 3/8$, of steel with $F_y = 46$ ksi, is used with an effective unbraced height of 12 ft. Find the maximum total factored axial load.
93. Using Table 5.11, select the lightest structural tubing column to carry an axial dead load of 30 kips and a live load of 34 kips if the effective unbraced height is 10 ft.
94. A double-angle compression member 8 ft long is composed of two A36 angles $4 \times 3 \times \frac{3}{8}$ in. with the long legs back to back. Determine the maximum factored axial compression load for the angles.
95. Using Table 5.12, select a double-angle compression member for an axial compression dead load of 25 kips and a live load of 25 kips if the effective unbraced length is 10 ft.
96. An A36 steel W-shape column supports loads as shown in Figure 5.39. Select a trial size for the column for the following data: column factored axial load from above = 200 kips, factored beam reaction = 30 kips, unbraced column height = 14 ft.
97. Check the column shape found in Problem 96 to see if it complies with the AISC interaction formulas for axial compression plus bending.

Chapter 6

98. A concrete beam with a rectangular cross section has $f_c' = 3000$ psi [20.7 MPa] and steel with $f_y = 40$ ksi [276 MPa]. Select the beam dimensions and reinforcement for a balanced section if the beam sustains a moment due to dead load of 60 kip-ft [81.4 kN-m] and a moment due to live load of 90 kip-ft [122 kN-m].
99. Same as Problem 98, except $f_c' = 4000$ ksi [27.6 MPa], $f_y = 60$ ksi [414 MPa], $M_{DL} = 36$ kip-ft [48.8 kN-m], and $M_{LL} = 65$ kip-ft [88.1 kN-m].
100. Find the area of steel reinforcement required and select the bars for the beam in Problem 98 if the section dimensions are $b = 16$ in. [406 mm] and $d = 32$ in. [813 mm].
101. Find the area of steel reinforcement required and select the bars for the beam in Problem 99 if the section dimensions are $b = 14$ in. [356 mm] and $d = 25$ in. [635 mm].
102. Find the area of steel einforcement required for a concrete T-beam for the following data: $f_c' = 3$ ksi [20.7 MPa], $f_y = 50$ ksi [345 MPa], $d = 28$ in. [711 mm], $t = 6$ in. [152 mm], $b_w = 16$ in. [406 mm], $b_f = 60$ in. [1520 mm], $M_u = 360$ kip-ft [488 kN-m].
103. Same as Problem 102, except $f_c' = 4$ ksi [27.6 MPa], $f_y = 60$ ksi [414 MPa], $d = 32$ in. [813 mm], $t = 5$ in. [127 mm], $b_w = 18$ in. [457 mm], $b_f = 54$ in. [1370 mm], $M_u = 500$ kip-ft [678 kN-m].
104. A concrete beam section with $b = 16$ in. [406 mm] and $d = 19.5$ in. [495 mm] is required to develop a bending moment strength of 400 kip-ft [542 kN-m]. Use of compressive reinforcement is desired. Find the required reinforcement. Use $f_c' = 4$ ksi [27.6 MPa] and $f_y = 60$ ksi [414 MPa].
105. Same as Problem 104, except required moment is 1000 kip-ft [1360 kN-m], $b = 20$ in. [508 mm], and $d = 27$ in. [686 kN-m].
106. Same as Problem 104, except moment is 640 kip-ft [868 kN-m].
107. Same as Problem 105, except moment is 1400 kip-ft [1900 kN-m].
108. A concrete slab is to be used for a simple span of 16 ft [4.88 m]. In addition to its own weight, the slab carries a dead load of 40 psf [1.92 kPa] and a live load of 100 psf [4.79 kPa]. Design the slab for minimum thickness using $f_c' = 3$ ksi [20.7 Mpa] and $f_y = 40$ ksi [276 MPa].
109. Same as Problem 108, except span is 18 ft [5.49 m], dead load is 50 psf [2.39 kPa], live load is 75 psf [3.59 kPa], $f_c' = 4$ ksi [27.6 MPa], $f_y = 60$ ksi [414 MPa],

110. A concrete beam similar to that shown in Figure 6.25 sustains a uniform live load of 1.5 klf and a uniform dead load of 1 klf on a span of 24 ft. Determine the layout for a set of No. 3 U-stirrups using $f_c' = 3$ ksi and $f_y = 40$ ksi [276 MPa]. The beam section dimensions are $b = 12$ in. and $d = 26$ in.
111. Same as Problem 110, except the span is 20 ft, $b = 10$ in., and $d = 23$ in.
112. Same as Problem 110, except the live load is 0.75 klf and the dead load is 0.5 klf.
113. Same as Problem 111, except the live load is 1.875 klf and the dead load is 1.25 klf.
114. A short cantilever is achieved as shown in Figure 6.28. Determine whether adequate development is achieved without hooked ends on the bars if $L_1 = 36$ in., $L_2 = 24$ in., overall beam height is 16 in., bar size is No. 4, $f_c' = 4$ ksi, and $f_y = 40$ ksi.
115. Same as Problem 114, except $L_1 = 40$ in., $L_2 = 30$ in., and the bar size is No. 5.
116. Find the development length required for the bars in Problem 114 if the bar ends in the support are provided with 90° hooks.
117. Find the development length required for the bars in Problem 115 if the bar ends in the support are provided with 90° hooks.
118. A one-way-spanning slab is to be used for a framing system similar to that shown in Figure 6.33. Column spacing is 36 ft, with regularly spaced beams at 12 ft on centers. Superimposed dead load is 40 psf and live load is 80 psf. Use $f_c' = 4$ ksi [27.6 MPa] and $f_y = 60$ ksi [414 MPa]. Determine the thickness for the slab and select bar sizes and spacings.
119. Same as Problem 118, except column spacing is 33 ft, beams are 11 ft on centers, dead load is 50 psf, and live load is 75 psf.
120. Using Figures 6.47 through 6.50, determine the minimum-size square tied column and its reinforcement for the following data:

	Concrete Strength (ksi)	Compressive Load (kips) Live	Compressive Load (kips) Dead	Bending Moment (kip-ft) Live	Bending Moment (kip-ft) Dead
(a)	5	80	100	30	25
(b)	5	100	140	40	60
(c)	5	150	200	100	100

121. From Figures 6.51 and 6.52, determine minimum-size rectangular tied columns for the same data as in Problem 120.
122. From Figure 6.53, pick the minimum-size round tied columns for the same data as in Problem 120.
123. Using concrete with a design strength of 3 ksi and Grade 40 bars with yield strength of 40 ksi, design a square column footing for a 14-in. square column with a dead load of 100 kips and a live load of 100 kips. Maximum soil pressure is 3000 psf.
124. Same as Problem 123, except column is 18 in. square, dead load is 200 kips, live load is 300 kips, and maximum soil pressure is 4000 psf.
125. Using concrete with a design strength of 2 ksi and Grade 40 bars with yield strength of 40 ksi, design a wall footing for the following data: wall thickness = 10 in., dead load on footing = 5 kips/ft, live load = 7 kips/ft, maximum soil pressure = 2000 psf.
126. Same as Problem 125, except wall thickness = 15 in., dead load = 6 kips/ft, live load = 8 kips/ft, and maximum soil pressure = 3000 psf.

Chapter 8

127. Given the value for the dry-unit weight assuming a specific gravity of 2.65, find the following for the samples of sand listed: (1) void ratio, (2) soil unit weight and water content if fully saturated, (3) water content if wet sample weighs 110 pcf. Samples: (a) dry weight = 90 pcf, (b) dry weight = 95 pcf, (c) dry weight = 100 pcf, (d) dry weight = 105 pcf.
128. Given the value for the saturated unit weight and assuming a specific gravity of 2.7, find the following for the samples: (1) void ratio, (2) water content of the saturated sample, (3) dry unit weight. Samples: (a) saturated unit weight = 90 pcf, (b) saturated unit weight = 100 pcf, (c) saturated unit weight = 110 pcf, (d) saturated unit weight = 120 pcf.
129. Given the following data from the grain size analysis, classify the following soil samples according to general type (sand or gravel) and nature of particle size gradation (uniform, well graded, gap graded). Samples: (sizes in mm) (a) $D_{10} = 1.00$, $D_{30} = 4.00$, $D_{60} = 50.00$; (b) $D_{10} = 0.20$, $D_{30} = 1.00$, $D_{60} = 2.00$; (c) $D_{10} = 0.10$, $D_{30} = 0.25$, $D_{60} = 0.40$; (d) $D_{10} = 0.06$, $D_{30} = 0.15$, $D_{60} = 7.00$.

Appendix A

130. Find the location of the centroid for the cross-sectional areas shown in the figure. Use the reference axes indicated, and compute the distances from the axes to the centroid as shown in figure (*c*).
131. Compute the moments of inertia about the indicated centroidal axes for the cross-sectional shapes in the figure.
132. Compute the moments of inertia with respect to the centroidal x–x axes for the built-up sections in the figure. Make use of any appropriate data from the tables of properties for steel shapes.

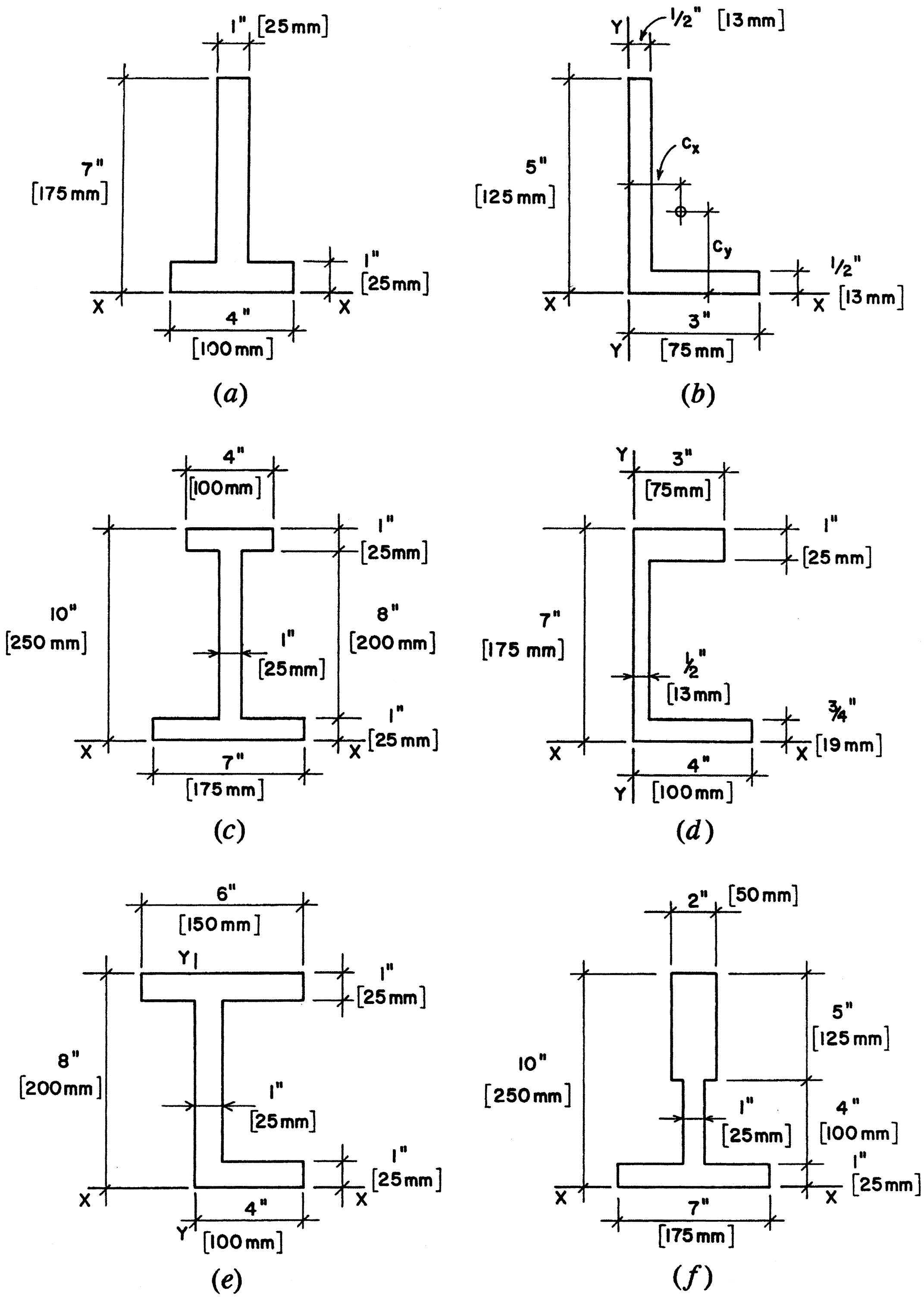


Figure C.13 Problem 130.

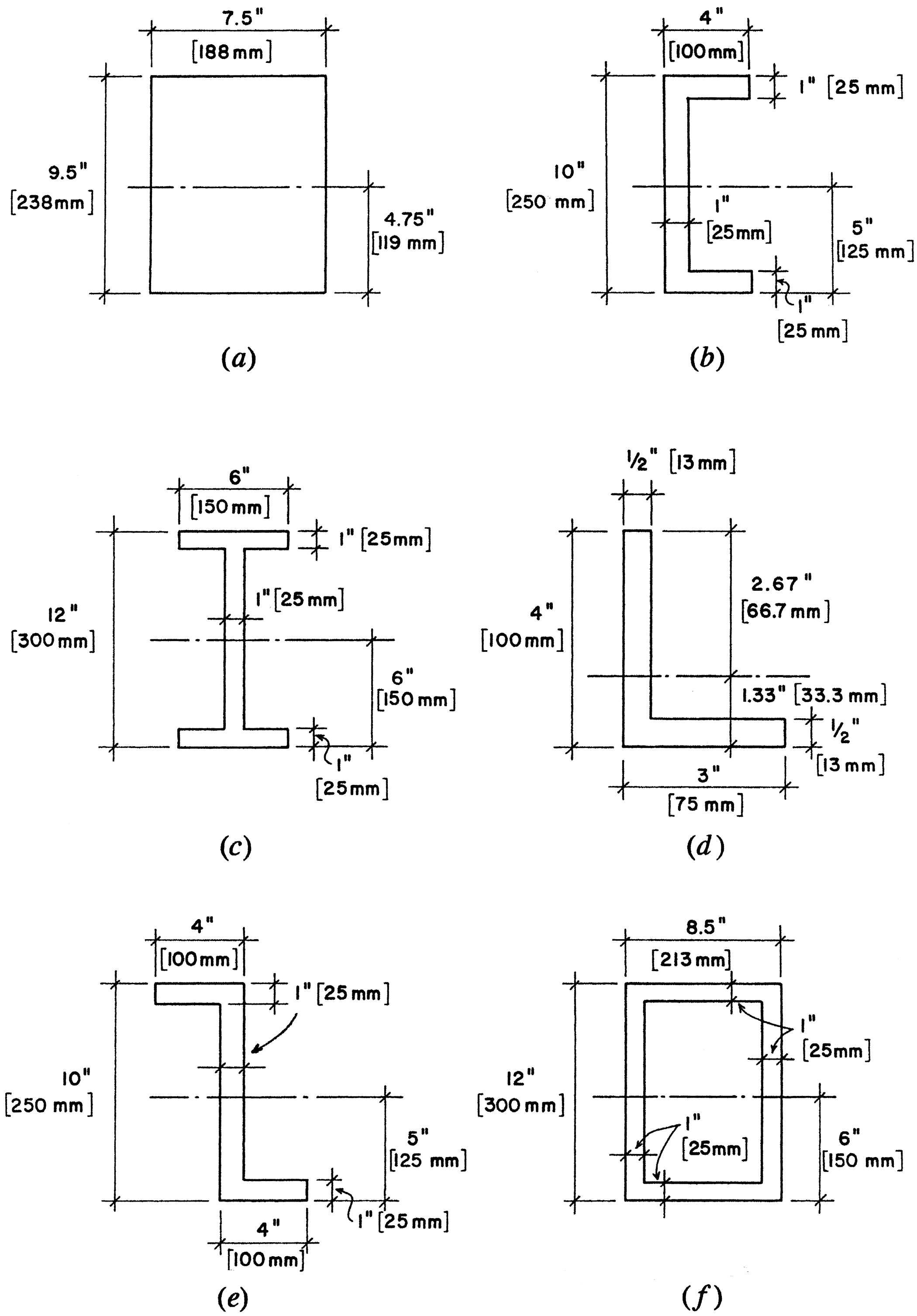


Figure C.14 Problem 131.

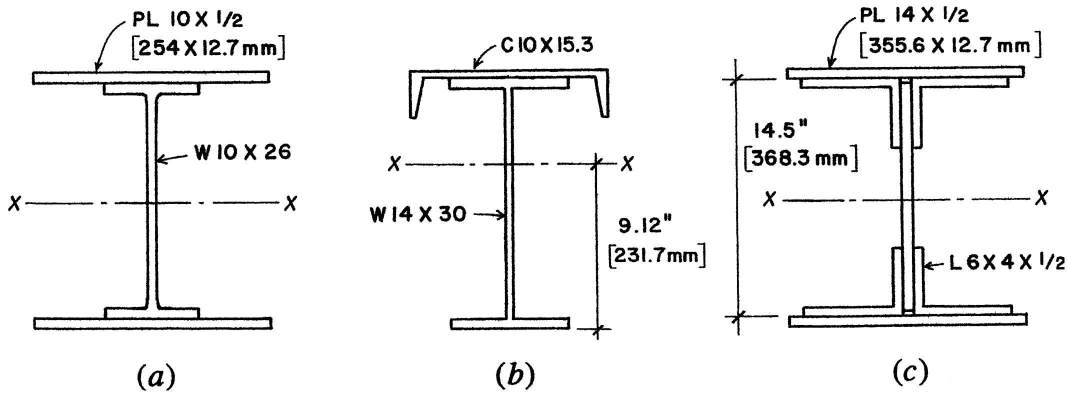


Figure C.15 Problem 132.

ANSWERS TO EXERCISE PROBLEMS

Chapter 2

14. (a) $F_v = F_h = 70.7$ lb
    (b) $F_v = 100$ lb, $F_h = 173.2$ lb
    (c) $F_v = 90.8$ lb, $F_h = 178.2$ lb
    (d) $F_v = 210$ lb, $F_h = 250.5$ lb
15. (a) $R = 141.4$ lb, $\theta = 45°$
    (b) $R = 111.8$ lb, $\theta = 63.4°$
    (c) $R = 74.6$ lb, $\theta = 54.8°$
    (d) $R = 135.4$ lb, $\theta = 20.31°$
16. (a) $R = 41.4$ lb, $\theta = 225°$
    (b) $R = 128.5$ lb, $\theta = 76.6°$
    (c) $R = 39.8$ lb, $\theta = 13.06°$
    (d) $R = 13.56$ lb, $\theta = 4.36°$
17. (a) $T = 1414$ lb
    (b) $T = 707$ lb
    (c) $T = 943$ lb
    (d) $T = 998$ lb
18. (a) Sample values: $CI = 2000$C, $IJ = 812.5$T, $JG = 1250$T
    (b) Sample values: $BI = 2828$C, $IJ = 1000$C, $IH = 2000$T
19. (a) Same as 18a
    (b) Same as 18b
20. (a) (1) $R_1 = 37.5$ lb, $R_2 = 62.5$ lb; (2) $R_1 = 60$ lb down, $R_2 = 160$ lb up
    (b) (1) $R_1 = 28.57$ lb, $R_2 = 71.43$ lb; (2) $R_1 = 40$ lb down, $R_2 = 140$ lb up
    (c) (1) $R_1 = 33.3$ lb, $R_2 = 66.7$ lb; (2) $R_1 = 50$ lb down, $R_2 = 150$ lb
    (d) (1) $R_1 = 41.75$ lb, $R_2 = 58.25$ lb; (2) $R_1 = 71.7$ lb down, $R_2 = 171.7$ lb
21. (a) $R_1 = 1128$ lb down to left, $R_2 = 2115$ lb up to left, $\theta_1 = 77.2°$; $\theta_2 = 83.2°$
    (b) $R_1 = 882$ lb down to left, $R_2 = 1863$ lb up to left, $\theta_1 = 73.5°$; $\theta_2 = 82.4°$
    (c) $R_1 = 1208$ lb down to left, $R_2 = 2100$ lb up, $\theta_1 = 65.6°$; $\theta_2 = 90°$
    (d) $R_1 = 983$ lb down to left, $R_2 = 1846$ lb up, $\theta_1 = 59.4°$; $\theta_2 = 90°$
22. 99.275 kips [442 kN]
23. 5.48 ft, or 5 ft, 6 in. [1.67 m]
24. 2.5 in.^{2} [1613 mm^2]
25. Required area is 115 in.^{2}, 12 by 12 has 132.25 in.^{2}
26. 3.47 in. [88 mm]
27. 6.185 kips [27.51 kN]
28. 231.4 kip-in or 19.3 kip-ft [26.15 kN-m]
29. 0.077 in. [1.95 mm]
30. 19.33 kips [83.3 kN]
31. (a) 1.85 in. [47 mm]
    (b) 3.08 ksi [21 MPa]
32. $f_c = 0.971$ ksi, $f_s = 4.693$ ksi
33. $f = 0.191$ ksi, $v = 0.110$ ksi
34. Read on section clockwise, starting with lower, down-slope corner, $f = +1338$ psi, -387 psi, -1338 psi, $+387$ psi
35. Using notation as shown in Figure 2.48, $f_A = -0.2279$ ksi, $f_B = +0.3581$ ksi, $f_C = +0.1627$ ksi, $f_D = +0.7487$ ksi, neutral axis measured from A toward B is 0.6.22 in. and measured from A toward C is 14.0 in
36. At neutral axis $f_v = 811.4$ psi; at junction of web and flange $f_v = 699.3$ psi in web and 174.8 psi in flange
37. 19.3°
38. Will not slide but will tip

Chapter 3

39. (a) $R_1 = 1200$ lb, $R_2 = 2400$ lb, maximum shear = 1500 lb, zero shear at 8 ft from left end, maximum $+M = 4800$ ft-lb, maximum $-M = 2700$ ft-lb, inflection at 16 ft from left end
    (b) $R_1 = R_2 = 2250$ lb, maximum shear = 1350 lb, zero shear at middle of center span, maximum $+M = 3375$ ft-lb, maximum $-M = 2700$ ft-lb, inflection at 2.3 ft from support
    (c) $R_1 = 133.3$ lb, $R_2 = 266.7$ lb, maximum shear = 133.3 lb, zero shear at 6 ft from left end, maximum

$+M = 500$ ft-lb, maximum $-M = 300$ ft-lb, inflection at 2.57 ft from right support
   (d) $R_1 = R_2 = 250$ lb, maximum shear = 100 lb, zero shear at midspan, maximum $+M = 300$ ft-lb, maximum $-M = 300$ ft-lb, inflection at 3 ft from support
40. (a) 13.46 kips
    (b) 9.43 kips
41. 112.5 kips
42. (a) 3.04 ksf
    (b) 5.33 ksf
43. (a) $R = 10$ kips up and 110 kip-ft counterclockwise
    (b) $R = 5$ kips up and 24 kip-ft counterclockwise
    (c) $R = 6$ kips toward the left and 72 kip-ft counterclockwise
    (d) Left $R = 4.5$ kips up, right $R = 4.5$ kips down and 12 kips toward the right
    (e) Left $R = 4.5$ kips down and 6 kips toward the left, right $R = 4.5$ kips up and 6 kips toward the left

Chapter 4

44. 2.695 ksi [18.6 MPa]
45. Required $S = 150$ in.^{3}, lightest is 6 by 14
46. 9.65 kip-ft [13.1 kN-m]
47. 70.2 psi [484 kPa]
48. Maximum shear stress is 62.6 psi [432 kPa], beam is OK
49. Maximum shear stress is 82.4 psi [568 kPa], beam is OK
50. Stress is 303 psi, allowable is 625 psi, beam is OK
51. Stress is 480 psi, allowable is 625 psi, beam is OK
52. $\Delta = 0.31$ in., allowable is 0.8 in., beam is OK
53. $\Delta = 0.19$ in., allowable is 0.6 in., beam is OK
54. $\Delta = 0.23$ in., allowable is 0.75 in., beam is OK
55. $\Delta = 0.30$ in., allowable is 0.8 in., beam is OK
56. Required I is 911 in.^{4}, lightest is 4 by 16
57. (a) 2 by 10
    (b) 2 by 8
    (c) 2 by 12
    (d) 2 by 12
58. (a) 2 by 6
    (b) 2 by 8
    (c) 2 by 10
    (d) 2 by 12
59. (a) 6800 lb or so, using graph
    (b) 15,700 lb or so, using graph
    (c) 21,300 lb or so, using graph
    (d) 53,000 lb or so, using graph
60. (a) 6 by 6, allowable load = 24.8 kips
    (b) 10 by 10, allowable load = 79.0 kips
    (c) 10 by 10, allowable load = 52.9 kips
    (d) 12 by 12, allowable load = 111 kips
61. For interaction, $0.094 + 0.785 = 0.879 < 1$, OK
62. For interaction, $0.094 + 0.962 = 1.056$, not OK
63. For interaction, $0.056 + 0.931 = 0.987 < 1$, OK
64. For interaction, $0.039 + 0.797 = 0.836 < 1$, OK
65. 1050 lb
66. 1700 lb
67. 1560 lb
68. 2064 lb

Chapter 5

69. 13.5%
70. 13.2%
71. (a) 849 kip-ft
    (b) 725 kip-ft
    (c) 520 kip-ft
72. (a) 192 kip-ft
    (b) 170 kip-ft
    (c) 106 kip-ft
73. W14 × 26
74. W21 × 44
75. (a) W30 × 90
    (b) W30 × 108
    (c) W27 × 114
76. (a) W24 × 55
    (b) W24 × 55
    (c) W24 × 62
77. (a) 220 kips
    (b) 78.5 kips
    (c) 54.8 kips
78. (a) (1) 0.8 in., (2) 0.9 in
    (b) (1) 0.692 in., (2) 0.7 in
    (c) (1) 0.829 in., (2) 0.8 in
    (d) (1) 0.881 in., (2) 0.9 in
79. (a) (1) W16 × 57, (2) W10 × 88
    (b) (1) W16 × 26, (2) W10 × 45
    (c) (1) W24 × 55, (2) W18 × 86
    (d) (1) W30 × 90, (2) W21 × 122
80. 26K7
81. 30K7
82. (a) 24K4
    (b) 22K6
83. (a) 20K3
    (b) 16K6
84. (a) WR20
    (b) IR22
    (c) WR18
    (d) WR22
    (e) IR22 or WR22
    (f) WR20
85. 338 kips [1606 kN]
86. 921 kips [4067 kN]
87. 401 kips [1957 kN]
88. 717 kips [3189 kN]
89. W8 × 31
90. W12 × 53
91. (a) 4-in. standard pipe
    (b) 5-in. standard pipe

(c) 6-in. standard pipe
(d) 8-in. standard pipe

92. 98 kips [436 kN]
93. HSS6 × 6 × 3/16
94. 104 kips [463 kN]
95. 4 × 3 × 5/16
96. W12 × 45
97. Complies

Chapter 6

98. Possible choice: b = 12 in., d = 14.5 in., requires 7.60 in.^{2} of steel, use 5 No. 11 bars. However, width required to get bars into one layer is critical; must use wider beam or two layers of bars.
99. Possible choice: b = 12 in., d = 10.76 in., requires 4.02 in.^{2} of steel, use 4 No. 9 bars
100. From work for Problem 101, this section is larger than a balanced section, try $a/d = 0.4$, required area is 2.81 in.^{2}, actual $a/d = 0.086$, new area is 2.34 in.^{2}, use 3 No. 9, width OK
101. Required area = 1.37 in.^{2}, use 2 No. 8
102. For bending moment approximate required area is 3.84 in.^{2}; however, minimum based on width of flange is 5.52 in.^{2}
103. Flexure requires 3.77 in.^{2}, but minimum is 5.46 in.^{2}
104. Section does not require compressive reinforcement, but if desired use 5 No. 9 for tension and 3 No. 7 for compression
105. Section does not require compressive reinforcement, but if desired use 6 No. 11 for tension and 3 No. 9 for compression
106. Section does not require compressive reinforcement, but if desired use 5 No. 11 for tension (width required is 17 in.) and 3 No. 9 for compression.
107. Section does not require compressive reinforcement, but if desired use 8 No. 11 (requires greater width or two layers) for tension and 4 No. 9 for compression.
108. For deflection use 8-in. slab, reinforce with No. 8 at 16 in., No. 7 at 12 in., No. 6 at 9 in., or No. 5 at 6 in.; use No. 4 at 12 for temperature.
109. Requires 11-in. slab, No. 5 at 11, No. 6 at 16 in., or No. 7 at 18 in.; No. 4 at 9 for temperature
110. Possible choice: 1 at 6, 8 at 13
111. Minimum reinforcement: 1 at 5, 8 at 11
112. Possible choice, 1 at 6, 5 at 13
113. Possible choice: 1 at 5, 9 at 11
114. Required length in cantilever is 17 in., 34 in. provided; required length in support is 13 in., 22 in. provided; anchorage as shown is adequate, but use hook anyway—just feels right
115. Lengths provided are adequate.
116. Required length is 4.9 in., but use full available length.
117. Required length is 5.6 in., but use full available length.
118. For deflection need 5.5-in. slab; referring to Figure 6.35, left to right, with all No. 4 bars, use spacings of 16, 18, 13, 20, 16 in.
119. Need 5-in. slab, referring to Figure 6.35, with all No. 4 bars, use spacings of 15, 15, 12, 15, 15 in.
120. (a) 12-in. square, 4 No. 8
(b) 14-in. square, 8 No. 9
(c) 18-in. square, 8 No. 11
121. (a) 12 by 16, 6 No. 7
(b) 12 by 16, 6 No. 7
(c) 14 by 20, 6 No. 9
122. (a) 14 in. diameter, 4 No. 11
(b) 16 in. diameter, 6 No. 10
(c) 20 in. diameter, 8 No. 11
123. From Table 6.13, possible choice is footing 9 ft square by 21 in. thick with 9 No. 8 each way; computation will permit footing 8 ft, 8 in. wide by 20 in. thick with 7 No. 8 each way.
124. Table yields 12 ft square by 32 in. thick with 11 No. 10 each way; computations will permit footing 11 ft, 9 in. square by 30 in. thick with 10 No. 10 each way.
125. Possible choice is footing 6 ft, 9 in. wide by 17 in. thick with No. 5 at 11 in. in short direction and 7 No. 5 in long direction.
126. Possible choice is footing 5 ft wide by 16 in. thick with No. 5 at 18 in. in short direction and 5 No. 5 in long direction.

Chapter 8

127. (a) (1) $e = 0.838$, (2) saturated weight = 118 pcf, (3) $w = 22\%$
(b) (1) $e = 0.739$, (2) saturated weight = 121.5 pcf, (3) $w = 16\%$
(c) (1) $e = 0.653$, (2) saturated weight = 125 pcf,
(d) (3) $w = 10\%$
(e) (d) (1) $e = 0.575$, (2) saturated weight = 128 pcf, (3) $w = 4.8\%$
128. (a) (1) $e = 2.85$, (2) $w = 105\%$, (3) dry weight = 44
(b) pcf
(c) (1) $e = 1.84$, (2) $w = 68\%$, (3) dry weight = 59 pcf
(d) (1) $e = 1.23$, (2) $w = 45\%$, (3) dry weight = 76 pcf
(e) (1) $e = 0.84$, (2) $w = 31\%$, (3) dry weight = 91.5 pcf
129. (a) Poorly graded sandy gravel, probably GP
(b) Well-graded sand, maybe SW
(c) Uniform fine sand, poorly graded, maybe SP
(d) Gap-graded silty sand, maybe SP or SM

Appendix A

130. (a) $c_y = 2.6$ in. [66 mm]
(b) $c_y = 1.75$ in. [43.9 mm], $c_x = 0.75$ in. [18.9 mm]
(c) $c_y = 4.2895$ in. [107.24 mm]
(d) $c_y = 3.4185$ in. [85.185 mm], $c_x = 1.293$ in. [32.2 mm]

(e) $c_y = 4.4375$ in. [110.9 mm], $c_x = 1.0625$ in. [26.6 mm]
(f) $c_y = 4.3095$ in. [107.7 mm]

131. (a) $\bar{I} = 535.86$ in.^{4} [223×10^6 mm^4]
(b) $I = 205.33$ in.^{4} [85.46×10^6 mm^4]
(c) $I = 447.33$ in.^{4} [186×10^6 mm^4]
(d) $I = 6.5325$ in.^{4} [2.72×10^6 mm^4]
(e) $I = 205.33$ in.^{4} [85.46×10^6 mm^4]
(f) $I = 682.33$ in.^{4} [284×10^6 mm^4]

132. (a) $I = 438$ in.^{4} [182×10^6 mm^4]
(b) $I = 420.1$ in.^{4} [175×10^6 mm^4]
(c) $I = 1672.49$ in.^{4} [696×10^6 mm^4]

APPENDIX

D

Study Aids

The materials in this section are provided for readers to use in order to test their general understanding of the book presentations. It is recommended that, upon completion of reading of an individual chapter, the materials here be used as a review. Materials include terms for which a definition and general understanding of the significance should be developed and general questions regarding various issues. Answers to the questions are provided following the questions. Use the Glossary and Index to find the applicable book materials.

TERMS

Chapter 1

Allowable stress design method
Arch
Beam
Bending
Building–ground relationship
Combination of loads
Curtain wall
Dispersion of loads
Dome
Ductile material
Dynamic load
Equilibrium
Filter functions of the building exterior
Flat-spanning structure
Form–scale relationships
Framed structure
Fundamental period
Geodesic dome
Lamella frame
Lintel
Mast structure
Modulus of elasticity
Monopod unit
One-way spanning structure
Overturning
Partition
Post
Reaction
Rigid frame
Safety factor
Service load
Strain
Strain hardening
Strength design method
Stress
Structural wall
Surface structure
Truss
Two-way spanning structure

Chapter 2

Bending
Coefficient of thermal expansion
Combined stress (shear plus direct, in beams)
Composite structural systems
Composition and resolution of forces
Cut section
Equilibrium
Equivalent static effect
Force
Force polygon
Force systems (arrangement)
Free-body diagram

Friction, sliding, coefficient of
Harmonic motion: amplitude, cycle, period, frequency, damping, resonance
Internal force
Kinematics: displacement, velocity, acceleration
Lateral deformation, shear
Load-deflected structural profile
Maxwell diagram
Modulus of elasticity
Moment
Motion: translation, rotation, rigid body, of deformable body
Properties of a force: magnitude, sense, direction
Resultant
Shear: force, stress (direct, in beam)
Shear center
Shear modulus (of elasticity)
Space diagram
Strain (and deformation)
Stress: direct, shear, compressive, tensile, bending, torsional, unit, allowable, ultimate
Torsion: moment, stress
Unsymmetrical bending

Chapter 3

Arches, type: tied, fixed, two hinged, three hinged
Beam, type: simple, cantilever, overhanging, continuous, restrained, indeterminate
Bearing wall
Bent
Buckling (of beams): lateral, torsional (rotational)
Cable
Combined stress: tension plus bending, compression plus bending
Column
Cracked section
Determinacy and stability of trusses
Diagrams: loading and support conditions, shear, moment, deflected shape
Effective area (for tension development)
Effective buckling length, column
Interaction, compression plus bending
Internal forces in beams: shear, bending
Investigation of trusses, for internal forces, methods of: graphic (Maxwell diagram), joints, sections, beam analogy
Kern
Members of trusses: chords, web
Net section
Panel point, in truss
P−delta effect
Pier
Reactions
Rigid frame
Rotation: of beam cross section, of beam end
Slenderness of compression member
Support conditions: rotation free (simple, pinned), rotation restrained (fixed)
Tie
Truss, form of: gabled, flat (parallel chorded), bent (rigid frame), Vierendeel
X-brace

Chapter 4

Axial compression
Bearing
Blocking
Board deck
Built-up member
Chord
Common wire nail
Dimensional stability
Edge distance
Engineered wood products
Fastener
Glued-laminated products
Grade of structural lumber
Joist
Lag screw
Lateral load on nail
Lateral support requirements: bridging, blocking, depth-to-width ratio
Load-to-grain orientation: parallel, perpendicular, angled
Lumber, structural
Modification factors for design reference values
Moisture conditions
Nail, common wire
Net section for tension
Penetration, of nail or screw
Pitch of fasteners
Plies (plywood), face, core
Plywood
Pole structure
Rafter
Repetitive stress use
Sandwich panel
Size factors for wood beams
Slenderness ratio, column
Solid-sawn wood
Spaced column
Species, of tree
Stud
Veneer
Withdrawal resistance, of nail or screw
Wood fiber products

Chapter 5

Bearing, beam
Bending factor, column
Buckling: of column, of beam (lateral, torsional), of beam web, of truss

Built-up sections, columns
Cold-formed shapes
Compact section
Double-angle compression member
Ductility
Effective column length (K factors for columns)
Formed-sheet steel products
Lateral unsupported length
Least-weight design choice
Light-gauge steel products
Open-web steel joist
Rolled shape
Slenderness ratio, column
Strut
Torsional buckling
Web crippling
For Bolted Connections:
Double shear
Edge distance
Effective area (in tension)
Fastener
Framed beam connection
Gauge, for angles
High-strength bolt
Pitch
Single shear
Tearing

Chapter 6

Admixture
Aggregate: fine, coarse, lightweight
Air-entrained concrete
Balanced section, of beam
Bearing wall
Cast-in-place concrete (sitecast)
Cement
Composite construction
Cover
Creep
Curing
Deformed bars (reinforcement)
Design strength (ultimate compressive), specified as f'_c
Development, of reinforcement
Development length
Diagonal tension stress
Doubly reinforced section
Dowel
Effective depth, of beam
Elastic ratio, n
Factored load
Flat plate
Flat slab
Footing: wall, column
Forming
Freestanding wall
Grade, of reinforcement steel
Grade wall
Hook: standard, equivalent development length of
Interaction, compression and bending
Minimum thickness of slab; depth of beam
One-way solid slab
Pedestal
Peripheral (punching) shear
Precast concrete
Reinforcement
Retaining wall
Shear wall
Shrinkage of concrete
Shrinkage reinforcement
Sitecast concrete
Slab–beam–girder system
Spacing of reinforcement
Specified compressive strength of concrete, f'_c
Spiral column
Splice
Standard hook
Stirrup
Strength design (LRFD)
Strength reduction factor, ϕ
T-beam
Temperature reinforcement
Tied column
Two-way solid slab
Waffle construction
Water–cement ratio
Workability, of wet concrete

Chapter 7

Adobe
Architectural terra cotta
Beam pocket
Bond beam
Brick wall terms: wall, pier, column, pedestal
Clay tile
CMU
Course
Grouted void
Header (brick)
Lintel
Masonry
Masonry type: cavity, solid, grouted, unreinforced, reinforced, veneer
Masonry unit
Nonstructural masonry
Pedestal
Reinforced masonry
Specified compressive strength of masonry, f'_m
Unreinforced masonry
Veneer
Wythe

Chapter 8

Abutment
Active lateral soil pressure
Allowable bearing pressure
Atterberg limits
Backfill
Belled pier
Braced cut
Cantilever: footing (strap), retaining wall
Clay
Cohesionless soil
Cohesive soil
Combined column footing
Compaction
Compressibility of soil
Consolidation
Curb
Cut
Density
Dewatering
Downhill frame
Drilled pier
Equalized settlement
Equivalent fluid pressure
Expansive soil
Fill
Fines
Footing: wall, column, combined, cantilever, rectangular
Foundation wall
Gap-graded soil
Grade beam
Grain shape
Grain size
Gravel
Key (shear)
Liquid limit
Moment-resistive footing
Passive lateral soil pressure
Penetration resistance, *N*
Permeability
Pier: deep foundation, pedestal, wall
Pile: cap, end bearing, friction, sheet
Plasticity index
Preconsolidation
Presumptive bearing pressure
Rectangular (oblong) footing
Retaining wall
Rock
Sand
Sheet piling
Shrinkage limit
Silt
Soil
Soil structure: loose, dense, compacted, honeycombed, flocculent
Specific gravity, of soil solids
Surcharge
Unconfined compressive strength, of cohesive soil
Void, in soil
Void ratio

Chapter 9

Acceleration
Accidental eccentricity
Base isolation
Braced frame
Center of stiffness, of vertical bracing
Collector
Diaphragm: horizontal, vertical
Diaphragm chord
Drift
Eccentric bracing
Fundamental period
Horizontal anchor
Horizontal movement due to vertical load
Horizontal thrust: gabled structure, arch, cable, rigid frame, cantilevered structure
K-bracing
Lateral force of: thermal expansion, shrinkage, moisture change, soil pressure
Lateral resistive systems: box, braced frame, rigid frame, self-stabilizing structure
Liquefaction, of soil
Load sharing
Multimassed building
Overturn
Perimeter bracing
Regularity of building and structural form
Relative stiffness of: horizontal diaphragm, vertical bracing system
Rigid frame
Seismic separation joint
Shear wall
Site–structure interaction
Soft story
Three-sided building, lateral bracing system
Tiedown (hold-down)
V-bracing
Vulnerable elements
Weak story
Wind actions on buildings: sliding, uplift, overturning, torsion, clean-off effect, basic design pressure
Wind effects (general, direct) on any stationary object in the wind path: direct pressure (positive), negative pressure (suction), drag (wind shear, friction), gust, harmonic effects
X-bracing
Zones of probability (from codes): for windstorms, for earthquakes

Chapter 10

Building code: model, legal (locally enforced)
Construction (building) permit
Design; designing; designers
Industry standards
Planning of structures
Process of design of building structures

Appendix A

Centroid
Moment of inertia
Parallel-axis theorem
Radius of gyration
Section modulus

QUESTIONS

Chapter 1

1. What is the most direct way of implementing concern for structural safety in the building design process?
2. Why is design for fire safety in buildings viewed as a race against time?
3. What is the essential difference between the structural design methods called ASD and LRFD?
4. Describe some ways in which the building exterior shell serves as a selective filter.
5. Describe some basic structural functions for walls.
6. What are the architectural functions and construction considerations that differentiate between structural and nonstructural walls?
7. Both roofs and floors require spanning structures. What are some differences in design requirements for the spanning structure, depending on whether it supports a roof or a floor?
8. With regard to the building structure, what changes in design requirements are implied by the following changes in the building's form or scale?
   (a) Single-space building; change from short to long span.
   (b) Multiple linear space building; change from regular repeated unit size to multiple sizes of spaces.
   (c) Multistory building; change from low rise to high rise.
9. Buildings are mostly built above ground. In comparison to the usual forms of construction, what are some differences in structural requirements for buildings built below ground surface level?
10. Structural tasks are viewed primarily in terms of loads. What is the basic nature of the load derived from the following sources?
    (a) Gravity
    (b) Wind
    (c) Seismic activity
11. What are some significant structural concerns implied by the following types of loads?
    (a) Live load
    (b) Dead load
    (c) Dynamic load
    (d) Concentrated load
    (e) Multiple potential load combinations
12. Both loads and reactions are visualized as exterior forces on a structure. What is the essential difference between them?
13. How are internal resistive forces generated in a structure?
14. What stress–strain response characterizes a material that is described as ductile?
15. What are the essential structural properties of a material that relate to design of building structures?
16. For classification of wood as a structural material, what is the significance of the following?
    (a) Species
    (b) Grade
    (c) Moisture content
17. Although steel is an essential material for most structures, what are some of the basic, unavoidable difficulties in its use?
18. Concrete is a highly variable material. Describe some of the basic structural and general physical properties of finished concrete that are subject to variation during the production process.
19. As a finished structural material, masonry is similar to concrete. What are some of the basic differences?
20. Describe some of the basic structural functions that are required of structures.
21. An arch and a draped catenary cable are essentially similar in basic form in profile. What is the basic difference between them as it relates to the definition of their structural tasks?
22. Describe some specific examples of structures that relate to the following categories.
    (a) Solid structures
    (b) Framed structures
    (c) Surface structures
23. Describe some ways in which walls may be stabilized against loads that are perpendicular to the wall plane.
24. What are the ways that a post-and-beam frame can be stabilized against lateral loads in the plane of the frame?
25. What is the basic factor that makes a rigid frame different from a simple post-and-beam frame?
26. Depth (vertical dimension) is a critical factor for a flat-spanning structure. How can depth be obtained, other than by an increase of the solid mass of material?
27. What are the two basic structural characteristics that identify a typical trussed structure?

28. What is the basic relationship between rise and thrust in an arch?
29. Compression surface structures require a structural property that differentiates them from tension surface structures. What is it?

Chapter 2

1. Investigation of structural behaviors have some specific uses. What are they?
2. What basic means are used for performing structural investigations?
3. What aspects of structural behavior are determined through the use of the following?
   (a) A free-body diagram
   (b) A cut section
4. What are the basic vector properties of a single force?
5. Geometric classification of force systems is based on what three considerations?
6. With regard to force analysis, what is the difference between the following?
   (a) Composition and resolution
   (b) Component and resultant
7. For a system of coplanar, concurrent forces, what does the closing of the force polygon signify?
8. In algebraic analysis, what are the necessary conditions for equilibrium of a coplanar, concurrent force system?
9. What is the basic geometric principle of structural analysis that permits the use of a Maxwell diagram for investigation of internal forces in a truss?
10. For a coplanar, parallel force system, it is possible to have the sum of all the forces equal to zero without having equilibrium. Why is this?
11. Explain the difference between force and stress.
12. With respect to actions on a cross section, what is the basic difference between direct stress (tension or compression) and shear stress?
13. What is the difference between unit strain and total deformation?
14. What is meant by the term "double shear"?
15. What direct stress condition exists at the neutral axis of a cross section in a structural member subjected to bending?
16. How is member deformation measured in a structure subjected to torsion?
17. What is the significance of the yield point in stress–strain behavior?
18. How can thermal change produce stress in a structural member?
19. The difference in what structural property results in the nonuniform distribution of stress in a composite structural member?
20. Shear stress developed in one direction results in a secondary shear stress in a perpendicular direction. How are these two stresses related in magnitude?
21. Shear stress produces diagonal tension and diagonal compression stresses. How is the magnitude of the maximum diagonal stress related to the magnitude of the shear stress?
22. In addition to bending, what occurs in a beam when loads are not in the plane of the beam cross section's shear center?
23. How does the distribution of shear stress on the cross section of a member subjected to direct shear force differ from that on a cross section in a beam?
24. What numerical relationship usually exists between a friction force and the force normal to the plane in which the friction is developed?
25. What generally distinguishes the difference between the fields of kinematics and kinetics?
26. How do the means of measurement differentiate motions of translation and motions of rotation?
27. How are work and energy related?
28. What is the usual reason for using the equivalent static force method for the investigation of dynamic actions on structures?

Chapter 3

1. With regard to load deformations in a beam, what is significant about the following support conditions?
   (a) Simple support
   (b) Restrained (fixed) support
   (c) Multiple supports (continuous beam)
2. What is indicated when a moment in a beam is described as either positive or negative?
3. What constitutes the complete external force system that operates on a beam?
4. What are the significant relationships between the shear diagram and the moment diagram for a beam?
5. What are the significant relationships between the moment diagram and the deflected shape for a beam?
6. What are the basic forms of buckling for an unbraced beam?
7. Pure axial tension action tends to produce what geometric responses in a linear structural member?
8. What is the significance of the net section in a tension member?
9. What is the basic relationship between the loads and the geometric profile of a flexible, draped, spanning cable?
10. What are the basic structural responses of short and long compression members?
11. How do end support conditions affect the buckling of columns?
12. The P–delta effect is sometimes described as being potentially accelerating. How does this occur?
13. What essential response characteristic of the material of a structure determines the development of a cracked section?
14. For planar trusses, what geometric condition is required in determining the arrangement of truss members to assure stability?

15. Why is it generally desired that trusses be loaded only at their panel points (joints)?
16. What essential interaction is required between members of a rigid frame?
17. A single-span rigid frame bent with fixed column bases is said to be indeterminate. Why is this?
18. The three-hinged arch is popular for medium-span buildings. What are some reasons for this?
19. What planning considerations make the use of two-way spanning systems feasible?

Chapter 4

1. How does moisture content affect design values for structural lumber?
2. What general qualification of loads permits the use of load duration modification factors for reference design values?
3. Why is the depth of a beam cross section more important than its width in determining bending resistance?
4. Why are heavy loads on short spans not often feasible for solid-sawn wood beams?
5. What is the purpose of bridging or blocking in joist and rafter construction?
6. Why is deflection of particular concern in the design of joists and rafters?
7. For structural purposes, what is the primary determining factor for the choice of thickness of a wood deck?
8. Why is the strength in compressive stress of the material not the major concern for a slender column?
9. Why is it not possible in most cases to simply invert the direct stress formula ($f = P/A$) for a solution in the design of a column?
10. Although the limitation for maximum slenderness of $L/50$ for a wood column would limit the height of a 2-by-4 stud to 75 in., what makes it possible to use 2-by-4 studs for greater heights?
11. When a column is subjected to bending, why is it not possible to simply add the maximum stresses due to axial compression and bending in order to find a load limit for the column?
12. Why is the two-member bolted joint not a favored form for development of force transfer?
13. Why is resistance to withdrawal not a reliable form of development of load resistance in nailed wood joints.
14. What is the reason for the difference in resistance of a bolted joint in wood based on the direction of the load with respect to the grain in the wood?
15. How is structural performance improved when a shear developer is added to a bolted joint between wood members?
16. Describe the factors that usually determine the general profile and the arrangements of members and joints for a truss.
17. What generally determines the selection of member form and the method of making joints for a truss?
18. What is structurally significant about the face plies of a plywood panel?
19. What advantages may be gained by the use of fabricated I-beams in place of solid-sawn rafters and joists?

Chapter 5

1. Why is the yield point generally of more concern than the ultimate strength of the steel for rolled structural products?
2. Why is the depth indicated in the designation for a W shape referred to as a nominal dimension?
3. Steel is generally considered to be vulnerable in exposed conditions. What are the primary concerns in this situation?
4. What single property of a beam cross section is most predictive of bending strength?
5. What is significant about the properties for a rolled W shape designated as L_p and L_r?
6. For shapes of A36 steel used as beams, what makes it possible to say that all beams of the same depth will have the same deflection at their limiting loads on a given span?
7. What is the most common means for dealing with torsion on a W-shape beam?
8. What single property of the cross section is most critical to the resistance of web crippling in a W-shape beam?
9. For evaluation of simple axial compression capacity, what are the most significant properties of the cross section of a steel column?
10. What makes it necessary to consider the effects of buckling on both axes of a W-shape steel column?
11. When so-called high-strength steel bolts are used for a lapped connection between steel members, what basic action develops the initial load resistance in the joint?
12. Tearing in a bolted connection is resisted by what combination of stress developments?
13. Other than spacing and edge distances, what basic dimension limits the number of bolts that can be used in a framed beam connection?
14. Why is it not desired to have supporting steel beams of the same depth as the beams they support?
15. What advantage is gained by the use of a so-called self-bracing form for a truss?
16. What are the usual reasons for using built-up sections, rather than single rolled shapes, for steel beams or columns?

Chapter 6

1. What is the primary structural limitation of concrete that generates the need for reinforcement?
2. What is the significance of having a well-graded range of size for the aggregate in concrete?

3. What is the primary determinate of the unit weight (density) of finished concrete?
4. What is the purpose of the deformations on the surface of steel bars used for reinforcing concrete?
5. What structural property is most significant for the grade of steel used for reinforcing bars for concrete?
6. During the curing period for concrete (after casting and before significant structural usage), what controls should be exercised?
7. What are the significant factors that determine required cover dimensions for concrete reinforcement?
8. Other than spacing limits and bar diameters, what factors establish the maximum number of bars that can be placed in a single layer in a reinforced concrete beam?
9. Why is compressive strength in the concrete generally not critical for T-beam sections in sitecast concrete construction?
10. For shear actions in concrete beams, what acts together with the vertical stirrups to resist the diagonal tension stresses?
11. Stirrups are designed to resist what portion of the total shear force in a concrete beam?
12. Why is development length generally less critical for small-diameter reinforcing bars?
13. Why are longer development lengths required for bars of higher grade steel?
14. In what form is the anchorage capacity of a hook expressed?
15. What is the basis for establishment of the required length of a lapped bar splice?
16. What are the usual considerations for determination of slab thickness in a slab–beam–girder framing system?
17. How is the minimum depth required for slabs and beams affected by span and support conditions?
18. What is the essential difference in structural action between concrete joist construction and waffle construction?
19. What structural improvements are achieved by the use of column capitals and drop panels in flat-slab construction?
20. What is the essential function of the shear developers (welded steel studs) in a composite structure with steel beams and a sitecast concrete deck?
21. With regard to effects on the vertical reinforcement, what is the primary purpose of the ties in a tied column?
22. With a column subjected to a large bending moment, why is a slightly higher moment possible with addition of a minor axial compression load?
23. What is the usual basis for limitation of the number of bars that can be placed in a spiral column?
24. Other than forming considerations, what favors the use of a square plan shape for a column footing?
25. What is the purpose of the longitudinal reinforcement (parallel to the wall) in a wall footing?
26. How is the choice of reinforcing bar size in a concrete column related to the thickness of the support footing?
27. How does the use of a pedestal result in the possibility of a thinner footing?

Chapter 7

1. Why is the quality of the mortar and the structural integrity of the units of less concern when concrete masonry units are used with reinforced construction?
2. What is the purpose of the brick headers in a multiple-whythe brick masonry wall?
3. Why are there usually fewer and larger voids in the concrete masonry units that are used for reinforced construction?
4. What is the basis for the old-time definition of mortar as a material used to keep masonry units *apart*?
5. What types of reinforcement or strength enhancement are possible with masonry wall construction in addition to the use of steel reinforcing bars?

Chapter 8

1. What are the basic purposes for site exploration to assist the design of building foundations?
2. What are the principal engineering properties of soils that most affect design of foundations?
3. What factors regarding a soil have major effects on building construction?
4. What two materials usually take up the void in a soil?
5. What condition makes it possible to estimate the void ratio of ordinary soils when only the dry unit weight is known?
6. What does it mean to say that a clay is "fat"?
7. Besides predominant grain size, what single property is most importane for distinguishing between silts and clays?
8. What condition qualifies a sand or gravel as clean?
9. Why is it desirable to keep the forces acting on a moment-resistive footing within the kern limit of the footing?
10. Why is a combined column footing sometimes not placed in a symmetrical position with respect to the columns it supports?
11. How does the depth of the footing below grade affect the general behavior of a laterally loaded, moment-resistive foundation?
12. What are the principal considerations that influence the decision to use a deep foundation instead of a shallow bearing foundation?
13. Why are piles usually used in groups?
14. What soil conditions make the installation of piles or drilled piers difficult?

15. What is the principal cause of the deterioration of the tops of wood or steel piles?
16. What is the difference between active lateral pressure and passive lateral pressure in soils?
17. How is frictional resistance determined for the following soils?
    (a) Sand
    (b) Clay
18. How is lateral soil pressure determined by the equivalent fluid pressure method?

Chapter 9

1. When wind forces act on a building, various elements of the building construction typically perform functions in the development of the building's response. Briefly describe the usual role of the following elements in this regard:
   (a) Exterior walls facing the wind
   (b) Exterior walls parallel to the wind direction
   (c) Roofs and upper level floors
   (d) Building foundations
2. Repeat question 1, except that the force is due to an earthquake, considered as horizontal movement in a single direction.
3. What basic forms of structural response are developed by the following bracing systems in resisting lateral loads?
   (a) Box system
   (b) Braced (trussed) frame
   (c) Rigid frame
4. Ordinary light wood frame construction may have some deficiencies with regard to resistance of seismic forces. Describe some of these potential weaknesses.
5. Why are horizontal, rather than vertical, effects of wind and earthquakes generally more critical for structural design?
6. What form of masonry construction is least desirable for resistance to earthquake forces?
7. Architectural design decisions greatly affect the determination of the character of response of a building to lateral loads. Describe some design features of buildings that may complicate or make difficult the design for lateral load responses.
8. What characterizes the structural nature of the so-called three-sided building?
9. What constitutes a so-called soft story?
10. Why are wind pressures greater on the upper portions of tall buildings?
11. What relationship exists between the magnitude of wind velocity and the resulting pressure on stationary objects in the wind's path?
12. How does wind or seismic force usually generate a torsional twisting effect on a building?
13. What basic dynamic properties of a building affect the building's response to an earthquake in terms of the magnitude of force exerted on the building?
14. What does a "seismic separation joint" usually separate?
15. What is generally required of a structural connection to qualify it as "positive"?
16. What does base isolation accomplish with regard to the total seismic force on a building?
17. What factors of the dynamic response of the bracing system are affected by an in-line shock absorber?
18. What basic function does a chord perform for a horizontal diaphragm?
19. Other than base anchorage, what resists overturn on a shear wall?
20. What is the significance of nail spacing at the edges of plywood panels in a horizontal diaphragm?
21. Why is a braced (trussed) frame considered to be a relatively stiff lateral bracing system?
22. What actions in the structure are added to the usual truss actions when knee bracing or K-bracing is used?
23. Why is the term "rigid" inappropriate in describing the so-called rigid frame bracing system for lateral loads?
24. When used in conjunction with a horizontal diaphragm, what does a "collector" usually collect?
25. From a structural viewpoint only, what is the usual simplest means for resolving horizontal thrust from an arch, a gabled frame, or a single-span rigid-frame bent?
26. Why are horizontal ties not required for domes or round suspension structures?
27. What is the basic method most often used to alleviate thermal expansion in a very long building?

Chapter 10

1. What is the function of a "model" building code?
2. Of what does the design dead load primarily consist?
3. What primary factor affects the percentage of live-load reduction?
4. Why is the achievement of optimal structural efficiency not always a dominant concern for control of the cost of building construction?

ANSWERS TO QUESTIONS

Chapter 1

1. Use of a safety factor in design computations.
2. Primary concerns are for getting the occupants safely out and letting firefighters work on the fire before the building collapses.
3. ASD is based on service conditions; LRFD is based on failure limits.
4. By controlling and/or preventing the passage of heat, air, light, people, and so on.
5. Major structural uses are for bearing walls, shear walls, and resistance of wind pressures (on exterior walls).

Nonstructural walls have many tasks—principally for enclosure and for definition and separation of interior spaces.

6. All walls have some form of structural task (such as holding themselves up), but use for major structural functions is another level of concern. See answer to question 5.
7. Roofs should slope for drainage; floors are usually dead flat. Floor live loads are usually greater. Floor spans are limited by deflection. Roof spans are sometimes the longest in building construction. Floors should not bounce; roofs are less critical in general to structural movements.
8. (a) Feasible systems change with span length; only a few, very efficient, systems are feasible for very long spans—arch, dome, cable, truss, etc.
   (b) Complicates choice of most feasible system for span; see answer to (a). Also makes use of a regular plan module difficult.
   (c) Columns and foundations become larger at ground and below-grade levels. Lateral bracing and resistance to overturn become critical factors for selection of structure, plan, and building form in general.
9. Roof with soil or terrace load becomes a very heavy structure, compared to the usual light roof structure. Wind is not a problem. Materials and details for building exterior (as basement walls) are drastically limited.
10. (a) Vertical only, of constant magnitude, enduring forever.
    (b) Fluid flow, producing aerodynamic effects: suction, drag, flutter, etc. Magnitude varies with height above ground. Upward force as well as horizontal. Major force is a short-duration gust, with a slamming effect.
    (c) Random in direction: up and down, back and forth in all directions. Cyclic, but irregular in magnitude. Occurs instantly, without warning. Static strength of building less important than nature of dynamic response: fundamental period, damping, etc.
11. (a) Random occurrence (magnitude and location); structure must accommodate various patterns of loading. Often quite arbitrary for quantification.
    (b) Long time effect, critical for sag of wood, creep of concrete, settlement of foundations.
    (c) Requires dynamic analysis for building response; static analysis very empirical.
    (d) May punch holes, cause major local effects.
    (e) Requires multiple analyses for load combination cases, with design for worst effects.
12. Loads are "active," basic source of forces on structure. Reactions—as the name implies—are "reactive," responding to need for equilibrium.
13. By development of stresses in the material.
14. Existence of a definite yield point (departure from pure elastic behavior) and a considerable range of deformation before fracture.
15. Major characteristics are strength, deformation resistance, hardness, time-dependent losses, uniformity of physical structure.
16. (a) Establishes basic identity of wood (family name), such as Douglas fir. Also establishes wood as softwood or hardwood.
    (b) Establishes identity for qualified design values for allowable stresses and modulus of elasticity.
    (c) Requires consideration for modification of design values.
17. Two major factors are rusting and rapid loss of strength when exposed to heat of fires. Also is very expensive, requiring attention to efficient use.
18. Strength is highly variable with changes in ingredients and curing conditions. Unit density (weight) is variable, mainly by choice of bulk aggregate (e.g., gravel). Also variable: color, surface finish, insulative character, porosity.
19. Masonry does not require forming but does involve extensive crafted hand labor. Dimensions of finished construction limited by modular sizes of units. Quality control of site work is critical.
20. Providing support, spanning, cantilevering, bracing, anchoring.
21. Same basic form (curved) but different in orientation to load. Arch is basically a compression element; draped cable is a tension element.
22. (a) Heavy pier or abutment, dam, massive retaining wall, Egyptian pyramid.
    (b) Multistory framework of columns and beams, trussed frame of transmission tower or bridge.
    (c) Shell-like dome, tent, inflated object, such as an automobile tire.
23. Add attached columns that span vertically; use braces (buttresses); use intersecting walls; curve or fold wall in plan.
24. Add moment-resistive connections to make a rigid frame; use X-bracing or other forms of trussing; fix column bases against rotation; use shear panels.
25. Use of moment-resistive connections and moment-capable columns.
26. By articulation of the cross section to obtain other than a solid profile. Stems to produce T-beams; hollow spaces to produce box; folded or corrugated profile (e.g., as steel deck).
27. Triangulation of the frame members: triangles in two dimensions, tetrahedrons in three dimensions. Use very little material with a lot of space between members to define a large structure with little mass of material.
28. The less the rise, the greater the horizontal thrust on supports.
29. Stiffness to resist buckling.

Chapter 2

1. To provide data for logical development of the structure and to assure the safety of its performance.
2. Visualization, mathematical modeling, physical modeling (scaled or full-size prototypes).
3. (a) Visualization of the support reactions or the general means required for stability of the body.
   (b) Visualization of internal force actions, stresses, and local deformations.
4. Magnitude, sense, and direction.
5. Determination as to whether the forces are coplanar, concurrent, or parallel—all three considered separately.
6. (a) Composition is the combination of a set of related forces to find their resultant; resolution is the breaking down of a single force into components.
   (b) A component is one of a set of forces that represents the partial effect of a force. A resultant is the single force that represents the total effect of a set of interrelated forces.
7. That the forces are in equilibrium.
8. The algebraic sum is zero; usually by adding separately the vertical and horizontal components.
9. Closing of a force polygon establishes equilibrium of the forces at a single truss joint. Close all the joint force systems and find all the internal forces.
10. The sum of the moments must also be zero.
11. Force is a cumulative, total effect. Stress is a unit of force, expressed as a total force divided by some area, to produce a quantity expressed as force per unit area.
12. Direct stress acts perpendicular to the cross section; shear acts in the plane of the section.
13. Unit strain expresses the percentage of material deformation in nondimensional terms; deformation refers to the total, cumulative effect, expressed as a total length change, rotation, or deflection.
14. The shear action involves two separate cross sections acting simultaneously to share a shear force.
15. Zero flexural stress.
16. As angular change (rotation).
17. It defines the end of the proportional stress–strain range and signals the onset of plastic deformation in a material that is ductile.
18. If the dimensional change is not allowed to freely occur.
19. Modulus of elasticity.
20. They are equal.
21. They are equal.
22. A torsional moment (twisting) on the beam.
23. Distribution is constant with direct shear; for a beam, it varies from zero at the edges of a section to a maximum value at the neutral axis.
24. In most cases the frictional force is considered to be developed in proportion to the magnitude of the normal force (the force that squeezes the two surfaces together). Lubrication messes this up, so foundations on wet, slippery clay are considered to have a single magnitude of friction resistance.
25. Kinematics involves only motion with time. Kinetics adds considerations for mass and force, with work accomplished and energy expended.
26. Translation is measured in length (inches, miles, etc.); rotation is measured as angular change.
27. Work requires, and uses up, energy; energy is quantified in units expressing capacity for accomplishing work.
28. The work for performing it is usually much easier, at least for hand computation. It is also easier to visualize combined load effects, such as gravity plus wind.

Chapter 3

1. (a) As a qualification of a single support, it indicates lack of resistance to rotation of the supported object. "Simply supported beam" is also used to describe the primary case of a single-span beam with no end restraint.
   (b) The support resists rotation, usually producing moment in the beam.
   (c) If the beam is continuous, it may rotate at the supports, but there will be moment in the beam at the supports where the beam is continuous.
2. Refers to the convention of relating the sense of the moment to bending stress conditions in the beam. For a horizontal beam, a positive moment indicates tension in the bottom of the beam; for a negative moment, tension is in the top.
3. The loads and the reactions.
4. Area of the shear diagram indicates moment changes, in both magnitude and sense. Points of zero shear locate peaks of moment magnitude.
5. Sign of moment relates to direction of curvature of the beam; zero moment indicates the location of inflection (change of direction of curvature).
6. Lateral (sideways) buckling of the compression side of the beam, in column-like action. Rotational (torsional) buckling at midspan or at supports.
7. Straightening of nonstraight elements. Deformation of lengthening (stretching).
8. It is usually the location of maximum tensile stress.
9. The highly flexible cable (rope, chain, etc.) forms a profile in direct response to the supports and loads.
10. Short members crush (a stress failure); long ones buckle sideways (a bending failure).
11. They may alter the effective buckling length of the column, either increasing or decreasing resistance to buckling. This is the basis for the K factor in KL/r.
12. The column deformation (the delta, for deflection) is usually a result of bending. The P–delta effect is the bending moment (P times delta) which adds to other bending and produces more delta—and may cause a continuation or progression that ends in failure.

13. Inability to resist tension due to a tension-weak material, or separation at the section which is actually a contact surface between two objects, such as a footing on the soil surface.
14. They must all form triangles.
15. To avoid shear and bending in truss members.
16. Transfer of bending moment from member to member through the connections.
17. Its structural response is not able to be determined by use of static equilibrium conditions alone.
18. For medium spans, the two parts may be shop fabricated or site assembled and erected, avoiding more difficult or costly procedures. Structural behavior is more easily and reliably determined.
19. Planning arrangements resulting in approximately square units for the two-way spans. Supports that avoid edge or corner concentrations of shear or bending. Form of the supports that reduces any punching shear effects.

Chapter 4

1. Tabular design values are based on a specific moisture content. If the structure is exposed to weather or other extreme moisture conditions, some values are reduced.
2. Modification relates to time of duration of the maximum effect of the load; the shorter the time, the less the load.
3. For bending stress, depth affects the section modulus by the second power (square) of the depth. For deflection it affects the moment of inertia by the third power (cube). Width affects both properties only in a linear proportionate manner.
4. These conditions imply very high shear forces, and wood beams are relatively weak in shear. Glue-laminated beams are stronger and steel beams the best.
5. They are required for lateral bracing.
6. Joists and rafters are often pushed to their span limits, and the lightly loaded, shallow-depth member is likely to be critical for deflection.
7. Spacing of the deck's supports, which determines the deck span.
8. Buckling, not stress, is critical. Resistance to buckling is essentially resistance to bending. Stiffness in bending is determined by the modulus of elasticity of the material and the radius of gyration of the member cross section.
9. Most columns are not designed for compression stress alone. Allowable stress that includes consideration for buckling is a function of the member stiffness, which is not known if the member is not known.
10. Wall-covering materials fastened to the studs usually brace the studs on their weak axis (1.5 in. direction). They are thus critical on the other axis, and the limit for the unbraced height is $50 \times 3.5 = 175$ in.
11. Bending and axial compression are two separate phenomena with separate allowable stresses and different behavior responses. The interaction formula allows for these factors.
12. Basic action of the joint involves a twisting of the joint. This response is not acceptable for major loads.
13. Shrinkage of the wood loosens the nails.
14. Different allowable values for the wood.
15. The connection is tighter, with less slippage under load; also, strength is greater.
16. Desired building form; loading conditions; type of span; size of truss.
17. Size of the truss and size of the members.
18. They contribute the most to bending resistance with spans in the grain direction.
19. Longer members with deeper cross sections are possible. Dimensional stability problems (e.g., warping) are reduced

Chapter 5

1. Deformation in ductile response is usually the practical limit for structures, so acceptable performance is based on yield stress.
2. True dimensions vary from this in most cases.
3. Rusting and failure in fire.
4. Section modulus
5. They indicate limits for forms of buckling response and allowable stresses.
6. In the elastic range, strain is proportional to stress, and stress is proportional to load magnitude. For the same depth, deflection will be proportional to beam depth and thus will be the same for all beams with the same limiting stress value.
7. Bracing of the beam to prevent its lateral rotation.
8. Thickness of the beam web.
9. Area and radius of gyration.
10. When KL/r is different for the two axes. The higher value must be used.
11. Friction between the two connected parts, created by the clamping action of the highly tensioned bolts.
12. Shear and tension.
13. Depth of the beam.
14. Both flanges of the supported beam must be cut back to achieve the connection; results in significant loss of shear capacity.
15. Possible elimination of the need for lateral bracing. Development of significant resistance to bending on more than one axis.
16. The desired shape or size of the section is not available as a single-piece, rolled shape.

Chapter 6

1. Low resistance to tensile stress, as generated mostly by bending and shear actions.
2. The aggregate, separately considered, will pack into the most dense bulk, with smaller pieces filling the voids between larger ones. This leaves the least void to be filled with cement and water, producing stronger and more economical concrete with less shrinkage.

3. The unit density (weight) of the concrete is primarily determined by the unit density of the coarse aggregate (usually gravel), which ordinarily constitutes from two-thirds to three-quarters of the volume of the concrete.
4. To enhance the grip of the surrounding concrete on the bar surface. This is critical to the anchorage of bars and to load sharing by the steel and concrete.
5. Yield strength.
6. Temperature, moisture content, lack of stress due to structural demands.
7. Size of the concrete member, fire resistance requirements, and exposure conditions for the concrete surface (to weather, soil contact, etc.).
8. Beam width, presence and size of stirrups, size of the aggregate (largest piece), and general code requirements for bar spacing.
9. Ordinary thicknesses of slabs and size of beam stems result in excessive compression area for development of stress necessary to balance the capacity of practical amounts of tension reinforcing.
10. Horizontal, tension reinforcing bars at the ends of the beam.
11. Shear in excess of that permitted to be resisted by the concrete.
12. Development occurs on the bar surface, to “develop” the strength of the bar in stress on its cross-sectional area. The smaller the bar, the greater the ratio of surface to area; increase the bar diameter and surface (circumference) increases only linearly, while area increases by the square of the diameter.
13. Required development relates to potential bar strength, which is greater if allowable stress (or yield strength) is higher.
14. In units of equivalent development length.
15. Development required for the bars being spliced.
16. Span of the slab, design loading, fire resistance requirements, and maybe the T-beam action of the framing beams.
17. These factors are the basis for recommended minimum dimensions in the ACI Code.
18. Joists form a one-way system; the waffle is a two-way system.
19. Slab clear span is slightly reduced and the bending and shear resistances of the slab are increased.
20. To make the steel beam and concrete slab work together as a single unit.
21. To prevent the bars from buckling and bursting sideways through the thin cover of concrete.
22. Code-imposed design limitations result in columns having yielding of the bars as the initial failure of the column. Adding a small axial compression load prestresses the bars in compression, permitting them to take some more tension to develop additional bending moment.
23. Spacing requirements for the bars.
24. Ease of placement of the reinforcing.
25. To resist shrinkage stresses and maybe develop some beam-spanning action over uneven soil.
26. Compression stress in the column bars must be developed by extension into the footing a distance as required for development length for the bars.
27. By reducing shear and bending in the footing and possibly relieving the development problem for the footing.

Chapter 7

1. The grout and reinforcement form a reinforced concrete frame structure inside the wall that may be very significant to the total structural capacity of the construction.
2. To tie the wythes together.
3. See answer to question 1. The role of the grout and reinforcement in the voids becomes a dominant concern.
4. Lack of faith in the bonding (tensile) capability of the mortar.
5. Concentration of mass (such as with a pilaster); use of form—arched or folded in plan; use of stronger units at strategic locations, such as corners and edges of openings.

Chapter 8

1. To determine the form of the site surface, the character and arrangement of surface and subsurface soils, and the condition of water in the ground.
2. Size of solid particles, in-place (undisturbed) density and hardness, water content, penetration resistance, potential instabilities, and presence of organic materials.
3. Ease of excavation, need for dewatering during construction, potential for use of excavated materials for fill, need for bracing of excavations, effects of construction work on soils.
4. Air and water.
5. Small range of the specific gravity of ordinary soil materials.
6. It is highly plastic (soft, oozing).
7. Plasticity.
8. Lack of significant amounts of fine soil particles (silt and clay).
9. To be able to use the full plan area of the footing for bearing and to keep soil pressure as uniform as possible.
10. The column loads are not always equal and the footing plan centroid should coincide with the column load center (resultant of the column loads).
11. The form of response will be more or less related to lateral passive soil pressure.
12. Limited usable bearing capacity or instability of soils near the ground surface; sensitivity of the building and its contents to settlements; ease of installation of deep foundation elements.

13. The precise location of the tops of driven piles cannot be controlled.
14. Presence of large rocks in the soil; effects of installation on neighboring properties (especially driving of piles); dewatering for drilled piers; highly unstable soils.
15. Fluctuating water level near the pile tops.
16. For active pressure the soil itself is the source as it pushes against restraints (basement walls, retaining walls, etc.). For passive pressure something else is pushing against the soil.
17. (a) On sand, friction is proportional to the load that produces bearing and is determined with a coefficient of friction.
    (b) On clay, friction is a fixed magnitude determined by the area of contact.
18. The soil is considered to act as a fluid with a density equal to some fraction of the soil weight. As for a fluid, pressure varies directly with the depth below the surface.

Chapter 9

1. (a) Exterior walls receive wind pressure directly, spanning between horizontal supports and delivering reaction forces to those supports (usually roof or floor edges).
   (b) Walls may act as shear walls. If not intended as shear walls, they may be structurally isolated from the bracing structure to prevent damage to them.
   (c) These normally function as horizontal diaphragms.
   (d) These receive the load from the building and transfer it to the supporting soil, mostly by development of lateral passive pressure in the soil.
2. (a) These are sources of lateral loads due to their weight. Load is transferred through the bracing system as for wind.
   (b) Walls are sources of lateral load; actions are as described for question 1.
   (c) These are sources of loads collected in the system; functions are as described for question 1.
   (d) As described for question 1. It is essential to tie the system together because of the rapid back-and-forth action of the earthquake. Base isolation—if any—will occur between the supported building and the foundations.
3. (a) Resistance to lateral effects in the planes of the diaphragms; edge-to-edge transfers between individual diaphragms.
   (b) Connected beams, columns, and diagonal members form planar trussed bents.
   (c) Connected columns and beams form planar rigid-frame bents through moment-resistive connections.
4. Anchorage of elements (roof to walls, walls to foundations, etc.) may not be adequate. Structural framing members may not be strong enough, have continuity through splices, or be attached sufficiently to diaphragm elements.
5. Buildings are routinely developed primarily in response to gravity loads.
6. Unreinforced masonry made with low-quality mortar and relatively soft and weak masonry units.
7. Unsymmetrical plans; abrupt changes in building form (setbacks, notched corners, etc.); use of rigid but nonstructural forms of construction (plaster, stucco, masonry veneer, etc.); creation of building profiles that produce soft stories; use of very heavy materials in upper levels of the building (tile roofs, concrete roof slabs, etc.).
8. Inability to develop lateral bracing in one exterior wall.
9. Relative stiffness of the vertical bracing is significantly less than the story above or below.
10. Wind velocity increases with distance above the ground, as ground surface drag decreases and the effects of sheltering by trees, hills, and surrounding buildings is less significant.
11. The magnitude of wind pressure is proportional to the square of the wind velocity (speed).
12. When the resultant of the horizontal force does not coincide with the center of stiffness of the lateral bracing system.
13. Mass, fundamental period of vibration, presence of damping sources, potential for resonance within the building bracing system, potential for resonant building–site interaction, effects of irregularity of building form.
14. Separate units of the building, when their interaction is potentially harmful during earthquake-induced movements.
15. Lack of potential for loosening or progressive fracture due to dynamic actions. Most critically during the repeated actions of seismic vibration or wind flutter.
16. Reduces the magnitude of movements and force transferred by the ground into the building mass and shortens the duration of continued vibration.
17. Same as base isolation, reduces force effects, reduces movements, and especially provides critical damping of single vibration cycle.
18. Development of tension and compression for resistance to the bending created by the spanning action of the diaphragm.
19. Dead weight of the wall and of any construction it supports or is attached to.
20. It indicates potential for transfer of diaphragm shear between adjacent panels.
21. Principal deformation of the frame is due to lengthening and shortening of the truss frame members—ordinarily very small dimensions. Trussed frames are therefore much stiffer than rigid frames, in which deformations are largely due to bending in the frame members. This relationship can change if truss

connections have a lot of deformation or rigid frame members are exceptionally stiff.

22. Essentially, rigid frame actions involving the bending of members.
23. See answer to question 21.
24. Shear in the diaphragm for transfer to vertical bracing.
25. A horizontal tie between the supports.
26. The circular edge support (ring) may be developed as a tension ring (for a dome) or a compression ring (for a draped cable system).
27. To isolate manageable units of the building with control joints that permit independent movement of units.

Chapter 10

1. To serve as a guide and reference for development of individual, local building codes.
2. Weight of the building construction.
3. Total area of the loaded surface carried by the member being considered
4. Structural efficiency, while an important basic concept for structural engineering design, may be less important economically than the effect of the structure on the cost of other elements of the construction and building services.

References

1. *Minimum Design Loads for Buildings and Other Structures*, SEI/ASCE 7-05, American Society of Civil Engineers, Reston, VA, 2005.
2. *International Building Code*, International Code Council, Inc., Country Club Hills, IL, 2009.
3. *National Design Specification for Wood Construction*, American Forest and Paper Association, Washington, DC, 2005.
4. *Timber Construction Manual*, 5th ed., American Institute of Timber Construction, Wiley, Hoboken, NJ, 2005.
5. *Structural Wood Design Solved Example Problems—ASD/LRFD*, American Forest and Paper Association, Washington, DC, 2005.
6. *Manual for Engineered Wood Construction—ASD/LRFD*, American Forest and Paper Association, Washington, DC, 2005.
7. *Engineered Wood Construction Guide*, APA—The Engineered Wood Association, Tacoma, WA, 2005.
8. *Commercial Design Guide No. 1048: Parallam PSL for Commercial Applications*, Trus Joist, Boise, ID.
9. *Introduction to Lateral Design*, APA—The Engineered Wood Association, Tacoma, WA, 2005.
10. *Manual of Steel Construction, Load and Resistance Factor Design*, 3rd ed., American Institute of Steel Construction, Chicago, IL, 2001.
11. *Standard Specifications, Load Tables, and Weight Tables for Steel Joists and Joist Girders*, Steel Joist Institute, Myrtle Beach, SC, 2002.
12. *Guide for Specifying Steel Joists with LRFD*, Steel Joist Institute, Myrtle Beach, SC, 2000.
13. *Steel Deck Institute Design Manual for Composite Decks, Form Decks, and Roof Decks*, Steel Deck Institute, St. Louis, MO, 2000.
14. J. McCormac and J. Nelson, *Structural Steel Design, LRFD Method*, 3rd ed., Pearson Education, Upper Saddle River, NJ, 2003.
15. C. M. Uang, S. W. Wattar, and K. M. Leet, "Proposed Revision of the Equivalent Axial Load Method for LRFD Steel and Composite Beam—Column Design," *Engineering Journal*, AISC, Fall 1990.
16. *Building Code Requirements for Structural Concrete*, ACI 318-08, American Concrete Institute, Detroit, MI, 2008.
17. J. McCormac and J. Nelson, *Design of Reinforced Concrete*, 6th ed., Wiley, Hoboken, NJ, 2005.
18. *CRSI Design Handbook 2002*, Concrete Reinforcing Steel Institute, Schaumburg, IL, 2002.
19. J. Ambrose and D. Vergun, *Simplified Building Design for Wind and Earthquake Forces*, 3rd ed., Wiley, Hoboken, NJ, 1995.
20. J. Ambrose, *Simplified Design of Building Foundations*, 2nd ed., Wiley, New York, 1988.
21. R. Peck, W. Hanson, and T. Thornburn, *Foundation Engineering*, 2nd ed., Wiley, New York, 1974.

Index